Shelving or developing?
Optimal policy for mergers with potential competitors*

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September 5, 2022

Abstract

A start-up and an incumbent negotiate over an acquisition price under asymmetric information about the start-up's ability to succeed in the market. The acquisition may result in the shelving of the start-up's project or the development of a project that would otherwise never reach the market. Despite this possible pro-competitive effect, the optimal merger policy commits to standards of review that prohibit high-price takeovers, even if they may be welfare-beneficial ex post. Ex ante this pushes the incumbent to acquire start-ups which are otherwise unviable to develop independently, and increases expected welfare.

Keywords: Optimal merger policy, selection effect, nascent competitors.
JEL Classification: L41, L13, K21

*Comments by Heski Bar-Isaac, Özlem Bedre-Defolie, Michele Bisceglia, Giacomo Calzolari, Florian Ederer, Paul Eliason, Bruno Jullien, Bruce Lyons, Matt Mitchell, Patrick Rey, Thomas Rønde, Nicolas Schutz, Giancarlo Spagnolo, Yossi Spiegel, Hal Varian, Lulu Wang and participants in the 2nd Cambridge Symposium on Competition Policy, Centre of Competition Policy Annual Conference, CEPR Conference on Applied Industrial Organization, CRESSE, Digital Economics Conference (Toulouse), EARIE, EFA, MaCCI Annual Conference, Oligo Workshop (Maastricht University), University of Bergamo IO Winter Symposium, and at the Autoridade da Concorrência, Cambridge University Judge Business School, CEPR Virtual Industrial Organization Seminar, Düsseldorf Institute for Competition Economics (DICE), European Commission (DG Competition), Imperial College London, Universidade de Pernambuco (Recife), University of Bern, University of East Anglia, University of Padova, University of Toronto, University of Vienna are gratefully acknowledged. Chiara Fumagalli acknowledges financial support from Baffi-Carefin Centre and IGIER (Università Bocconi). Massimo Motta gratefully acknowledges financial aid from the Spanish Agencia Estatal de Investigación (AEI) and FEDER (project ECO2016-76998-P) and from "Ayudas Fundación BBVA a Equipos de Investigación Cientifica 2019" (project on "Digital platforms: effects and policy implications"). None of the authors has done consulting work on cases related to this topic. Massimo Motta was exposed to a few such cases when he served as Chief Competition Economist of the European Commission (2013-2016).

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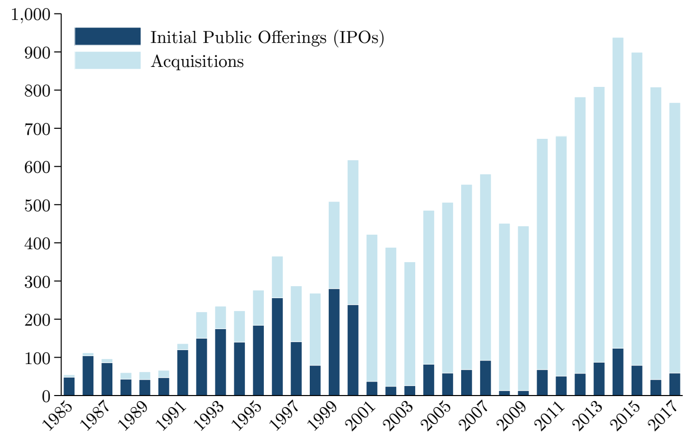
1 Introduction

Potential competitors are firms that currently do not exert competitive pressure but might do so in the future. The acquisition of these firms is a widespread phenomenon. As Figure 1 shows, since the mid-90s there has been a dramatic shift in the exit strategy of venture-capital backed start-ups, from IPOs towards acquisitions. In the digital economy alone, hundreds of start-ups have been bought in the last few years by incumbents such as Alphabet (Google), Amazon, Apple, Meta (Facebook) and Microsoft (The Economist, 2018; The Wall Street Journal, 2019; The New York Times, 2020), but this phenomenon extends beyond the digital industries. Similar patterns prevail in, among the others, the healthcare and the pharmaceutical industries, as documented by Eliason et al. (2020) and Cunningham et al. (2021), respectively. In the vast majority of cases, such

Figure 1: Venture capital start-up exit by type



The figure is drawn from Pellegrino (2021). It plots the number of successful exits of venture-capital backed start-ups in the US by year and type (Initial Public Offering vs. Acquisition). The data is sourced from the National Venture Capital Association (NVCA).

acquisitions did not trigger mandatory pre-merger notification requirements because the latter are based on the turnover of the merging parties. This leads to stealth consolidation (see Wollmann, 2019). [1]When Antitrust Agencies (AAs) did open an investigation, they authorised them, with the exception of the recent Facebook/Giphy decision by the UK's CMA. As a result, many have asked for stricter antitrust action, alarmed by the possible anti-competitive consequences arising from the elimination of future competition (see, e.g., Crémer et al., 2019; Furman et al., 2019; Scott Morton et al., 2019; Lemley and McCreary 2020; Motta and Peitz 2021).

The traditional approach to the analysis of horizontal mergers trades off the costs of market power and the benefits of cost efficiencies (see, among many others, Williamson, 1968; Farrell and Shapiro, 1990; McAfee and Williams, 1992). The acquisition of potential competitors triggers an

[1]Eliason et al. (2020) show that most of the acquisitions of small dialysis facilities conducted by large national chains fall outside the scope of current antitrust laws. Similarly, Cunningham et al. (2021) also document empirically that, in the pharmaceutical sector, incumbents conduct acquisitions that do not trigger the Federal Trade Commission reporting requirements.

additional trade-off. On the one hand, the incumbent may acquire the start-up to then shelve the start-up's project. This would be a "killer acquisition" as documented by Cunningham et al. (2021) in the pharma industry. On the other hand, the acquisition may allow for the development of a project that would otherwise never reach the market. This may happen because the incumbent has availability of resources – managerial skills such as implementation skills, market opportunities, capital – that the target firm lacks. Given this trade-off, we ask: What merger policy should an antitrust authority follow? The literature offers little guidance regarding the answer to this question. The existing models mainly analyse the impact of these acquisitions on investment, but do not consider product market competition. We study both the decision to develop and its effect on competition in a model that embeds the trade-offs above. Another thorny aspect of these acquisitions concerns which notification criteria should be used when the target is a nascent firm. Our theory proves that the acquisition price plays an important role. in this respect.

We propose a model where a start-up owns a project with positive net present value that, if developed, will allow it to compete with an incumbent firm. However, the start-up may be *viable* – that is, it has the necessary (managerial, financial, etc.) resources to develop the project independently – or *unviable* – that is, it cannot develop the project on its own. In the latter case, the only chance for society that the (beneficial) project sees the light is that the incumbent takes over the start-up and develops the project itself. The incumbent is assumed to have the resources to develop the project. However, cannibalisation of existing profits may weaken investment incentives and the incumbent may shelve a project that a viable target firm would carry out.

In our base model, the incumbent can acquire the start-up prior to project development. We model the bargaining over the takeover price as a non-cooperative game. With some probability it is the incumbent that makes a take-it-or-leave-it offer to the start-up; with complementary probability it is the start-up. This probability captures agents' relative bargaining power. If the acquisition proposal is made, the AA will decide whether to approve or block it, consistently with the standards of review to which it commits at the beginning of the game.[2] Finally, we assume that players are not equally informed about the start-up's ability to succeed in the absence of the acquisition. In our base model, the start-up knows its own type, whereas the incumbent and the AA only know the probability distribution that the start-up is viable.[3]

These assumptions imply that the takeover price is determined in a non-cooperative bargaining

[2]We follow the literature and assume commitment to a merger policy (see, e.g., Sørgard, 2009; Nocke and Whinston, 2010 and 2013). Given that AAs may take hundreds of merger decisions every year, and that precedents matter in competition law, the credibility of the commitment in this context is not an issue.

[3]This approach is consistent with the empirical literature documenting the presence of informational frictions between acquirers and targets, especially when the latter is a knowledge-based R&D intensive firm, and thus difficult to value (Officer et al., 2009). An anecdote suggesting that incumbents may have difficulties in assessing the start-ups' ability to succeed on their own is the negotiation between Excite and Google regarding a possible takeover in 1999 (CNBC, 2015). It was Google to approach Excite, which at the time was a big player in the search engine market, while Google was a new, small, player. After factoring the uncertainty surrounding Google's ability to grow on its own, Excite decided to pass on the offer. In the words of George Bell, the Excite's CEO at the time (emphasis added): "I think the decision we made at the time, *with what we knew*, was a good decision. It's laughable to say that now." We obtain similar results when we revert this assumption (see below).

game with asymmetric information and alternative bargaining power allocation. When we restrict the analysis to equilibria in pure strategies, the game can feature two types of equilibrium offers: either a high takeover price, such that any start-up would accept (or offer) that price, irrespective of its type; or a low takeover price, targeting only the unviable start-ups.

If a low price is accepted, the AA infers that the start-up is unviable and will authorise the deal. This is the case in which the takeover is (weakly) beneficial: if the incumbent develops, a new product will reach the market; if it shelves, the takeover has no impact on competition, because the start-up would not have been able to develop the product anyway. A high price does not reveal additional information on the start-up's type to incumbent and AA. The AA will authorise the deal if the prior probability that the start-up is viable is low enough. If so, the scenario in which the takeover is welfare detrimental, because of the suppression of product market competition and, when the incumbent shelves, also of project development, is sufficiently unlikely. However, the incumbent needs to be willing to pay a high price for the start-up. By doing so, it is certain to appropriate the project and suppress product market competition, but it may overpay for an unviable start-up. This is a risk worth taking when the prior probability that the start-up is viable is large enough.

Based on these considerations, we derive the optimal merger policy. We show that it exerts a "selection effect": for configurations of the parameters for which a high-price offer is profitable for the incumbent, the anticipation that the deal will be blocked leaves no other option than offering a low price. In this case, the merger policy pushes towards acquisitions that target only unviable start-ups and are thus preferable in terms of welfare. Moreover, the more stringent the standards of review, the stronger the selection effect and the more likely that a low-price takeover replaces a high-price takeover at the equilibrium. This is one important message of the paper: despite the possible pro-competitive effect of high-price takeovers when the incumbent develops, the optimal merger policy should not allow them. In equilibrium, the optimal policy might then commit to standards of review that prohibit high-price takeovers that ex post are welfare-beneficial. By forcing the switch to a low-price takeover, such a merger policy makes expected welfare even higher and achieves the first best.

Another important message of the paper is that, when acquisitions involve potential competitors, the takeover price conveys key information. Differently from the case in which the target is a mature firm, there exists considerable uncertainty about the ability of the start-up to succeed in the market. In this environment, a high takeover price signals that there are high chances that the takeover is not indispensable for the success of the start-up and that, therefore, is likely to raise anti-competitive concerns. This insight can be applied more broadly: in our model, the synergies produced by the merger consist in the possibility that I's assets, by complementing the assets of the start-up, may enable development. In practice, acquisitions of nascent and mature firms might produce synergies of different nature. Also in those cases a high transaction price may reflect that the start-up does not need those synergies to grow and have success in the market, and that the takeover may harm competition.

When we focus on the pure-strategy equilibria of the bargaining game, there cannot exist an equilibrium in which the two start-ups (viable and unviable) are acquired at different positive prices. The reason is that the unviable start-up would always have an incentive to mimic the viable one. This may lead to an inefficiency, since there may not be a takeover of the unviable start-up even when such takeover would increase industry profits (and welfare). Mixed-strategy equilibria alleviate this inefficiency. We characterise the conditions for the existence of such equilibria. We show that the optimal merger policy obtained relying on the pure-strategy equilibria of the bargaining game is still optimal when one allows for mixed strategies.

From a policy perspective, our results question the current laissez-faire approach towards acquisitions of potential competitors. It supports the use of a transaction-value threshold as an additional test to identify mergers that are potentially anti-competitive and thus deserve a closer look. The current notification thresholds, which are mostly based on both merger parties having a sufficiently high turnover, prevent AAs from investigating the vast majority of acquisitions involving potential competitors. Our results echo the proposals made by various AAs to add the transaction value as notification requirement in the revision of the approach towards mergers in digital markets (see, e.g., CMA, 2021:51). However, we show that the validity of this approach extends beyond the digital markets. It can be applied to screen any merger involving a potential competitor.[4]

We extend the base model to test the robustness of our conclusions. The main model assumes that the start-up has superior information regarding own type. We reverse the information asymmetry and assume that the incumbent knows whether the start-up is able to develop the project, whereas the start-up and the AA do not. As in the base model, also in this setting a high takeover price signals that the takeover is detrimental to welfare because it reveals that the start-up is viable with certainty. Therefore, the optimal merger policy based on pure-strategy equilibria prohibits such takeovers. Differently from the case in which the start-up has superior information, though, the AA cannot achieve the first-best outcome in which only unviable start-ups are acquired.

When extending the analysis to mixed-strategy equilibria, if the incumbent is better informed about the type of the start-up, there exist specific circumstances in which the optimal merger policy authorises high-price takeovers. In particular, authorising high-price takeovers may sustain an hybrid equilibrium in which also unviable start-ups can be acquired, and for this reason generate a welfare gain. Overall, though, the key takeaway of the base model goes through: high-price takeovers are generally welfare detrimental, and should be allowed only if authorising them increases the probability that unviable start-ups are acquired.

In a second extension, we allow the incumbent to have a second chance to acquire the start-up, *after* product development, and accordingly for the AA to have two different standards of review: not only one for *early* takeovers — that is, acquisitions of potential competitors — as in the base

[4]The Austrian and the German AAs require that acquisitions whose transaction value is above a certain threshold must be notified, regardless of the merging parties' sector.

model, but also a possibly different one for *late* takeovers — that is, acquisitions of committed entrants.[5]

While allowing for acquisitions of committed entrants does not change the optimal policy regarding early takeovers, we show that the optimal merger policy establishes a possibly more lenient treatment towards acquisitions of committed entrants than acquisitions of potential competitors, a result which may seem paradoxical but that holds under specific cumulative conditions. First, the anticipation of a late takeover must increase the viability of the start-up: for instance, if the start-up can appropriate some of the gains from higher industry profits created by the acquisition, it may have a higher chance to develop the project. For example, this can happen because a late takeover relaxes the start-up's financial constraints; or because the perspective of a future career within the incumbent firm may make it easier for a start-up to attract managerial skills. Second, it must be that the incumbent shelves the project after an early takeover. Finally, the sacrifice of allocative efficiency caused by late takeovers must be dominated by the benefit due to the higher probability of development.

Literature review Our paper contributes to several literatures at the intersection between industrial organization, competition policy and innovation economics.

A recent literature has analysed theoretically how the acquisitions of potential competitors affect the target's project development.[6] Cunningham et al. (2021) determine the conditions under which, after acquiring the potential entrant, the incumbent has incentives to shelve the entrant's project. In this way, the monopoly avoids the cannibalisation of own existing products' sales. Differently from them, our model features asymmetric information regarding the start-up's type. Moreover, we model the AA as a strategic player, and derive the optimal merger policy accordingly. Wang (2021) shows that blocking a merger can exacerbate financial constraints and lead to underinvestment in project development in a model with adverse selection à la Myers and Majluf (1984). This is similar to our insight that approving late acquisitions can relax financial constraints. However, our analysis is different in two important dimensions. First, we model the link between investments and product market competition. Second, since the AA is a strategic player of the game, we show that the optimal merger policy gives rise to a selection effect in the choice of the takeovers that occur in equilibrium. Another difference with respect to both papers is that we also distinguish between the acquisitions of potential competitors and those of

[5]This is a relevant distinction in practice. The US Horizontal Merger Guidelines explicitly distinguish between potential entrants and committed entrants. The potential entrants are those that are "likely [to] provide [...] supply response" in the event the conditions allow them to compete on the market. The committed entrants are firms that are "not currently earning revenues in the relevant market, but that have committed to entering the market in the near future."

[6]There is also a literature that emphasizes the effect of acquisitions of potential competitors on innovation and on the direction of innovation (e.g., among others, Rasmusen, 1988; Norbäck and Persson, 2012; Bryan and Hovenkamp, 2020; Letina et al., 2020; Katz, 2020; Cabral, 2021; Bisceglia et al., 2021; Denicolò and Polo, 2021; Gilbert and Katz, 2021; Kamepalli et al., 2021; Motta and Shelegia, 2021; Callander and Matouschek, 2022). Our paper is related to this literature in that it shows the possible ex-ante effects of late takeovers: the expectation of a future acquisition may make it easier to bring the new product to the market. (This literature mostly focuses on the incentives for entrepreneurs to innovate in the first place.)

committed entrants.

There is a literature that studies the interaction between late and early takeovers. In particular, in Norbäck and Persson (2009), incumbents engage in a pre-emptive early acquisition to avoid "excessive" investment in project development by the independent start-up which, in turn, is due to the prospect of a late acquisition. Arora et al. (2021) study the interaction from the perspective of the start-up. Their model analyses the trade-off faced by the start-up, between capturing more value being acquired late, when the business is more mature, and running a greater risk of failing due to lacking assets. In these papers, however, there is no AA taking decisions on the mergers, and they do not investigate merger policy.

We also contribute to the vast industrial organization literature on horizontal mergers (see, e.g., Farrell and Shapiro, 1990; Besanko and Spulber, 1993; Armstrong and Vickers, 2010; Nocke and Whinston, 2010; Nocke and Whinston, 2013), by determining the optimal merger policy in the presence of asymmetric information. We show that even in a relatively rich setting as ours, the AA can formulate a simple "information-free" policy that does not require the knowledge of whether the start-up is viable or not, of the bargaining power allocation or of whether the incumbent has an incentive to shelve the project.

In equilibrium, a selection effect shapes the AA's optimal merger policy, which forbids high-price takeovers in order to induce parties towards acquisitions that target unviable start-ups, and are thus preferable in terms of welfare. To obtain this outcome, the AA may have to block some welfare-increasing mergers. A similar selection effect arises in Nocke and Whinston (2013), where the AA also optimally commits to blocking welfare-increasing mergers in equilibrium. However, the information problems in the two papers are different. They consider mergers involving actual competitors, and assume that the AA knows the impact of proposed mergers on welfare, but has limited information on the alternatives that can be proposed in case the merger is turned down. We consider takeovers targeting potential competitors, for which the information problem concerns the welfare effects of the merger under investigation.

The paper proceeds as follows. Section 2 presents the ingredients of the base model and, as a benchmark, studies a simplified version with symmetric information. Section 3 presents the equilibria in pure strategies and Section 4 identifies the optimal merger policy in this context. Section 5 characterises the equilibria when one allows for mixed strategies and shows that the optimal merger policy is the same as under pure strategies. Section 6 offers possible micro-foundations for the assumptions about viability of the firms. Section 7 analyses the case where it is the incumbent which has superior information. Section 8 extends the model to consider the case where takeovers may take place either before or after product development. Section 10 concludes the paper.

2 The model

There are three players in our game: an Antitrust Authority (AA), which commits to a merger policy and later enforces it; a monopolist (I)ncumbent; and a (S)tart-up.[7] The start-up owns a project that, if developed, leads to a substitute to the incumbent's good (or to a more efficient process to produce that good).

The start-up may be 'viable' $(S = S_v)$ or 'unviable' $(S = S_u)$. A viable start-up will invest and bring successfully the project to the market even if independent (that is, absent takeover). An unviable start-up, instead, would not be able to develop the new product. Different from a viable start-up, an unviable one lacks critical resources – managerial, market opportunities, capital – that are necessary to make it into the market. The incumbent is assumed to own such resources; thus, it is able to develop the start-up's project, provided it has incentive to do so.

The incumbent may acquire the start-up, conditional on the AA's approval. We assume that the takeover involves a negligible but positive transaction cost. This assumption serves as a tie-breaking rule when the profits of the agent making the offer are the same with and without the takeover (gross of the transaction costs). If the takeover takes place, the incumbent will decide whether to develop the project or shelve it.

We consider a merger policy consistent with the approach currently adopted in most jurisdictions: the AA commits to a standard of review, that we denote as $\bar{H}$: $\bar{H}$ indicates the maximum level of "harm" that the AA is ready to tolerate. If $\bar{H} > 0$, the AA commits to approving even mergers that produce a reduction in expected welfare, to the extent that the expected harm is lower than the tolerated one $\bar{H}$.[8] If $\bar{H} = 0$, the AA commits to approving only mergers that are expected to be welfare beneficial. If $\bar{H} < 0$, even a welfare beneficial merger can be blocked, if the expected increase in welfare is lower than the minimum level $\bar{H}$ that the AA requires. The expected impact on welfare of a proposed merger consists of the difference between the expected welfare in case the merger goes ahead, and in the counterfactual where it does not take place (derived of course by correctly anticipating the continuation equilibrium of the game).

Payoffs If the project has not been developed, the incumbent remains a monopolist with its existing product/technology, and earns profits π_I^m. Welfare is W^m.[9] If the viable start-up develops the project, then it will offer a substitute good that competes with the incumbent's product. S and I will make duopoly profits, π_S^d and π_I^d, respectively, with $\pi_I^d < \pi_I^m$. Associated (gross) welfare is W^d. If I develops the project, it will obtain higher monopoly profits $\pi_I^M > \pi_I^m$, due to

[7] In the base model we follow Cunningham et al. (2021) and consider a single acquirer I. In Section 8 we discuss the case of competing offers for the start-up.

[8] In practice, $\bar{H}$ is usually strictly positive for several reasons: the law prescribes that only mergers that *significantly* affect competition can be prohibited; some mergers may not even be reviewed because they do not meet notification criteria (e.g., in most jurisdictions the merger has to be notified only if the combined turnover goes beyond certain thresholds); the law (or the courts) assigns the burden of proving that the merger is anti-competitive to the AA, and sets a high standard of proof.

[9] Our analysis is qualitatively the same independently of whether the AA's objective is consumer or total surplus. For a discussion of the merits of consumer v. total surplus as standards in antitrust, see Farrell and Katz (2006).

the additional product (or the use of a more efficient technology).[10] Gross welfare is W^M.

The ranking of (gross) welfare is $W^m < W^M < W^d$. This assumption reflects the role of market competition: $W^M < W^d$. Moreover, $W^m < W^M$, due to consumers' love for variety (or to the more efficient production process).

Development has a positive net present value (NPV) for the start-up:

$$\pi_S^d > K, \tag{A1}$$

thus, it is privately efficient to develop the project.

Industry profits are assumed to be higher under monopoly (when the incumbent I develops the project) than under duopoly:

$$\pi_I^M > \pi_I^d + \pi_S^d. \tag{A2}$$

This is the "efficiency effect" (Gilbert and Newbery, 1982), which ensures that there is always room for an acquisition. We also assume that

$$\pi_S^d > \pi_I^M - \pi_I^m, \tag{A3}$$

which corresponds to the well-known Arrow's "replacement effect" (Arrow, 1962): an incumbent has less incentive to innovate than a potential entrant because the innovation would cannibalise its current profits.

While, absent the acquisition, the viable start-up always invests (Assumption A1), the incumbent does not always have the incentive to do so because the increase in its profits may be insufficient to cover the investment cost (Assumption A3). Specifically, the incumbent invests in development if (and only if) the following condition is satisified

$$\pi_I^M - \pi_I^m \geq K. \tag{1}$$

If this condition is not satisfied, the incumbent decides to shelve a project that an independent start-up would develop. If Assumption A3 did not hold, then not only shelving would never take place, but the incumbent might even develop projects that a viable entrant would find unprofitable.

Finally, the development of the project is assumed to be beneficial for society whether undertaken by the incumbent,

$$W^M - W^m > K, \tag{A4}$$

or by the start-up, $W^d - W^m > K$, which follows from Assumption A4.

Takeover game and information We model the bargaining over the takeover price as a non-cooperative game with asymmetric information. With probability α it is the incumbent I that

[10]Since the investment is costly, this assumption represents a necessary (but not sufficient) condition for the incumbent to invest. If $\pi_I^M < \pi_I^m$, the incumbent would always shelve the project after an acquisition.

makes a take-it-or-leave-it offer to the start-up S; with probability $1-\alpha$ it is S that makes the take-it-or-leave-it offer. Parameter α proxies I's bargaining power relative to S.

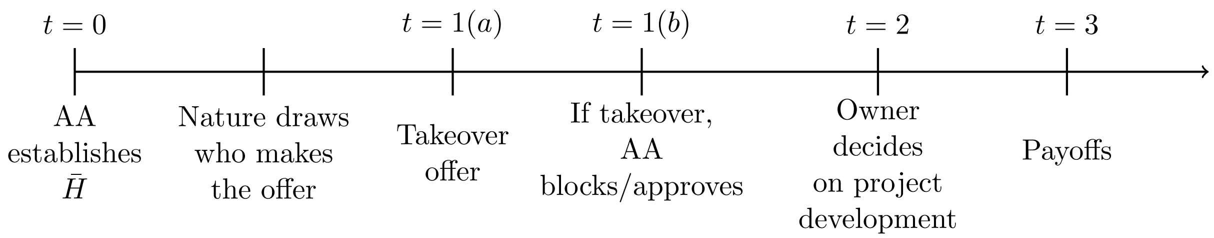
In the base model, the start-up knows its own type, but the incumbent and the AA only know the probability p that the start-up is viable, so that $\Pr(S = S_v) = p$ and $\Pr(S = S_u) = 1 - p$. In Section 6, we discuss possible micro-foundations for the sources of asymmetric information on the start-up's type. In Section 7, we consider the case where the incumbent has superior information on the start-up's type.

All the rest is common knowledge, so that when the AA establishes the merger policy and when it decides on a takeover proposal, it knows the investment cost K and it anticipates the payoffs in the different continuation games. Moreover, when it reviews a proposed takeover, the AA observes the takeover price and who has made the offer. Finally, all agents are risk neutral.

Timing Next, we describe the timing of the game (see also Figure 2).

- At $t = 0$, the AA commits to the standards for merger approval, $\bar{H}$. Subsequently, nature draws who makes the take-it-or-leave-it offer at $t = 1(a)$.
- At $t = 1(a)$, there is the 'takeover game': either I or S makes a takeover offer, which can be accepted or rejected by the recipient.
- At $t = 1(b)$, if a takeover is agreed upon, the AA approves or blocks it on the basis of the policy previously decided.
- At $t = 2$, the firm that owns the project decides whether to develop or shelve it.
- At $t = 3$, active firms sell in the product market, payoffs are realised and contracts are honoured.

Figure 2: Timeline



In what follows, we assume without loss of generality that:

$$\bar{H} \geq -(W^M - K - W^m). \tag{A5}$$

In our setting a takeover cannot produce a welfare gain higher than $W^M - K - W^m$; therefore if $\bar{H} < -(W^M - K - W^m)$ not even the most beneficial takeover would be approved.

