Patent privateering, litigation, and R&D incentives

Jorge Lemus and Emil Temnyalov*

August 13, 2015

Abstract

Patent Assertion Entities (PAEs) play an increasingly important role in business strategy, innovation, and litigation. Their strategic advantage in litigation comes from the ability to fend off counter-suits. We develop a model of R&D competition, bargaining and litigation to study the channels through which 'patent privateering' (whereby a PAE asserts patents bought from a producing firm) affect the incentives of operating companies to invest in R&D and enforce intellectual property. We find that PAEs increase the offensive value of patents, thus enhancing firms' incentives to invest in R&D. On the other hand, they lower these incentives by reducing the defensive value of patents, and also by decreasing total industry profits. We show that the welfare effect of PAEs on firms and consumers may be positive, even when they increase litigation threats, lower industry profits, and acquire patents only for monetization reasons.

Keywords: Patent assertion entities, privateering, counter-suits, litigation, R&D, innovation.
JEL: D2, K41, L4, L13, O3

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*Department of Economics, Northwestern University: jlemus@u.northwestern.edu and emil@u.northwestern.edu. We are grateful to Michael Whinston and William Rogerson for their guidance, support and encouragement throughout this project. We are also very thankful to Jeffrey Ely and Benjamin Jones for helpful suggestions and advice. We appreciate comments from attendees and discussants at the IIOC 2015, the Munich Conference on Innovation and Competition (MCIC 2014), EconCon 2014 at Princeton University, and seminar audiences at Analysis Group, Bates White, CRA, MIT, Northwestern, Notre Dame, and UIUC. We have also benefited from conversations with Kevin Bryan, Gonzalo Cisternas, Erik Hovenkamp, Elliot Lipnowski, Robert Porter and Aviv Nevo. This paper was previously entitled 'Outsourcing Patent Enforcement: The Effect of "Patent Privateers" on Litigation and R&D Investments.'

1 Introduction

Patent Assertion Entities (PAEs), also known as non-practicing entities (NPEs), or "patent trolls," have risen to prominence by buying up significant numbers of patents, bringing alleged infringements to court, and using the threat of litigation to extract license payments. The majority of these companies do not invest in R&D nor they use the acquired patents to make new products. The recent increase of PAEs-related activities has led to much public and academic debate on their merits and effect on innovation and litigation. The impact of PAE activities on the patent system can be gauged from the multitude of bills that have recently been passed or proposed in Congress,[1] and in President Obama's public stance on the issue.[2]

In this paper we study the practice of "patent privateering," where a PAE buys patents from producing firms and uses them to sue other producing firms, typically rivals of the original patent owner.[3] This phenomenon has recently attracted significant attention: in 2014 the Federal Trade Commission began an economic study of the business practices and impact of PAEs, while on April 5, 2013, Google, BlackBerry, EarthLink, and Red Hat sent a letter to the Federal Trade Commission and the Department of Justice asking for more scrutiny, specifically on patent privateering.[4] An extract of this letter makes clear the importance of this patent monetization strategy:

"PAEs impose tremendous costs on innovative industries. These costs are exacerbated by the evolving practice of operating companies employing PAE privateers as competitive weapons. The consequences of this marriage on innovation are alarming. Operating company transfers to PAEs create incentives that undermine patent peace. [...] We therefore urge the antitrust agencies to study carefully the issue of operating company patent transfers to PAEs."

Examples of producing firms that have sold significant numbers of patents to PAEs include Alcatel-Lucent, British Telecom, Digimarc, Ericsson, Kodak, Micron Technology, Microsoft,

[1]These include for example the SHIELD Act, the Patent Quality Improvement Act, America Invents Act, and the End Anonymous Patents Act, among others.

[2]See "Patent Assertion and U.S. Innovation," published in June 2013 by the Executive Office of the President and prepared by the CEA, NEC & OSTP.

[3]In practice PAEs vary significantly in their business strategies: see for example Risch (2012). One relevant difference is the source of the patents they own. In recent years these sources have included universities (including deals whereby a PAE buys the rights to future patents), individual inventors, companies which have at some point invested in R&D but do not (or no longer) produce commercial products using those patents, as well as actively producing firms. The latter source of patents is the most relevant for our paper.

[4]http://patentlyo.com/media/docs/2013/06/pae-0047.pdf (Visited on August 20, 2014)

Nokia, and Sony. Nokia and Sony, for example, sold some of their portfolios to MobileMedia, a PAE which subsequently sued Apple, HTC, and Research In Motion.[5]Another example is Micron Technology, a multinational corporation and one of the largest memory chip makers in the world.[6] Micron has sold at least 20% of its patent portfolio to Round Rock, in multiple transactions between 2009 and 2013. Round Rock, a PAE, asserted these patents against SanDisk.[7] Although many examples can be found in the high-tech industry, the patent privateering phenomenon is also found in other industries. For instance, in 2006, Nike sold part of its patent portfolio to a company called Cushion Technologies, LLC, which later sued several rivals of Nike in the running shoe market.

We build a theory of patent privateering to assess the effect of this practice on innovation, licensing, and litigation. Broadly speaking, firms invest in R&D to acquire patents and then bargain bilaterally with PAEs and rival operating firms over patent trade and licensing, under the threat of litigation. We incorporate key features of the patent system today, especially relevant to high tech industries where PAEs have been most active: litigation is costly and is often resolved through settlement; firms counter-sue using their patents when they are accused of infringement; PAEs cannot be counter-sued (since they do not produce); patent enforcement is noisy; and products use multiple patentable components. The latter point is fundamental for our results and it is often observed in reality. For instance, Apple holds nearly 1,300 patents protecting the iPhone, including software, hardware, and design patents.[8]

Our model endogenizes both the innovation and the litigation processes. Firms decide how much to invest in R&D in anticipation of the rewards to patenting, which can come in the form of product sales, patent trade, licensing, or litigation revenue. This is in contrast to most existing papers that study PAEs, which take R&D investments as exogenous and look at litigation and licensing incentives in a fixed patent landscape (for example, Choi and Gerlach (2013)).

Our main contribution is to identify two effects of outsourcing patent monetization to PAEs. First, since patent monetization involves transaction costs, when producing firms do not have access to PAEs the threat of countersuits and the cost of litigation dampen ex-ante innovation

[5]"Patent Privateers Sail the Legal Waters Against Apple, Google," by Susan Decker (Bloomberg; January 10, 2013).

[6]Micron has recently been named one of Thomson Reuters's top 100 global innovators.

[7]"Patent 'Troll' Tactics Spread," by Asbhy Jones (WSJ; July 8, 2012). Also, see http://www.law360.com/articles/520387/sandisk-accuses-round-rock-of-patent-antitrust-plot.

[8]Lloyd et al. (2011) shows evidence of the large amount of patents involved in the legal protection of one product.

incentives. PAEs help producing firms to overcome these transaction costs, enhancing ex-ante incentives to invest in R&D. Second, PAEs also reduce R&D incentives by decreasing the marginal value of patents that are used defensively, and by extracting rents from the market.

The first effect can be explained by understanding patent enforcement without PAEs. In their absence, competitors with similarly-sized patent portfolios will often engage in a tacit "IP truce," whereby neither firm is willing to sue its rivals for infringing their patents, as the rivals' portfolios act as a deterrent. Since going to court is costly for both parties, even when one firm has more patents than its rival, the net benefit from enforcing them may be less than the expected cost of a potential counter-suit plus the legal fees. This "mutually assured destruction" scenario implies that some of the value of a large patent portfolio is lost.[9] Since PAEs cannot be counter sued, their litigation threats are stronger than those of an operating firm, conditional on having the same patents. Thus, by enforcing patents, PAEs can extract higher licensing payments compared to a producing firm. However, PAEs change the bargaining position of the firms.By selling patents to the PAE, a producing firm has fewer patents to use in a counter-suit, so it is more vulnerable to lawsuits. We show that, overall, the interaction with PAEs always benefits the firm with more patents and harms the firm with fewer. In fact, when the PAE increases patent monetization there are two effects: 1) the firm with the smaller portfolio loses more compared to the tacit "IP truce" equilibrium in the absence of PAEs; and 2) the firm with the larger portfolio can capture some of the extra surplus generated by the PAE, determined by its bargaining power, while the rest goes to the PAE as rents. Notice that both of these effects push the incentives for patenting in the same direction: they both make it more profitable to be the firm with a larger portfolio. Thus, PAEs help overcome transaction costs generated by the thread of countersuits and the legal costs, which can lead to larger ex-ante incentives to invest in R&D.[10]

The second effect of PAEs on R&D incentives comes via two channels: reduction of the marginal value of *defensive portfolios* and rent extraction. A firm's portfolio is defensive when it is not large enough to profitably start a lawsuit, even if its rival will not counter sue. When PAEs monetize patents they eliminate the value of defensive the portfolios (because PAE cannot be counter sued), which lowers ex-ante incentives to invest in R&D. Also, if firms are unable to sign licensing agreements before bilaterally trading patents with PAEs, some

[9]Counter-suing plays a crucial role in litigation strategy when firms can use their patents as defensive weapons. Some salient examples are Apple vs HTC, where HTC counter sued with 2 patents, or Yahoo vs Facebook, where Facebook counter sued with 10 patents.

[10]In fact, we show that when firms enter the market with patent portfolios of similar size, the firms payoff under the presence of PAEs are equivalent to those in an economy with no legal costs.

rents will be extracted by PAEs, lowering total industry profits. These two effects reduce the incentives to invest in R&D.

In general, whether PAEs increase or decrease innovation activity depends on which of these effects dominates. We show that, under fairly general conditions, the effect of PAEs on innovation is to increase equilibrium R&D investments of the firms, even though they lower the total surplus of producing firms.

In our model, the social benefit of R&D is to reduce the delay of the introduction of the final product in the market. Firms' R&D investments determine the random arrival time of the invention of each component of the final product. These stochastic arrivals determine two elements: the expected time to discover all technologies that are necessary for production, and the patent portfolio of each firm. A firm that invests in R&D more than its rival is more likely to discover and patent more components of the final product. A larger R&D investment also speeds up innovation, which implies that both firms and consumers can capture the rewards from commercialization sooner. We characterize conditions under which the firms under-invest and over-invest in equilibrium (in the absence of PAEs) relative to the social planner's first- and second-best outcome. The R&D equilibrium features under-investment when firms are impatient, when consumer surplus in the final product market is large, when the product is more complex (i.e. involves more pieces of technology), or when patent protection is weak. Overall, for a broad range of parameter values, PAEs may enhance welfare even though they extract rents.

The paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we introduce a model of R&D, licensing, and litigation. In Sections 4 and 5, we solve the licensing and litigation game and present our main results. In Section 6, we characterize the endogenous R&D investment equilibrium. In Section 7, we discuss the welfare implications of our results. In Section 8, we discuss some extensions of our model, which also serve as robustness checks for our main results. Finally, in Section 9, we summarize our findings and discuss some policy implications.

2 Literature review

This paper relates most closely to the growing theoretical literature on the effect of patent assertion entities on R&D and litigation. Choi and Gerlach (2013) study the aggregation of large patent portfolios affect patent litigation and product development. They consider an exogenous landscape of existing patents and analyze how the strength of firms' portfolios determines licensing and settlement terms, depending on whether they are producing or non-producing entities. They also consider how litigation in the presence of large portfolios affects the development of new products, and also study the incentives of producing firms and PAEs to acquire patents from a third party. Unlike Choi and Gerlach (2013), we study an endogenous patent landscape where firms obtain patents as a result of investments in R&D, and then engage in trade, licensing and litigation. Thus, we can discuss how the effect of PAEs on licensing and litigation affects the incentives of firms to produce patents.

PAE business strategies vary significantly and different companies have found different ways of monetizing patent portfolios. There is no "one-size-fits-all" patent monetization strategy. Lemley and Melamed (2013) discuss different strategies for patent assertion by practicing and non-practicing firms. In a related paper, Scott Morton and Shapiro (2014) study different strategies employed by PAEs to monetize patents. They provide a reduced form model of PAE intermediation between an individual inventor and an operating company. In their baseline case, the individual inventor cannot monetize its patent and the operating company infringes on the patent to produce the final product. The PAE acquires the patent from the individual inventor and has the ability to enforce it. They show that if the PAE does not transfer enough rents to the original inventor, PAEs will have a negative impact on welfare. Our paper instead considers a richer model of R&D investment and licensing, where PAEs can buy patents from producing firms. Cohen et al. (2014) present a model of PAE formation. In their model, initially two firms try to enter the market and each firm owns one invention of exogenously given quality. If one firm has very low quality, it will stay out of the market and it will act as a PAE. In our model, we endogenize the strenght of the patent portfolio, and also allow for entry decisions. Cosandier et al. (2014) study defensive patent acquisition services, a strategy that is utilized by, for example, RPX Corporation.

There is a sparse but growing empirical literature on the business practices and impact of PAEs, which is mainly limited by the lack of extensive data. Khan (2005) shows that the commercialization of patents is not a phenomenon particularly tied to high technology products. Companies whose sole business is the monetization of patents have existed for a long time and were called "patent sharks" rather than "patent trolls" in the past. In recent years the proliferation of companies focused on the assertion, rather than the commercialization, of patents has opened an important debate. Chien (2010) studies this proliferation of PAEs and the rise of strategic management of patents. In particular, she emphasizes the importance of holding large portfolios to sustain "patent peace" among operating companies through the threat of counter-suing.

The majority of the empirical studies have been restricted to a small number firms for which data is available, and some arguments against PAEs have been based on anecdotal evidence or isolated cases. Risch (2012) and Fischer and Henkel (2012) have tried to shed light on the practices of PAEs by analyzing the patent portfolios of a sample of firms. An important finding of these papers is that in their sample PAEs acquire patents of relatively good quality (in terms of validity), which goes against the commonly held belief that PAEs try to enforce low quality patents. Shrestha (2010) finds similar results when comparing the forward citations of patents acquired by PAEs versus those acquired by operating companies. An important empirical finding is that PAEs do not acquire all their patents from individual inventors. Fischer and Henkel (2012) find that about 65% of the patents acquired by PAEs came from operating companies with more than 100 employees.

Bessen et al. (2011) estimate the cost imposed by PAEs on operating companies. Analyzing stock market events around NPE lawsuit filings, they find a loss of about half a trillion dollars to defendants over the period 1990-2010. Bessen and Meurer (2014) estimate that the direct costs of PAE assertions (not including diversion of resources, delays in new products, and loss of market share) was about $29 billion in 2011. These studies, highly cited in the media for the large amount of rents extracted by PAEs, are not without caveats and critiques to their methodology. Risch (2014) and Cotropia et al. (2014), for example, question the sensitivity of these results to definition used for PAEs.[11] Schwartz and Kesan (2014) claim that the findings in Bessen and Meurer (2014) are based on a biased sample, and that majority of the $29 billion correspond to settlements and licensing, which are transfers and not costs.

Our model of R&D contributes to the literature on patent races and contests. Standard models study contests for a single prize or patent, including the literature on patent races started by Loury (1979) and Lee and Wilde (1980), and the literature on rent-seeking contests– for example, the survey in Corchón (2007). We develop a novel model of "contests for bundles," where firms compete to discover and patent multiple complementary technologies. This relates to the models of Fu and Lu (2012) and Clark and Riis (1998). However, our setting differs from those papers in that firms care (non-linearly) about the bundle of patents they obtain. To our knowledge this area of research is still fairly undeveloped.

[11]In particular, Cotropia et al. (2014) provides a finer classification of PAEs by different types (universities, individual inventor, IP holding companies, etc.) for all patent litigation cases in the years 2010 and 2012.

3 Model

Two firms (A and B) race to discover N pieces of technology, which we call *components*, in order to produce and sell a final product that incorporates all of them. The timing of the model, depicted in Figure 1, is as follows: first, firms invest in R&D and patent their discoveries; second, observing the realization of patent portfolios after the R&D stage, firms have the option to buy or sell patents; third, firms decide whether to enter the final product market; fourth, once firms have entered the market, PAEs may acquire patents; fifth, patent owners and producing firms engage in patent licensing in the shadow of litigation; sixth, if a firm has entered the product market without patents or licenses on all N components, it faces the risk of being sued patent infringement.

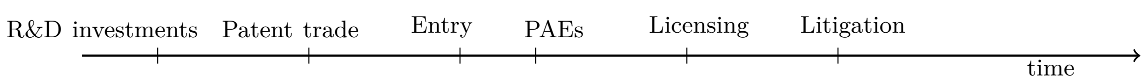

Figure 1: Timing of the events in the model.

In the first stage firms simultaneously make sunk R&D investments to discover the N components, and the discovery of a particular component arrives stochastically, given the R&D investments. We assume as in Loury (1979) that R&D investment is a one-time fixed investment, rather than a flow investment that can be revised upon the realization of uncertainty as in Lee and Wilde (1980). The cost of investing z units of R&D for the firms is $c_I(z)$, where $c_I(\cdot)$ is increasing, convex, differentiable, and $c_I(0) = 0$. Fixing the firm's R&D investments x and y, respectively for Firm A and B, Firm A is the first to discover any one particular component independently with probability $p(x, y) = \frac{h(x)}{h(x)+h(y)}$, where $h(\cdot)$ is increasing, concave, differentiable, and $h(0) = 0$. This probability is derived from independent exponential arrivals. If, for a given level of R&D z, each component $i \in \{1, .., N\}$ arrives independently at time $\tau_i(z) \sim \exp(h(z))$, then firm A discovers a component first if $\tau_i(x) < \tau_i(y)$ which occurs with probability $\frac{h(x)}{h(x)+h(y)}$. Hence, the number of patented components for a particular firm follows a binomial distribution, and the probability that Firm A discovers exactly k out of the N components is given by

$$P(k; x, y) = \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k}.$$

Discoveries are publicly observable, and the firm which discovers a component immediately and costlessly obtains a patent on it.[12] At the end of the R&D stage the patent portfolio of each firm is fixed. The expected time to complete the R&D stage is endogenously determined by the level of investment of the firms. The time at which a particular component $i \in \{1, ..., N\}$ is discovered is given by $\hat{\tau}_i(x, y) = \min\{\tau_i(x), \tau_i(y)\}$. Production can take place only when *every* component has been discovered by some firm, since firms require every component to produce. The time at which firms will enter the market and produce is therefore given by $\tau(x, y) = \max_{i=1,...,N}\{\hat{\tau}_i(x, y)\}$, which is distributed according to $F(\tau; x, y)$.