Finally, in what follows we will say that a price is "high", and thus that a policy prohibits "high-price" takeovers, if the price P of the transaction is larger than the outside payoff of the viable start-up $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$. The outside payoff of the unviable start-up is $\pi_S^{\emptyset}(S_u) = 0$.

DEFINITION 1 (High- and low-price takeovers)**.**
A high-price takeover features a transaction price $P \geq \pi_S^{\emptyset}(S_v) = \pi_S^d - K$. *A low-price takeover features* $P < \pi_S^{\emptyset}(S_v) = \pi_S^d - K$.

Before solving the game by backward induction, we consider a version of the model in which only the AA cannot observe the start-up's type. Within such a simplified environment some of the main forces at work still emerge. In particular, a key insight of our analysis, which is that the bargaining outcome between incumbent I and start-up S reveals key information for the AA.

2.1 Benchmark: symmetric information between Incumbent and Start-up

In the base model, the incumbent I does not know the start-up's type. Here, we suppose instead that the incumbent has the same information as the start-up, whereas the AA only knows the prior probability that a start-up is viable ($p = \Pr(S = S_v)$). To simplify further, we assume that the incumbent has the bargaining power and that it develops the project upon acquiring the start-up (so that condition (30) is satisfied).

Let us look for the optimal merger policy, $\bar{H}$. First, if $\bar{H} \geq W^d - W^M$, the AA would approve any acquisition: the takeover of a viable start-up entails a welfare loss $H_v = W^d - K - (W^M - K) > 0$, whereas that of an unviable start-up results in a gain: $H_u = W^m - (W^M - K) < 0$ (Assumption A4). Since $\bar{H} \geq H_v > H_u$, both takeovers would be approved. The incumbent knows it and it will offer the outside payoffs $P = \pi_S^{\emptyset}(S_v) = \pi_S^d - K$ to the viable start-up, and $P = \pi_S^{\emptyset}(S_u) = 0$ to the unviable one. They will both accept. Expected welfare will be $W^M - K$.

Next, let $\bar{H} < W^d - W^M = H_v$. If the AA infers the acquisition involves a viable start-up, it will block it. Hence, there exists no equilibrium where the incumbent offers a higher price to a viable start-up S_v: the AA would correctly infer when the merger involves a viable start-up and would block it.

Since a viable start-up accepts to sell out only if $P \geq \pi_S^d - K$, any candidate equilibrium where the incumbent offers the same price independently of the start-up's type must feature an offer equal to $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$. However, the incumbent has an incentive to deviate from this offer when it faces an unviable start-up. By offering $\pi_S^d - K$, its profit is $\pi_I^M - \pi_S^d$. By not making such an offer, its profit is $\pi_I^m > \pi_I^M - \pi_S^d$ (by Assumption A3).

Therefore, if $\bar{H} < W^d - W^M$, then at the perfect Bayesian equilibrium, the incumbent makes no offer to the viable start-up (the AA would block it), and it offers $P = 0$ to the unviable one, which accepts it; the AA then approves the acquisition. Expected welfare under such a policy is $p(W^d - K) + (1 - p)(W^M - K)$, which is higher than expected welfare when $\bar{H} \geq W^d - W^M$. Hence, the optimal policy is to set $\bar{H} < W^d - W^M$.

This policy achieves the first best (conditional on development).[11] Note also that if the AA knew the start-up's type, it would simply block acquisitions targeting the viable start-up and approve acquisitions of unviable ones, achieving exactly the same outcome, i.e. the first best.

This analysis shows that the AA can make inference on the type of start-up acquired by observing the transaction price. Hence, it sets the welfare standard so as to ban welfare-detrimental high-price takeovers that suppress competition, and approve the low-price takeovers, which are welfare-beneficial. We shall show that, with asymmetric information between incumbent and start-up, the AA may need to set a stricter standard ($\bar{H} < 0$) in order to achieve the same first-best outcome.

3 Takeover game ($t = 1$)

After having solved a simplified version of it, let us solve our main model. At $t = 1$ the parties decide whether to engage in a takeover. The incumbent and the AA take their decisions facing imperfect information about whether, absent the takeover, the start-up can develop the project independently. They have the same prior belief p on S's type and will update their belief based on the information that is public at the time they take a decision: the offered price and the acceptance decision, who holds the bargaining power and the incumbent's incentive to develop or shelve after the takeover takes place. Section 3.1 describes the AA's decision at $t = 1(b)$, for given beliefs about the start-up. Section 3.2 illustrates the equilibrium takeover offer and acceptance decision, together with I's and AA's belief update processes. It also shows how the AA's conditions for approval affect the outcome of the bargaining game. In this section we present the equilibria in pure strategies. In Section 5 we will allow for equilibria in mixed strategies.

DEFINITION 2 (Perfect Bayesian equilibrium in pure strategies).
Let $s_j \in \{\emptyset, P_j\}$ be the pure-strategy profile of agent $j \in \{I, S_v, S_u\}$ that formulates the takeover offer, where $\emptyset$ denotes that no takeover offer is made and $P_j \in \mathbb{R}$ is the price offered by agent j. Let $r_{\neg j} \in \{Accept\ P_j,\ Reject\ P_j\}$ be the pure-strategy profile of the agent $\neg j$ that receives the price offer, with $\neg j \neq j$. Finally, let $\phi_I(\Omega) = \phi_{AA}(\Omega) = \phi(\Omega) = \Pr(S = S_v|\Omega) \in [0,1]$ be the incumbent and the AA's beliefs that $S = S_v$ given their information set Ω. If the incumbent makes the offer, $\Omega = \{s_I, r_{S_v}\}$ or $\Omega = \{s_I, r_{S_u}\}$. If S makes the offer, $\Omega = \{s_{S_v}, r_I\}$ or $\Omega = \{s_{S_u}, r_I\}$. The beliefs are computed using Bayes' rule whenever possible. A perfect Bayesian equilibrium (PBE) in pure strategies is denoted by $\{s_I, r_{S_v}, r_{S_u}; \phi(\{s_I, r_{S_v}\}), \phi(\{s_I, r_{S_u}\})\}$ if I makes the offer; $\{s_{S_v}, s_{S_u}, r_I; \phi(\{s_{S_v}, r_I\}), \phi(\{s_{S_u}, r_I\})\}$ if S makes the offer.

We characterize the PBE by specifying the (posterior) beliefs $\phi(\Omega)$ at each information set on the equilibrium path. We assume that, off equilibrium, the posterior beliefs of I and AA coincide with their priors, $\phi(\Omega) = p$, if the offer or acceptance decisions do not disclose additional information on the type of the target. If the acquisition goes through, we denote by $\pi_I^A =$

[11] In this environment, if the incumbent shelved the project, or the start-up makes the offers, the result that the AA's optimal policy achieves the first best still holds.

$\max(\pi_I^M - K, \pi_I^m)$ the incumbent's profits, gross of the takeover price, where $\pi_I^A = \pi_I^M - K$ if I develops; it is $\pi_I^A = \pi_I^m$ if it shelves.

3.1 Decision on merger approval ($t = 1(b)$)

LEMMA 1 (Decision on merger approval).
There exists a threshold $F_W(\pi_I^A, \bar{H}) \geq 0$ such that the AA authorises the takeover if and only if:

$$\phi(\Omega) \leq F_W(\pi_I^A, \bar{H}). \tag{2}$$

The threshold $F_W(\pi_I^A, \bar{H})$ is: (i) strictly increasing in $\bar{H}$; (ii) higher if $\pi_I^A = \pi_I^M - K$ than if $\pi_I^A = \pi_I^m$.

Proof. See Appendix A.1 Q.E.D.

The AA authorises a takeover if it assigns a sufficiently low probability that the start-up is viable. If the start-up is unviable, the takeover is either welfare neutral (when I shelves), because the project would die anyway; or it is welfare beneficial (when I develops), because it allows the project to reach the market. Instead, if the start-up is viable, the takeover is welfare detrimental: it suppresses product market competition and, when I shelves, it also suppresses project development. For a given $\bar{H}$, the AA approves the takeover if the scenario in which the takeover is welfare detrimental is sufficiently unlikely, that is when the probability that the start-up is viable is low enough.

Intuitively, Lemma 1 also shows that the AA is the more likely to approve a takeover: (i) the more lenient the standard for approval $\bar{H}$ (i.e. the higher the tolerated harm); (ii) when the incumbent develops than when it shelves.

Corollary 1 describes the AA's decision in some specific cases:

COROLLARY 1.

1. *When the incumbent develops, the AA* always *approves a takeover if it assigns probability one to the start-up being unviable (i.e. $\phi(\Omega) = 0$).*

2. *When the incumbent shelves, no takeover is approved if the merger policy commits to blocking any welfare detrimental takeover (i.e. $\bar{H} < 0$).*

Proof. See Appendix A.2. Q.E.D.

3.2 Equilibrium offers at $t = 1(a)$

Having established when the acquisition will be approved or prohibited, we move backwards to study the price offers. In Section 3.2.1, we assume that the incumbent makes a take-it-or-leave-it offer. In Section 3.2.2 we assume that it is the start-up that makes it.

3.2.1 The incumbent holds the bargaining power

We find the following:

LEMMA 2 (PBE of the bargaining game when I makes the offer)**.**
Let:

$$F_I(\pi_I^A) \equiv \frac{\pi_S^d - K}{\pi_I^A - \pi_I^d} \in (0, 1]. \tag{3}$$

When I makes a take-it-or-leave-it offer:

1. *If $\pi_I^A = \pi_I^m$ and either $p \leq F_I$ or $p > \max(F_W, F_I)$, no takeover occurs at the equilibrium.*

2. *For any π_I^A, if $p \in (F_I, \max(F_W, F_I)]$, the PBE is: $\{s_I^* = \pi_S^{\emptyset}(S_v), r_{S_v}^* = r_{S_u}^* = r_S^* =$ Accept $\pi_S^{\emptyset}(S_v)$; $\phi(\{s_I^*, r_S^*\}) = p\}$, with $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$.*

3. *If $\pi_I^A = \pi_I^M - K$ and either $p \leq F_I$ or $p > \max(F_W, F_I)$, the PBE is: $\{s_I^* = 0, r_{S_v}^* =$ Reject $0, r_{S_u}^* =$ Accept $0; \phi(\{s_I^*, r_{S_v}^*\}) = 1, \phi(\{s_I^*, r_{S_u}^*\}) = 0\}$.*

Proof. See Appendix A.3. Q.E.D.

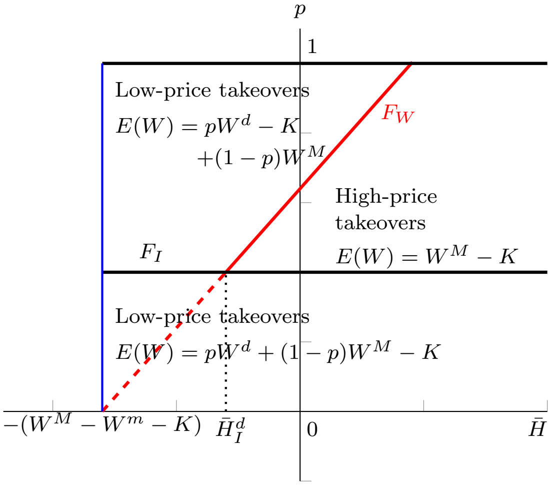
Lemma 2 hinges on the agreed takeover price conveying key information to the AA. Since the takeover price is lower than the outside option of the viable start-up, $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$, the AA infers that a start-up that accepts the offer is unviable ($\phi(\Omega) = 0$). Consistent with Definition 1, this is a low-price takeover. If the takeover price is $\pi_S^d - K$ or higher, it cannot be excluded that the start-up accepting the offer is viable. In this case, the posterior beliefs coincide with the priors: $\phi(\Omega) = p$. This is a high-price takeover under Definition 1. The anticipation of all this affects the incumbent's decision on which takeover price to offer.

Let us consider first the case in which the incumbent plans to develop the project. The incumbent anticipates that, if it offers a low price, the deal will be authorised (from Corollary 1). Instead, if it offers a high price, the takeover will be blocked, unless the a priori probability that it involves a viable start-up is sufficiently low (i.e. for all $\phi(\Omega) = p \leq F_W$ as defined by Lemma 1). Moreover, with a high-price offer, the incumbent is certain to appropriate the project and to avoid product market competition, but it might overpay for an unviable start-up. The latter is a risk worth taking when the a priori probability that the start-up is viable is sufficiently high (i.e. for all $\phi(\Omega) = p > F_I$).

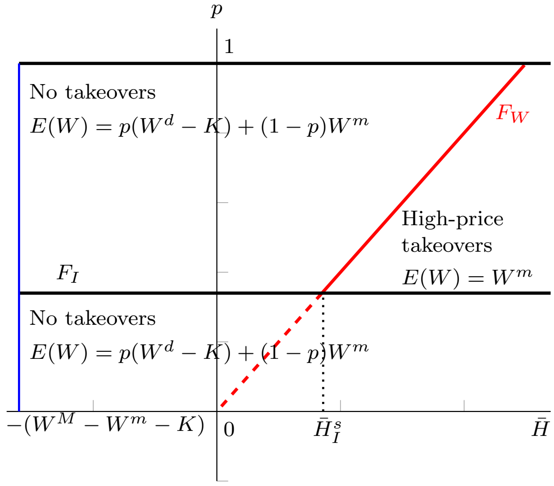
It is only when both conditions are satisfied simultaneously that the incumbent's preferred choice is also approved by the AA, so that a high-price takeover occurs at the equilibrium (part 2 of Lemma 2). A low-price takeover occurs otherwise, either because it is the incumbent's preferred option (when $p \leq F_I$), or because the incumbent anticipates that a high-price takeover would not be authorised and it has to settle for a low-price offer (when $p > \max(F_W, F_I)$), as claimed in part 3 of the lemma.

Figure 3 (left panel) displays the equilibrium takeovers and the expected welfare at $t = 0$, as a function of the merger policy, $\bar{H}$, and the a priori probability that the start-up is viable, p. It

Figure 3: Equilibrium takeovers when I holds the bargaining powers and associated welfare expected at $t = 0$.



(a) The incumbent develops



(b) The incumbent shelves

Notes: On the axes, $\bar{H}$ is the merger standard of review (level of tolerated harm); p is the prior probability that the start-up is viable. Below F_I the incumbent is unwilling to pay a high price since the probability of acquiring a viable firm is too low. Above F_W the AA would not approve a high price transaction because the probability of the acquired firm being viable is too high. $\bar{H}_I^d$ and $\bar{H}_I^s$, the values of $\bar{H}$ such that F_W and F_I cross when I develops, and respectively when I shelves, will be central to the determination of the optimal merger policy studied in Section 4. When the incumbent develops, $\bar{H}_I^d$ may be negative, a case displayed in this figure. With shelving, $\bar{H}_I^s$ is necessarily positive.

shows the regions where high- and low-price takeovers emerge at the equilibrium. Let us focus on the region in which $p > F_I$, so that the incumbent would want to make a high-price offer, and $p > F_W$, so that the AA would block such a takeover. Anticipating the AA's prohibition decision, the incumbent will then make a low-price offer. This illustrates the "selection effect" of the merger policy, that pushes the incumbent towards acquisitions that target only unviable start-ups, which are better for welfare. Since F_W increases in $\bar{H}$, as established by Lemma 1, the figure also shows that the stricter the merger policy (that is, the lower $\bar{H}$), the stronger the selection effect and the more likely that a low-price takeover occurs at the equilibrium instead of a high-price takeover. When the merger policy is strict enough, that is when $\bar{H} < \bar{H}_I^d$ in the figure, a high-price takeover would be blocked whenever it is the incumbent's preferred option, and only low-price takeovers occur at equilibrium. The cut-off level $\bar{H}_I^d$ is the value of $\bar{H}$ such that $F_W = F_I$, as shown in the figure.

The underlying mechanisms are similar when the incumbent plans to shelve (i.e. $\pi_I^A = \pi_I^m$). However, in this case offering a low price and acquiring an unviable start-up is equivalent to not engaging in a takeover: the project would be suppressed anyway, either by the incumbent or because of S's inability to invest. Since the takeover involves a negligible but positive transaction cost, when a high-price takeover is not the incumbent's best option (i.e. when $p \leq F_I$) or it is prohibited by the AA (i.e. when $p > F_W$), no takeover occurs at equilibrium (part 1 of the lemma).

Equilibrium takeovers with shelving are displayed in Figure 3 (right panel), with the associated welfare expected at $t = 0$. Also in the case of shelving, a sufficiently strict merger policy, that is $\bar{H} < \bar{H}_I^s$, implies that a high-price takeover would be blocked whenever it is the incumbent's preferred option and it never occurs at the equilibrium. The cut-off level $\bar{H}_I^s$ is the value of $\bar{H}$ such that $F_W = F_I$, as shown in the figure. However, when the incumbent shelves, a high-price takeover cannot be welfare beneficial in expected terms. Hence, differently from the case of development, the cut-off level $\bar{H}_I^s$ is necessarily positive.

3.2.2 The start-up holds the bargaining power

We now analyse the case in which the start-up makes a take-it-or-leave-it offer. The equilibrium of the takeover game is as follows:

LEMMA 3 (Pure-strategy PBE of the bargaining game when S makes the offer)**.**
Let:

$$F_S(\pi_I^A) \equiv \frac{\pi_S^d - K + \pi_I^m - \pi_I^A}{\pi_I^m - \pi_I^d} \in (0, 1]. \tag{4}$$

When $S \in \{S_v, S_u\}$ makes a take-it-or-leave-it offer:

1. *If $\pi_I^A = \pi_I^m$ and either $p \leq F_S$ or $p > \max(F_W, F_S)$, no takeover occurs at the equilibrium.*
2. *For any π_I^A, if $p \in (F_S, \max(F_S, F_W)]$, the PBE is: $\{s^*_{S_u} = s^*_{S_v} = \bar{P}, r_I^* =$ Accept $\bar{P}$; $\phi(\{\bar{P},$ Accept $\bar{P}\}) = p\}$, with $\bar{P} = \pi_I^A - \pi_I^m + p(\pi_I^m - \pi_I^d)$.*
3. *If $\pi_I^A = \pi_I^M - K$ and either $p \leq F_S$ or $p > \max(F_W, F_S)$ the PBE is: $\{s^*_{S_v} = \emptyset, s^*_{S_u} = P_L, r_I^* =$ Accept $P_L; \phi(\{P_L,$ Accept $P_L\}) = 0\}$, with $P_L = \pi_I^M - K - \pi_I^m > 0$.*

Proof. See Appendix A.4. Q.E.D.

The start-up makes an offer that leaves the incumbent indifferent between accepting and rejecting the deal given its beliefs about the start-up's type. Hence, equilibrium prices are higher than in the case in which the incumbent has the bargaining power. Apart from this consideration, the qualitative nature of the results and the underlying intuitions are similar to those in Lemma 2. Consider an equilibrium in which both start-ups offer the same price $\bar{P}$, which is (strictly) larger than the outside option of the viable start-up. This is a high-price takeover under our Definition 1. Observing such a price, the incumbent and the AA do not learn S's type, and assign the a priori probability p to the start-up being viable. The incumbent must be at least indifferent between paying that price and rejecting the offer. For this to be the case, the a priori probability must be high enough (i.e. $p > F_S$) because, in that case, the risk of overpaying for an unviable start-up is relatively low. However, the a priori probability must be sufficiently low for the AA to authorise the deal ($p \leq F_W$). It is only when both conditions are simultaneously satisfied that a high-price equilibrium exists (as claimed in part 2 of Lemma 3).

When, instead, either the incumbent is not willing to pay a price $P > \pi_S^\emptyset(S_v)$ (i.e. when $p \leq F_S$), or the incumbent would accept that offer but it is the AA that would block the deal (i.e. when $p > F_W$), then the viable start-up does not make any offer and the unviable one offers a price equal to $\pi_I^A - \pi_I^m < \pi_S^{\emptyset}(S_v)$. This is a low-price takeover under our Definition 1. The incumbent and the AA, observing this price, infer that the start-up is unviable. Since it is indifferent, the incumbent accepts the offer. When the incumbent develops ($\pi_I^A = \pi_I^M - K$), the AA authorises the deal (from Corollary 1), as claimed in part 3 of Lemma 3. If the incumbent shelves ($\pi_I^A = \pi_I^m$), the highest price that the incumbent is willing to pay is equal to zero. Since engaging in the takeover involves a positive transaction cost, the unviable start-up does not make any offer either and no takeover occurs at the equilibrium (part 1 of Lemma 3). Finally, an equilibrium in pure strategies in which the viable start-up also makes an (acceptable, but different) offer cannot exist, because the unviable start-up would have an incentive to mimic the viable one.

With development, a "selection effect" of the merger policy arises: when a high-price offer is accepted by the incumbent (that is, when $p > F_S$) but the AA blocks the deal (because $p > F_W$), the viable start-up will refrain from making an offer, while the unviable one will offer a low price. The merger policy, then, pushes towards acquisitions that target only unviable start-ups and are superior in terms of welfare. Moreover, the stricter the merger policy (that is, the lower $\bar{H}$) the stronger the selection effect and the more likely that a low-price takeover replaces a high-price takeover at the equilibrium.

If S holds the bargaining power, the figures displaying the equilibrium takeovers as function of $\bar{H}$ and p are similar to those in Section 3.2.1, with F_S replacing F_I, $\bar{H}_S^i$ replacing $\bar{H}_I^i$, and $i = s, d$ depending on whether I shelves (s) or develops (d) (see Figure B.1, Appendix B).

4 Optimal merger policy ($t = 0$)

In this section, we study the optimal merger policy at $t = 0$, when the AA commits to the threshold of tolerated harm, $\bar{H}$. The AA's choice is taken before nature draws who makes the take-it-or-leave-it offer – α is the probability that I holds the bargaining power. The AA knows whether the incumbent shelves or develops the project of an acquired start-up. The optimal policy will be derived considering the pure-strategy equilibria of the bargaining game at $t = 1$ (we shall also build on Figure 3 and Figure B.1 for an intuitive explanation of the optimal policy). In Section 5 we extend our analysis of the bargaining game to consider mixed-strategy equilibria, and then derive the optimal merger policy for all the admissible equilibria of the continuation game. We shall show that the merger policies identified in this section are optimal also when allowing for mixed-strategy equilibria.

PROPOSITION 1 (Optimal merger policy with pure-strategy equilibria)**.**
The optimal merger policy commits to a standard of review that prevents high-price takeovers (such that $P \geq \pi_S^{\emptyset}(S_v)$) at the equilibrium:

(a) If $\pi_I^A = \pi_I^M - K$, there exists a threshold level of $\bar{H}$, $\bar{H}^d > -(W^M - W^m - K)$, such that all $\bar{H} \leq \bar{H}^d$ are optimal for any value of α.

(b) If $\pi_I^A = \pi_I^m$, there exists a threshold level of $\bar{H}$, $\bar{H}^s > 0$ such that all $\bar{H} \leq \bar{H}^s$ are optimal for any value of α.

(c) All $\bar{H} \leq \min(\bar{H}^d, \bar{H}^s)$ are optimal for any value of α and π_I^A.

Proof. See Appendix A.5. Q.E.D.

Figures 3 (and Figure B.1 in Appendix B) show that high-price takeovers are the least desirable outcome for welfare. Hence, the merger policy that maximises welfare expected at $t = 0$ is the one that commits to standards of review that remove the possibility that high-price takeovers occur at the equilibrium.

Consider first the case of shelving. A high-price takeover is a killer acquisition that deprives society of the project and (strictly) decreases welfare relative to the no-takeover scenario. A merger policy that commits to prohibiting all takeovers that, at the moment in which they are reviewed, are welfare detrimental screens such takeovers out. As the right panels of Figures 3 and B.1 show, to avoid that high-price takeovers occur at the equilibrium, it suffices to commit to a *sufficiently low tolerated level of harm*, such that a high-price takeover is prohibited whenever it is the preferred choice of the agent that makes the offer.

There is a continuum of optimal policies, all equivalent in terms of expected welfare: any $\bar{H} \leq \bar{H}_j^s$, with $\bar{H}_j^s > 0$ and $j = I, S$ depending on whether the incumbent or the start-up makes the offer, is optimal.[12] By taking the minimum value of the cut-offs across the two cases, that is $\bar{H}_s = \min(\bar{H}_I^s, \bar{H}_S^s) > 0$, Proposition 1, part (b), characterises the policy that is optimal irrespective of the bargaining power allocation.

Consider next the case in which the incumbent develops. Lemma 1 shows that, for given standards of review, the AA is more likely to approve a high-price takeover when I develops than when it shelves: the takeover remedies to the start-up's inability to develop, when it is not viable; and it does not kill the innovation, when the start-up is viable. Therefore, in order to prevent high-price takeovers from arising, the optimal merger policy might need to commit to a more stringent standard of review than in the case of shelving. The cut-off $\bar{H}_i^d$ (such that $F_W = F_i$, with $j = I, S$ depending on who makes the offer) below which high-price takeovers will not occur at the continuation equilibrium might be negative, as depicted in the left panels of Figures 3 and B.1. In turn, the upper bound that characterises the optimal policies irrespective of who makes the offer – that is $\bar{H}^d \equiv \min(\bar{H}_I^d, \bar{H}_S^d)$ – may also be negative (Proposition 1, part (a)). When this is the case the optimal merger policy commits to prohibiting takeovers that are welfare beneficial, if their expected welfare gain is low enough.

Why is it optimal to commit to prohibiting a takeover that is expected to increase welfare? The reason is that, under the optimal policy, if the incumbent has the bargaining power, it will anticipate that high-price takeovers will not be authorised. Hence, it will have no other option than offering a low price. If the start-up has the bargaining power, the unviable start-up will

[12] Recall from Section 3.2 that $\bar{H}_j^s > 0$ is the cut-off level of $\bar{H}$ such that $F_W = F_i$, with $i = I, S$ depending on who makes the offer.

switch to a low-price offer, while the viable start-up will refrain from making an offer. In either case, the start-up will be acquired only when unviable, and society will benefit from intensified competition when the start-up is, instead, viable.

Since we assume that it can compute the relevant cut-offs in the various cases, at $t = 0$ the AA can also commit to an "information-free" merger policy. This policy is not contingent on the incumbent's decision to shelve or develop, and on the allocation of bargaining power, as indicated in Proposition 1, part (c).

Finally, comparing the values of expected welfare in Figures 3 and B.1, it turns out that the optimal policy generates the first best, which is equal to $p(W^d - K) + (1-p)(W^M - K)$ with development and $p(W^d - K) + (1-p)W^m$ with shelving.

5 Equilibrium analysis with mixed strategies

The previous analysis showed that, when we focus on pure-strategy equilibria, there cannot exist an equilibrium in which the viable start-up formulates a higher price offer than the unviable one, and the incumbent accepts both offers, because the unviable start-up would always have an incentive to mimic the viable start-up. The only way to avoid the unviable start-up's incentive to mimic the viable one is to have the latter refrain from making an offer. Hence, the equilibrium comes with an inefficiency from the firms' perspective: the viable start-up is not acquired even though the takeover would increase the joint profits of target and acquirer.

If S holds the bargaining power, such an inefficiency is alleviated when one allows for equilibria in mixed strategies, whose analysis is the object of this section.[13] Lemma 4 below shows that an equilibrium may exist where the viable start-up offers the high price P_H with certainty, while the unviable one randomises between P_H and a lower price P_L. When observing P_H the incumbent cannot be sure that the offer originates from a viable start-up, and does not always accept. This reduces the unviable start-up's incentive to mimic the viable one.

We will show that allowing for mixed strategies does not modify the conclusions reached in Section 4: the merger policy delineated in Proposition 1 will still be the optimal one.

DEFINITION 3 (Perfect Bayesian equilibrium in mixed strategies)**.**
Let $\gamma_k^H = \Pr(P_{S_k} = P_H|S_k)$ *and* $1 - \gamma_k^H = \Pr(P_{S_k} = P_L|S_k)$ *be the probability that* S_k *assigns to actions* $P_S = P_H$ *and* $P_S = P_L$*, respectively, with* $k \in \{v, u\}$ *and* $P_H, P_L \in \mathbb{R}$*. Then,* (γ_v^H, γ_u^H) *is the mixed-strategy profile of agent* S*. Let* $\beta^H = \Pr(\textit{Accept } P_H)$ *and* $\beta^L = \Pr(\textit{Accept } P_L)$ *be the probability that* I *assigns to action* Accept P_S *when* S *plays* $P_S = P_H$ *and* $P_S = P_L$*, respectively. Then,* (β^H, β^L) *is the mixed-strategy profile of agent* I*. A perfect Bayesian equilibrium in mixed strategies is denoted by* $\{\gamma_u^H, \gamma_v^H, \beta^H, \beta^L; \phi(P_H), \phi(P_L)\}$.

Lemma 4 describes the equilibria in mixed strategies and specifies the conditions for their existence.

[13] If I makes the take-it-or-leave-it offer, the bargaining game boils down to a standard problem of monopolistic screening. Hence, as is well-known, there cannot exist mixed-strategy equilibria.

LEMMA 4 (Hybrid PBE of the bargaining game when S makes the offer).
If $\pi_I^A = \pi_I^M - K$, $p \leq F_S$, there exist hybrid PBE featuring:

- $P_L = \pi_I^M - K - \pi_I^m$ *and* $P_H \in (\pi_S^d - K, \hat{P}_H(\bar{H})]$, *with* $P_H > \pi_S^{\emptyset}(S_v) > P_L > 0$, *and* $\hat{P}_H(\bar{H}) < \pi_I^M - K - \pi_I^d$ *increasing in* $\bar{H}$;
- $\phi(P_H) \leq F_W$ *for all* $P_H \in (\pi_S^d - K, \hat{P}_H(\bar{H})]$;
- S_u *offering* P_H *with probability:*

$$\gamma_u^H = \frac{p}{(1-p)} \frac{(\pi_I^M - K - \pi_I^d - P_H)}{(P_H - \pi_I^M + K + \pi_I^m)} \in (0,1)$$

  (strictly) decreasing in P_H;
- S_v *offering* P_H *with probability* $\gamma_v^H = 1$;
- I *accepting* P_H *with probability* $\beta^H = P_L/P_H \in (0, \beta^L)$, *(strictly) decreasing in* P_H, *and accepting* P_L *with probability* $\beta^L = 1$;
- *posterior beliefs:*

$$\phi(P_H) = \frac{p}{\gamma_u^H(1-p)+p} = \frac{P_H - \pi_I^M + K + \pi_I^m}{\pi_I^m - \pi_I^d} \in (0,1) \text{ and } \phi(P_L) = 0,$$

  with $\phi(P_H) > p$, *and* $\phi(P_H)$ *(strictly) increasing in* P_H.

Proof. See Appendix A.6. Q.E.D.

Mixed-strategy equilibria feature a unique low price P_L, which takes the same value as in the pure-strategy equilibrium of Lemma 3, part 3. The high price P_H belongs to an interval whose lower bound is $\pi_S^d - K$, and whose upper bound $\hat{P}_H(\bar{H})$ is determined by the merger policy.

When it observes a high takeover price P_H, the AA (as well as I) updates the priors p, and assigns a higher probability $\phi(P_H)$ to the start-up being viable. For the takeover to be approved the posterior probability $\phi(P_H)$ must be lower than the threshold that governs the AA's decision F_W. Lemma 4 shows that $\phi(P_H)$ is strictly increasing in P_H. The posterior probability makes the incumbent indifferent between accepting and rejecting an offer involving the price P_H. The higher the price, the less profitable for the incumbent to accept the offer, and the less profitable it must be to reject, so as to ensure indifference. Rejecting the offer is less profitable the higher the (posterior) probability that the start-up is viable, because the incumbent has higher chances to face competition in the final market. Hence, given $\bar{H}$, for the deal to be approved the price P_H must be sufficiently low. Moreover, a more stringent merger policy (i.e. a lower $\bar{H}$), by decreasing the threshold for approval F_W, also decreases the upper bound $\hat{P}_H(\bar{H})$ of the prices P_H that can be supported at the equilibrium in mixed strategies. These are key considerations for the analysis of the optimal merger policy of the next section.

5.1 Optimal merger policy with pure- and mixed-strategy equilibria

The earlier analysis has shown that multiple equilibria may arise when S makes the offer, I plans to develop and $p \leq F_S$. Namely, the pure-strategy equilibrium in Lemma 3 (part 3) and mixed-strategy equilibria in Lemma 4.

The equilibrium in pure strategies in which a low-price takeover occurs exists for any feasible $\bar{H}$. Since a start-up is acquired only if it is unviable, expected welfare at $t = 0$ is given by:

$$E(W^{ps}) = p(W^d - K) + (1-p)(W^M - K).$$

Expected welfare at $t = 0$ with the equilibria in mixed strategies is:

$$\begin{aligned} E(W^{ms}) &= p[W^d - K - \beta^H(W^d - W^M)] \\ &+ (1-p)[W^M - K - \gamma_u^H(1-\beta^H)(W^M - K - W^m)]. \end{aligned}$$

The first term in the expression of $E(W^{ms})$ refers to the case in which the start-up is viable, which occurs with probability p: in that case expected welfare is given by $W^d - K$ – i.e. welfare when the start-up remains independent and reaches the final market giving rise to a duopoly – minus the loss $W^d - W^M$ caused to welfare when the high-price offer is accepted (which occurs with probability β^H), the start-up is acquired and product market competition is suppressed. The second term refers to the case in which the start-up is unviable, which occurs with probability $1-p$: expected welfare is given by $W^M - K$ – i.e. welfare when the start-up is acquired and the incumbent develops the project – minus the loss caused to welfare when the unviable start-up offers a high price and that offer is rejected (which occurs with probability $\gamma_u^H \times (1-\beta^H)$) and the project cannot be developed.

The comparison between $E(W^{ps})$ and $E(W^{ms})$ shows that the equilibrium in pure strategies dominates, in terms of welfare, any equilibrium in mixed strategies: first, because a viable start-up is never acquired and competition never suppressed; second, because an unviable start-up is always acquired and it is never the case that the project fails to reach the final market. These considerations are summarised in the following lemma:

LEMMA 5.
The expected welfare in the equilibrium in pure strategies featuring low-price takeovers (such that $P < \pi_S^{\emptyset}(S_v)$) is higher than in any mixed-strategy equilibrium.

Proof. It follows from the discussion above. Q.E.D.

The optimal merger policy must prevent mixed-strategy equilibria from arising. This goal can be achieved by setting a sufficiently strict standard of review so that the posterior probability $\phi(P_H)$ is (strictly) higher than the threshold that governs the decision of the AA, F_W, for all feasible $P_H > \pi_S^d - K$. This ensures that the AA blocks the takeover whenever it observes a transaction price $P_H > \pi_S^d - K$.

We now provide the intuition for the reason why the standards of review that are optimal when one focuses on equilibria in pure strategies are optimal also when one allows for mixed strategies, as stated by Proposition 2.

For the case in which the incumbent develops and the start-up makes the offer, the optimal standards of review $\bar{H}$ characterised in Proposition 1 are such that $F_W \leq F_S$: they make sure that, whenever the incumbent is willing to accept a high price $\bar{P}$ – i.e. whenever the posterior probability $\phi(\bar{P}) = p > F_S$ – the AA blocks the transaction because $\phi(\bar{P}) = p > F_W$. F_S is the cut-off value of the posterior that makes the incumbent indifferent between accepting and rejecting the offer $\bar{P} = \pi_S^d - K$ (see the proof of Lemma 3).

In the mixed strategy equilibria, the posterior probability makes the incumbent indifferent between accepting and rejecting an offer involving the price P_H. Therefore, as $P_H \to \pi_S^d - K$, the posterior $\phi(P_H)$ approaches F_S (from above), and $\phi(P_H) > F_S$ for all the prices $P_H > \pi_S^d - K$ that are feasible at the mixed strategy equilibria. As a consequence, the standards of review that ensure $F_W \leq F_S$ also ensure that $\phi(P_H) > F_W$: the AA blocks any transaction involving a high-price offer $P_H > \pi_S^d - K$ and mixed-strategies equilibria cannot exist.

PROPOSITION 2 (Optimal merger policy with pure- and mixed-strategy equilibria)**.**
Under the merger policy described in Proposition 1 the game admits no mixed strategy equilibria, hence the policy remains optimal also when equilibria in mixed strategies are allowed for.

Proof. See Appendix A.7. Q.E.D.

6 Sources of asymmetric information on the start-up's type

In the baseline model, we assume that the incumbent and the AA do not observe the start-up's type. Here, we discuss possible sources of asymmetric information that are particularly relevant in the context of potential competition.

The first relies on the possibility that the investment cost depends on non-financial resources, like innovations or managerial skills, that, different from the incumbent, the start-up may lack. Suppose that the cost of project development for the start-up is $\tilde{c}K$, with $\tilde{c} \in \{1, \bar{c}\}$. If S has access to technological innovations or enough managerial skills then its project-development cost is K, with $\tilde{c} = 1$. If it lacks such resources or skills, S's development cost is $\bar{c}K$, with $\bar{c}K > \pi_S^d$. That is, the low-cost start-up would be able to profitably bring the project to the market absent the takeover, while the high-cost start-up would not. Project development costs K to the incumbent.

To recast this setting within our model, we can assume that, different from S (which knows its type $\tilde{c}$), I and the AA can only observe $p = \Pr(\tilde{c} = 1)$. This is in line with what is commonly assumed in the literature on innovation economics, in which the innovator is more informed than the acquirer about the value of the innovation. This can happen because the start-up is an innovator that is privately informed about the value of its invention (Anton and Yao, 1994), or about the existence of relevant patents or patent applications (Ganglmair and Tarantino, 2014) that may lower the cost of development, or about the skills of its managers.

The second relies on the presence of financial constraints for the start-up. There is ample evidence that financial constraints impede start-ups' growth.[14] We then assume that the start-up and the incumbent differ in their ability to fund the investment. Whereas I is endowed with sufficient own assets to pay K, S holds cash $A \geq 0$, and needs to search for funds $K - A > 0$ in competitive capital markets. Following Holmström and Tirole (1997), the project is developed with certainty if and only if the start-up exerts non-contractible effort. Without effort, the project fails and yields no profit, but the start-up obtains private benefits $B > 0$. The financiers will fund the start-up's project if and only if $B \leq \bar{B} = \pi_S^d - K + A$, which corresponds to the NPV of the project with effort.

This framework can be nested within our baseline model by assuming that B is observed by S and the financiers, not by I and the AA. The latter observe $p = F(\bar{B})$, with $F(\bar{B}) = \Pr(B \leq \bar{B})$. This assumption reflects the different skills of the players in the game. While it is the core business of financiers to establish the financial merits of a company, it is not the key expertise of incumbents and regulators (Tirole, 2006). Moreover, financiers can inspect S's banking records and history of debt repayment, while I and the AA typically do not have access to this information (or may lack the financial skills that is necessary to interpret the relevant data).

7 The incumbent has an informational advantage

One may think that the incumbent, being already in the market, might have better insights as to whether a start-up is able to successfully develop its project. In this section, then, we assume that the incumbent has perfect information on whether $S = S_v$ or $S = S_u$, whereas the start-up and the AA assign prior probability p to the start-up being of type $S = S_v$. We also assume that after the takeover proposal and the AA's decision, and before investing (if it has not been bought by the incumbent), the start-up learns its type.

For shortness, we consider only the case where the incumbent prefers to invest if it acquires the start-up: $\pi_I^A = \pi_I^M - K \geq \pi_I^m$ (condition (30) is satisfied). Otherwise, the game is the same as in the rest of the paper.

Initially, we present (perfect Bayesian) equilibria in pure strategies characterized as in Definition 2, with the difference that we denote by $s_I(S)$ and $r_I(S)$ the pure-strategy profile of I when it formulates the offer to type $S \in \{S_v, S_u\}$, and when it receives the offer from type $S \in \{S_v, S_u\}$, respectively. We then study the resulting optimal merger policy. Finally, we allow for mixed strategies in the bargaining game and formulate the optimal merger policy accordingly. We directly analyse the takeover game of $t = 1(a)$, because the decision of the AA on the merger approval in $t = 1(b)$ is the same as in Section 3.1.

[14]The Federal Reserve Bank of New York (2017) finds that 69% of US start-up applicants obtained less than the amount of funding they sought, compared to 54% of mature applicants. The Survey on the Access to Finance of Enterprises reaches similar conclusions in the sample of European small and medium firms (European Central Bank, 2019).

7.1 Equilibria in pure strategies

We first consider the case where the incumbent has the bargaining power, then the one where the start-up has it.

7.1.1 The incumbent holds the bargaining power

LEMMA 6 (PBE of the bargaining game when I makes the offer and develops)**.**
Let $\pi_I^A = \pi_I^M - K$ *and*

$$F_{II} \equiv \frac{\pi_I^M - K - \pi_I^m}{\pi_S^d - K} \in [0, 1). \tag{5}$$

When I makes a take-it-or-leave-it offer:

1. *If* $p \leq \min(F_W, F_{II})$*, the PBE is:* $\{s_I^*(S_u) = s_I^*(S_v) = s_I^* = p\pi_S^{\emptyset}(S_v), r_{S_v}^* = r_{S_u}^* = r_I^* = Accept\ p\pi_S^{\emptyset}(S_v); \phi(\{s_I^*, r_S^*\}) = p\}$*, with* $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$*.*

2. *If* $\bar{H} < W^d - W^M$ *and* $p > \min(F_W, F_{II})$*, no takeover occurs at the equilibrium.*

3. *If* $\bar{H} \geq W^d - W^M$ *and* $p > F_{II}$*, the PBE is:* $\{s_I^*(S_u) = \emptyset, s_I^*(S_v) = \pi_S^{\emptyset}(S_v),\ r_{S_v}^* = Accept\ \pi_S^{\emptyset}(S_v); \phi(\{s_I^*(S_v), r_{S_v}^*\}) = 1, \phi(\{s_I^*(S_u), r_{S_u}^*\}) = 0\}$*.*

Proof. See Appendix A.8. Q.E.D.

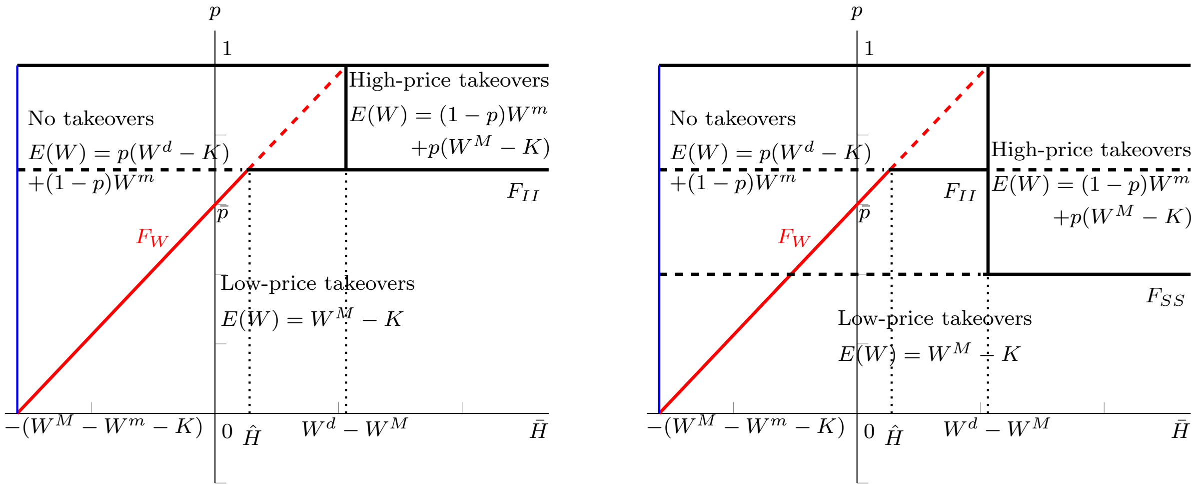
Part 1 of the lemma lists the conditions for the existence of an equilibrium where the incumbent offers the price $P = p\pi_S^{\emptyset}(S_v) = p(\pi_S^d - K)$, equal to the expected profits of a start-up that does not know its type, both to a viable and to an unviable start-up. First, this equilibrium exists if the prior probability is sufficiently low, i.e. $p \leq F_{II}$, so that the expected profitability of the start-up is low enough to make the offer $P = p(\pi_S^d - K)$ profitable irrespective of S's type. Moreover, for the equilibrium to exist the AA must approve the deal. This is the case if the AA assigns sufficiently low probability to the scenario in which the takeover is welfare detrimental, i.e. if $p \leq F_W$. When $p \leq \min(F_W, F_{II})$ both conditions are satisfied and a low-price takeover – such that the price $P = p\pi_S^{\emptyset}(S_v) < \pi_S^{\emptyset}(S_v)$ is lower than the outside option of the viable start-up – occurs at the equilibrium.

When offering $P = p(\pi_S^d - K)$ to an unviable start-up is unprofitable, i.e. $p > F_{II}$, an equilibrium in which the incumbent acquires any S does not exist. At the equilibrium the incumbent acquires only a viable start-up, by offering a high price equal to $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$ to $S = S_v$, and abstaining from making an offer to $S = S_u$. The equilibrium exists if merger control is so lenient to approve a takeover involving a viable start-up. This is the case if $\bar{H} \geq W^d - W^M$.

In all the other cases no takeover occurs at the equilibrium because the AA either blocks a high-price deal (namely, when $p > F_{II}$ and $\bar{H} \leq W^d - W^M$) or it blocks a low-price deal featuring $P = p\pi_S^{\emptyset}(S_v)$ (namely, when $p \in (F_W, F_{II})$.

Differently from the setting in which the start-up knows its type (Section 3.2.1), now that the incumbent has superior information a low-price takeover does not target only unviable start-ups.

Figure 4: Equilibrium takeovers when I holds the bargaining powers and associated welfare expected at $t = 0$.



(a) The incumbent makes the offer

(b) The start-up makes the offer

Notes: $\bar{H}$ is the merger standard of review (level of tolerated harm); p is the prior probability that the start-up is viable.

Any start-up is acquired at the low-price price $P = p(\pi_S^d - K) < \pi_S^{\emptyset}(S_v)$ at the equilibrium. Therefore, in this setting, a pure-strategy equilibrium in which only an unviable start-up is acquired cannot exist, because also the viable start-up accepts a low price if the prior probability p is sufficiently low. It is the high price $P = \pi_S^{\emptyset}(S_v) = \pi_S^d - K$ offered by I to S_v in part 3 of the lemma that reveals that the start-up is viable.

Figure 4 (left panel) displays the takeovers (and the respective expected welfare) occurring at the equilibrium when the incumbent makes the offer, depending on the prior probability p and the standard of review $\bar{H}$.

7.1.2 The start-up holds the bargaining power

We turn here to the case where the start-up makes take-it-or-leave-it offers at the takeover stage.

LEMMA 7 (PBE of the bargaining game when S makes the offer and I develops)**.**
Let $\pi_I^A = \pi_I^M - K$ *and*

$$F_{SS} \equiv \frac{\pi_I^M - K - \pi_I^m}{\pi_I^M - K - \pi_I^d} \in (0,1), \tag{6}$$

with $F_{SS} \leq F_{II}$*.*

When S *makes a take-it-or-leave-it offer:*

1. *If either* $\bar{H} < W^d - W^M$ *and* $p \leq \min(F_W, F_{II})$ *or* $\bar{H} \geq W^d - W^M$ *and* $p \leq F_{SS}$*, the PBE is:* $\{s_S^* = P_L, r_I^*(S_v) = r_I^*(S_u) = r_I^* = \textit{Accept } P_L; \phi(\{s_S^*, r_I^*\}) = p\}$*, with* $P_L = \pi_I^M - K - \pi_I^m$*.*
2. *If* $\bar{H} < W^d - W^M$ *and* $p > \min(F_W, F_{II})$*, no takeover occurs at the equilibrium.*

3. *If $\bar{H} \geq W^d - W^M$ and $p > F_{SS}$, the PBE is:* $\{s_S^* = \pi_I^M - K - \pi_I^d, r_I^*(S_v) = Accept\ \pi_I^M - K - \pi_I^d,$ $r_I^*(S_u) = Reject\ \pi_I^M - K - \pi_I^d$; $\phi(\{s_S^*, r_I^*(S_v)\}) = 1, \phi(\{s_S^*, r_I^*(S_u)\}) = 0\}$.

Proof. See Appendix A.9 . Q.E.D.

The start-up has two options: either offering a low price P_L that the incumbent is willing to pay irrespective of S's type, or a higher price that the incumbent is willing to pay only for a viable start-up. In the latter case the offer is accepted only with probability p. Hence, offering a low price is the start-up's preferred option if the prior probability that S is viable is sufficiently low, i.e. $p \leq F_{SS}$. However, the AA has to approve such a takeover, which occurs if $p \leq F_W$. Hence, it is only when $p \leq \min(F_W, F_{SS})$ that a low-price takeover, where the incumbent acquires any start-up, occurs at the equilibrium.

When $p > F_{SS}$, the start-up's preferred option is to offer a high price $P = \pi_I^M - K - \pi_I^d > \pi_S^{\emptyset}(S_v)$ such that I accepts only if $S = S_v$. However, the AA approves a takeover at which a viable start-up is sold out only if the tolerated level of harm is high enough, i.e. $\bar{H} > W^d - W^M$. When these two conditions are satisfied, a high-price takeover occurs at the equilibrium.

When $p > F_{SS}$ but the standard of review is not so lenient (i.e. $\bar{H} \leq W^d - W^M$), the start-up's preferred option is blocked by the AA. Hence, S will settle for a low-price takeover, when being acquired at the price $P_L = \pi_I^M - K - \pi_I^m$ is more profitable than going ahead independently (which occurs if $p \leq F_{II}$) and the AA approves the deal (which occurs if $p \leq F_W$). Thus, the merger policy, by forcing S to choose its second-best option, exerts a selection effect.

In all the other cases no takeover occurs at the equilibrium, either because the AA blocks a low-price takeover (when $p \in (F_W, F_{II}]$) or because S finds it more profitable to go ahead independently than being acquired at a low price (when $p > F_{II}$).

Figure 4 (right panel) displays the takeovers (and the respective expected welfare) occurring at the equilibrium when S makes the offer.

7.1.3 The optimal merger policy

PROPOSITION 3 (The optimal merger policy when I has superior information)**.**
When the incumbent has superior information and $\pi_I^A = \pi_I^M - K$, it is optimal for the AA to commit to a standard of review such that only welfare increasing takeovers are approved (i.e., $\bar{H}^* = 0$) for any value of α. This standard of review prevents high-price takeovers at the equilibrium.

Proof. See Appendix A.10. Q.E.D.

Differently from the results obtained with the model of Section 2, when the incumbent has superior information, the first-best outcome, i.e. the case in which only unviable start-ups are acquired, cannot be achieved by the merger policy, because it is not a feasible equilibrium of the bargaining game (see Figure 4).

However, similarly to the results obtained when S holds superior information, the optimal merger policy forbids high-price takeovers, that are inferior in terms of welfare both to low-price takeovers (in the latter case also unviable start-ups are acquired and society benefits from project development) and to the no-takeover equilibrium (in which the viable start-up develops independently and society benefits from intensified competition).

Once high-price takeovers are prohibited, either low-price takeovers (such that start-ups are acquired irrespective of their type) or no takeovers are possible equilibrium outcomes. In the former case viable start-ups are acquired and competition softened, but development is promoted. It is therefore optimal to choose a merger policy that authorised low-price takeovers whenever they are welfare beneficial, i.e. $\bar{H} = 0$.

7.2 Mixed-strategy equilibria

Under the pure-strategy PBE analysed so far, unviable start-ups may not be acquired at the equilibrium, even though such acquisitions would increase joint profits and welfare. We now show that, if I holds the bargaining power, this inefficiency can be attenuated when considering mixed-strategy equilibria of the bargaining game.[15]

DEFINITION 4 (PBE in mixed strategies when I holds superior information)**.**
Let $\gamma^L(S_k) = \Pr(P_I = \tilde{P}_L|S_k)$ and $1 - \gamma^L(S_k) = \Pr(P_I = \tilde{P}_L|S_k)$ be the probability that I assigns to offering $P_I = \tilde{P}_L$ and $P_I = \tilde{P}_H$, respectively, to start up S_k, with $k = v, u$, and $\tilde{P}_L, \tilde{P}_H \in \mathbb{R}$. Then, $(\gamma^L(S_v), \gamma^L(S_u))$ is the mixed-strategy profile of agent I. Let $\beta^L = \Pr(Accept\ \tilde{P}_L)$ and $\beta^H = \Pr(Accept\ \tilde{P}_H)$ be the probability that S assigns to action Accept P_I *when I plays $P_I = \tilde{P}_L$ and $P_I = \tilde{P}_H$, respectively. Then, (β^H, β^L) is the mixed-strategy profile of agent $S \in \{S_v, S_u\}$. A perfect Bayesian equilibrium in mixed strategies is denoted by $\{\gamma^H(S_v), \gamma^H(S_u), \beta^H, \beta^L; \phi(\tilde{P}_L), \phi(\tilde{P}_H)\}$.*

LEMMA 8 (Hybrid PBE of the bargaining game when I has superior information)**.**
Let $\pi_I^A = \pi_I^M - K$. If $p > F_{II}$, there exist hybrid PBE such that:

- $\tilde{P}_L \in (0, \pi_I^M - K - \pi_I^m]$ *and* $\tilde{P}_H = \pi_S^d - K$, *with* $\tilde{P}_H = \pi_S^{\emptyset}(S_v) > \tilde{P}_L > 0$;
- *I offers the price $\tilde{P}_L$ to S_u with probability $\gamma^L(S_u) = 1$;*
- *I offers $\tilde{P}_L$ with probability*
$$\gamma^L(S_v) = \frac{\tilde{P}_L(1-p)}{p(\pi_S^d - K - \tilde{P}_L)}$$
and $\tilde{P}_H$ with probability $1 - \gamma^L(S_v)$ to S_v;
- *if the start-up receives the offer $\tilde{P}_H$, it accepts with probability $\beta^H = 1$;*

[15] As in the main model, there cannot exist mixed-strategy equilibria of the bargaining game when the less-informed party (S in this case) makes the take-it-or-leave-it offer.