Once patent portfolios are determined, in stage two, firms can engage in patent trade. We assume that the original inventor of a patent always retains a license for his invention, even after assigning the patent to a new firm. In consequence, the original assignor of a patent cannot infringe on that patent, even after it no longer owns it.

Given the patent portfolios after the patent trade, in stage 3, firms simultaneously decide whether to enter the final product market. Entry is not blocked by the lack of patents or licenses for some components, since firms can freely and immediately imitate any component discovered by any other firm. Industry profits (before accounting for license and litigation costs) are modeled in reduced form: if both firms enter the market, each one of them makes profit $\pi > 0$ by selling the final product.[13] If only one firm enters, it monopolizes the market and obtains π_m in the final product market.

If PAEs are present, in stage 4, firms can decide to trade patents with a PAE after entry and before they license with their rivals. Next, in stage 5, firms engage in patent licensing with any patent owner (including, possibly, PAEs), and licenses are determined under the threat of litigation. We assume that license prices are set through Nash bargaining over the surplus that is generated by a licensing deal, relative to the firms' outside option of not licensing and potentially going to court.[14]

Finally, in stage 6, if a firm has entered the product market and does not have a license or patent protection for some of the N components, it may be sued for infringement. The court will decide whether unlicensed components infringe on the final product. We assume that patents are valid ("strong"), but probabilistic: a firm's product infringes on each patent with probability $\beta > 0$, which is independent across patents. We further assume that going to

[12] For simplicity, we assume away the possibility of trade secrets or strategic delay in patenting.

[13] Notice that firms have the same market size, despite having potentially asymmetric patent portfolios.

[14] See Spier (2007) for a discussion of the role of bargaining power in settlement outcomes. In a separate paper, Lemus and Temnyalov (2013), we study how bargaining power and injunctions affect the incentives for litigation in the presence and absence of a PAE.

court is costly: each side must pay $c > 0$ in legal fees per lawsuit, and that the defendant may bring counter-claims (defensive countersuing) to the court at no additional cost.[15] If the court determines a firm is infringing, that firm must pay to the patent-owner a per-patent infringement fee $R > 0$.[16]

In the next section we solve the model by backward induction. We first derive the continuation payoffs from the last stage in an economy without PAEs, and then we re-derive the continuation payoffs in an economy with a PAE. We then examine the equilibrium of the R&D contest when firms anticipate these continuation payoffs.

4 Patent trade, entry, licensing and litigation without PAEs

We solve the game by backward induction, assuming first that PAEs do not exist. We start by solving the licensing and litigation stages, and we move backwards to entry and patent trade. We then solve the game incorporating PAEs and study their effect on continuation payoffs.

4.1 Licensing & litigation

We analyze the licensing and litigation stages, for any given distribution of patents, after one or both firms has entered the final product market.

Licensing and litigation payoffs when both firms enter

Consider the situation in which firms A and B sell the final product, which incorporates the N components previously developed in the research stage. After the resolution of uncertainty, Firm A patented n components, while Firm B patented m, with $n + m = N$. Notice that $0 < n < N$ implies that both firms have entered the product market with incomplete patent protection. When Firm A sues Firm B using its complete patent portfolio, given that each infringement claim is evaluated independently, the probability that Firm B's product infringes

[15] We assume that c is independent of the number of patents being litigated, which is relaxed in section 8.

[16] Because we model industry profits in reduced form, we also focus on per-patent royalties, rather than per-sale royalties. This avoids the complication of how royalties themselves affect pricing, which is not central to this paper.

on exactly k out of the n patents owned by Firm A is given by

$$\binom{n}{k}\beta^k(1-\beta)^{n-k},$$

and the expected payment in royalties received by A is given by

$$\sum_{k=0}^{n}\binom{n}{k}\beta^k(1-\beta)^{n-k}(Rk) = Rn\beta.$$

Thus, in our model, the expected benefit of going to court is proportional to the number of patents asserted. Since countersuing is free, Firm B will counter-sue using its entire portfolio to obtain expected royalties of $Rm\beta$. Thus, Firm A's expected payoff from going to litigation is $R\beta(n-m)-c$, and Firm B's is $R\beta(m-n)-c$. In this situation, should licensing negotiations fail, Firm A is willing to initiate litigation if and only if $R\beta(n-m) > c$, while Firm B is willing to initiate litigation if and only if $R\beta(m-n) > c$ (i.e., these are the cases where one side has a positive-expected-value suit, as discussed for example in Shavell (1982) and Nalebuff (1987)).

If one firm has a credible threat of litigation, firms will bargain to avoid losing a total of $2c$ in joint surplus due to litigation costs. For simplicity, we assume equal bargaining power in the Nash bargaining solution at this stage. Defining $V \equiv R\beta$, $\hat{c} \equiv \left[\frac{c}{V}\right]$, and the function

$$T(n,m) = \begin{cases} V\cdot(n-m) & \text{if } |n-m| > \hat{c} \\ 0 & \text{if } |n-m| \leq \hat{c} \end{cases},$$

we have three relevant cases to analyze:

1. If Firm A has a credible litigation threat: $n-m > \hat{c}$, firms cross license and Firm A receives the transfer $T(n,m) = V\cdot(n-m) - c + \frac{1}{2}(2c) = V\cdot(n-m)$.
2. If Firm B has a credible litigation threat: $m-n > \hat{c}$, firms cross license and Firm B receives the transfer $T(m,n) = V\cdot(m-n) - c + \frac{1}{2}(2c) = V\cdot(m-n)$.
3. If no firm has a credible litigation threat: $-\hat{c} \leq n-m \leq \hat{c}$, and $T(n,m) = 0$.

This means that for fixed portfolio sizes (n,m) either one of the following two cases occur: one of the firms has a relatively large enough portfolio, so the litigation threat is credible and that firm receives a payment from cross-licensing portfolios; or firms have portfolios of similar sizes, so litigation is not credible and firms produce while tacitly agreeing not to sue. The figure below depicts the function $T(n,m)$ for all possible combinations of patent portfolios.

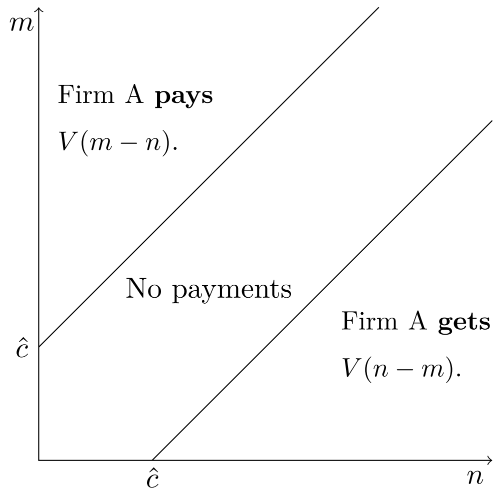

Figure 2: Licensing tranfers are shown for different portfolio configurations. Cross-licensing agreements featuring positive transfers occur only when $|n-m| > \hat{c}$.

We denote by $U^A_{E,E}(n,m)$ the payoff of Firm A after both firms entered the product market ("E" stands for entry) and bargained over their patent portfolios, whose sizes are n and m for Firm A and Firm B, respectively. Then, by definition, $U^A_{E,E}(n,m) = \pi + T(n,m)$, and by symmetry, $U^B_{E,E}(m,n) = U^A_{E,E}(n,m)$.

Licensing and litigation when only one firm enters

Consider now the case in which only one firm enters the product market, say Firm A, and A has n patents, and B has m patents. Because B is not producing anything, Firm A can't use its portfolio to sue Firm B, while Firm B's monetizes its patents as long as it is profitable to do so, which is the case when $m > \hat{c}$. In this case, the negotiated license fees are given by $T(m,0) = Vm$ and Firm B actually operates as a PAE (although it invested in R&D).

If $m \leq \hat{c}$, Firm B cannot monetize its portfolio and Firm A produces as a monopolist without any credible threat of litigation. The total payoffs when only Firm A enters the product market, denoted by $U^A_{E,NE}(n,m)$ and $U^B_{E,NE}(n,m)$, are therefore given by

$$U^A_{E,NE}(n,m) = \begin{cases} \pi_m & \text{if } m \leq \hat{c}, \\ \pi_m - Vm & \text{if } m > \hat{c} \end{cases}, \quad U^B_{E,NE}(n,m) = \begin{cases} 0 & \text{if } m \leq \hat{c}, \\ Vm & \text{if } m > \hat{c} \end{cases}.$$

4.2 Entry decisions

We now analyze the optimal product market entry strategies, given the continuation payoffs described above, for a fixed patent portfolio, where Firm A has n patents and Firm B has m patents, and $n + m = N$. Since the problem is symmetric for both firms, we focus on Firm B's optimal entry decision, taking the decision of Firm A as given. We assume that duopoly profits are larger than the cost of litigation, i.e. that $\pi > c$, and that monopoly profits are larger than twice duopoly profits, i.e. that $\pi_m > 2\pi$.

Lemma 1. *When $\pi > NV$ it is a dominant strategy for each firm to enter the final product market.*

The interesting case for patent privateering is precisely when both firms enter the market and there are credible litigation threats for some patent portfolio configurations. For this reason, we assume $\pi > NV$ and $N > \hat{c}$ for the remainder of the paper.

4.3 Patent trade

We now analyze the possibility of patent trade among firms A and B after the R&D stage is over. Recall that an original patentee always retains a license, even after selling the patent to another party. That is, the first firm to discover a component will always have protection over it. We assume that firms cannot sign a contracts to monopolize the market, even when this is be profitable to do. In other words, we assume that the antitrust authorities will be vigilant and prevent firms from signing these type of contracts.

Under these conditions firms are indifferent about trading patents given that they can license in the following stage. Since the continuation game is efficient conditional on both firms entering the market, there are no gains from patent trade before entry.

4.4 The return to R&D without PAEs

To summarize, in the absence of PAEs, firms do not trade patents, they both enter, and they reach a licensing agreement prior to litigation. When Firm A has discovered and patented n components, and Firm B has discovered the remaining m components, the continuation payoffs are $U^A_{E,E}(n, m) = \pi + T(n, m)$ for Firm A, and $U^B_{E,E}(n, m) = \pi - T(n, m)$ for Firm B. Since $n+m = N$ and N is fixed, we can write the payoff of a firm that enters the market with

n patents as $U(n) = \pi + T(n, N-n)$. Notice that $|n-m| > \hat{c}$ is equivalent to $|2n-N| > \hat{c}$, and in that case $T(n, N-n) = (2n-N)V$. Firm A's continuation payoff as a function of n is depicted in Figure 3.

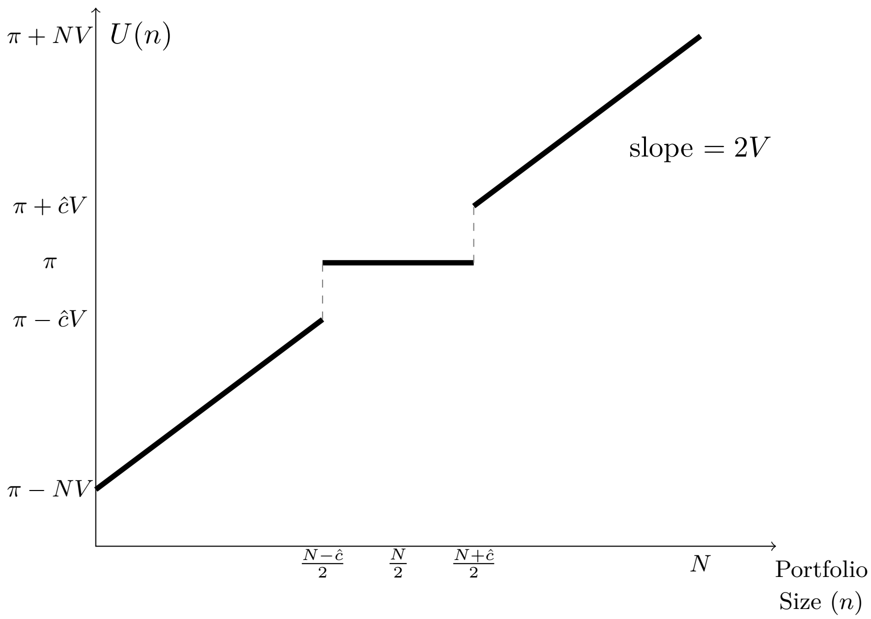

Figure 3: Continuation payoff without PAEs after the R&D stage for a firm that discovered n components, while its rival discovered $N-n$.

When the difference in the size of the patent portfolios is large, the marginal value of a patent is $2V$. If a firm has less than $\frac{N-\hat{c}}{2}$ patents, the marginal value derives from two sources: the defensive value of a patent (used in a countersuit, the firm expect to get V); and its appropriation value (since the total number of components is fixed, the rival firm has one less patent to use offensively, which saves V in licenses). Similarly, if a firm has more than $\frac{N+\hat{c}}{2}$ patents, the marginal value derives from two sources: the offensive value of a patent (one more patent in the lawsuit increases the expected payment by V); and its appropriation value (the rival firm has one less patent to use defensively, which saves V in counter-suit payments).

When firms have patent portfolios of similar size, that is $\frac{N-\hat{c}}{2} \leq n < \frac{N+\hat{c}}{2}$, the marginal value of patents is zero. Having one more patent does not make a difference, since transaction costs imply that the "mutually assured destruction" scenario would still prevail.

5 Patent trade, entry, licensing and litigation with PAEs

In this section, we introduce a PAE into the model, study its effects on licensing and derive the returns to R&D with a PAE. The PAE is strategically different from producing firms- it cannot be sued for patent infringement because it does not make or sell products in the market. Therefore, producing firms have no tools to defend themselves against PAE's lawsuits. The only risk that PAEs face in litigation is the randomness of court decisions. In our model the PAE begins the game with no patents, since it does not invest in R&D. The only way for the PAE to acquire patents is to buy them from firms that invested in R&D, once the research stage is over, and after the entry decisions have been made. When the PAE acquires n' patents from a producing firm, that firm is granted a license for the patents it sold, so the PAE cannot sue the original inventor. A producing firm is more vulnerable to lawsuits after selling some of its patent portfolio, since those patents can no longer be used defensively; the upside of selling is the additional revenue from the price that the PAE would be willing to pay for the patents. Patents can be used offensively and without the countersuing threat by PAEs, so they have a higher monetization value for PAEs compared to producing firms.

For modeling purposes, our model has only one PAE that will bilaterally and simultaneously bargain with both firms for the acquisition of patents. In this negotiation, we allow for arbitrary bargaining power for the PAE. Also, we assume that the PAE cannot commit to negotiate with one firm only. Under these assumptions, our results are almost identical to a model with multiple competing PAEs offering contracts (under passive beliefs) to firms to acquire patents their patents.

5.1 Licensing and litigation

Suppose a PAE has acquired $n' > \hat{c}$ patents from Firm A and is planning to sue Firm B. The expected payoff from litigation for the PAE is $V \cdot n' - c$, and for Firm B is $-V \cdot n' - c$. We again assume symmetric bargaining power in the negotiation of licenses between the PAE and a producing firm. When the litigation threat is credible ($n' > \hat{c}$), firms will bargain over the surplus gained by avoiding litigation. To avoid litigation, Firm B is willing to pay the PAE[17]

$$T(n', 0) = V \cdot n' - c + \frac{1}{2}(2c) = Vn'.$$

We adopt the following notation: n are the number of patents originally invented by Firm A, n' are the number of patents sold by Firm A to the PAE, m are the number of patents

[17] The PAE earns its payoff in the event negotiations fail ($Vn' - c$) plus half of the gains from trade $\frac{1}{2} \cdot (2c)$.

originally invented by Firm B and m' are the number of patents sold by Firm B to the PAE. The values n' and m' will be endogenously determined in equilibrium, which is fully characterized in section 5.3.

Licensing and litigation payoffs when both firms enter

Consider a fixed allocation of patents (n, m, n', m') after both firms have entered the final product market and traded patents with the PAE. We denote by $p_A(n')$ the price paid by the PAE for the n' patents acquired from Firm A, and $p_B(m')$ the price paid by the PAE for the m' patents acquired from B.[18] The payoffs from licensing and litigation are:

$$\pi_A = \pi + T(n-n', m-m') - T(m', 0) + p_A(n'),\ \pi_B = \pi - T(n-n', m-m') - T(n', 0) + p_B(m'),$$

$$\pi_{PAE} = T(n', 0) + T(m', 0) - p_A(n') - p_B(m').$$

After selling to the PAE Firm A owns $n - n'$ patents and Firm B $m - m'$. The transfer $T(n - n', m - m')$ reflects the license payment among producing firms. $T(n', 0)$ is what the PAE gets from Firm B by asserting the patents acquired from A against B, and $T(m', 0)$ is what the PAE gets from Firm A asserting Firm B's patents.