- *if the start-up receives the offer* $\tilde{P}_L$*, it accepts with probability*

$$\beta^L = \frac{\pi_I^M - \pi_I^d - \pi_S^d}{\pi_I^M - K - \pi_I^d - P_L} \in (0,1);$$

- *when receiving* $\tilde{P}_L$*,* $\phi(\tilde{P}_L) < p$*, with* $\phi'(\tilde{P}_L) > 0$*;*

- *when receiving* $\tilde{P}_H$*,* $\phi(\tilde{P}_H) = 1$*; thus,* $\phi(\tilde{P}_H) = 1 \leq F_W$ *if and only if* $\bar{H} \geq W^d - W^M$*.*

Proof. See Appendix A.11. Q.E.D.

With pure-strategy equilibria, if $p > F_{II}$ the incumbent is not able to acquire the unviable start-up because the price that it is willing to offer to both types is lower than S's expected profits computed based on the prior probability p (and equal to $p\pi_S^{\emptyset}(S_v)$).

If mixed strategies are allowed for, the lemma shows that, when $p > F_{II}$, the incumbent offers a low price $\tilde{P}_L$ with certainty when it observes an unviable start-up, while it randomises between a low price ($\tilde{P}_L$) and a high price ($\tilde{P}_H$) when it observes a viable start-up. Hence, a start-up receiving a low-price offer revises downwards the probability that it assigns to being unviable: $\phi(P_L) < p$. This makes it willing to accept such an offer. In turn, S randomizes between accepting and rejecting a low-price offer: this makes the incumbent indifferent between offering a low price (accepted with some probability) and a high price (accepted with certainty) to a viable start-up.

This means that, if $p > F_{II}$ and the AA authorizes takeovers that target viable start-ups (i.e. if $\bar{H} \geq W^d - W^M$), there also exist equilibria where both start-ups may be acquired.

7.3 Optimal merger policy with pure- and mixed-strategy equilibria

Assume the incumbent has the bargaining power and $p > F_{II}$. If $\bar{H} \geq W^d - W^M$, two types of equilibria may arise: the equilibrium in pure strategies where I acquires a viable start-up at a high price equal to $\pi_S^{\emptyset}(S_v) = \pi_S^d - K$ (Lemma 6, part 3) and the equilibria in mixed strategies (Lemma 8).

In the former case, only viable start-ups are acquired, whereas unviable start-ups are never acquired and fail to develop. Expected welfare at $t = 0$ is:

$$E(W^{ps}) = p(W^M - K) + (1-p)W^m.$$

In the latter case, both viable and unviable start-ups may be acquired. Expected welfare is:

$$\begin{aligned} E(W^{ms}) &= p[(\gamma^L(S_v)\beta^L + 1 - \gamma^L(S_v))W^M + \gamma^L(S_v)(1-\beta^L)W^d - K] \\ &\quad +(1-p)[\beta^L(W^M - K) + (1-\beta^L)W^m]. \end{aligned}$$

If $\bar{H} < W^d - W^M$, no takeover occurs at the equilibrium. Since viable start-ups are able to develop whereas unviable ones fail to develop, expected welfare is:

$$E(W^{no}) = p(W^d - K) + (1-p)W^m.$$

As already established in Section 7.1.3, expected welfare is higher under the no-takeover equilibrium than under the pure-strategy equilibrium in which high-price takeovers occur. Hence, if the pure-strategy equilibrium arises whenever $\bar{H} \geq W^d - W^M$, the AA optimally prohibits high-price takeovers as in Proposition 3.

However, if a mixed-strategy PBE occurs whenever $\bar{H} \geq W^d - W^M$, a trade-off arises. On the one hand, the no-takeover equilibrium entails two welfare gains relative to the mixed-strategy equilibrium: the first is when the viable start-up is acquired in any mixed-strategy equilibrium, i.e. when the low price $\tilde{P}_L$ is offered to a viable start-up and such an offer is accepted, which occurs with probability $p\gamma^L(S_v)\beta^L$; the second is when a high price $\tilde{P}_H$ is offered to a viable start-up, which occurs with probability $p(1-\gamma^L(S_v))$. On the other hand, the no-takeover equilibrium entails a welfare loss when the unviable start-up is acquired in the mixed-strategy equilibrium and, therefore, the takeover enables development, i.e. when the low-price $\tilde{P}_L$ offer is accepted by an unviable start-up, which occurs with probability $(1-p)\beta^L$.

Expected welfare at the no-takeover equilibrium is higher than in the mixed-strategy equilibrium when the welfare gains dominate the welfare loss, that is when the probability that the start-up is viable is large enough:

$$\begin{aligned} & p(\gamma^L(S_v)\beta^L + 1 - \gamma^L(S_v))(W^d - W^M) > (1-p)\beta^L(W^M - K - W^m) \\ \iff \quad & p > \frac{\beta^L(W^M - K - W^m)}{\beta^L(W^M - K - W^m) + (\gamma^L(S_v)\beta^L + 1 - \gamma^L(S_v))(W^d - W^M)} \equiv F_{WW} \in (0,1). \end{aligned}$$

Hence, under specific cumulative conditions, listed in the proposition below, the optimal merger policy may authorise high-price takeovers:

PROPOSITION 4 (Optimal merger policy with pure- and mixed-strategy equilibria)**.** *Let $\pi_I^A = \pi_I^M - K$. The optimal merger policy may authorise high-price takeovers, i.e., $\bar{H} \geq W^d - W^M$) if (and only if) the following conditions are jointly satisfied:*

(i) the incumbent has superior information regarding the start-up's type;

(ii) the incumbent has the bargaining power;

(iii) $p \in (F_{II}, F_{WW}]$;

(iv) the mixed-strategy equilibrium arises whenever $\bar{H} \geq W^d - W^M$.

The merger policy $\bar{H}^* = 0$ is optimal if otherwise.

Proof. Follows from the above discussion. Q.E.D.

Note that the optimal merger policy may authorise high-price takeovers not because the acquisition of viable start-ups is beneficial per se. Rather, it is because authorising high-price takeovers sustains an equilibrium in which also unviable start-ups may be acquired. When the prior probability p takes intermediate values, the mixed strategy equilibrium exists and the expected gain from the acquisition of unviable start-ups is large enough to make the optimal policy lenient.

8 Late takeovers

In this section, we allow the incumbent to acquire the start-up either before *or after* product development. We will denote the former, which involve *potential competitors* as "early takeovers" and the latter, which involve *committed entrants* as "late" takeover.

First, we allow the AA to commit to two different standards of review for early and late takeovers, denoted as $\bar{H}_1$ and $\bar{H}_2$, respectively. Second, and more important, we allow for the possibility that *the expectation of a late takeover to increase the viability of a start-up.* A natural way to rationalise this property comes from the imperfect financial market micro-foundation of the model (see Section 6): if the start-up has some bargaining power in the negotiation for the late takeover, the anticipation that it will appropriate some of the rents generated by the acquisition makes it easier to obtain external funding. In this case the expectation of a late takeover relaxes financial constraints and increases the probability that the start-up is viable. (The extended model with late takeovers and imperfect financial markets is solved in the Online Appendix). One could also think of alternative mechanisms whereby late takeovers may affect viability ex ante. For instance, viability might depend on the availability of managerial skills. The anticipation of a late takeover may make it easier for a start-up to hire high-skilled personnel, attracted by the possibility of a future career at the incumbent firm. In this case, the likelihood of viability may increase with expected late takeovers independently of the distribution of bargaining power.

While allowing for acquisitions of committed entrants does not change the optimal policy regarding early takeovers (which remains the same as in Proposition 1), the new insight is that the optimal merger policy may establish a more lenient treatment towards late acquisitions than early acquisitions, a result which may seem paradoxical but that holds under specific cumulative conditions. Not only the anticipation of a late takeover must increase the viability of the start-up, but it must also be that the incumbent shelves the project after an early takeover, and that the sacrifice of allocative efficiency caused by late takeovers must be dominated by the benefit due to the higher probability of development.

Consider first the case in which the incumbent develops the project in case of an acquisition. When late takeovers are authorised, only low-price takeovers occur at the early stage: given that a viable start-up can be acquired later, there is no point for the incumbent in overpaying for an unviable start-up in an early acquisition. Since takeovers targeting unviable stat-ups are always authorised, irrespective of the standard of review, the merger policy regarding early takeovers is immaterial. At the equilibrium, unviable start-ups are acquired early and viable start-ups are acquired later. As already proved in Section 4, when late takeovers are blocked the early merger policy matters, and it is optimal to prohibit high price takeovers. Also in this case, therefore, only low-price takeovers occur at the early stage. Relative to the case in which late takeovers are authorised, however, the probability that the start-up is *unviable*, and that is taken over early, is *higher*, for instance due to the relaxation of financial constraints. Since the incumbent develops the project following the acquisition, there is no welfare loss in this. Had that start-up been viable because late takeovers are authorised, it would have developed the project and it would have been acquired later anyway. Rather, blocking late takeovers promotes allocative efficiency by intensifying product market competition when the start-up is viable under either policy. Therefore, when the incumbent develops, the optimal policy blocks late takeovers.

Consider now the case in which the incumbent shelves the project in case of an acquisition. Different from the case with development, the incumbent may be willing to engage in a high-price early takeover also when late takeovers are authorised: from the perspective of the incumbent, developing the project is an inefficient investment which cannot be avoided if the viable start-up remains independent; hence, the incumbent may be willing to overpay for an unviable start-up at the early stage. Such takeovers are killer acquisitions and the optimal merger policy prohibits them, irrespective of whether late takeovers are blocked or authorised. In either case no takeover occurs at the early stage (recall that transaction costs make a low-price takeover less preferable than no takeover, in the case of shelving) and only viable start-ups reach the final market. For this reason, with shelving, a policy that blocks late takeovers, by increasing the probability that the start-up is unviable, produces *a loss* relative to the case in which late takeovers are authorised. The unviable start-up fails to develop. Had that start-up been viable because late takeovers are authorised, it would have developed and the innovation would have reached the final market. When the increase in the likelihood of viability induced by the anticipation of the late takeover is large enough, such a loss dominates the gain in allocative efficiency caused by a policy that blocks late takeovers, and authorising late takeovers is optimal.

9 Concluding remarks

The acquisition of potential competitors has been a particularly debated issue in the last few years, due especially to research showing that they have led to killer acquisitions (Cunningham et al., 2021) and to the vast number of unchallenged mergers with start-ups in the digital industries. Commentators and policymakers have been invoking stricter merger control, and as we write, legislative initiatives as well as changes in enforcement standards are being considered in several jurisdictions. For instance, the US agencies have announced the review of the Horizontal Merger Guidelines,[16] and that they may challenge acquisitions of potential competitors, a departure from previous policy.[17] In the UK, the CMA issued revised merger guidelines in July 2021,[18] announcing a stricter merger enforcement across sectors. In November 2021 it also prohibited Facebook's acquisition of Giphy, the first time a merger by one of the Big Tech companies has ever been blocked.

While much of the emphasis in this debate has arguably been on such mergers being "killer acquisitions", we have investigated an environment in which they may in principle have both detrimental and beneficial effects. The former consist in the possible suppression of innovation, and the elimination of competition. The latter in higher (potential) ability to invest due to the acquirer's better resources.

From a policy perspective, the main result of our analysis is that the optimal merger policy should *not* be lenient towards acquisitions of potential competitors. The optimal merger policy commits to standards of review that are sufficiently strict to prohibit high-price takeovers, that is takeovers where the start-up might be able to invest and compete with the incumbent, even though ex-post they may be welfare-beneficial. Such a policy exerts a selection effect: it pushes towards acquisitions that target only unviable start-ups and that, therefore, increase welfare more. The policy does not imply blocking all acquisitions of potential competitors: low-price merger transactions can involve only start-ups which would not be viable, and hence would not be able to become independent competitors. Such takeovers should be approved.

Moreover, our analysis suggests that AAs should use the information conveyed by the takeover price when reviewing acquisitions of potential competitors, both for the initial screening, to identify mergers that deserve a closer look, and for the investigation, to assess the counterfactual to the merger and the effects on competition.

These results offer support to the idea that the value of a merger transaction may signal possible anti-competitive effects, and should be at the very least carefully investigated by Antitrust Authorities (AAs). The CMA in the UK is considering the introduction of a transaction value threshold to screen acquisitions by firms in the digital sector with "strategic market status" and identify those that deserve a closer look. The effect of this change would be to capture "com-

[16]See, e.g., https://www.ftc.gov/news-events/press-releases/2022/01/ftc-and-justice-department-seek-to-strengthen-enforcement-against-illegal-mergers.

[17]See e.g. the speech delivered by AAG Vanita Gupta at Georgetown Law's 15th Annual Global Antitrust Enforcement Symposium Washington, DC, September 14, 2021.

[18]Competition and Markets Authority, "Merger Assessment Guidelines", 18 March 2021.

petitively significant mergers (as signaled by having a high transaction value)" (CMA, 2021:51) which may otherwise fail to meet the current review tests based on firms' market presence. Our results also suggest that the validity of this approach extends beyond the digital sector, and thus should be used to screen any merger involving potential competitors. They also suggest that the information conveyed by a high transaction value should be used not only for the initial screening, but also for the assessment of the effects of the mergers that are investigated.

To sum up, our analysis confirms that the laissez-faire approach towards acquisition of potential competitors, which AAs around the world have been following for a long time, should be scrapped, and it supports the current proposals towards stricter enforcement of these mergers.

Finally, it may seem paradoxical that, contrary to the current merger practice, the optimal policy prescribes a possibly more lenient treatment towards *acquisitions of committed entrants* – that is, of start-ups which have already developed an innovation allowing them to compete with the incumbent – than acquisitions of potential competitors. Nevertheless, the optimal policy establishes a lenient treatment of takeovers of committed entrants only when a number of cumulative conditions hold. First, the anticipation of a late takeover must increase the viability of the start-up. Second, it must be that the incumbent shelves the project after an early takeover. Finally, the sacrifice of allocative efficiency caused by late takeovers must be dominated by the benefit due to the higher probability of development.

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A Appendix

To ease the exposition, whenever possible, in the proofs we suppress the functional notation.

A.1 Proof of Lemma 1

Two cases must be considered.

Case 1: The incumbent plans to shelve (i.e. $\pi_I^A = \pi_I^m$). If the start-up is unviable, the takeover does not affect welfare, because the project cannot be developed by S. If instead the start-up is viable, the takeover leads to the suppression of a project that the start-up would develop, and of competition. The takeover is authorised if (and only if) the expected harm, $H = \phi[W^d - K - W^m] > 0$ is lower than the tolerated harm, $\bar{H}$, i.e. if and only if:

$$\phi \leq \frac{\bar{H}}{W^d - K - W^m} = F_W(\pi_I^m, \bar{H}).$$

Case 2: The incumbent plans to develop (i.e. $\pi_I^A = \pi_I^M - K$). A takeover creates expected harm $H = (1-\phi)[W^m - (W^M - K)] + \phi[W^d - K - (W^M - K)]$: if the start-up is unviable, the takeover is now beneficial, because it allows the project to reach the market; if the start-up is viable, the takeover is detrimental because of the suppression of product market competition. The takeover is authorised if and only if:

$$\phi \leq \frac{\bar{H} + W^M - W^m - K}{W^d - K - W^m} = F_W(\pi_I^M - K, \bar{H}).$$

Note that (i) F_W increases with $\bar{H}$ and (ii) is higher when the incumbent develops than when it shelves. This follows from $W^M - W^m - K > 0$.

Moreover, by Assumption A4, $F_W \geq 0$ when $\pi_I^A = \pi_I^M - K$.

A.2 Proof of Corollary 1

(1) Since $F_W(\pi_I^M - K, \bar{H}) \geq 0$, condition (2) is always satisfied when $\phi(\Omega) = 0$. (2) Since $F_W(\pi_I^m, \bar{H}) < 0$ when $\bar{H} < 0$, condition (2) is never satisfied.

A.3 Proof of Lemma 2

If $S = S_u$, the start-up's outside payoff when rejecting I's offer is $\pi_S^\emptyset(S_u) = 0$; if $S = S_v$, it is $\pi_S^\emptyset(S_v) = \pi_S^d - K > 0$ from Assumption A1. The incumbent will then offer either a price $P_I = 0$ such that only the unviable start-up $S = S_u$ will accept, or a price $P_I = \pi_S^d - K > 0$ such that both types of start-up will accept. In the former case, observing that the offer is accepted allows the incumbent and the AA to update their beliefs and infer that the start-up is unviable: $\phi(\{0, \mathit{Accept}\ P_I\}) = 0$. In the latter case the acceptance decision of the start-up does not reveal its type, and the posteriors coincide with the priors: $\phi(\{\pi_S^d - K, \mathit{Accept}\ P_I\}) = p$. From Lemma 1, the deal is authorised if and only if $p \leq F_W$. Finally, there cannot exist an equilibrium in which both start-ups are acquired at a different positive price: the start-up receiving the lower price offer would pretend to be the type receiving the higher price offer, thus breaking the equilibrium.

If I does not make any offer ($\emptyset$), its expected profit is:

$$p\pi_I^d + (1-p)\pi_I^m. \tag{7}$$

If I offers $P_I = 0$ and the deal is authorised (i.e. if $\phi(\Omega) = 0 \leq F_W$, a condition that is always satisfied if the incumbent develops, from Corollary 1), I's expected profit (gross of the transaction cost) is:

$$p\pi_I^d + (1-p)\pi_I^A. \tag{8}$$

If I offers $P_I = \pi_S^d - K > 0$ and the deal is authorised (i.e. if $\phi(\Omega) = p \leq F_W$), its expected profit (gross of the transaction cost) is:

$$\pi_I^A - (\pi_S^d - K). \tag{9}$$

By comparing the expressions in (8) and (9) one obtains that offering $P_I = 0$ is more profitable for the incumbent than offering $P_I = \pi_S^d - K > 0$ if and only if $p \leq F_I$, where F_I is defined in equation (3).

However, it must also be the case that making an offer is more profitable than not engaging in the takeover. If $\pi_I^A = \pi_I^m$ (i.e. the incumbent shelves) and $p \leq F_I$, the comparison between (7) and (8) and the existence of the positive transaction cost reveal that I's equilibrium decision is not to engage in the takeover. The same equilibrium decision is taken when $\pi_I^A = \pi_I^m$ and $p > \max(F_W, F_I)$: I would prefer to offer a high price, but the AA would not authorise the deal. Since offering a price $P_I = 0$ is dominated by making no offer, a takeover does not occur at the equilibrium. This concludes part 1 of the lemma.

If $p \in (F_I, \max(F_W, F_I)]$, the equilibrium offer involves a price $P_I = \pi_S^d - K > 0$, as the incumbent's preferred choice is authorised by the AA. The posteriors coincide with the priors as stated in part 2 of the lemma.

If $\pi_I^A = \pi_I^M - K$ (i.e. the incumbent develops) and either $p \leq F_I$ or $p > \max(F_W, F_I)$, $P_I = 0$ is offered at the equilibrium, and the incumbent and the AA update their beliefs based on whether the start-up accepts, as stated in part 3 of the lemma. When $p > \max(F_W, F_I)$ the incumbent would prefer to offer a price $P_I = \pi_S^d - K > 0$. However, anticipating that the AA would not authorise the transaction, the incumbent has to settle for a second-best offer featuring $P_I = 0$.

Finally, Assumption A1 implies that $\pi_I^A - \pi_I^d > 0$ and $\pi_S^d - K > 0$. Therefore $F_I > 0$. Moreover, $F_I < 1$ if (and only if) the joint payoff of I and S_v in the absence of a takeover is strictly lower than their joint payoff when the takeover occurs. Assumption A2 ensures that this is the case.

A.4 Proof of Lemma 3

Consider a candidate equilibrium in which the start-up, irrespective of whether it is viable or not, offers price $P_{S_u} = P_{S_v} = P$. For this to be an equilibrium, price P must satisfy the start-ups' participation constraints:

$$P > \pi_S^{\emptyset}(S_u) = 0 \quad (10)$$
$$P > \pi_S^{\emptyset}(S_v) = \pi_S^d - K, \quad (11)$$

where the strict inequality is used because of the small but positive transaction cost.

The price P must also satisfy the incumbent's participation constraint:

$$\pi_I^A - P \geq p\pi_I^d + (1-p)\pi_I^m, \quad (12)$$

where the incumbent's posterior beliefs on the viability of the start-up coincide with the priors. Since the constraint (11) is more binding than the constraint (10), P must satisfy:

$$\pi_S^{\emptyset}(S_v) < P \leq \pi_I^A - \pi_I^m + p(\pi_I^m - \pi_I^d) \equiv \bar{P}.$$

Since $\pi_I^m - \pi_I^d > 0$, a necessary condition for the existence of an equilibrium featuring $P = \bar{P}$ is:

$$p > F_S, \quad (13)$$

where F_S is defined in equation (4).

Finally, if the offer $P = \bar{P}$ is accepted the AA does not revise its priors and it authorises the deal if (and only if) $p \leq F_W$. Combining the above conditions one obtains part 2 of the lemma. An equilibrium with $\tilde{P} \in (\pi_S^{\emptyset}(S_v), \bar{P})$ does not exist because $S \in \{S_u, S_v\}$ would have an incentive to deviate and increase the price: following an out-of-equilibrium offer $P' \in (\tilde{P}, \bar{P}]$, I and AA would attach the prior probability to the start-up being viable. Since $p \leq F_W$ the AA would authorise the deal; since $P' \leq \bar{P}$, I would accept. The deviation would be profitable. Hence, $P = \bar{P}$ is the unique equilibrium price such that $P_{S_u} = P_{S_v}$.

Consider now a candidate equilibrium in which $S = S_u$ offers $P_{S_u} = \pi_I^A - \pi_I^m$, $S = S_v$ does not make any offer ($\emptyset$), and the incumbent accepts P_{S_u}. From Assumptions A1 and A3, $\pi_I^A - \pi_I^m < \pi_S^d - K = \pi_S^{\emptyset}(S_v)$. Therefore, observing such an offer both I and the AA infer that the start-up is unviable (i.e. $\phi(\Omega) = 0$). Then, I is indifferent between accepting and rejecting P_{S_u}. For this to be an equilibrium, S must have no incentive to deviate. To start with, $S = S_u$ must find it unprofitable not to make an offer, which, because of the transaction cost, requires that

$$P_{S_u} > \pi_S^{\emptyset}(S_u) = 0. \quad (14)$$

Let us focus on the case in which I develops and $P_{S_u} = P_L \equiv \pi_I^M - K - \pi_I^m > 0$. Since

$\pi^{\emptyset}_S(S_v) > P_L$, then $S = S_v$ has no incentive to deviate and offer P_L. Clearly, $S = S_u$ has no incentive to decrease its offer. Has it an incentive to offer $P' > P_L$? As long as $P' \leq \pi^{\emptyset}_S(S_v)$, the incumbent infers that the start-up is unviable and rejects the deviation offer. The deviation is unprofitable. If, instead, $P' > \pi^{\emptyset}_S(S_v)$, the incumbent attributes the offer to a viable start-up with probability p. The deviation is unprofitable either if I would reject the offer, i.e. if $\pi^{\emptyset}_S(S_v) \geq \bar{P}$ which is satisfied if $p \leq F_S$; or if I would accept the deviation offer but the AA would not authorise the deal, i.e. if $p > \max(F_S, F_W)$. For the same reason, it is not profitable for $S = S_v$ to offer $P' \geq \pi^{\emptyset}_S(S_v)$. In sum, when I develops and either $p \leq F_S$ or $p > \max(F_S, F_W)$ the proposed one is an equilibrium, as stated in part 3 of the lemma. Note that there cannot exist an equilibrium in which $S = S_u$ offers $P_L < \pi^M_I - K - \pi^m_I$: $S = S_u$ would have an incentive to deviate and offer $P' = \pi^M_I - K - \pi^m_I$, since I would accept the offer and the AA would authorise the deal.

Finally, there cannot exist an equilibrium in which $S = S_v$ offers $P_{S_v} = \tilde{P} > \pi^{\emptyset}_S(S_v)$, $S = S_u$ offers $P \neq \tilde{P}$, the incumbent accepts the former and rejects the latter. If the AA authorises the deal, $S = S_u$ would always have an incentive to mimic S_v and offer $\tilde{P}$ instead. For a similar reason, there cannot exist an equilibrium in which $S = S_u$ offers $P_{S_u} = P_L$, $S = S_v$ offers $P_{S_v} \in (\pi^{\emptyset}_S(S_v), \pi^M_I - K - \pi^d_I]$ and I accepts both offers.

Let us consider now the case in which I shelves. Since $\pi^A_I = \pi^m_I$, then $P_{S_u} = 0$. Hence, condition (14) cannot be satisfied and the proposed one is not an equilibrium. Other equilibria in which each start-up is traded at a different price do not exist, for the same reasoning developed above. Therefore, if $\pi^A_I = \pi^m_I$ and either $p \leq F_S$ or $p > \max(F_W, F_S)$, there is no early takeover in equilibrium, as stated in part 1. of the lemma.

To conclude, $\pi^m_I > \pi^d_I$, $\pi^{\emptyset}_S(S_v) = \pi^d_S - K > 0$ and Assumption A3 imply $F_S > 0$. Moreover, $F_S < 1$ if (and only if) the joint payoff of I and S_v in the absence of a takeover is strictly lower than their joint payoff when a takeover occurs, which holds true by Assumption A2.

A.5 Proof of Proposition 1

Case 1: The incumbent plans to develop (i.e. $\pi^A_I = \pi^M_I - K$).

Let us consider the case in which the incumbent makes a take-it-or-leave-it offer at $t = 1(a)$. Lemma 2 implies that two sub-cases must be considered:

1. If either $p \leq F_I$ or $p > \max(F_W, F_I)$, I offers $P_I = 0$ in $t = 1(a)$ and only type $S = S_u$ accepts. Expected welfare is $E(W) = p(W^d - K) + (1-p)(W^M - K) > W^M - K$.