Licensing and litigation when only one firm enters

Suppose that only Firm A enters the final product market. In this case, Firm B effectively acts as a PAE when monetizing its patents. The payoffs from licensing and litigation in this case are:

$$\pi_A = \pi_m - T(m, 0) - T(m', 0) + p_A(n'),\ \pi_B = T(m, 0) + p_B(m'),\ \pi_{PAE} = T(m', 0) - p_A(n') - p_B(m').$$

5.2 Entry decisions

Analogously to section 4.2, we focus on the case where both firms enter the final product market, because this is the only interesting case in which PAEs affect litigation and R&D incentives. Specifically, as in Lemma 1, we impose the conditions $\pi > NV$ and $N > \hat{c}$, which guarantees that entry is a dominant strategy for both firms across all possible patent allocations, and there is scope for credible litigation threats.

[18] Rigorously, we should write $p_A(n, m, m', n', s)$. We omit the rest of the arguments for the sake of exposition.

In the next section, we solve find the equilibrium values of n' and m', which are the solution to the bilateral bargaining game between the operating firms and the PAE.

5.3 Patent acquisition by the PAE

We adopt the simultaneous and symmetric Nash bargaining approach of Horn and Wolinsky (1988). The PAE simultaneously bargains with firms A and B over the outcomes. An outcome corresponds to the set of patents acquired by the PAE from the producing firms and the prices at which they were bought. Let $S_{PAE}(n', m')$, $S_A(n', m')$, and $S_B(n', m')$ be the payoffs from licensing in the shadow of litigation for the PAE, Firm A, and Firm B, respectively, after the PAE acquires n' patents from A and m' from B, at prices $p_A(n')$ and $p_B(m')$, respectively. The result of each negotiation is the solution to Nash Bargaining, given the deal reached between the PAE and the other producing firm. We allow for arbitrary bargaining power $s \in [0, 1]$ for producing firms, when they negotiate patent sales with the PAE.

For a given initial patent allocation (n, m), with $n + m = N$, we find the equilibrium of this bargaining game between the producing firms and the PAE, and we focus on the case $n \geq m$ (the other case is symmetric).

Formally, the equilibrium in the bargaining game is one in which, taking m' as given, Firm A and the PAE bargain over their outcome à la Nash

$$(n', p_A) \in \max_{(z,p)} (S_{PAE}(z, m') - p - S_{PAE}(0, m'))^{1-s}(S_A(z, m') + p - S_A(0, m'))^s, \tag{1}$$

and taking n' as given, Firm B and the PAE bargain over their outcome

$$(m', p_B) \in \max_{(z,p)} (S_{PAE}(n', z) - p - S_{PAE}(n', 0))^{1-s}(S_B(n', z) + p - S_B(n', 0))^s. \tag{2}$$

We denote by $J_{A,PAE}(z, m')$ the joint surplus of Firm A and the PAE from licensing under the threat of litigation, when Firm A transfers z patents to the PAE and Firm B has sold m' patents to the PAE. We define $J_{B,PAE}(z, m')$ analogously for Firm B and the PAE. A standard result in bargaining games with lump sum payments is the following:

Lemma 2. *The outcome of the bilateral negotiation between an operating Firm And the PAE maximizes their joint surplus, for a fixed deal between the rival Firm And the PAE.*

Bilaterally, an operating Firm And the PAE trade patents to maximize their joint surplus. Once the allocation of patents maximizes the joint surplus, a monetary transfer splits the surplus between the parties according to their bargain power. Thus, to find the number of patents traded between producing firms and the PAE, we need to examine the allocations that simultaneously maximize the joint surplus of each pair.

Proposition 1. *It is an equilibrium for each firm to sell its whole portfolio to the PAE in the bargaining game described above. In that equilibrium, the PAE extracts no rents from the producing firms. The equilibrium payoffs are*

$$\pi_A = \pi + T(n,0) - T(m,0), \quad \pi_B = \pi - T(n,0) + T(m,0), \quad \pi_{PAE} = 0. \tag{3}$$

Proposition 1 shows one equilibrium outcome of the bargaining game in which both producing firms are selling their entire portfolio to the PAE. In this equilibrium, the PAE will not extract rents from the producing firms, despite the fact that it allows for more patent monetization. The reason for behind this result is that when Firm B sells its entire portfolio to the PAE, it loses the ability to use patents defensively in a countersuit. This implies that the maximum surplus that Firm A and the PAE can achieve jointly equals what Firm A can achieve on its own. The PAE does not offer any strategic advantage to Firm A once its rival has sold everything to the PAE. Therefore, it does not increase their joint surplus.

Depending on the size of the patent portfolios after the R&D stage, the bargaining game may have multiple equilibria. The following lemmas are used to characterize all the equilibria in this game. Lemma 3 examines the case in which Firm B has enough patents to be monetized by the PAE, while Lemma 4 examines the case in which Firm B's portfolio cannot be monetized by the PAE.

Lemma 3. *Suppose $n > m > \hat{c}$. In any equilibrium of the bargaining game Firm B sells all of its patents to the PAE.*

Intuitively, when Firm B holds on to some patents, Firm A and the PAE have a strategy that prevents Firm B from monetizing those patents. When Firm A and the PAE are playing this strategy, Firm B and the PAE are losing value on the patents held by Firm B, since the PAE could monetize them. Thus, when $n > m > \hat{c}$ any equilibrium must have Firm B selling everything to the PAE.

Lemma 4. *When $m \leq \hat{c}$, in every equilibrium of the bargaining game Firm A sells all of its patents to the PAE.*

If Firm B does not have enough patents to be monetized by the PAE, Firm A is "safe" by selling all of its patents to the PAE. Even better, by selling, Firm A avoids counterclaims that would be brought by Firm B, had Firm A sued directly.

Lemmas 3 and 4 characterize the unique equilibrium behavior of one of the firms in the game. The multiplicity arises from the different strategies that the other firm can play. In Proposition 2, we characterize the payoffs for all the equilibria of the game.[19]

Proposition 2. *The equilibrium payoffs of the game are:*

1. *When $n = m$ there are multiple equilibria, and in all these equilibria the payoffs are:*

$$\pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0.$$

2. *When $n > m > \hat{c}$, Firm B sells its entire portfolio. There are multiple equilibria, but any equilibrium has the following payoffs:*

$$\pi_A = \pi + V(n-m), \quad \pi_B = \pi - V(n-m), \quad \pi_{PAE} = 0.$$

3. *When $m \leq \hat{c}$ Firm A sells its entire portfolio to the PAE and Firm B is indifferent between selling any amount $m' \in [0, m]$. The equilibrium payoffs depend on how many patents Firm B is selling, as they change the outside option in the bilateral bargaining of Firm A and the PAE.*

$$\pi_A = \pi + T(n, m-m') + s[Vn - T(n, m-m')], \quad \pi_B = \pi - Vn, \quad \pi_{PAE} = (1-s)[Vn - T(n, m-m')].$$

Figure 4 summarizes the effect of the PAE on the equilibrium continuation payoffs. From the figure it is easy to see that the PAE affects the firms' continuation payoffs only in three ways: *monetization* (M), *shielding* (S), and *monetization and shielding* (MS). To explain the reason for the changes we focus on the case $n > m$.

[19] The equilibrium allocations are described in the Appendix, in the Proof of Proposition 2.

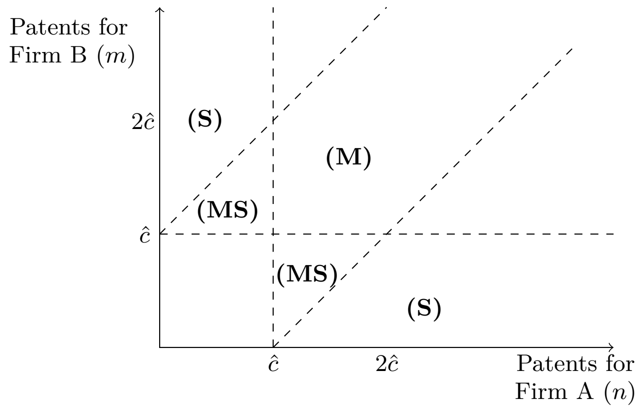

Figure 4: Changes produced by the PAE on firms' continuation payoffs for different patent portfolio configurations. The changes in payoffs occur only in three regions.

1. **Region (M):** When $|n-m| \leq \hat{c}$, $n > m > \hat{c}$, without PAEs firms did not have a credible litigation threats. However, when the PAE acquires all the patents it has two individually rational lawsuits $n > \hat{c}$ and $m > \hat{c}$, against Firm A and B, respectively. The effect of the PAE on this region is to allow for patent monetization, generating a positive total surplus of $V(n-m)$, which is captured by the firm with the largest portfolio. Thus, comparing to the case of no PAEs, the PAE increases Firm A's continuation payoff by $V(n-m)$, which equals the decrease in the continuation payoff for Firm B. The PAE is not able to extract surplus in this case, because in equilibrium the "weak" player (the firm with fewer patents) sells everything to the PAE. This implies that Firm A's outside option is to monetize all its patents without a threat of countersuing, which is the same benefit that the PAE can offer.