2. If $p \in (F_I, \max(F_W, F_I)]$, I offers $P_I = \pi^d_S - K$ in $t = 1(a)$ and both $S = S_v$ and $S = S_u$ accept. Expected welfare is $E(W) = W^M - K$. This case arises if and only if $F_W > F_I$.

After comparing the two sub-cases, the optimal policy is the one that avoids high-price takeovers featuring $P_I = \pi^{\emptyset}_S(S_v) = \pi^d_S - K$ from arising at the equilibrium. This can be ensured by setting $\bar{H}$ such that $F_W \leq F_I$: in this way, for all the values of p such that the incumbent finds it profitable to offer a high price, the takeover is blocked.

If $\pi_I^A = \pi_I^M - K$, $F_I = (\pi_S^d - K)/(\pi_I^M - K - \pi_I^d) \in (0,1)$ from Assumptions A1 and A2. Since F_W is strictly increasing in $\bar{H}$ (from Lemma 1 (i)), $F_W = 0$ if $\bar{H} = -(W^M - W^m - K)$ and $F_W \geq 1$ for all $\bar{H} \geq W^d - W^M$, there exists a unique cut-off $H_I^d \in (-(W^M - W^m - K), W^d - W^M)$ such that $F_W \leq F_I$ for all $\bar{H} \leq H_I^d$. Hence, all $\bar{H} \leq \bar{H}_I^d$ in the set of admissible values of $\bar{H}$ are optimal. The set of admissible values of $\bar{H}$ is such that $\bar{H} \geq -(W^M - K - W^m)$ (Assumption A5), and $F_W \geq 0$ for all $\bar{H} \geq -(W^M - W^m - K)$. This ensures that low-price takeovers are authorised under the optimal policy. Moreover, H_I^d is not necessarily positive: $H_I^d < 0$ if $F_W > F_I$ at $\bar{H} = 0$.

We reach similar conclusions when considering the case in which S makes a take-it-or-leave-it offer at $t = 1(a)$ (so that the bargaining outcomes in Lemma 3 apply). The optimal policy avoids high-price takeovers from arising at the equilibrium. Hence, all $\bar{H} \leq \bar{H}_S^d$ in the set of admissible values are optimal, where $\bar{H}_S^d \in (-(W^M - W^m - K), W^d - W^M)$ is the unique cut-off such that, when $\pi_I^A = \pi_I^M - K$, $F_W(\pi_I^M - K, \bar{H}_S^d) = F_S = (\pi_S^d - \pi_I^M + \pi_I^m)/(\pi_I^m - \pi_I^d)$, with $F_S \in (0,1)$ from Assumptions A2 and A3.

Optimal $\bar{H}$ when I develops. Since $\pi_I^M - K > \pi_I^m$, $F_S > F_I$ and $H_I^d < H_S^d$. Hence, a policy $\bar{H} \leq \bar{H}^d \equiv H_I^d$ in the set of admissible values ensures that high-price takeovers are blocked for any value of α, and is optimal irrespective of α, as stated in Proposition 1 part (a).

Case 2: The incumbent plans to shelve (i.e. $\pi_I^A = \pi_I^m$).

Let us start with the case in which I makes a take-it-or-leave-it offer at $t = 1(a)$. Lemma 2 implies that two sub-cases must be considered:

1. If either $p \leq F_I$ or $p > \max(F_W, F_I)$, no early takeover occurs at the equilibrium. Expected welfare is $E(W) = p(W^d - K) + (1-p)W^m > W^m$.

2. If $p \in (F_I, \max(F_W, F_I)]$, I offers $P_I = \pi_S^d - K$ in $t = 1(a)$ and both types $S = S_v$ and $S = S_u$ accept. Expected welfare is $E(W) = W^m$.

Comparing these two sub-cases, we conclude that the optimal policy avoids high-price takeovers featuring $P_I = \pi_S^{\emptyset}(S_v) = \pi_S^d - K$ from arising at the equilibrium. This can be ensured by setting $\bar{H}$ such that $F_W \leq F_I$. If I shelves ($\pi_I^A = \pi_I^m$), $F_I = (\pi_S^d - K)/(\pi_I^m - \pi_I^d) \in (0,1)$ from Assumptions A1 and A2. Since F_W is strictly increasing in $\bar{H}$, $F_W = 0$ if $\bar{H} = 0$ and $F_W \geq 1$ for all the values of $\bar{H} \geq W^d - W^m - K$, there exists a unique cut-off $H_I^s \in (0, W^d - W^m - K)$ such that $F_W \leq F_I$ for all the values of $\bar{H} \leq H_I^s$.

Let us consider now the case in which S makes a take-it-or-leave-it offer at $t = 1(a)$ (so that the bargaining outcomes in Lemma 3 apply). The threshold F_I is substituted by F_S. Therefore,

1. If either $p \leq F_S$ or $p > \max(F_W, F_S)$, no takeover occurs at the equilibrium. Expected welfare is $E(W) = p(W^d - K) + (1-p)W^m > W^m$.

2. If $p \in (F_S, \max(F_W, F_S)]$, both types $S = S_v$ and $S = S_u$ offer $\bar{P} \geq \pi_S^{\emptyset}(S_v) = \pi_S^d - K$ (defined in Lemma 3) in $t = 1(a)$ and I accepts. Expected welfare is $E(W) = W^m$.

The comparison between these sub-cases allows us to conclude that, the optimal policy is the one that avoids high-price takeovers featuring a price $\bar{P}$ from arising at the equilibrium. This can be ensured by setting $\bar{H}$ such that $F_W \leq F_S$.

When $\pi_I^A = \pi_I^m$, $F_S = F_I = (\pi_S^d - K)/(\pi_I^m - \pi_I^d) \in (0, 1)$ from Assumptions A1 and A2. As shown above, setting any value of $\bar{H}$ such that $\bar{H} \leq \bar{H}_I^s$ is optimal.

Optimal $\bar{H}$ when I shelves. The cut-off level $\bar{H}_I^s = \bar{H}_S^s$ is positive. Hence the policy $\bar{H} \leq \bar{H}^s \equiv \bar{H}_I^s > 0$ ensures that high-price early takeovers are blocked and is, therefore, optimal, irrespective of the value of α, as stated in Proposition 1 part (b).

Optimal $\bar{H}$ (irrespective of shelving or developing). All $\bar{H} \leq \min(\bar{H}^d, \bar{H}^s)$ in the set of admissible values are optimal irrespective of the value of π_I^A and α, as stated in Proposition 1 part (c).

A.6 Proof of Lemma 4

We construct the equilibrium through a sequence of intermediate results.

Lemma A.1.
In any mixed-strategy PBE, the probability γ_u^H is given by:

$$\gamma_u^H = \frac{\gamma_v^H p}{(1-p)} \frac{(\pi_I^A - \pi_I^d - P_H)}{(P_H - \pi_I^A + \pi_I^m)}. \tag{15}$$

Proof. First, we compute $\phi(P_H)$ by Bayes' rule:

$$\phi(P_H) = \frac{\gamma_v^H p}{\gamma_u^H (1-p) + \gamma_v^H p}.$$

I is indifferent between accepting and rejecting a price offer featuring P_H if and only if:

$$\pi_I^A - P_H = \phi(P_H)\pi_I^d + (1 - \phi(P_H))\pi_I^m.$$

Plugging the formula for $\phi(P_H)$, and simplifying, we obtain the expression for γ_u^H in equation (15). Since $\pi_I^m > \pi_I^d$, $\gamma_u^H > 0$ if (and only if) $\pi_I^A - \pi_I^m < P_H < \pi_I^A - \pi_I^d$. Q.E.D.

We next define β^H.

Lemma A.2.
In any mixed-strategy PBE, the probability β^H is given by $\beta^H = \frac{P_L \beta^L}{P_H}$.

Proof. S_u's indifference between a price offer featuring $P_{S_u} = P_H$ and one featuring $P_{S_u} = P_L$ requires that:

$$\beta^H P_H = P_L \beta^L \iff \beta^H = \frac{P_L \beta^L}{P_H}.$$

with $\beta^H > 0$ if (and only if) $P_L > 0$ and $\beta_L > 0$. Q.E.D.

In the next lemma, we prove several results. First, that a necessary condition for the existence of a mixed-strategy equilibrium is that I does not shelve the project of the start-up; second, that such mixed-strategy equilibrium is a hybrid equilibrium featuring S_v offering P_H with certainty and I accepting P_L with probability $\beta^L > \beta^H$. Finally, when P_H is observed, the posterior probability assigned to the start-up being viable must be (weakly) higher than the a priori probability. Moreover the posterior probability is strictly increasing in P_H.

Lemma A.3.
In any mixed-strategy PBE:

1. *S_v offers P_H with certainty (i.e., $\gamma_v^H = 1$) for all $P_H > P_L > 0$ and $P_H > \pi_S^d - K$.*
2. *If $\pi_I^A = \pi_I^m$, there does not exist a mixed-strategy PBE in which I acquires S. Moreover, $\phi(P_L) = 0$*
3. *If $\pi_I^A = \pi_I^M - K$, I accepts any offer featuring a price $P_L \leq \pi_I^A - \pi_I^m$ with probability $\beta^L > \beta^H$.*
4. *When P_H is observed, it cannot be that $\phi(P_H) < p$.*
5. *$\phi(P_H)$ is strictly increasing in P_H and $\phi(P_H) < 1$ for any $\pi_S^d - K < P_H < \pi_I^A - \pi_I^d$.*

Proof. We start from claim 1 S_v prefers offering $P_{S_v} = P_H$ to offering a price at which there is no acquisition if (and only if):

$$P_H\beta^H + (1-\beta^H)(\pi_S^d - K) > \pi_S^d - K \iff P_H > \pi_S^d - K.$$

Moreover, S_v prefers offering P_H to P_L if (and only if):

$$\begin{aligned} P_H\beta^H + (1-\beta^H)(\pi_S^d - K) &> P_L\beta^L + (1-\beta^L)(\pi_S^d - K) \\ \iff \beta^H &> \frac{\beta^L(P_L - \pi_S^d + K)}{P_H - \pi_S^d + K}, \end{aligned}$$

which is always satisfied if $P_H > P_L$ and $P_H > \pi_S^d - K$. Hence, $\gamma_v^H = 1$ for all $P_H > P_L > 0$ and $P_H > \pi_S^d - K$.

Let us turn to claim 2. From $\gamma_v^H = 1$ it follows that $\phi(P_L) = 0$. Then, for I not to reject P_L with certainty it must be:

$$\pi_I^A - P_L \geq \pi_I^m \iff P_L \leq \pi_I^A - \pi_I^m. \tag{16}$$

If $\pi_I^A = \pi_I^m$, $\pi_I^A - \pi_I^m = 0$. Since it must be that $P_L > 0$, a mixed-strategy equilibrium in which a takeover takes place does not exist when I shelves (claim 2). If $\pi_I^A = \pi_I^M - K$, instead, $\pi_I^A - \pi_I^m > 0$ is the upper bound of P_L such that I will be willing to accept, with $\pi_I^A - \pi_I^m < \pi_S^d - K$. From this it follows that $P_H > P_L$ and $\beta^H < \beta^L$ (claim 3).

From claim 1 (i.e. $\gamma_v^H = 1$), it follows that:

$$\phi(P_H) = \frac{p}{\gamma_u^H(1-p)+p}. \tag{17}$$

Since $\gamma_u^H(1-p)+p \leq 1$, then $\phi(P_H) \geq p$ (claim 4).

From claim 1, it also follows that:

$$\gamma_u^H = \frac{p}{(1-p)} \frac{(\pi_I^A - \pi_I^d - P_H)}{(P_H - \pi_I^A + \pi_I^m)}, \tag{18}$$

which is strictly decreasing in P_H when $\pi_S^d - K < P_H < \pi_I^A - \pi_I^d$, where $\pi_S^d - K > \pi_I^A - \pi_I^m$. Hence, $\phi(P_H)$ is strictly increasing in P_H (claim 5).

Since $\gamma_u^H > 0$ for any $\pi_S^d - K < P_H < \pi_I^A - \pi_I^d$, $\phi(P_H) < 1$ for any $\pi_S^d - K < P_H < \pi_I^A - \pi_I^d$. This concludes claim 5. Q.E.D.

Finally, we determine the values of P_H and P_L that can be sustained as part of the hybrid PBE.

Lemma A.4.
Let $\pi_I^A = \pi_I^M - K$:

1. *If $p \leq F_S$, there exists a continuum of hybrid PBE featuring: $P_L = \pi_I^M - K - \pi_I^m$ and $P_H \in (\pi_S^d - K, \hat{P}_H(\bar{H})]$, with $P_H > \pi_S^{\emptyset}(S_v) > P_L > 0$, $\phi(P_H) \leq F_W$ and $\hat{P}_H(\bar{H}) < \pi_I^M - K - \pi_I^d$ increasing in $\bar{H}$;*

2. *$\gamma_u^H \in (0,1)$, $\beta^L \in (0,1]$ and $\beta^H \in (0, \beta^L)$.*

Proof. Throughout the proof, we set $\pi_I^A = \pi_I^M - K$ (the incumbent develops).

We start with claim 1. A mixed-strategy PBE exists if the high-price offer is approved by the AA, which occurs if and only if $\phi(P_H) \leq F_W$.

Consider an offer $P_L \in (0, \pi_I^M - K - \pi_I^m)$. Such an offer cannot be sustained in equilibrium because S_u would have an incentive to deviate and offer $P' \in (P_L, \pi_I^M - K - \pi_I^m)$. Since $P' < \pi_I^M - K - \pi_I^m < \pi_S^d - K$, I would attribute the deviation offer to S_u with certainty and would accept. The AA would authorise the deal (see Corollary 1 part 1). The deviation would be profitable.

Consider now $P_L = \pi_I^M - K - \pi_I^m$. S_u has no incentive to deviate and offer $P' \in (P_L, \pi_S^d - K]$: I would attribute the deviation offer to S_u with certainty and would reject. S_u has no incentive to deviate and offer $P' > \max(\bar{P}, \pi_S^d - K)$ (with $P' \neq P_H$ and $\bar{P}$ defined in Lemma 3): I would attribute the deviation offer to S_v with probability p and would reject. Consider now $P' \in (\pi_S^d - K, \bar{P}]$, a possibility that arises if (and only if) $p > F_S$. I would attribute the deviation offer to S_v with probability p and would accept. From Lemma A.3 (part 4), $p \leq \phi(P_H)$. Hence, from $\phi(P_H) \leq F_W$ it follows that $p \leq F_W$: the deal would be authorised by the AA and the deviation would be profitable. Thus, offering $P_L = \pi_I^M - K - \pi_I^m$ is part of the equilibrium if and only if $p \leq F_S$.

Consider then an offer $P_H \in (\pi_S^d - K, \pi_I^M - K - \pi_I^d)$. For it to be sustained at the equilibrium S must not have an incentive to deviate and offer $P' > P_H$. Since $P' > \pi_S^d - K$, I attributes the deviation offer to S_v with probability p. Moreover, from $p \leq F_S$ it follows that $\pi_S^d - K \geq \bar{P}$. Therefore $P' > \bar{P}$ and I would reject the deviation offer. We then determine which prices P_H, within the interval $(\pi_S^d - K, \pi_I^M - K - \pi_I^d)$, are such that the AA approves the deal because $\phi(P_H) \leq F_W$.

From Lemma 1, when $\pi_I^A = \pi_I^M - K$, $F_W = (\bar{H} + W^M - W^m - K)/(W^d - K - W^m)$. Moreover, substituting the expression for γ_u^H (equation (18)) into the expression for $\phi(P_H)$ (equation (17)), one obtains:

$$\phi(P_H) = \frac{P_H - \pi_I^M + K + \pi_I^m}{\pi_I^m - \pi_I^d}, \tag{19}$$

with $\phi(P_H) < 1$ for any $P_H < \pi_I^M - K - \pi_I^d$.

Therefore, we can distinguish the following cases:

1. $\bar{H} \geq W^d - W^M$. In this case $F_W \geq 1$. Therefore, $\phi(P_H) < 1 \leq F_W$ for any $P_H < \pi_I^M - K - \pi_I^d$. This means that, when the standard of review regarding early takeovers is sufficiently lenient, any $P_H \in (\pi_S^d - K, \pi_I^M - K - \pi_I^d)$ can be supported at the PBE in mixed strategies.

2. $\bar{H} < W^d - W^M$. In this case $F_W < 1$. Moreover, $F_W \geq 0$ for any feasible value of $\bar{H}$, i.e. for any $\bar{H} \geq -(W^M - K - W^m)$. Since $\phi(P_H) = 1$ if $P_H = \pi_I^M - K - \pi_I^d$, $\phi(P_H) = 0$ if $P_H = \pi_I^M - K - \pi_I^m$, and $\phi(P_H)$ is strictly increasing in P_H, for any $\bar{H} \in [-(W^M - K - W^m), W^d - W^M)$ there exists a $\hat{P}_H(\bar{H}) \in [\pi_I^M - K - \pi_I^m, \pi_I^M - K - \pi_I^d)$ such that $\phi(P_H) \leq F_W$ for any $P_H \leq \hat{P}_H(\bar{H})$. Moreover, since F_W is strictly increasing in $\bar{H}$, also $\hat{P}_H(\bar{H})$ is strictly increasing in $\bar{H}$. If $\bar{H} = -(W^M - K - W^m)$, $F_W = 0$ and $\hat{P}_H(\bar{H}) = \pi_I^M - K - \pi_I^m < \pi_S^d - K$ (from Assumption A3). Since $\pi_S^d - K < \pi_I^M - K - \pi_I^d$ and $\hat{P}_H(\bar{H})$ is strictly increasing in $\bar{H}$, there exists a cut-off level of $\bar{H}$, $\bar{H}^m \in (-(W^M - K - W^m), W^d - W^M)$ such that $\hat{P}_H(\bar{H}) \leq \pi_S^d - K$ for any $\bar{H} \leq \bar{H}^m$. Therefore:
   (a) No $P_H \in (\pi_S^d - K, \pi_I^M - K - \pi_I^d)$ can be supported at the PBE in mixed strategies when $\bar{H} \leq \bar{H}^m$, i.e. when the standard of review regarding early takeovers is sufficiently strict.
   (b) Any $P_H \in (\pi_S^d - K, \hat{P}_H(\bar{H})]$ can be supported at the PBE in mixed strategies when $\bar{H} \in (\bar{H}^m, W^d - W^M)$.

We conclude with claim 2. Given $\gamma_v^H = 1$ and $P_H > \pi_S^d - K > \bar{P}$, $\gamma_u^H < 1$. Therefore $\phi(P_H) > p$. Moreover, $0 < P_L < P_H$ and $\beta^L \in (0, 1]$ implies $\beta^H \in (0, \beta^L)$. Q.E.D.

A.7 Proof of Proposition 2

From the proof of Lemma A.4, it follows that when S makes the offer, $\pi_I^A = \pi_I^M - K$ and $p \leq F_S$, setting $\bar{H} \leq \bar{H}^m$, with $\bar{H}^m \in (-(W^M - K - W^m), W^d - W^M)$, ensures that no hybrid PBE exists. Therefore only an equilibrium featuring a low-price exists, which is superior in terms of welfare:

$$\begin{aligned} EW^{ps} &= p(W^d - K) + (1-p)(W^M - K) > \\ EW^{ms} &= p[W^d - K - \beta^H(P_H)(W^d - W^M)] \\ &+ (1-p)[W^M - K - \gamma_u^H(P_H)(1-\beta^H(P_H))(W^M - K - W^m)]. \end{aligned}$$

From the proof of Lemma A.4, $\bar{H}^m$ is such that $\hat{P}_H = \pi_S^d - K$ and, therefore, $F_W = \phi(P_H = \pi_S^d - K)$. From equation (19), $\phi(P_H = \pi_S^d - K) = (\pi_S^d - \pi_I^M - \pi_I^m)/(\pi_I^m - \pi_I^d) = F_S$. From the proof of Proposition 1, $\bar{H}_S^d$ is such that $F_W = F_S$. Hence, $\bar{H}_S^d = \bar{H}^m$.

It follows that when S makes the offer and the incumbent develops, setting $\bar{H} \leq H_S^d$ prevents high-price takeovers from arising not only at the equilibrium in pure strategies, but also at any hybrid PBE. Hence, when S makes the offer and the incumbent develops all $\bar{H} \leq H_S^d$ are optimal, also when one allows for equilibria in mixed strategies.

The proof of Proposition 1 shows that $\bar{H}^d = \bar{H}_I^d < \bar{H}_S^d$. Hence, when the incumbent develops, setting $\bar{H} \leq H^d$ prevents high-price takeovers from arising not only at the pure-strategy equilibrium, but also at the hybrid PBE, irrespective of who makes the offer. Therefore, all $\bar{H} \leq H^d$ are optimal for any value of α, also when one allows for equilibria in mixed strategies at $t = 1(a)$. It also follows that $\bar{H} \leq \min(\bar{H}^d, \bar{H}^s)$ is optimal for any value of π_I^A.

A.8 Proof of Lemma 6

Given a price P, the start-up's acceptance decision depends on its belief on its ability to develop the project successfully. If the posterior probability coincides with the prior, the start-up will accept the offer if the takeover price $P \geq p(\pi_S^d - K) + (1-p)0 = p\pi_S^{\emptyset}(S_v)$. If the incumbent's offer reveals that the start-up is of the viable type $S = S_v$, S will accept if $P \geq \pi_S^d - K$.

Consider a pooling equilibrium in which the incumbent offers $p\pi_S^{\emptyset}(S_v) = p(\pi_S^d - K)$ to any start-up, independently of the type, $\phi = p$ and the start-up accepts the offer. This is an equilibrium if the incumbent finds it profitable to offer $p(\pi_S^d - K)$ not only to a viable start-up, so that $\pi_I^M - K - p(\pi_S^d - K) > \pi_I^d$, but also to an unviable start-up: $\pi_I^M - K - p(\pi_S^d - K) \geq \pi_I^m$. The latter constraint (which is the more binding) is satisfied if (and only if) $p \leq (\pi_I^M - K - \pi_I^m)/(\pi_S^d - K) \equiv F_{II}$. Moreover, for this to be an equilibrium, the AA must approve the deal, which requires $\phi = p \leq F_W$. Hence, as stated in Lemma 6, part 1, both conditions must be satisfied.

Are there profitable deviations for I and S? Given $\phi = p$, any start-up $S \in \{S_v, S_u\}$ makes an expected profit equal to the takeover price. Hence, it cannot do better than accepting the offer.

The incumbent has no incentive to offer a higher price. It has no incentive to decrease the price either: observing $P' < p(\pi_S^d - K)$ the start-up would continue assigning the prior probability p to being viable and would reject the offer.

If $p > F_{II}$, a start-up that is offered $p(\pi_S^d - K)$ infers that it is of a viable type, because for the incumbent it is not profitable to offer that price to an unviable type of start-up. Hence S rejects the offer.

Consider next an equilibrium in which the incumbent offers $P = \pi_S^d - K$ to $S = S_v$ and does not make any offer to $S = S_u$. The incumbent has no incentive to deviate and offer $P' \in (\pi_I^M - K - \pi_I^m, \pi_S^d - K)$ to S_v, where $\pi_I^M - K - \pi_I^m$ is the highest price that the incumbent is willing to offer to an unviable start-up. The start-up that receives the takeover offer would infer that it is viable, and would reject P'. Consider now a deviation to a price $P' \leq \pi_I^M - K - \pi_I^m$. The start-up that receives the offer would not update its belief, because the incumbent has an incentive to make such an offer both to the viable and the unviable start-up. Hence, for the deviation to be unprofitable it must be $p > F_{II}$: this ensures that $\pi_I^M - K - \pi_I^m < p(\pi_S^d - K)$ and that the start-up would reject the offer. Moreover, for the equilibrium to exist, the AA must approve the deal. Since $P = \pi_S^d - K$ reveals that the start-up is viable, the AA updates the prior beliefs to $\phi = 1$. Thus, condition 2 requires that for the deal to be approved, $F_W \geq 1$, which is satisfied if (and only if) $\bar{H} \geq W^d - W^M$.

For a similar reasoning an equilibrium where the incumbent offers $P = \pi_S^d - K$ to $S = S_v$ and offers 0 to $S = S_v$, with both S_v and S_u accepting the offer, cannot exist. Since the incumbent would find it profitable to offer a price equal to 0 irrespective of the type, the start-up receiving 0 should not update its prior and would reject the offer. Hence, as stated in Lemma 6, part 3, both conditions ($p > F_{II}$ and $\bar{H} \geq W^d - W^M$) must be satisfied.

In all the other cases, no takeover occurs at the equilibrium. When $\bar{H} < W^d - W^M$, the equilibrium in which only $S = S_v$ is acquired at $P = \pi_S^d - K$ does not exist because the AA would block the deal. An equilibrium in which both start-ups are acquired at the same price does not exist either, either because $p > F_{II}$ so that the start-up would reject an offer involving a price $P = p(\pi_S^d - K)$ that the incumbent profitably offers irrespective of the nature of the start-up (when $p > F_{II}$) or because $p \in (F_W, F_{II}]$, so that the AA blocks the deal involving that price.

A.9 Proof of Lemma 7

One possible equilibrium entails S offering the price $P_L = \pi_I^M - K - \pi_I^m$, knowing it will be accepted, irrespective of the type of the start-up: this is the highest price that the incumbent is willing to pay when it observes that the start-up is unviable. For this to be an equilibrium, S must not have an incentive to deviate and offer the price $P' = \pi_I^M - K - \pi_I^d > \pi_S^\emptyset(S_v)$, i.e. the highest price that the incumbent is willing to pay when it observes that the start-up is viable. Such an offer will be accepted with probability p. Hence, for the deviation not to be profitable, it must hold that $P_L = \pi_I^M - K - \pi_I^m \geq p(\pi_I^M - K - \pi_I^d)$, which is satisfied if (and only if) $p \leq (\pi_I^M - K - \pi_I^m)/(\pi_I^M - K - \pi_I^d) \equiv F_{SS} \in (0, 1)$. Moreover, the deal must be approved by the AA, which requires $p \leq F_W$.

When $p > F_{SS}$, S finds it more profitable to offer the price $P = \pi_I^M - K - \pi_I^d$ that the incumbent accepts only when it observes that the start-up is viable. However, the deal must be authorised by the AA. After observing this price the AA updates its beliefs. Hence, the deal will be authorised only when $1 \leq F_W$, or $\bar{H} \geq W^d - W^M$.

When $p > F_{SS}$ and $\bar{H} < W^d - W^M$, the high-price offer featuring $P \geq \pi_I^M - K - \pi_I^d = \pi_S^{\emptyset}(S_v)$ is blocked by the AA. S might therefore make the second-best offer $P_L = \pi_I^M - K - \pi_I^m$. It will do so, when making such an offer is more profitable than making no offer, which requires $P_L = \pi_I^M - K - \pi_I^m \geq p(\pi_S^d - K)$ or, equivalently, $p \leq F_{II}$. Moreover, the AA must authorise the low-price takeover, which requires that $p \leq F_W$.