2. **Region (S):** When $|n-m| > \hat{c}$, $m \leq \hat{c}$, and although Firm A had a credible litigation without the PAE, the PAE can offer to "cancel out" Firm B's portfolio, which increases Firm A's surplus and the PAE is able to extract rents. The effect of the PAE is to shield firm A from countersuits. Equilibrium payoffs increase for Firm A's from $V(n-m)$ to $V(n-m) + sVm + (1-s)Vm'$, and they decrease for Firm B by Vm.

3. **Region (MS):** When $|n-m| \leq \hat{c}$, $m \leq \hat{c}$, and the PAE did not exist, firms were not suing each other. Even when $n > \hat{c}$, Firm A did not have the ability to monetize its patents, because of the fear of retaliation from Firm B. If the PAE exists, it allows Firm A to monetize its patents (when $n > \hat{c}$) and to "canceling out" Firm B's portfolio. This strategic advantage offered by the PAE comes from its ability to avoid countersuing. Thus, the PAE allows for patent monetization and shields the producing firm with the

largest portfolio from countersuits.[20] The effect of the PAE on equilibrium payoffs is to increase Firm A's payoff from 0 to $sVn + (1 - s)T(n, m - m')$, and decrease Firm B's payoffs by Vn. ling out" Firm B's portfolio. This strategic advantage offered by the PAE comes from its ability to avoid countersuing. Thus, the PAE allows for patent monetization and shields the producing firm with the largest portfolio from countersuits.[21] The effect of the PAE on equilibrium payoffs is to increase Firm A's payoff from 0 to $sVn + (1 - s)T(n, m - m')$, and decrease Firm B's payoffs by Vn.

4. **Other regions:** For the rest of the cases, either firms A and B were already monetizing all their patents and shielding is not possible, or no firm had enough patents to start a lawsuit. Hence, in all the other regions, the PAE does not change the continuation payoffs in any equilibrium.

5.4 The returns to R&D with PAEs

To summarize, when firms can sell patents to a PAE, the subgame perfect equilibrium of this game has both firms entering the final product market, trading with the PAE, and licensing is determined by bilateral bargaining between all parties. Proposition 2 characterizes the payoffs of both firms as a function of the number of components they patent.

Consider the case where $N > 3\hat{c}$ and the equilibrium where any firm which discovers fewer than $\hat{c}$ components retains all of its patents. Note that, as discussed in Proposition 2, this is the equilibrium with the largest payoff for the PAE, i.e. where the PAE extracts the most rents and features the lowest total industry profits among the producing firms. The payoff of a firm that enters with n patents is depicted in Figure 5.

[20] Notice that although Firm B cannot monetize its patents, its decision of how many patents to keep has an impact on the equilibrium payoffs, as they determine the outside option of Firm A in the bilateral bargain with the PAE.

[21] Notice that although Firm B cannot monetize its patents, its decision of how many patents to keep has an impact on the equilibrium payoffs, as they determine the outside option of Firm A in the bilateral bargain with the PAE.

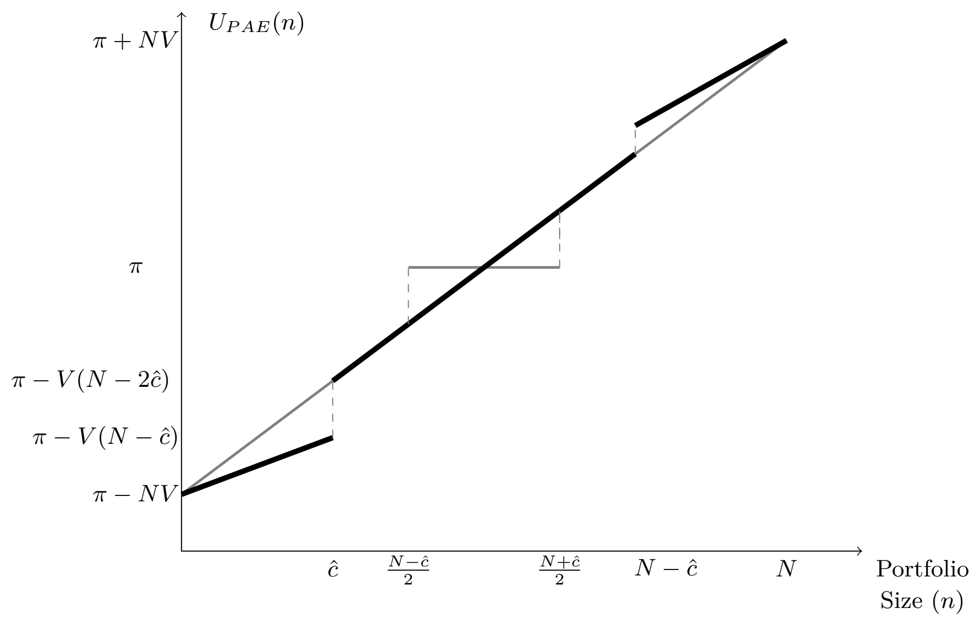

Figure 5: Continuation payoff for a firm that enters with n patents in the game with a PAE (in bold). Superimposed (in gray) is the payoff of the game without a PAEs. The figure depicts the case $N > \hat{c}$ for the equilibrium where any firm which discovers fewer than $\hat{c}$ components retains all of its patents.

The PAE has three effects on the continuation payoffs of the producing firms. First, when firms enter with roughly half of the patents –to be precise, $\frac{N-\hat{c}}{2} \leq n \leq \frac{N+\hat{c}}{2}$– the payoff has positive slope. In other words, each patent has positive marginal value while in the absence of PAEs the marginal value of a patent in this region is 0 (in Figure 5, where the superimposed payoff function is flat). This is precisely the *monetization* effect of a PAE. Second, the payoff function is flatter in the two extreme regions, implying that the marginal value of a patent in those regions is lower, compared to what it would be in the absence of a PAE. Absent of PAEs, a firm with fewer than $\hat{c}$ patents can only use its portfolio defensively, but when its rival's portfolio is monetized by a PAE the patents have no defensive value. This explains why, in Figure 5, the payoff is flatter on the left side of the graph. The slope is also flatter on the right side, compare to the payoff without PAEs wher the marginal value of each extra patent is $2V$, as explain in Section 4.4. Since the defensive value of patents is destroyed, the *appropriation* value of a firm with the largest portfolio is also destroyed by PAEs. Thus, the marginal value for the firm with the largest number of patents decreases, although the level of payoff increases. Thus, the *shielding* effect increases the payoff for the firm with the largest portfolio and decreases it for the firm with fewer patents, but the marginal value of a patent is decreased for both firms. Finally, the PAE reduces the total expected profit of a firm, and hence total industry profits among the producing firms, by extracting some of the total surplus as bargaining rents, characterized in Proposition 2.

In a nutshell, the effects of the PAE are: weakly decrease (increase) the level of payoff for the firm with fewer (more) patents; increase the marginal value of patents when firms have patent portfolios of similar size; and to decrease the marginal value of patents for both firms when the size of patent portfolios is very asymmetric.

The case where $N \in [2\hat{c}, 3\hat{c})$ is qualitatively similar to $N > 3\hat{c}$. The other important case is, $N \in [\hat{c}, 2\hat{c})$. In this case, the PAE is ineffective to monetize patents when both firms individually have fewer than $\hat{c}$ patents, but it can still help monetization when one firm has more than $\hat{c}$ patents. In the latter case, the PAE still shields the firm with the largest portfolio from litigation. The details for these two cases are in Appendix B.

In the next section we study the firms' R&D investment problem, taking as given the equilibrium continuation payoffs above. For the remaining sections of the paper, we focus our analysis on the case where $N > 3\hat{c}$.

6 Endogenous R&D investments

We now turn to the optimal R&D investments when firms rationally anticipate the subsequent payoffs from entry, trade with the PAE, licensing and litigation. After each firm makes its investments, discoveries arrive stochastically and are immediately patented. Once all of the N components are discovered, firms play their optimal strategies in the entry, licensing, and litigation stages of the game. We focus on the case where $\pi > NV > 3c$, which guarantees that Lemma 1 holds.

The returns to R&D depend on the number of components discovered by each firm. In the game without a PAE, we denote by $U(k)$ the continuation payoff of a firm that discovers (and patents) k out of N components (see Figure 3). In the game with a PAE, we denote the continuation payoff by $U_{PAE}(k)$ (see Figure 5).

Our model requires a generalized rent-seeking contest, because firms derive utility for a *bundle* of objects and their investments also determine the 'size of the pie' through the discount factor. Although competition over multiple prizes has been studied before(for example, Clark and Riis (1998)) the setting differs from ours in an important way, since in our formulation firms care non-linearly for the *bundle* of components they obtain. Figures (3) and (5) show that the continuation payoff is weakly increasing, non-linear, and not concave. Besides this difficulty, the expected discount factor for $N > 1$ is not a concave function of the R&D investment (See Appendix B). As a consequence the R&D decision problem is not generally well-behaved (in particular, the objective function is not pseudo-concave), which does not allow us to use standard results for existence and comparative statics for symmetric games.