Finally, no takeover occurs when $\bar{H} < W^d - W^M$ and $p > \min(F_W, F_{II})$: either the AA blocks also a low-price takeover featuring P_L, or S finds it more profitable not to engage in any takeover than offering P_L.

A.10 Proof of Proposition 3

Let us consider first $p > F_{II} = (\pi_I^M - K - \pi_I^m)/(\pi_S^d - K)$. If $\bar{H} \geq W^d - W^M$, a high-price takeover occurs at the equilibrium, which entails expected welfare $p(W^M - K) + (1-p)W^m$. If $\bar{H} < W^d - W^M$, no takeover occurs at the equilibrium, which entails expected welfare $p(W^d - K) + (1-p)W^m$. The latter is larger because $W^d > W^M$. In order to maximise expected welfare, high-price takeovers should be prohibited. This can be achieved by choosing any $\bar{H} \in [-(W^M - K - W^m), W^d - W^M)$.

Let us consider, now, $p \in [0, F_{II}]$. If $p \leq \min(F_W, F_{II})$, a low-price takeover occurs at the equilibrium, in which the start-up is acquired irrespective of its type. Expected welfare is $W^M - K$. If $p \in (F_W, F_{II}]$, no takeover occurs at the equilibrium, entailing expected welfare $p(W^d - K) + (1-p)W^m$. Expected welfare is higher under a low-price takeover if and only if $p \leq (W^M - K - W^m)/(W^d - K - W^m) \equiv \bar{p}$.

To identify the optimal choice of the standard of review $\bar{H} \in [-(W^M - K - W^m), W^d - W^M)$, two cases must be distinguished:

1. $\bar{p} \leq F_{II}$. Expected welfare is maximised if low-price takeovers are authorised whenever $p \in [0, \bar{p}]$ and blocked when $p \in (\bar{p}, F_{II}]$. Since $\bar{p} = F_W(\pi_I^M - K, \bar{H} = 0)$, $\bar{H} = 0$ ensures that this is the case. The optimal merger policy is unique.

2. $\bar{p} > F_{II}$. For any $p \in [0, F_{II}]$, expected welfare is maximised if low-price takeovers are authorised. Since $F_{II} \in [0, 1)$, F_W is strictly increasing in $\bar{H}$ (from Lemma 1 (i)), $F_W = 0 \leq F_{II}$ if $\bar{H} = -(W^M - W^m - K)$ and $F_W > F_{II}$ if $\bar{H} = 0$, there exists a unique value $\hat{H} \in [-(W^M - W^m - K), 0)$ such that $F_W(\pi_I^M - K, \hat{H}) = F_{II}$. It follows that any $\bar{H} \geq \hat{H}$ ensures that low-price takeovers are authorised for any $p \in [0, F_{II}]$.

Combining the optimal merger policies in all the cases considered above, one can conclude that setting $\bar{H} = 0$ is optimal for any p (and for any distribution of the bargaining power).

Finally, when $\bar{p} > F_{II}$ and $p \in [F_{II}, \bar{p}]$, no takeover occurs at the equilibrium. Expected welfare would be higher under a low-price takeover, but the parties do not have any private incentive to engage in a low-price takeover. Hence, the merger policy is powerless.

A.11 Proof of Lemma 8

For the construction of the mixed-strategy equilibria when I has superior information, we build on the steps of the proof of Lemma 4, mutatis mutandis.

Consider first the start-up's decision. A start-up would never reject a price offer featuring $\tilde{P}_H > \pi_S^{\emptyset}(S_v)$. Under our mixed-strategy equilibrium, a start-up that receives an offer $\tilde{P}_L < \pi_S^{\emptyset}(S_v)$, instead, must be indifferent between accepting and rejecting it. In other words, it must hold that:

$$\tilde{P}_L = \phi(\tilde{P}_L)(\pi_S^d - k). \tag{20}$$

We now compute by Bayes' rule the probability that the start-up assigns to being valuable given that it receives an offer $\tilde{P}_L$:

$$\phi(\tilde{P}_L) = \frac{\gamma^L(S_v)p}{\gamma^L(S_v)p + \gamma^L(S_u)(1-p)}. \tag{21}$$

After plugging the last expression into equation (20), we obtain:

$$\gamma^L(S_v) = \frac{\gamma^L(S_u)\tilde{P}_L(1-p)}{p(\pi_S^d - K - \tilde{P}_L)}. \tag{22}$$

Let us now turn to the incumbent. It will (weakly) prefer to offer $\tilde{P}_L$ to an unviable start-up rather than not making it an offer if the following holds:

$$\beta^L(\pi_I^M - K - \tilde{P}_L) + (1-\beta^L)\pi_I^m \geq \pi_I^m. \tag{23}$$

This inequality can be simplified to get:

$$\beta^L(\pi_I^M - K - \tilde{P}_L - \pi_I^m) \geq 0, \tag{24}$$

which is satisfied for any $\beta^L \geq 0$ and $\tilde{P}_L \leq \pi_I^M - K - \pi_I^m$.

In the candidate equilibrium we consider, I offers $\tilde{P}_H \equiv \pi_S^{\emptyset}(S_v) = \pi_S^d - K$. For the incumbent to be indifferent between offering $\tilde{P}_L$ and $\tilde{P}_H$ to S_v, it must hold that:

$$\beta^L(\pi_I^M - K - \tilde{P}_L) + (1-\beta^L)\pi_I^d = \pi_I^M - \pi_S^d, \tag{25}$$

from which we obtain

$$\beta^L = \frac{\pi_I^M - \pi_S^d - \pi_I^d}{\pi_I^M - K - \pi_I^d - \tilde{P}_L}. \tag{26}$$

That $\beta^L \in (0,1)$ follows from $\pi_I^M > \pi_S^d + \pi_I^d$ (Assumption A2) and $\tilde{P}_L < \pi_S^d - K$.

We now determine the equilibrium values of $\tilde{P}_L$ and $\tilde{P}_H$. Recall that we assume that S, when receiving an out-of-equilibrium offer, updates the probability assigned to being viable by using Bayes' rule (whenever possible). If the offer does not convey information on the type, the start-up maintains the prior belief.

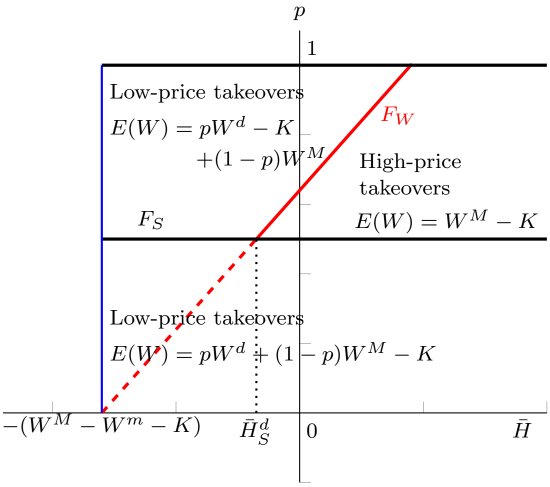
Clearly, the incumbent has no incentive to increase $\tilde{P}_H$ above $\pi_S^{\emptyset}(S_v)$. Assume instead that it offers $P' < \tilde{P}_H = \pi_S^{\emptyset}(S_v)$. If $P' > \pi_I^M - K - \pi_I^m$, the start-up infers that it is viable, because I has no incentive to offer that price to an unviable start-up. However, since $P' < \pi_S^d - K$, the viable type rejects the offer and the deviation is not profitable. If $P' \leq \pi_I^M - K - \pi_I^m$, no additional information is conveyed by the price offer and the start-up assigns probability p to being viable. Hence, if it also holds that $P' \geq p(\pi_S^d - K)$, the start-up accepts the offer and the deviation is profitable. It follows that offering $\tilde{P}_H = \pi_S^d - K$ is an equilibrium if (and only if) $\pi_I^M - K - \pi_I^m < p(\pi_S^d - K)$, i.e. $p > F_{II}$. A fortiori, there cannot exist an equilibrium in mixed strategies if $p \leq F_{II}$.

Consider now the case in which the incumbent deviates from $\tilde{P}_L$ to offer some $P' \in (\tilde{P}_L, \tilde{P}_H)$. Upon receiving a price offer featuring $P' \in (\pi_I^M - K - \pi_I^m, \tilde{P}_H)$, the start-up infers to be viable and rejects the offer. The deviation is not profitable. If $P' \in (\tilde{P}_L, \pi_I^M - K - \pi_I^m]$, the start-up assigns probability p to being viable. Since $P' \leq \pi_I^M - K - \pi_I^m < p(\pi_S^d - K)$ for all $p > F_{II}$, the start-up rejects the offer and the deviation is not profitable. The reasoning is similar if $P' < \tilde{P}_L$. Hence, there is a continuum of values of $\tilde{P}_L$ that can be sustained at equilibrium: $\tilde{P}_L \in (0, \pi_I^M - K - \pi_I^m]$.

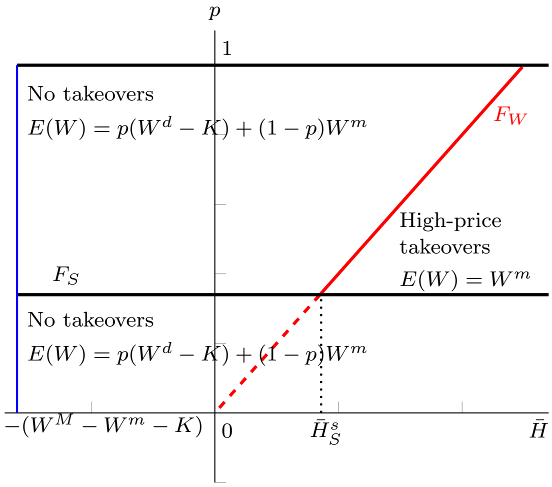
We now conclude the proof by noting that, since $\tilde{P}_L \in (0, \pi_I^M - K - \pi_I^m]$, with $\pi_I^M - K - \pi_I^m < p(\pi_S^d - K)$ for all $p > F_{II}$, the following holds: $\gamma^L(S_v) \in (0,1)$, $\gamma^L(S_u) = 1$ (condition (24) is always satisfied) and $\phi(\tilde{P}_H) = 1$ (only a viable start-up can receive an offer featuring $\tilde{P}_H$). Moreover, $\gamma^L(S_v) < 1$ implies that $\phi(\tilde{P}_L) < p$: when the start-up observes that it is offered a price $\tilde{P}_L$, it assigns a probability lower than the prior to being viable. Finally, the lower $\tilde{P}_L$, the lower $\gamma^L(S_v)$, the lower $\phi(\tilde{P}_L)$ and the lower β^L. Finally, $\phi(\tilde{P}_H) = 1$ implies that a hybrid PBE exists if and only if $\phi(\tilde{P}_H) = 1 \leq F_W$, or $\bar{H} \geq W^d - W^M$.

B Additional Figures

Figure B.1: Equilibrium takeovers when S makes take-it-or-leave-it offers, and associated welfare expected at $t = 0$.



(a) The incumbent develops



(b) The incumbent shelves

Notes: On the axes, $\bar{H}$ is the merger standard of review (level of tolerated harm); p is the a priori probability that the start-up is viable. F_S and F_W represent the cut-off values of the prior probability that govern the decision regarding the takeover price and, respectively, the approval decision of the AA. The left panel refers to the case in which the incumbent develops: $\bar{H}_S^d$ is the value of $\bar{H}$ such that F_W and F_S cross in this case, and may be negative as displayed in this figure. The right panel refers to the case in which the incumbent shelves: $\bar{H}_S^s$ is the value of $\bar{H}$ such that F_W and F_I cross with shelving. The cut-off $\bar{H}_S^s$ is necessarily positive.

C Online Appendix: The model, with late takeovers

In this Online Appendix we extend the baseline model by allowing the incumbent to acquire the start-up either before or after product development. Consistently, we allow the AA to commit to two different standards of intervention, denoted as $\bar{H}_1$ (which corresponds to $\bar{H}$ in the base model), and $\bar{H}_2$, respectively for mergers involving a potential competitor (that is, early takeovers occurring before development), and for mergers involving a committed entrant (that is, late takeovers, occurring after development).

More importantly, we also allow the expectation of a late takeover to increase the viability of the start-up. In this Online Appendix we solve the extended model *under the imperfect financial market micro-foundation*, in which this property arises naturally when the start-up has some bargaining power in the negotiation for the takeover: as we will show in Section C.2.1, the start-up anticipates that it will appropriate some of the rents created by the late acquisition; this makes it easier to obtain funding for its project. Therefore, the probability that the start-up is viable is higher when late takeovers are authorised than in the case in which they are not.

Funding of the project and information The development of the prototype requires a fixed investment K, which can be undertaken either by the start-up or by the incumbent, if the latter acquires the start-up at the beginning of the game. The start-up and the incumbent differ in their ability to fund the investment. Whereas I is endowed with sufficient own assets to pay the fixed cost K if it wanted to, S holds insufficient assets $A \geq 0$ to cover this initial outlay: $A < K$. Thus, S will search for funding in perfectly competitive capital markets.

Following Holmström and Tirole (1997), we assume that the probability that the prototype is developed successfully depends on the non-contractible effort exerted by the start-up. In case of effort the project succeeds with probability one, whereas in case of no effort it fails with probability one and yields no profit, but the start-up obtains private benefits $B > 0$. B proxies the start-up's agency costs. There are various ways in which management may not act in the firm's best interest. For example, it could take actions that are suboptimal, like relying on inefficient suppliers, or have diverging interests vis-à-vis lenders, for example preferring projects with less commercial value but stronger academic impact (as documented in biotech by Lerner and Malmendier, 2010).[19] As in Holmström-Tirole, the financial contract signed by the start-up and lenders takes the form of a sharing rule that specifies the income transferred to the start-up in the case of success (R_S^s) and failure (R_S^f). The investors' claim can be thought of as being either debt or equity. In other words, as shown in Tirole (2006), there is no difference between

[19]This framework with moral hazard is a natural choice to study a situation in which a project with positive net present value might fail to materialise because the start-up lacks resources that, instead, are available to the acquirer, thereby creating the scope for early acquisitions to be welfare beneficial. Alternatively one may assume adverse selection on the project type. This alternative setting is typically used by the literature in finance to determine optimal capital structure (Tirole, 2006). It would give rise to partitions of start-up types depending on their ability to secure funding. To the extent that inefficient credit rationing emerges also in this alternative model, and that the incumbent and the AA lack precise information on start-ups' access to finance, we would expect that such an alternative setting would give rise to qualitatively similar results to ours.

risky debt and equity in this model.

The assumption that the incumbent has enough internal resources to pay the investment cost implies that, in case of an acquisition, the management always exerts effort. An alternative but equivalent formulation would assume that also the incumbent needs to raise external funds, but it has active monitoring skills that remove the moral hazard problem when the acquisition takes place. Therefore, the incumbent is never financially constrained.

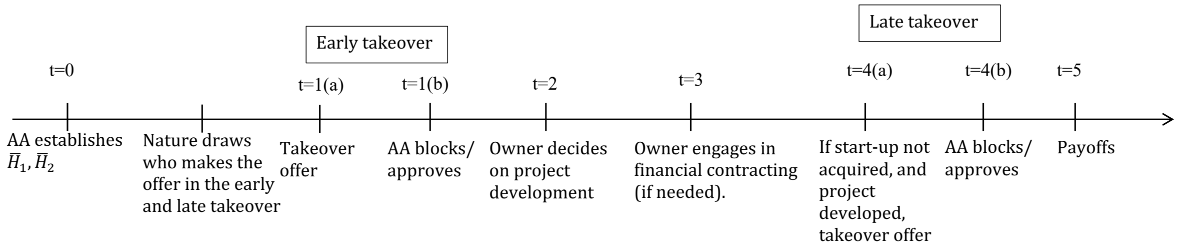
Before the game starts, B, the private benefit, is drawn from a continuous CDF $F(B)$, with $B \in [0, \pi_S^d]$. S and external financiers observe the value of B, while I and the AA do not. Our assumptions on the observability of B reflect the different skills of the various players in the game (Tirole, 2006). While it is the core business of financiers to establish the financial merits of a company, it is not the key expertise of incumbents and regulators. Moreover financiers can inspect S's banking records and history of debt repayment, while incumbents and AA typically do not have access to this information. Moreover, AAs generally lack the sophisticated financial ability necessary to interpret the relevant data, should they be able to access them. Hence, the lenders can conduct "backward looking" speculative monitoring that allows them to measure the value of B with certainty. Instead, the incumbent I, as well as the AA, cannot engage in such a speculative monitoring and only know the distribution $F(B)$ when they take their decisions.

Timing Next, we describe the timing of the game.

- At $t = 0$, the AA commits to the standards for merger approval, $\bar{H}_1, \bar{H}_2$. Subsequently, nature draws who makes the take-it-or-leave-it offer both at $t = 1(a)$ and $t = 4(a)$.
- At $t = 1(a)$, there is the 'early takeover game': either I or S makes a takeover offer, which can be accepted or rejected by the recipient.
- At $t = 1(b)$, the AA approves or blocks the takeover proposal.
- At $t = 2$, the firm that owns the prototype decides whether to develop or shelve it.
- At $t = 3$, the owner of the prototype engages in financial contracting (if needed).
- At $t = 4(a)$, there is the 'late takeover game': either I or S make a takeover offer (if the takeover did not already occur at $t = 1$, and if the project was developed).
- At $t = 4(b)$, the AA approves or blocks the takeover proposal.
- At $t = 5$, active firms sell in the product market, payoffs are realised and contracts are honoured.

We solve the game by backward induction.

Figure B.2: Timeline



C.1 Late takeover game (t=4)

Note that the late takeover game is — unlike the early takeover — one with perfect information, because all the relevant information has been revealed by the time it takes place. At $t = 3$ there exists scope for a late takeover if the start-up, that has not been acquired at $t = 1$, managed to develop the project. If so, absent the takeover, there would be a duopoly, with welfare $W^d - K$. Instead, a late takeover would lead to welfare $W^M - K$.[20] Since $W^d > W^M$, the AA will block the late takeover unless $\bar{H}_2 \geq W^d - W^M$.

If $\bar{H}_2 < W^d - W^M$, no late takeover occurs. Firms' profits at $t = 4$ are:

$$\pi_S^{\emptyset}(S_v, \bar{H}_2 < W^d - W^M) = \pi_S^d - A - R_l; \quad \pi_I^{\emptyset}(S_f, \bar{H}_2 < W^d - W^M) = \pi_I^d, \tag{27}$$

where S's profits are net of the internal resources invested in the project (A) and of the financial obligations to external investors R_l (where l stands for "lenders"). In these expressions, $\emptyset$ indicates that no takeover occured at $t = 1$ and $S = S_v$ that the start-up managed to obtain funding and is, therefore, viable.

If $\bar{H}_2 \geq W^d - W^M$, the AA authorises the late takeover. From Assumption $A1$, the takeover increases industry profits, implying that I and S are always willing to merge. When I makes the take-it-or-leave-it offer, it pays the price that leaves S with its threat point payoff. When the acquisition occurs, the incumbent also takes over the financial obligations of the start-up. Hence, I offers a price equal to $\pi_S^d - R_l$ appropriating the entire increase in joint profits produced by the takeover. Conversely, when S makes the offer, it requires to be paid a price equal to $\pi_I^M - \pi_I^d - R_l$ and leaves I with its threat-point payoff. Net profits are given by:

$$\begin{aligned} \pi_S^{\emptyset}(S_v, \bar{H}_2 \geq W^d - W^M) &= \mathbb{1}\pi_S^d + (1 - \mathbb{1})(\pi_I^M - \pi_I^d) - A - R_l; & (28) \\ \pi_I^{\emptyset}(S_v, \bar{H}_2 \geq W^d - W^M) &= (1 - \mathbb{1})\pi_I^d + \mathbb{1}(\pi_I^M - \pi_S^d). & (29) \end{aligned}$$

where $\mathbb{1}$ is an indicator function equal to 1 when I makes the offer, and 0 otherwise. Finally, a late takeover cannot take place if the start-up did not manage to obtain external funding and is

[20]Once the project has been developed, the incumbent will always market it: $\pi_I^M > \pi_I^m$.

unviable ($S = S_u$). In this case, the project would not be developed and firms' profits are:

$$\pi_S^{\emptyset}(S_u) = 0; \quad \pi_I^{\emptyset}(S_u) = \pi_I^m.$$

Table 1 summarises the profits of the incumbent and the start-up, when no early takeover occurs, depending on whether late takeovers are authorised and whether the start-up was viable or not. The profits of the latter are gross of the investment cost and are denoted with a capital letter.

Table 1: Firms' profit when no early takeover occurs

	Profit if $S = S_v$	Profit if $S = S_u$
Late takeover prohibited: $\bar{H}_2 < W^d - W^M$	$\Pi_S^{\emptyset} = \pi_S^d$ $\pi_I^{\emptyset} = \pi_I^d$	$\pi_S^{\emptyset} = 0$ $\pi_I^{\emptyset} = \pi_I^m$
Late takeover authorised: $\bar{H}_2 \geq W^d - W^M$	$\Pi_S^{\emptyset} = \pi_I^M - \pi_I^d$ $\pi_I^{\emptyset} = \pi_I^d$	$\pi_S^{\emptyset} = 0$ $\pi_I^{\emptyset} = \pi_I^m$

Table 1 shows that, when late takeovers are authorised, either the incumbent or the start-up that receives funding (S_v), depending on the one who makes the offer, seizes the the increase in industry profits due to the takeover. Hence, the one who makes the offer is better off than in the scenario in which late takeovers are blocked. The anticipation of this will affect financial contracting, as shown in the next section.

C.2 Investment decision and financial contracting

C.2.1 Financial contracting

If no takeover took place at $t = 1(b)$, a start-up that wants to develop the project looks for funding. Lemma 9 illustrates the outcome of the contracting game. Because of moral hazard, the start-up may be unable to obtain external funding even though the NPV of the project is positive. This is the case when the agency cost B is sufficiently high because the rent that is left to the borrower, once external financiers are repaid, is insufficient to induce the borrower to exert effort. Therefore, the parties cannot find an agreement that *both* induces effort and allows the lenders to break even. More importantly, the lemma shows that the merger policy targeting late takeovers affects the severity of financial constraints. This is because the start-up expects to obtain higher profits from the development of the project when late takeovers are authorised than when they are blocked (as shown in Section C.1). This makes it easier to incentivise effort and, therefore, to raise external funds.

LEMMA 9 (Financial contracting)**.**
There exists a threshold $\bar{B}(\bar{H}_2) = \Pi_S^{\emptyset}(S_v, \bar{H}_2) - K + A > 0$ *of the start-up's private benefit such that:*

(*i*) *If* $B > \bar{B}(\bar{H}_2)$*, the start-up does not obtain funding* ($S = S_u$)*.*

(*ii*) *If* $B \leq \bar{B}(\bar{H}_2)$*, the start-up is funded* ($S = S_v$)*. Its expected profit net of development costs is* $\pi_S^{\emptyset}(S_v, \bar{H}_2) = \Pi_S^{\emptyset}(S_v, \bar{H}_2) - K$*.*

(*iii*) *If the start-up holds the bargaining power, authorising late takeovers relaxes financial constraints:* $\bar{B}(\bar{H}_2 \geq W^d - W^M) > \bar{B}(\bar{H}_2 < W^d - W^M)$*.*

Proof. The financial contract stipulates the way gross profits from the development of the project are shared between S and the lenders. Both the start-up and the lenders correctly anticipate that, if funded and if effort is made, the project will be successful. If no effort is exerted, the project will fail and will produce 0 profits. Hence, the borrower's limited liability implies that both sides receive 0 in case of failure. In case of success, denote by R_l how much goes to external financiers. The financial contract must induce S to exert effort, because otherwise the lenders cannot break even:

$$\Pi_S^{\emptyset}(S_v, \bar{H}_2) - R_l \geq B. \tag{ICC}$$

Since the lenders are assumed to behave competitively, the zero-profit condition requires that:

$$R_l = K - A. \tag{PC}$$

Substituting the investor's participation constraint (PC) in the start-up's incentive compatibility constraint, one obtains that (ICC) holds if (and only if):

$$B \leq \bar{B}(\bar{H}_2) \equiv \Pi_S^{\emptyset}(S_v, \bar{H}_2) - (K - A),$$

with $\bar{B}(\bar{H}_2 < W^d - W^M) > 0$ by Assumption A1 and $A \geq 0$. If $B < \bar{B}(\bar{H}_2)$, the start-up is not funded (S_u) and cannot develop the project even though the NPV of the project is positive (part (i) of the lemma). We will say that it is credit constrained. If, instead, $B \geq \bar{B}(\bar{H}_2)$, the start-up obtains funding (S_v) – we will say that it is unconstrained. Substituting $R_l = K - A$ in equations (27) and (28), one obtains the net payoff indicated in part (ii).

In Section C.1, we showed that, if $\bar{H}_2 \geq W^d - W^M$, $\Pi_S^{\emptyset}(S_v, \bar{H}_2) = \mathbb{1}\pi_S^d + (1-\mathbb{1})(\pi_I^M - \pi_I^d)$, where $\mathbb{1}$ is an indicator function equal to 1 when the incumbent makes the offer in the takeover game; if $\bar{H}_2 < W^d - W^M$, $\Pi_S^{\emptyset}(S_v, \bar{H}_2) = \pi_S^d \leq \mathbb{1}\pi_S^d + (1-\mathbb{1})(\pi_I^M - \pi_I^d)$. Then, if $\bar{H}_2 \geq W^d - W^M$ and the start-up makes the offer in the takeover game, $\bar{B}(\bar{H}_2)$ is strictly larger than if $\bar{H}_2 < W^d - W^M$. Instead, if the incumbent makes the offer, $\bar{B}(\bar{H}_2)$ does not vary with $\bar{H}_2$. Q.E.D.

Finally, if the start-up was acquired by I at $t = 1$, no financial contracting takes place because I has enough resources to invest.

C.2.2 The investment decision

The investment decision is the same as in the baseline model:

LEMMA 10 (Investment decision)**.**

- *A start-up that obtains financing always invests in the development of the project.*
- *The incumbent invests if (and only if):*

$$\pi_I^M - \pi_I^m \geq K. \tag{30}$$

Proof. Follows from assumptions $A1$ and $A3$.

Q.E.D.

C.3 Early takeover game ($t = 1$)

At $t = 1$ the parties decide whether to engage in an early takeover. The results of the base model hold through in the extended setting. We will state them and we will provide the formal proof. We will discuss only the new insights due to the fact that the parties anticipate that in the continuation game a late takeover may take place.