We use a simple R&D model inspired by Loury (1979). In our model, firms simultaneously make a one-time R&D denoted by x and y for firms A and B, respectively. The cost of z units of R&D is the same for both firms, given by $c_I(z)$. Firm A discovers any one particular component first (independently) with probability $p(x,y) = \frac{h(x)}{h(x)+h(y)}$, which implies a binomial distribution for the total number of discovered components.

$$P(k; x, y) = \binom{N}{k} p(x,y)^k (1 - p(x,y))^{N-k}.$$

R&D investments not only determine the distribution of patents, but also the time at which production begins. Since only the first version of a component is patentable, the time at which component $i \in \{1, ..., N\}$ is available is given by $\hat{\tau}_i(x,y) = \min\{\tau_i(x), \tau_i(y)\}$, where $\tau_i(x)$ and $\tau_i(y)$ are random arrival times for component i for firms A and B, respectively. Production can take place only when *every* component has been discovered, since firms either invent (and get a patent) or imitate. The time at which firms will enter the market and produce is given by $\tau(x,y) = \max\limits_{i=1,\dots,N}\{\hat{\tau}_i(x,y)\}$, distributed according to $F(\tau; x, y)$. Firms discount profits at rate r.

6.1 The R&D investment problem without PAEs

Each firm chooses its R&D investment to maximize its expected payoff, given the investment level chosen by its rival. Since firms are ex-ante symmetric, they solve:

$$\max_{x \geq 0} \; \mathbf{E}_{\tau,k}[e^{-r\tau} U(k) | x, y] - c_I(x). \tag{4}$$

Lemma 5. *The random variables k and τ are independent for all x and y.*

Lemma 5, derived from the properties of the exponential distribution, implies that $e^{-r\tau}$ and $U(k)$ are independent, so we can write the firm's problem as:

$$\max_{x \geq 0} \; \underbrace{\mathbf{E}_{\tau}[e^{-r\tau} | x, y]}_{G(x,y)} \cdot \underbrace{\mathbf{E}_{k}[U(k) | x, y]}_{\Pi(x,y)} - c_I(x).$$

Given R&D investments x and y, $G(x,y)$ is the expected discount rate at the time of production, and $\Pi(x,y)$ the expected continuation payoff.

In Appendix C, we derive explicit formulas for $G(x,y)$, $\Pi_x(x,y)$ and show their properties. Note that $\Pi(x,x) = \pi$ because, given symmetric R&D investments, firms expect symmetrically each portfolio allocation, expected licensing transfers for each firm net to zero, and the expected reward of entering the market is π.

Under a stability condition, similar in spirit to the one in Lee and Wilde (1980), we can show that the problem has a unique interior solution $x^* > 0$ that solves

$$G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) = c'(x^*). \tag{5}$$

This first order condition in equation 5 characterizes how the payoffs from the continuation game determine the incentives to innovate. Investing one more unit of R&D has two effects. First, it brings the expected continuation payoff earlier. This effect is given by the term $G_x(x^*, x^*)\pi$. Because more R&D today brings this payoff sooner, it will be discounted at a smaller rate, captured by the marginal change in the expected discount rate, G_x. Second, as the firm with the largest portfolio can capture weakly more rents than π through licenses in the continuation stage, firms race to be the firm that discovers more components. This is represented by the second term $G(x^*, x^*)\Pi_x(x^*, x^*)$. The marginal gain $\Pi_x(x,y)$ is positive for all x and y, by first order stochastic dominance.[22]

Our analysis in the remainder of the paper will restrict attention to parameter values such that a symmetric pure strategy equilibrium exists. In some cases, a symmetric pure strategy equilibrium with a positive level of investment might fail to exists. This happens when π is too small or r is too large, in which case there is not much of an incentive to invest in R&D since the expected discounted continuation profits are too small. For an extended discussion of existence and uniqueness of equilibrium, see Appendix D.

6.2 The R&D investment problem with a PAE

When firms are allowed to trade patents with the PAE the payoffs to R&D are the ones described in Proposition 2. We focus our the analysis on the case $N > 3\hat{c}$, for which the continuation payoffs are shown in Figure 5.

[22] $\Pi_x(x,y) = \frac{\partial \Pi(p)}{\partial p}\frac{\partial p}{\partial x} = \frac{\partial \Pi(p)}{\partial p}\frac{h'(x)}{h(x)}p(1-p)$. By properties of the binomial distribution (FOSD), and using the fact that $U(k)$ is weakly increasing in k, we have $\frac{\partial \Pi(p)}{\partial p} > 0$.

Lemma 6. *The problem with a PAE is equivalent to*

$$\max_{x \geq 0} \; G(x,y)\Pi(x,y) - c_I(x) + \Delta(x,y),$$

where $\Delta(x,y) = G(x,y)D(x,y)$ *and* $D(x,y) = \mathbf{E}_k[U_{PAE}(k) - U(k)|x,y]$.

The marginal effect of the PAE on incentives is given by

$$\Delta_x(x,y) = \underbrace{G_x(x,y)D(x,y)}_{\text{Rent Extraction}} + \underbrace{G(x,y)D_x(x,y)}_{\text{Rent Seeking}}.$$

The first term, the *rent extraction* effect, corresponds to the marginal effect of R&D investments on the time at which firms obtain the change in *levels* of expected payoffs, due to the presence of the PAE. If a firm invests too little relative to its rival, it is likely that will end up with a small portfolio. In that case, the PAE decreases its payoff compared to the case without PAEs. Similarly, a firm that invests a lot more than its rival, expect to obtain a large portfolio and the presence of PAE increases its payoff relative to the case without PAEs. This effect is illustrated by the difference in the *level* of payoffs in Figure 5 (bold vs gray lines).

The second term, the *rent seeking* effect, corresponds to the change in the marginal benefit from obtaining more patents with a PAE. As we can see in Figure 5, the marginal effect of a PAE is positive as long as firm end up with similar number of patents after the R&D stage, and negative if their portfolios are very asymmetric. This effect is illustrated by the difference in the *slopes* of payoffs in Figure 5 (bold vs gray lines).

Since firms are ex-ante identical, we focus on a symmetric equilibrium. Intuitively, since in a symmetric equilibrium firms expect to end up with similar number of patents, the effect that dominates is the monetization effect of the PAE. Proposition 3 shows that the overall marginal effect of the PAE is positive.

Proposition 3. *When* $N > 3\hat{c}$*, for symmetric R&D investments* $x = y = x^*_{PAE}$ *we have:*

(a) The rent extraction effect (RE) is weakly negative *and equal to*

$$RE(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{h'(x^*_{PAE})}{h^2(x^*_{PAE})} \frac{r\ln(N)}{2^{N+2}} \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \eta(k;s),$$

where $\eta(k;s) = -(1-s)V\ell_k$*,* ℓ_k *is the amount of patents retained by the firm with the smaller portfolio, and* $(1-s)$ *is the PAE's bargaining power.*

(b) The rent seeking (RS) effect is strictly positive *and equal to*

$$RS(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{h'(x^*_{PAE})}{2^{N+1} h(x^*_{PAE})} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)[U_{PAE}(k) - U(k)].$$

(c) The rent seeking effect is larger than the rent extraction effect if

$$h(x^*_{PAE}) > \frac{r \ln(N)}{2}.$$

The above proposition characterizes the PAE effect in a symmetric equilibrium. First, it shows that the rent extraction effect is weakly negative in a symmetric equilibrium, because the PAE extracts some of the total industry surplus as bargaining rents. This reduces the firm's incentive to try to discover all components earlier and bring payoffs earlier. Second, the proposition shows that the PAE has a positive impact on each firm's rent-seeking incentive: despite the fact that the continuation payoff becomes "flatter" for very asymmetric patent positions, it becomes "steeper" in the middle (see Figure 5). In a symmetric equilibrium being in the middle region is more likely and this outweighs the potential negative effect on incentives at the extremes. Finally, the last result in the proposition provides a sufficient condition which characterizes the total effect of the PAE. The two effects push the incentive to invest in R&D in opposite directions: the rent extraction effect decreases it, while the winner premium effect increases it. Overall, when $h(x) > \frac{r \ln N}{2}$, we show that $\Delta_x(x, x) > 0$. In other words, in a symmetric equilibrium the rent seeking effect dominates the rent extraction effect when the equilibrium R&D investment is larger than some threshold.

Having understood the marginal effect of the PAE, we can study how the equilibrium R&D investments compare in the games with and without the PAE. Under existence and uniqueness conditions, discussed in Appendix D, there exists a unique interior symmetric equilibrium without the PAE, x^*, such that

$$foc(x^*) \equiv G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) - c_I'(x^*) = 0.$$

In the game with a PAE this condition becomes

$$foc(x^*_{PAE}) + \Delta_x(x^*_{PAE}) = 0$$

Appendix D also shows that under the condition $h(x) > \frac{r \ln N}{2}$, R&D investments are strategic substitutes locally, around the symmetric equilibrium levels.