Section C.3.1 describes the AA's decision at $t = 1(b)$, for given beliefs that the start-up obtains financing. Sections C.3.2 and C.3.3 illustrate the equilibrium takeover offer and acceptance decision, together with I's and AA's belief update processes.

C.3.1 Decision on merger approval

Lemma 11 generalises Lemma 1 of the base model to the case where late takeovers might be approved. Lemma 11 also shows that the AA is the more likely to approve a takeover when late takeovers are authorised because, absent the early takeover, the viable start-up would be acquired ex-post and product market competition would be suppressed anyway.

LEMMA 11 (Decision on merger approval)**.**
Let $\phi(\Omega)$ be the probability that the AA assigns to the start-up being viable, given the information set Ω. There exists a threshold $F_W(\pi_I^A, \bar{H}_1, \bar{H}_2) \geq 0$ such that the AA authorises the takeover iff:

$$\phi(\Omega) \leq F_W(\pi_I^A, \bar{H}_1, \bar{H}_2). \tag{31}$$

The threshold $F_W(\pi_I^A, \bar{H}_1, \bar{H}_2)$ is: (i) strictly increasing in $\bar{H}_1$; (ii) higher when $\pi_I^A = \pi_I^M - K$ than when $\pi_I^A = \pi_I^m$; (iii) higher when $\bar{H}_2 \geq W^d - W^M$ than when $\bar{H}_2 < W^d - W^M$.

Proof. Two cases must be considered. For simplicity, throughout this proof, we omit the functional notation for ϕ.
Case 1: The incumbent plans to shelve (i.e. $\pi_I^A = \pi_I^m$).

- Assume late takeovers are blocked, i.e. $\bar{H}_2 < W^d - W^M$. In this case, an early takeover creates expected harm $H = \phi[W^d - K - W^m] > 0$. If the start-up cannot obtain financing in $t = 3$, the early takeover does not affect welfare, because the project would die anyway. However, if the start-up is funded, the early takeover leads to the suppression of a project that the start-up would manage to develop independently. Hence, the early takeover prevents the project from reaching the market and ex-post competition from developing. The takeover is authorised if (and only if) the expected harm is lower than the tolerated harm, i.e. iff:
$$\phi \leq \frac{\bar{H}_1}{W^d - K - W^m} = F_W(\pi_I^m, \bar{H}_1, \bar{H}_2 < W^d - W^M).$$
- Assume late takeovers are authorised, i.e. $\bar{H}_2 \geq W^d - W^M$. If the start-up cannot obtain financing in $t = 3$, the early takeover does not affect welfare. If the start-up is financed in $t = 3$, the early takeover is welfare detrimental; however, since it would be acquired anyway at $t = 4$, the harm is lower than in the case in which late takeovers are blocked because a monopoly rather than a duopoly would arise in the market absent the takeover. The takeover is authorised iff the expected harm $H = \phi[W^M - K - W^m] > 0$ is lower than the tolerated harm, i.e. iff:.
$$\phi \leq \frac{\bar{H}_1}{W^M - K - W^m} = F_W(\pi_I^m, \bar{H}_1, \bar{H}_2 \geq W^d - W^M).$$

Case 2: The incumbent plans to develop (i.e. $\pi_I^A = \pi_I^M - K$).

- If $\bar{H}_2 < W^d - W^M$, an early takeover creates expected harm $H = (1-\phi)[W^m - (W^M - K)] + \phi[W^d - K - (W^M - K)]$: if the start-up is constrained, the early takeover is now beneficial, because it makes up for financial constraints and allows the project to reach the market; when the start-up is unconstrained, the early takeover is detrimental because of the suppression of product market competition. The takeover is authorised iff:
$$\phi \leq \frac{\bar{H}_1 + W^M - W^m - K}{W^d - K - W^m} = F_W(\pi_I^M - K, \bar{H}_1, \bar{H}_2 < W^d - W^M).$$
- If $\bar{H}_2 \geq W^d - W^M$, an early takeover creates expected harm $H = (1-\phi)[W^m - (W^M - K)] < 0$, i.e. an early takeover is welfare beneficial. Since late takeovers are authorised, the unconstrained start-up would be acquired anyway and so a monopoly would arise, irrespective of whether the early takeover goes through; however, when the start-up is constrained, the early takeover is beneficial. In this case early takeovers are authorised iff:
$$\phi \leq \frac{\bar{H}_1 + W^M - W^m - K}{W^M - K - W^m} = F_W(\pi_I^M - K, \bar{H}_1, \bar{H}_2 \geq W^d - W^M).$$

A comparison of the cut-off levels of the probability that the start-up is unconstrained, denoted by F_W, in the different cases reveals that:

i. Given π_I^A, F_W is higher if later takeovers are authorised than in the case in which they are blocked (when F_W is positive). This follows from $W^d > W^M$.

ii. Given $\bar{H}_2$, F_W is higher when the incumbent develops than when the incumbent shelves. This follows from $W^M - W^m - K > 0$.

Moreover, $\bar{H}_1 \geq -(W^M - W^m - K)$ implies that $F_W \geq 0$ when $\pi_I^A = \pi_I^M - K$.

Q.E.D.

Corollary 2 is identical to the base model:

COROLLARY 2.

(i) *When the incumbent develops, the AA* always *approves an early takeover if it assigns probability one to the start-up being constrained (i.e. $\phi(\Omega) = 0$).*

(ii) *When the incumbent shelves, no early takeover is approved if the merger policy commits to blocking any welfare detrimental takeover (i.e. $\bar{H}_1 < 0$).*

Proof. (i) Since $F_W(\pi_I^M - K, \bar{H}_1, \bar{H}_2) \geq 0$, condition (31) is always satisfied when $\phi_{AA}(\Omega) = 0$. (ii) Since $F_W(\pi_I^m, \bar{H}_1, \bar{H}_2) < 0$ when $\bar{H}_1 < 0$, condition (31) is never satisfied.

Q.E.D.

C.3.2 Equilibrium offers at t=1(a): the incumbent holds the bargaining power

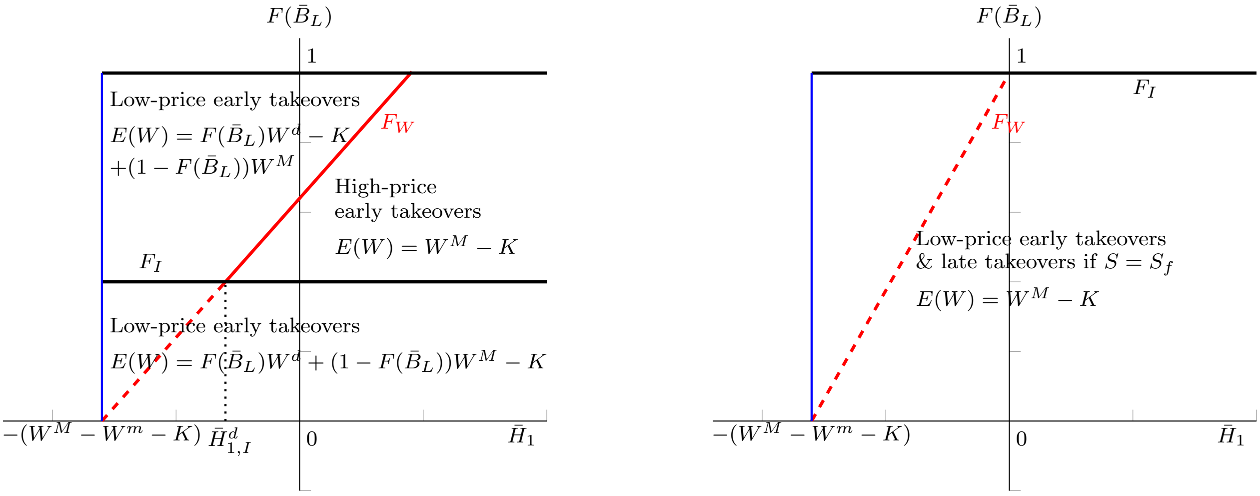
We first analyse the case in which the incumbent makes a take-it-or-leave-it offer. When this is the case, the incumbent fully appropriates the surplus produced by the late takeover and, when no early takeover occurs, the unconstrained start-up obtains the same payoff irrespective of whether the late takeover is authorised or blocked: $\pi_S^{\emptyset}(S_v, \bar{H}_2) = \pi_S^d - K$ for any $\bar{H}_2$. An implication of this is that the threshold level of B that determines whether a start-up is financially constrained does not depend on the merger policy regarding late takeovers: $\bar{B}(\bar{H}_2) = \pi_S^d - K + A$ for any $\bar{H}_2$ (see Lemma 9). We denote this threshold as $\bar{B}_L$.

The PBE of the bargaining game are described in Lemma 12. For the case of development, Figure B.3 displays the equilibrium takeovers and the expected welfare at $t = 0$ as a function of the merger policy regarding early takeovers $\bar{H}_1$ and the prior probability that the start-up is unconstrained $F(\bar{B}_L)$. The left panel refers to the case in which late takeovers are blocked and is the same as in the base model. The right panel refers to the case in which late takeovers are authorised (i.e. $\bar{H}_2 \geq W^d - W^M$). In such a case only low-price early takeovers occur at the equilibrium: given that an unconstrained start-up can be acquired at $t = 2$, there is no point for the incumbent in overpaying for a constrained start-up at the early stage.[21]

Figure B.4 refers to the case of shelving. Differently from the case of development, with shelving the incumbent may be willing to engage in a high-price takeover also when late takeovers

[21] When I develops and $\bar{H}_2 \geq W^d - W^M$, $F_I = 1$ so that it cannot be that $F(\bar{B}_L) > F_I$.

Figure B.3: Equilibrium takeovers when I develops (and holds the bargaining powers), and associated welfare expected at $t = 0$.

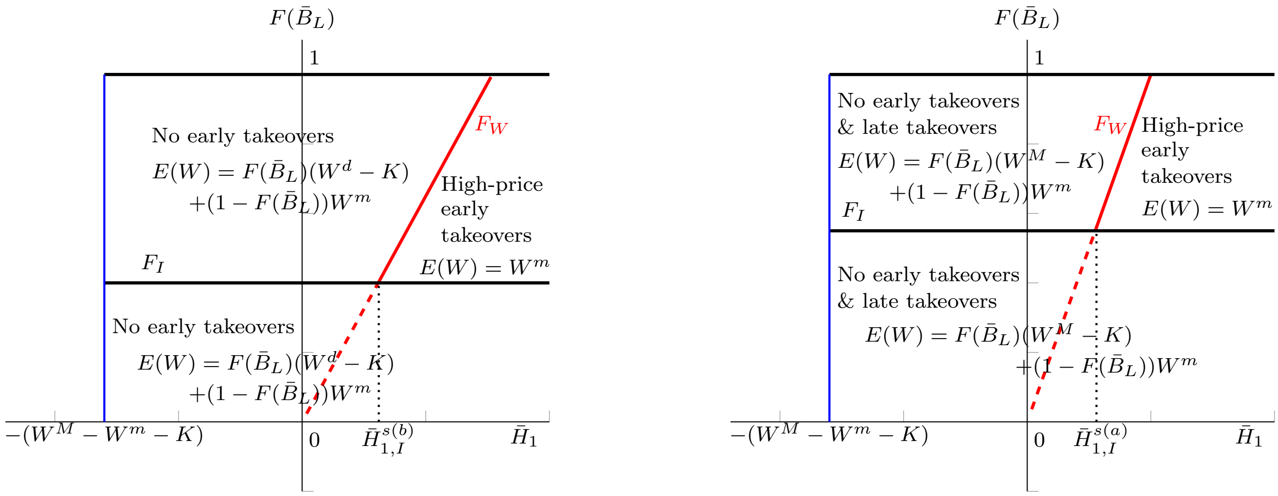


(a) Late takeovers blocked (b) Late takeovers authorised

On the axes, $\bar{H}_1$ is the standard of review (level of tolerated harm) for early takeovers; $F(\bar{B}_L)$ is the a priori probability that the start-up is unconstrained. F_I and F_W represent the cut-off values of the a priori probability that govern the decision of the incumbent regarding the takeover price and, respectively, the approval decision of the AA. The left panel refers to the case in which late takeovers are blocked (i.e. $\bar{H}_2 < W^d - W^M$). The right panel refers to the case in which late takeovers are authorised (i.e. $\bar{H}_2 \geq W^d - W^M$). $\bar{H}^d_{1,I}$, that is the value of $\bar{H}_1$ such that F_W and F_I cross, will be central to the determination of the optimal merger policy. When the incumbent develops, $\bar{H}^d_{1,I}$ may be negative, a case displayed in this figure.

are authorised (right panel of the figure): from the perspective of the incumbent, developing the project is an inefficient investment which cannot be avoided if the unconstrained start-up remains independent; hence, the incumbent may be willing to overpay for a constrained start-up at the early stage.

Figure B.4: Equilibrium takeovers when I shelves (and holds the bargaining power) and associated welfare expected at $t = 0$.



(a) Late takeovers blocked

(b) Late takeovers authorised

On the axes, $\bar{H}_1$ is the standard of review (level of tolerated harm) for early takeovers; $F(\bar{B}_L)$ is the a priori probability that the start-up is unconstrained. F_I and F_W represent the cut-off values of the a priori probability that govern the decision of the incumbent regarding the takeover price and, respectively, the approval decision of the AA. The left panel refers to the case in which late takeovers are blocked (i.e. $\bar{H}_2 < W^d - W^M$). The right panel refers to the case in which late takeovers are authorised (i.e. $\bar{H}_2 \geq W^d - W^M$). $\bar{H}_{1,I}^{s(j)}$, with $j = b, a$ depending on whether late takeovers are blocked or authorised, is the value of $\bar{H}_1$ such that F_W and F_I cross, and will be central to the determination of the optimal merger policy. Differently from the case of development, with shelving $\bar{H}_{1,I}^{s(j)}$ is necessarily positive.

LEMMA 12 (PBE of the bargaining game when I makes the offer)**.**

Let:

$$F_I(\pi_I^A, \bar{H}_2) \equiv \frac{\pi_S^d - K}{\pi_I^A - \pi_I^{\emptyset}(S_v, \bar{H}_2)} \in (0, 1]. \tag{32}$$

When I makes a take-it-or-leave-it offer:

1. *If $\pi_I^A = \pi_I^m$ and either $F(\bar{B}_L) \leq F_I$ or $F(\bar{B}_L) > \max(F_W, F_I)$, no early takeover occurs at the equilibrium.*

2. *For any π_I^A, if $F(\bar{B}_L) \in (F_I, \max(F_W, F_I)]$, the PBE is: $\{(s_I^* = \pi_S^d - K, r_{S_v}^* = r_{S_u}^* =$ Accept $\pi_S^d - K); \phi(\{s_I^*, r_S^*\}) = F(\bar{B}_L)\}$.*

3. *If $\pi_I^A = \pi_I^M - K$ and either $F(\bar{B}_L) \leq F_I$ or $F(\bar{B}_L) > \max(F_W, F_I)$, the PBE is: $\{(s_I^* = 0, r_{S_v}^* =$ Reject 0$, r_{S_u}^* =$ Accept 0$); \phi(\{s_I^*, r_{S_v}^*\}) = 1, \phi(\{s_I^*, r_{S_u}^*\}) = 0\}$.*

Proof. If $S = S_u$, the start-up's payoff when rejecting I's offer is $\pi_S^{\emptyset}(S_u) = 0$; if $S = S_v$, it is $\pi_S^{\emptyset}(S_v, \bar{H}_2) = \pi_S^d - K > 0$ from Assumption $A1$.[22] The incumbent will then offer either a low price $P_I = 0$, and only the constrained start-up $S = S_u$ will accept, or a high price $P_I = \pi_S^d - K > 0$ and both types of start-up will accept. In the former case, observing that the offer is accepted allows the incumbent and the AA to update their beliefs and infer that the start-up is financially constrained: $\phi(\{0, \textit{Accept } P_I\}) = 0$. In the latter case (as well as in the case in which no offer is made) the acceptance decision of the start-up does not reveal its type. Then the posteriors coincide with the priors: $\phi(\{\pi_S^d - K, \textit{Accept } P_I\}) = F(\bar{B}_L)$. From Lemma 11, the deal is authorised iff $F(\bar{B}_L) \leq F_W$. Finally, there cannot exist an equilibrium in which both start-ups are acquired at a different positive price: the start-up receiving the lower price offer would pretend to be the type receiving the higher price offer, thus breaking the equilibrium.

If I does not make any offer, its expected profit is:

$$F(\bar{B}_L)\pi_I^{\emptyset}(S_v, \bar{H}_2) + (1 - F(\bar{B}_L))\pi_I^m. \tag{33}$$

If I offers a low price and the deal is authorised (i.e. if $\phi(\Omega) = 0 \leq F_W$, a condition that is always satisfied if the incumbent develops, from Corollary 2 (i)), I's expected profit (gross of the transaction cost) is:

$$F(\bar{B}_L)\pi_I^{\emptyset}(S_v, \bar{H}_2) + (1 - F(\bar{B}_L))\pi_I^A. \tag{34}$$

If I offers a high price and the deal is authorised (i.e. if $\phi(\Omega) = F(\bar{B}_L) \leq F_W$), its expected profit (gross of the transaction cost) is:

$$\pi_I^A - (\pi_S^d - K). \tag{35}$$

By comparing the expressions in equations (34) and (35) one obtains that offering a low price is more profitable for the incumbent than offering a high price iff $F(\bar{B}_L) \leq F_I$, where F_I is defined

[22] For the sake of the exposition, throughout the proof, we drop the functional notation for F_I and F_W.

in equation (32). However, it must also be the case that making an offer is more profitable than not engaging in the takeover.

Therefore, when $\pi_I^A = \pi_I^m$ (i.e. the incumbent shelves) and $F(\bar{B}_L) \leq F_I$, the comparison between (33) and (34) and the existence of the positive transaction cost involved in the takeover reveal that I's equilibrium decision is not to engage in the takeover. The same equilibrium decision is taken when $\pi_I^A = \pi_I^m$ and $F(\bar{B}_L) > \max(F_W, F_I)$: I would prefer to offer a high price, but the AA would not authorise the deal. Since offering a low price is dominated by making no offer, an early takeover does not occur at the equilibrium. This concludes part 1. of the lemma.

If $F(\bar{B}_L) \in (F_I, \max(F_W, F_I)]$, the equilibrium offer involves a high price, as the incumbent's preferred choice is authorised by the AA. The posteriors coincide with the priors as stated in part 2. of the lemma.

Finally, if $\pi_I^A = \pi_I^M - K$ (i.e. the incumbent develops) and either $F(\bar{B}_L) \leq F_I$ or $F(\bar{B}_L) > \max(F_W, F_I)$, $P_I = 0$ is offered at the equilibrium, and the incumbent and the AA update their beliefs based on whether the start-up accepts, as stated in part 3. of the lemma. When $F(\bar{B}_L) > \max(F_W, F_I)$ the incumbent would prefer to offer a high price. However, anticipating that the AA would not authorise the transaction, the incumbent has to settle for a second-best low-price offer.

Assumption $A1$ implies that $\pi_I^A - \pi_I^\emptyset(S_v, \bar{H}_2) > 0$ and $\pi_S^d - K > 0$. Therefore $F_I > 0$. Moreover, $F_I < 1$ if (and only if) the joint payoff of I and S_f in the absence of an early takeover is strictly lower than their joint payoff when the early takeover occurs. Assumption A3 ensures that this is the case when $\bar{H}_2 < W^d - W^M$ and late takeovers are blocked. This is also the case when late takeovers are authorised and the incumbent shelves. Instead, when late takeovers are authorised and the incumbent develops the project, the joint payoff of I and S_v is the same irrespective of whether the takeover occurs early or at a later stage and $F_I = 1$.

Q.E.D.

C.3.3 Equilibrium offers at t=1(a): the start-up holds the bargaining power

We now analyse the case in which the start-up makes a take-it-or-leave-it offer. Differently from the case in which the incumbent holds the bargaining power, now it is the start-up that appropriates the whole surplus produced by a late takeover. The outside option of the unconstrained start-up now *does* depend on the merger policy regarding late takeovers and is higher when late takeovers are authorised (see Table 1). As a consequence, from Lemma 9 (iii), authorising late takeovers alleviates financial constraints: when $\bar{H}_2 \geq W^d - W^M$, $\bar{B}(\bar{H}_2) = \pi_I^M - \pi_I^d - K + A \equiv \bar{B}_H$, which is larger than the threshold $\bar{B}(\bar{H}_2) = \pi_S^d - K + A \equiv \bar{B}_L$ associated to $\bar{H}_2 < W^d - W^M$.

Apart from this consideration, the qualitative nature of the results and the underlying intuitions are similar to the case in which the incumbent has the bargaining power. The figures displaying the equilibrium takeovers as a function of $\bar{H}_1$ and $F(\bar{B}(\bar{H}_2))$ are also similar to those presented in Section C.3.2, with F_S substituting F_I, $F(\bar{B}_H)$ substituting $F(\bar{B}_L)$ when late takeovers are authorised, and $\bar{H}_{1,S}^i$ substituting $\bar{H}_{1,I}^i$, with $i = s, d$ depending on shelving or development (see Figure B.5). The PBE is described in Lemma 13.

LEMMA 13 (Pure-strategy PBE of the bargaining game when S makes the offer).

Let:

$$F_S(\pi_I^A, \bar{H}_2) \equiv \frac{\pi_S^{\emptyset}(S_v, \bar{H}_2) + \pi_I^m - \pi_I^A}{\pi_I^m - \pi_I^d} \in (0, 1]. \tag{36}$$

When $S \in \{S_v, S_u\}$ makes a take-it-or-leave-it offer:

1. *If $\pi_I^A = \pi_I^m$ and either $F(\bar{B}(\bar{H}_2)) \leq F_S$ or $F(\bar{B}(\bar{H}_2)) > \max(F_W, F_S)$, no early takeover occurs at the equilibrium.*

2. *For any π_I^A, if $F(\bar{B}(\bar{H}_2)) \in (F_S, \max(F_S, F_W)]$, the PBE is: $\{(s^*_{S_u} = s^*_{S_v} = \bar{P}, r^*_I = \text{Accept } \bar{P}); \phi(\{\bar{P}, \text{Accept } \bar{P}\}) = F(\bar{B}(\bar{H}_2))\}$, with $\bar{P} = \pi_I^A - \pi_I^m + F(\bar{B}(\bar{H}_2))(\pi_I^m - \pi_I^d)$.*

3. *If $\pi_I^A = \pi_I^M - K$ and either $F(\bar{B}(\bar{H}_2)) \leq F_S$ or $F(\bar{B}(\bar{H}_2)) > \max(F_W, F_S)$ the PBE is: $\{(s^*_{S_v} = \emptyset, s^*_{S_u} = P_L, r^*_I = \text{Accept } P_L); \phi(\{P_L, \text{Accept } P_L\}) = 0\}$, with $P_L = \pi_I^M - K - \pi_I^m > 0$.*

Proof. Consider a candidate equilibrium in which the start-up, irrespective of whether it is constrained or not, offers $P_{S_u} = P_{S_v} = P$. For this to be an equilibrium, P must satisfy the start-ups' participation constraints:[23]

$$\begin{aligned} P &> \pi_S^{\emptyset}(S_u) = 0 \qquad (37)\\ P &> \pi_S^{\emptyset}(S_v, \bar{H}_2). \qquad (38) \end{aligned}$$

P must also satisfy the incumbent's participation constraint:

$$\pi_I^A - P \geq F(\bar{B}(\bar{H}_2))\pi_I^d + [1 - F(\bar{B}(\bar{H}_2))]\pi_I^m, \tag{39}$$

where the incumbent's posterior beliefs on the probability that the start-up is unconstrained coincide with the priors. Since constraint (38) is more binding than constraint (37), P must satisfy:

$$\pi_S^{\emptyset}(S_v, \bar{H}_2) < P \leq \pi_I^A - \pi_I^m + F(\bar{B}(\bar{H}_2))(\pi_I^m - \pi_I^d) \equiv \bar{P}.$$

Since $\pi_I^m - \pi_I^d > 0$, a necessary condition for the existence of an equilibrium featuring $P = \bar{P}$ is:

$$F(\bar{B}(\bar{H}_2)) > F_S, \tag{40}$$

where F_S is defined in equation (36).

Finally, it must be that the AA authorises the deal, if the offer $P = \bar{P}$ is accepted. This is the case if (and only if) $F(\bar{B}(\bar{H}_2)) \leq F_W$. (Given our assumptions, the AA's posterior beliefs on the probability that the start-up is unconstrained coincide with the priors.) Combining the above

[23]For the sake of the exposition, throughout the proof, we drop the functional notation for F_S and F_W.

conditions one obtains part 2. of the lemma. An equilibrium with $P \in (\pi_S^{\emptyset}(S_f, \bar{H}_2), P_p)$ does not exist because $S \in \{S_v, S_u\}$ would have an incentive to deviate and increase the price: following an out-of-equilibrium offer $P' \leq \bar{P}$, I and AA would attach the prior probability to the start-up being unconstrained. Since $F(\bar{B}(\bar{H}_2)) \leq F_W$ the AA would authorise the deal; since $P' \leq \bar{P}$, I would accept. The deviation would be profitable. Hence, $P = \bar{P}$ is the unique equilibrium price such that $P_{S_v} = P_{S_u}$.

Consider now a candidate equilibrium in which $S = S_u$ offers $P_{S_u} = P_L = \pi_I^A - \pi_I^m$, $S = S_v$ does not make any offer, and the incumbent accepts P_L. From Assumptions $A1$ and $A3$, $\pi_I^A - \pi_I^m < \pi_S^d - K \leq \pi_S^{\emptyset}(S_v, \bar{H}_2)$. Therefore, observing such an offer both I and the AA infer that the start-up is constrained (i.e. $\phi(\Omega) = 0$). Then, I is indifferent between accepting and rejecting P_L. For this to be an equilibrium, S must have no incentive to deviate.

To start with, $S = S_u$ must find it unprofitable not to make an offer:

$$P_L > \pi_S^{\emptyset}(S_u) = 0 \tag{41}$$

with the inequality being strict because of the existence of a negligible but positive transaction cost associated with the takeover offer.

Let us focus on the case in which I develops and $P_L = \pi_I^M - K - \pi_I^m > 0$. Since $\pi_S^{\emptyset}(S_v, \bar{H}_2) > P_L$, then $S = S_v$ has no incentive to deviate and offer P_L. Clearly, $S = S_u$ has no incentive to decrease its offer. Has it an incentive to offer $P' > P_L$? As long as $P' \leq \pi_S^{\emptyset}(S_v, \bar{H}_2)$, the incumbent infers that the start-up is constrained and rejects the deviation offer. The deviation is unprofitable. If, instead, $P' > \pi_S^{\emptyset}(S_v, \bar{H}_2)$, the incumbent attributes the offer to an unconstrained start-up with probability $F(\bar{B}(\bar{H}_2))$. The deviation is unprofitable either if I would reject the offer, i.e. if $\pi_S^{\emptyset}(S_v, \bar{H}_2) \geq \bar{P}$ which is satisfied if $F(\bar{B}(\bar{H}_2)) \leq F_S$; or if I would accept the deviation offer but the AA would not authorise the deal, i.e. if $F(\bar{B}(\bar{H}_2)) > \max(F_S, F_W)$. For the same reason, it is not profitable for $S = S_f$ to offer $P' \geq \pi_S^{\emptyset}(S_v, \bar{H}_2)$. Of course, S_v has no incentive to deviate and offer $P' < \pi_S^{\emptyset}(S_v, \bar{H}_2)$. In sum, when I develops and either $F(\bar{B}(\bar{H}_2)) \leq F_S$ or $F(\bar{B}(\bar{H}_2)) > \max(F_S, F_W)$ the proposed one is an equilibrium, as stated in part 3. of the lemma.

Note that there cannot exist an equilibrium in which $S = S_u$ offers $P_L < \pi_I^M - K - \pi_I^m$. $S = S_u$ would have an incentive to deviate and offer $P' = \pi_I^M - K - \pi_I^m$, since I would accept the offer and the AA would authorise the deal.

Finally, there cannot exist an equilibrium in which $S = S_v$ offers $P_{S_f} = \tilde{P} > \pi_S^{\emptyset}(S_v, \bar{H}_2)$, $S = S_u$ offers $P \neq \tilde{P}$, the incumbent accepts the former and rejects the latter. If the AA authorises the deal, $S = S_u$ would always have an incentive to mimic S_v and offer $\tilde{P}$ instead. For a similar reason, there cannot exist an equilibrium in which $S = S_u$ offers $P_{S_u} = P_L$, $S = S_v$ offers $P_{S_v} \in (\pi_S^{\emptyset}(S_v, \bar{H}_2), \pi_I^M - K - \pi_I^d]$ and I accepts both offers.

Let us consider now the case in which I shelves. Since $\pi_I^A = \pi_I^m$, then $P_L = 0$. Hence, condition (41) cannot be satisfied and the proposed one is not an equilibrium. Other equilibria in which each start-up is traded at a different price do not exist, for the same reasoning developed above. Therefore, if $\pi_I^A = \pi_I^m$ and either $F(\bar{B}(\bar{H}_2)) \leq F_S$ or $F(\bar{B}(\bar{H}_2)) > \max(F_W, F_S)$, there is no early takeover in equilibrium, as stated in part 1. of the lemma.

Note that $\pi_I^m > \pi_I^d$, $\pi_S^{\emptyset}(S_v, \bar{H}_2) \geq \pi_S^d - K > 0$ (from the analysis in Section C.1 and assumption A2) and assumption A3 imply $F_S > 0$. Moreover, $F_S < 1$ if (and only if) the joint payoff of I and S_f in the absence of an early takeover is strictly lower than their joint payoff when the early takeover occurs. Assumption A1 ensures that this is the case when $\bar{H}_2 < W^d - W^M$. This is also the case when late takeovers are authorised and the incumbent shelves. Instead, when late takeovers are authorised and the incumbent develops the project, the joint payoff of I and S_f is the same irrespective of whether the takeover occurs early or at a later stage and $F_S = 1$.

Q.E.D.

C.4 The optimal merger policy

In this section, we study the optimal merger policy at $t = 0$, when the AA commits to the two thresholds of tolerated harm, $\bar{H}_1$ and $\bar{H}_2$, respectively for early takeovers and late takeovers.

Allowing for late takeovers does not change the optimal policy regarding early takeovers: it is still the case that high-price takeovers are prohibited. However, under some specific cumulative conditions, specified in Proposition 5, it is optimal to adopt a lenient approach that approves late takeover.

The optimal policy will be derived considering the pure-strategy equilibria of the bargaining game at $t = 1$. It can be shown that the mixed strategy equilibrium does not exist when late takeovers are authorised. Therefore, the result of Proposition 2 is still valid in this context.

PROPOSITION 5 (The optimal merger policy)**.**

i. *The optimal merger policy regarding early takeovers commits to standards of review that prevent early high-price takeovers at the equilibrium:*

   (a) *If $\pi_I^A = \pi_I^M - K$, there exists a threshold level of $\bar{H}_1$, $\bar{H}_1^d > -(W^M - W^m - K)$, such that all $\bar{H}_1 \leq \bar{H}_1^d$ in the admissible set are optimal for any value of α.*

   (b) *If $\pi_I^A = \pi_I^m$, there exists a threshold level of $\bar{H}_1$, $\bar{H}_1^s > 0$ such that all $\bar{H}_1 \leq \bar{H}_1^s$ in the admissible set are optimal for any value of α and for any $\bar{H}_2$.*

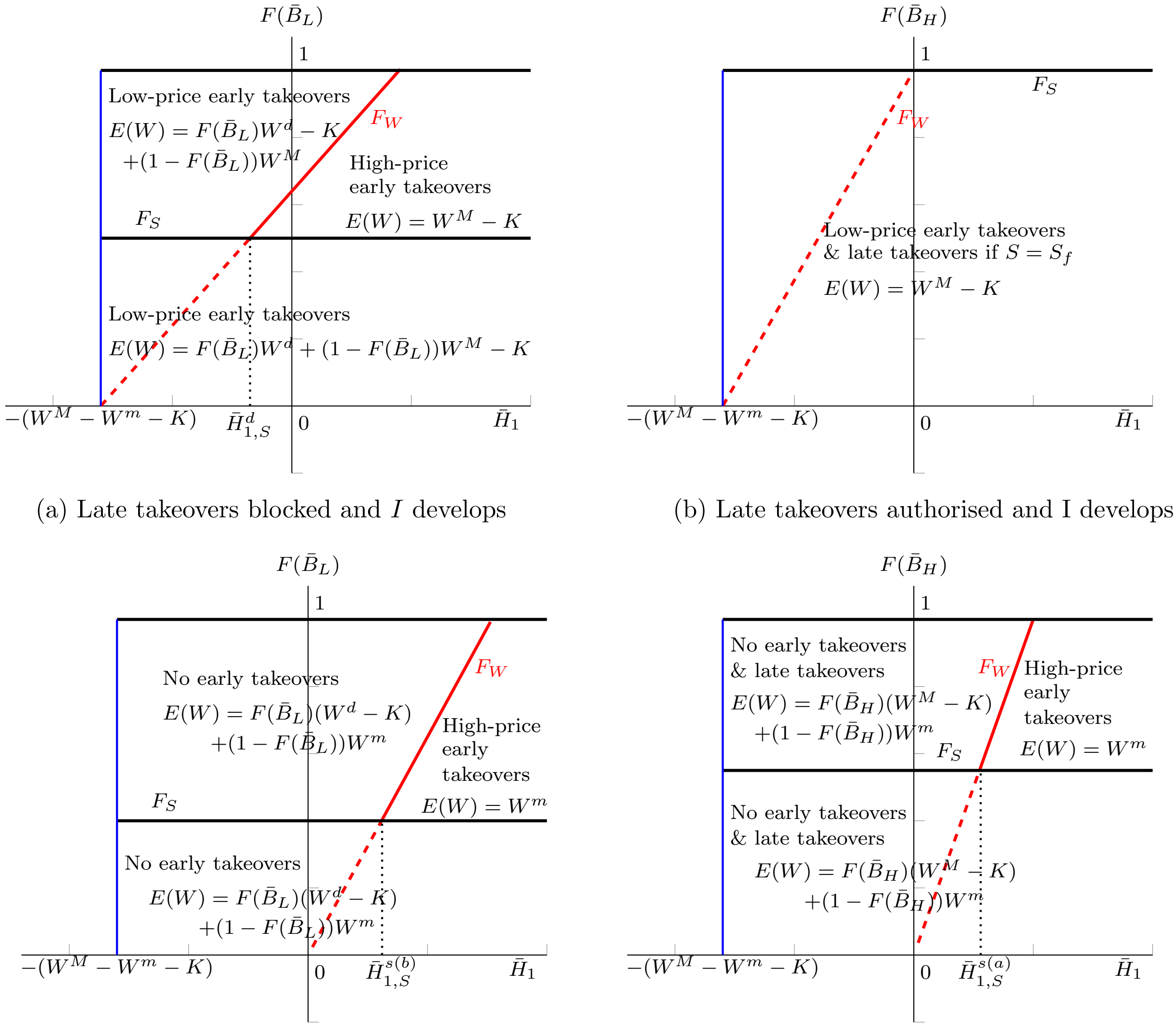
   (c) *All $\bar{H}_1 \leq \min(\bar{H}_1^d, \bar{H}_1^s)$ in the admissible set are optimal for any value of α, π_I^A and $\bar{H}_2$.*

ii. *The optimal merger policy regarding late takeovers when the start-up has the bargaining power is:*

   (a) *Lenient, i.e. all $\bar{H}_2 \geq W^d - W^M$ are optimal, if (and only if) $\pi_I^A = \pi_I^m$, $\alpha < \hat{\alpha}$ (with $\hat{\alpha} > 0$), and*

$$\frac{p(\bar{H} \geq W^d - W^M)}{p(\bar{H} < W^d - W^M)} > \frac{W^d - K - W^m}{W^M - K - W^m}. \tag{42}$$

Figure B.5: Equilibrium takeovers when S makes take-it-or-leave-it offers, and associated welfare expected at $t = 0$.



(a) Late takeovers blocked and I develops

(b) Late takeovers authorised and I develops

(c) Late takeovers blocked and I shelves

(d) Late takeovers authorised and I shelves

On the axes, $\bar{H}_1$ is the standard of review (level of tolerated harm) for early takeovers; $F(\bar{B}_L)$ or $F(\bar{B}_H)$ is the a priori probability that the start-up is unconstrained. F_S and F_W represent the cut-off values of the a priori probability that govern the decision regarding the takeover price and, respectively, the approval decision of the AA. The left panels refer to the case in which late takeovers are blocked (i.e. $\bar{H}_2 < W^d - W^M$). The right panels refer to the case in which late takeovers are authorised (i.e. $\bar{H}_2 \geq W^d - W^M$).The top panels refer to the case in which the incumbent develops. $\bar{H}^d_{1,S}$ is the value of $\bar{H}_1$ such that F_W and F_S cross and may be negative as displayed in this Figure. The bottom panels refer to the case in which the incumbent shelves and $\bar{H}^{s(j)}_{1,S}$, with $j = b, a$ depending on whether late takeovers are blocked or authorised, is the value of $\bar{H}_1$ such that F_W and F_I cross. $\bar{H}^{s(j)}_{1,S}$ is necessarily positive.

(b) $\bar{H}_2 < W^d - W^M$ *are optimal, otherwise.*

Proof. Case 1: The incumbent plans to develop (i.e. $\pi_I^A = \pi_I^M - K$).[24]

Let us consider the case in which the incumbent makes a take-it-or-leave-it offer at $t = 1(a)$.

[24]For the sake of the exposition, throughout the proof, we drop the functional notation for F_I, F_S and F_W.

In this case the threshold $\bar{B}(\bar{H}_2) = \pi_S^d - K + A = \bar{B}_L$ for all $\bar{H}_2$ because I has all the bargaining power (see Section C.3.2).

If $\bar{H}_2 \geq W^d - W^M$, expected welfare is the same for any feasible value of $\bar{H}_1$ (i.e. for any $\bar{H}_1 \geq -(W^M - W^m - K)$): in $t = 1(a)$, the incumbent offers a low-price, which is accepted by type $S = S_u$, and the acquisition is authorised by the AA. A start-up of the type $S = S_v$ is acquired in $t = 4(a)$. In either case, the expected welfare is $W^M - K$.

Let $\bar{H}_2 < W^d - W^M$. Lemma 12 implies that two sub-cases must be considered:

1. If either $F(\bar{B}_L) \leq F_I$ or $F(\bar{B}_L) > \max(F_W, F_I)$, I offers $P_I = 0$ in $t = 1(a)$ and only type $S = S_u$ accepts. Expected welfare is $E(W) = F(\bar{B}_L)(W^d - K) + (1 - F(\bar{B}_L))(W^M - K) > W^M - K$.

2. If $F(\bar{B}_L) \in (F_I, \max(F_W, F_I)]$, I offers $P_I = \pi_S^d - K$ in $t = 1(a)$ and both $S = S_v$ and $S = S_u$ accept. Expected welfare is $E(W) = W^M - K$. This case arises if and only if $F_W > F_I$.

Since $E(W)$ is strictly larger when $\bar{H}_2 < W^d - W^M$ than when $\bar{H}_2 \geq W^d - W^M$ for all the values of $F(\bar{B}_L)$ such that the first sub-case arises, and it is the same for all the values of $F(\bar{B}_L)$ such that the second sub-case arises, the welfare-maximizing value of $\bar{H}_2$ is such that late takeovers are blocked, i.e. any $\bar{H}_2 < W^d - W^M$ is optimal.

Regarding early takeovers, comparing the two sub-cases, we conclude that the optimal policy is the one that avoids high-price early takeovers from arising at the equilibrium. This can be ensured by setting $\bar{H}_1$ such that $F_W \leq F_I$: in this way, for all the values of $F(\bar{B}_L)$ such that the incumbent finds it profitable to offer a high price, the takeover is blocked.

When $\pi_I^A = \pi_I^M - K$ and $\bar{H}_2 < W^d - W^M$, $F_I = \frac{\pi_S^d - K}{\pi_I^M - K - \pi_I^d} \in (0,1)$ from Assumptions $A3$ and $A1$. Since F_W is strictly increasing in $\bar{H}_1$ (from Lemma 11 (i)), $F_W = 0$ if $\bar{H}_1 = -(W^M - W^m - K)$ and $F_W \geq 1$ for all $\bar{H}_1 \geq W^d - W^M$, there exists $H_{1,I}^d \in (-(W^M - W^m - K), W^d - W^M)$ such that $F_W \leq F_I$ for all $\bar{H}_1 \leq H_{1,I}^d$. Hence, all $\bar{H}_1 \leq \bar{H}_{1,I}^d$ in the set of admissible values of $\bar{H}_1$ are optimal.

Notice that the set of admissible values of $\bar{H}_1$ is such that $\bar{H}_1 \geq -(W^M - K - W^m)$, and $F_W \geq 0$ for all $\bar{H}_1 \geq -(W^M - W^m - K)$. This ensures that low-price early takeovers are authorised under the optimal policy. Moreover, note that $H_{1,I}^d$ is not necessarily positive. Indeed, $H_{1,I}^d < 0$ if $F_W > F_I$ at $\bar{H}_1 = 0$.

We reach similar conclusions when considering the case in which S makes a take-it-or-leave-it offer at $t = 1(a)$ (so that the bargaining outcomes in Lemma 13 apply). Also in this case the optimal policy regarding late takeovers is strict (the reasoning follows the same logic as in the case in which I makes the take-it-or-leave-it offers outlined above): any $\bar{H}_2 < W^d - W^M$ is optimal. Since late takeovers are blocked, the cut-off level of B is $\bar{B}_L = \pi_S^d - K + A$. The cut-off level of the prior $F(\bar{B}_L)$ that characterises the cases where the start-up offers a high or a low price is now F_S.

As in the case in which I has bargaining power, the optimal policy avoids high-price early takeovers from arising at the equilibrium. Hence, all $\bar{H}_1 \leq \bar{H}^d_{1,S}$ in the set of admissible values are optimal, where $\bar{H}^d_{1,S} \in (-(W^M - W^m - K), W^d - W^M)$ is such that, when $\pi_I^A = \pi_I^M - K$ and $\bar{H}_2 < W^d - W^M$, $F_W = F_S = \frac{\pi_S^d - \pi_I^M + \pi_I^m}{\pi_I^m - \pi_I^d}$, with $F_S \in (0,1)$ from Assumptions $A2$ and $A3$.

Optimal $\bar{H}_1$ and $\bar{H}_2$:
If I develops, $\pi_I^M - K > \pi_I^m$. Hence $F_S > F_I$ and $H^d_{1,I} < H^d_{1,S}$. A policy $\bar{H}_1 \leq \bar{H}^d_1 \equiv H^d_{1,I}$ in the set of admissible values ensures that high-price early takeovers are blocked for any value of α, and is optimal irrespective for any value of α, as stated in Proposition 5 (i.a).

We have shown above that, irrespective of who makes the offer, it is optimal to block late takeovers. Hence, when I develops, setting $\bar{H}_2 < W^d - W^M$ is optimal for any α, as stated in Proposition 5 (ii.b)

Case 2: The incumbent plans to shelve (i.e. $\pi_I^A = \pi_I^m$).

Let us start with the case in which I makes a take-it-or-leave-it offer at $t = 1(a)$ so that $\bar{B}(\bar{H}_2) = \pi_S^d - K + A = \bar{B}_L$ for all $\bar{H}_2$ (see Section C.3.2).

Lemma 12 implies that two sub-cases must be considered:

1.a If either $F(\bar{B}_L) \leq F_I$ or $F(\bar{B}_L) > \max(F_W, F_I)$, no early takeover occurs at the equilibrium. Expected welfare is $E(W) = F(\bar{B}_L)(W^d - K) + (1 - F(\bar{B}_L))W^m > W^m$ if late takeovers are blocked, and $E(W) = F(\bar{B}_L)(W^M - K) + (1 - F(\bar{B}_L))W^m > W^m$ if late takeovers are authorised.

2.a If $F(\bar{B}_L) \in (F_I, \max(F_W, F_I))]$, I offers $P_I = \pi_S^d - K$ in $t = 1(a)$ and both types $S = S_v$ and $S = S_u$ accept. Expected welfare is $E(W) = W^m$. This case arises if and only if $F_W > F_I$.

Comparing sub-cases 1.a and 2.a, we conclude that the optimal policy regarding early takeovers avoids high-price early takeovers from arising at the equilibrium, irrespective of whether late takeovers are authorised or not. This can be ensured by setting $\bar{H}_1$ such that $F_W \leq F_I$.

When $\pi_I^A = \pi_I^m$ and $\bar{H}_2 < W^d - W^M$, $F_I = \frac{\pi_S^d - K}{\pi_I^m - \pi_I^d} \in (0,1)$ from Assumptions $A1$ and $A2$. Since F_W is strictly increasing in $\bar{H}_1$, $F_W = 0$ if $\bar{H}_1 = 0$ and $F_W \geq 1$ for all the values of $\bar{H}_1 \geq W^d - W^m - K$, there exists a cut-off value $H^{s(b)}_{1,I} \in (0, W^d - W^m - K)$ such that $F_W \leq F_I$ for all the values of $\bar{H}_1 \leq H^{s(b)}_{1,I}$. The apex b in the cut-off level of $\bar{H}_1$ indicates that late takeovers are blocked.

When $\pi_I^A = \pi_I^m$ and $\bar{H}_2 \geq W^d - W^M$, $F_I = \frac{\pi_S^d - K}{\pi_I^m - (\pi_I^M - \pi_S^d)} \in (0,1)$ from Assumptions $A1$ and $K > \pi_I^M - \pi_I^m$. Since F_W is strictly increasing in $\bar{H}_1$, $F_W = 0$ if $\bar{H}_1 = 0$ and $F_W \geq 1$ for all the values of $\bar{H}_1 \geq W^M - W^m - K$, there exists a cut-off value $H^{s(a)}_{1,I} \in (0, W^M - W^m - K)$ such that $F_W \leq F_I$ for all the values of $\bar{H}_1 \leq H^{s(a)}_{1,I}$. The apex a in the cut-off level of $\bar{H}_1$ indicates that late takeovers are authorised.

Let us consider now the case in which S makes a take-it-or-leave-it offer at $t = 1(a)$ (so that the bargaining outcomes in Lemma 13 apply). The threshold F_I is substituted by F_S. More importantly, the relevant cut-off level of B depends on whether late takeovers are authorised: $\bar{B}(\bar{H}_2 \geq W^d - W^M) = \pi_I^M - \pi_I^d - K + A = \bar{B}_H > \bar{B}_L = \pi_S^d - K + A = \bar{B}(\bar{H}_2 < W^d - W^M)$ as established in Lemma 9.

Therefore,

1.b If either $F(\bar{B}(\bar{H}_2)) \leq F_S$ or $F(\bar{B}(\bar{H}_2)) > \max(F_W, F_S)$, no early takeover occurs at the equilibrium. Expected welfare is $E(W) = F(\bar{B}_L)(W^d - K) + (1 - F(\bar{B}_L))W^m > W^m$ if late takeovers are blocked, and $E(W) = F(\bar{B}_H)(W^M - K) + (1 - F(\bar{B}_H))W^m > W^m$ if late takeovers are authorised.

2.b If $F(\bar{B}(\bar{H}_2)) \in (F_S, \max(F_W, F_S)]$, both types $S = S_v$ and $S = S_u$ offer $\bar{P}$ in $t = 1(a)$ and I accepts. Expected welfare is $E(W) = W^m$. This case arises if and only if $F_W > F_S$.

Regarding early takeovers, the comparison between sub-cases 1.b and 2.b allows us to conclude that, irrespective of whether late takeovers are authorised or blocked, the optimal policy is the one that avoids high-price early takeovers from arising at the equilibrium. This can be ensured by setting $\bar{H}_1$ such that $F_W \leq F_S$.

When $\pi_I^A = \pi_I^m$ and $\bar{H}_2 < W^d - W^M$, $F_S = F_I = \frac{\pi_S^d - K}{\pi_I^m - \pi_I^d} \in (0,1)$ from Assumptions $A1$ and $A3$. As shown above, setting any value of $\bar{H}_1$ such that $\bar{H}_1 \leq \bar{H}_{1,I}^{s(b)}$ is optimal.

When $\pi_I^A = \pi_I^m$ and $\bar{H}_2 \geq W^d - W^M$, $F_S = \frac{\pi_I^M - \pi_I^d - K}{\pi_I^m - (\pi_I^m - \pi_I^d)} \in (0,1)$ from Assumptions $A1$, $A3$ and from $K > \pi_I^M - \pi_I^m$. Since F_W is strictly increasing in $\bar{H}_1$, $F_W = 0$ if $\bar{H}_1 = 0$ and $F_W \geq 1$ for all the values of $\bar{H}_1 \geq W^M - W^m - K$, there exists $H_{1,S}^{s(a)} \in (0, W^M - W^m - K)$ such that $F_W \leq F_S$ for all $\bar{H}_1 \leq H_{1,S}^{s(a)}$.

Optimal $\bar{H}_1$ and $\bar{H}_2$:

Note that the cut-off levels $\bar{H}_{1,I}^{s(b)}$, $\bar{H}_{1,I}^{s(a)}$ and $\bar{H}_{1,S}^{s(a)}$ are all positive. Hence the policy $\bar{H}_1 \leq \bar{H}_1^s = \min(\bar{H}_{1,I}^{s(b)}, \bar{H}_{1,I}^{s(a)}, \bar{H}_{1,S}^{s(a)}) > 0$ ensures that high-price early takeovers are blocked and is, therefore, optimal, irrespective of the value of α and of $\bar{H}_2$, as stated in Proposition 5 (i.b).

Let us consider now the policy regarding late takeovers. Since the optimal policy prevents high-price takeovers from arising and, because the incumbent would shelve, no takeover is always more profitable than a low-price early takeover, no early takeover occurs at the equilibrium.

When the incumbent makes the offer at $t = 1(a)$, which occurs with probability α, expected welfare is $F(\bar{B}_L)(W^d - K) + (1 - F(\bar{B}_L))W^m$ if late takeovers are blocked, and $F(\bar{B}_L)(W^M - K) + (1 - F(\bar{B}_L))W^m$ if late takeovers are authorised. Hence, authorising late takeovers causes a welfare loss equal to $F(\bar{B}_L)(W^d - W^M)$.

When the start-up makes the offer at $t = 1(a)$, which occurs with probability $1 - \alpha$, expected welfare is $F(\bar{B}_L)(W^d - K) + (1 - F(\bar{B}_L))W^m$ if late takeovers are blocked, and $F(\bar{B}_H)(W^M - K) + (1 - F(\bar{B}_H))W^m$ if late takeovers are authorised. Since $\bar{B}_H > \bar{B}_L$, authorising late takeovers is not necessarily welfare detrimental.

When condition (42) does not hold, authorising late takeovers causes a welfare loss also when the start-up makes the offer. Hence, it is optimal to block late takeovers for any α, as stated in Proposition 5 (ii.b).

When, instead, condition (42) holds, authorising late takeovers causes a welfare gain when the start-up makes the offer. In $t = 0$, the AA will authorise late takeovers if and only if the gain enjoyed when S makes the offer dominates the loss suffered when I makes the offer:

$$\Delta(\alpha) = (1-\alpha)[F(\bar{B}_H)(W^M - K - W^m) - F(\bar{B}_L)(W^d - K - W^m)] - \alpha[F(\bar{B}_L)(W^d - W^M)] > 0.$$

Since $\Delta(0) > 0$ if condition (42) is satisfied, $\Delta(1) < 0$ and $\Delta(\alpha)$ is strictly decreasing in α, there exists a threshold level of α, $\hat{\alpha} \in (0,1)$, such that $\Delta(\alpha) > 0$ if (and only if) $\alpha < \hat{\alpha}$.

To sum up, when $\pi_I^A = \pi_I^m$, condition (42) holds and $\alpha < \hat{\alpha}$, the optimal policy is to authorise late takeovers, as stated in Proposition 1 (ii.a). In all the other cases, the optimal policy is to block late takeovers, as stated in Proposition 1 (ii.b).

Recall that $B \in [0, \pi_S^d]$. Moreover, if $A = K$, $B_L = \pi_S^d$ and $B_H = \pi_I^M - \pi_I^d > \pi_S^d$. Hence, if $A = K$, $F(\bar{B}_H) = F(\bar{B}_L) = 1$, the l.h.s. of condition (42) is equal to 1, and condition (42) is not satisfied. As A decreases in $[K - (\pi_I^M - \pi_I^d - \pi_S^d), K]$, $F(\bar{B}_H) = 1$ because B_H is still higher than π_S^d, whereas $F(\bar{B}_L) < 1$ and decreases as A decreases. Hence, the l.h.s. of condition (42) increases as A decreases $[K - (\pi_I^M - \pi_I^d - \pi_S^d), K]$ and condition (42) will not be satisfied for A sufficiently close to K.

To conclude, we now derive a policy that is optimal also irrespective of whether I shelves or develops the project after the early takeover.

Optimal $\bar{H}_1$ (irrespective of shelving or developing):

All $\bar{H}_1 \leq \min(\bar{H}_1^d, \bar{H}_1^s)$ in the set of admissible values are optimal irrespective of the value of π_I^A, α and $\bar{H}_2$, as stated in Proposition 5 (i.c).

Q.E.D.