Proposition 4. *Suppose symmetric equilibria exist in the games with and without the PAE, and the equilibrium investments are such that* $h(x) > \frac{r \ln N}{2}$. *Total R&D investments are larger in the equilibrium with the PAE than in the equilibrium without the PAE.*

Proposition 4 establishes that the PAE can *increase* the equilibrium level of R&D investment, even if it extracts rents.

7 Welfare analysis

In this section we compare the symmetric equilibrium solution to what a planner would do if it could control the level of investment in each firm. Notice that with or without PAEs, the continuation game between the firms (and the PAE) is a zero-sum game with total industry profits 2π. The planner does not care about the allocation of patents, as long as both firms enter, even if one of the firms owns all the patents. Hence the first best solution, in which the planner controls the investment level of the firms and grants a license to every firm, coincides with the second best solution, in which the planner just controls the investment levels and once the patents are allocated among firms they will bargain over licenses in the shadow of litigation, possibly through a PAE.

The social planner chooses investment levels x and y to maximize the total surplus generated by the commercialization of the final product. Let W be the consumer surplus generated by discovering all the components and selling the final product. Then, the planner solves

$$\max_{x\geq 0,\ y\geq 0} (2\pi + W)G(x,y) - c_I(x) - c_I(y).$$

Lemma 7. *The unique planner solution is symmetric and characterized by*

$$(2\pi + W)G_x(x_P, x_P)\pi = c_I'(x_P)$$

The symmetric equilibrium conditions can be written as:

$$\begin{aligned} G_x(x_P,x_P)\pi - c_I'(x_P) + G_x(x_P,x_P)(\pi + W) &= 0 \quad \text{(Planner problem)} \\ G_x(x^*,x^*)\pi - c_I'(x^*) + G(x^*,x^*)\Pi_x(x^*,x^*) &= 0 \quad \text{(Equilibrium)} \end{aligned}$$

By comparing $G_x(x,x)(\pi+W)$ and $G(x,x)\Pi_x(x,x)$ we find when the planner invests more or less relative to the firm equilibrium.

The term $G_x(x,x)(\pi+W)$, which we call the *planner incentive*, represents the marginal benefit of higher investment that is internalized by the planner but not the firms. It corresponds to the marginal expected discounted consumer surplus and payoff of the rival firm. The term $G(x,x)\Pi_x(x,x)$, which we call the *competition incentive*, represents the winner premium which is taken into account by firms, but not by the planner. These two effects misalign the incentives to invest between the planner and the firms. The next lemma shows the interaction of these two different effects.

Lemma 8. *The planner incentive is larger than the competition incentive if and only if $x > x_M$, where x_M is defined by:*

$$h(x_M) = \frac{2^{N-1}(\pi + W) r \ln(N)}{V \cdot \sum_{k:\, |2k-N| \geq \hat{c}} \binom{N}{k} (2k - N)^2}$$

Lemma 8 shows that the planner incentive is larger than the competition incentive for levels of R&D investment larger than x_M. Moreover, the comparative statics of this cutoff point are straightfoward: it increases with consumer welfare increases, duopoly profits, discount rate, $\hat{c}$, and decreases with V. The intuition for this result is simple. The planner incentive only depends on bringing the payoff earlier.

Using lemma 8 we can compare the symmetric equilibrium without PAEs with the planner solution.

(1) *Under-investment in R&D:* If $x^* < x_M$, then $x^* < x_P$.

(2) *Over-investment in R&D:* If $x^* > x_M$, then $x^* > x_P$.

Firms will under-invest only when x_M is large. The main reason why x_M may be large is that W is likely to be large, to capture R&D spillovers and consumer welfare. Although measuring private versus social returns of R&D is not an easy task, Jones and Williams (1998) and Hall (1996) find that private R&D investment is lower than the socially optimal level of investment. Hence it is likely that x_M is large and firms are under-investing relative to the social optimum. In that case the welfare effect of the PAE is positive, i.e. the PAE effect attenuates the under-investment problem.

8 Extensions

In this section, we change our baseline modeling assumptions and discuss the impact on our results. We allow for: fixed plus variable litigation cost (as a function of the number of patents asserted); costly counter-suits; asymmetric cost between PAEs and producing firms; *injunctions* to producing firms; and more players, either more PAEs or more producing firms.

8.1 Litigation costs

In our baseline model producing firms and the PAE pay the same fixed fee c to go to court and producing firms can counter-sue for free. Suppose that when asserting patents firms not only pay the fixed legal cost c, but also a variable component c_{pp} per patent involved in the lawsuit. We need to assume that $V > c_{pp}$, otherwise litigation never occurs. Under these assumptions, Firm A has a credible litigation threat if

$$Vn > c + c_{pp}(n+m) + Vm \Leftrightarrow n > \gamma m + \hat{c}_p,$$

where $\gamma = \dfrac{V + c_{pp}}{V - c_{pp}} > 1$, and $\hat{c}_p = \frac{c}{V - c_{pp}} > \hat{c}$. A PAE with ℓ patents has a credible litigation thread if:

$$V\ell > c + c_{pp}\ell, \Leftrightarrow \ell > \hat{c}_p.$$

Adding a variable cost changes the shape of the region where litigation occurs, as shown in Figure 6. The main intuition and result of the paper does not change, since the monetization region, where patent portfolios are of similar size, still exists.

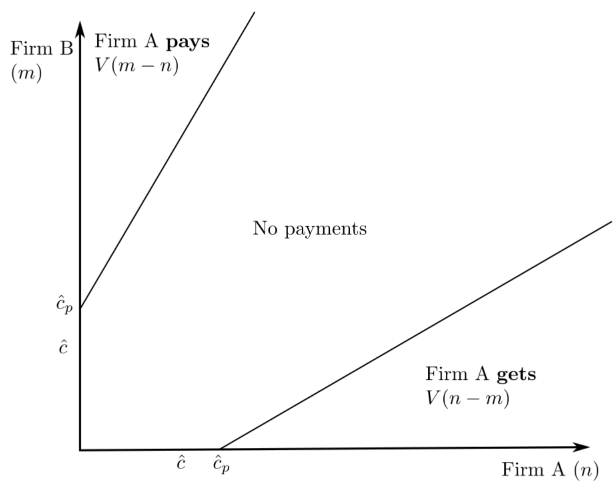

Figure 6: Change in the litigation region, assuming that firms have to pay a fix cost plus a variable cost that depends on the number of patents involved in the lawsuit.

Similarly, assuming that the PAE and producing firms have different litigation costs, would merely change the shape of the litigation region, but the main result would still hold.

Another assumption in our model is that producing firms can bring counter-suits for free. To relax this assumption, suppose that a counter-suit increases the litigation cost by a fixed amount c_{cs}. In this case, the firm with the smallest portfolio will not always use its patents to counter-sue, as it does in the baseline model. In fact, a firm is willing to use m patents to counter-sues only if $Vm > c_{cs}$. Thus there will be a new region, where $Vm \leq c_{cs}$, in which a firm with m patents does not counter-sue. Thus if the firm with the smallest portfolio has m patents and its rival has n patents, the firm with the largest portfolio has a credible litigation threat only if $Vn > c$. In this new region, PAEs do not have any effect because the firm with the largest portfolio never gets counter-sued. When the firm with the smallest portfolio is willing to counter-sue, that is $Vm > c_{cs}$, the firm with the largest portfolio sues if $Vn > Vm + c + c_{cs}$. In that case, the PAE has even stronger incentives to litigate, since its costs is effectively lower. Figure 7 below shows the new litigation region under the assumption of costly countersuits. With a large number of components N, the main result will still hold.

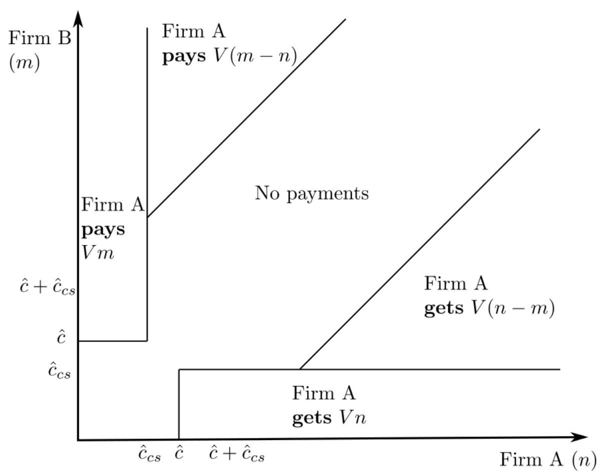

Figure 7: Change in the litigation region, assuming that counter-suits are costly.

8.2 Injunctions

An important strategic reason to start a lawsuit is the possibility of getting an injunction. In some cases, when producing firms have received irreparable damage from the infringer, courts can award an injunction to force the exit of the rival firm. Since the eBay vs MercExchange, L.L.C. case it has been almost impossible for PAEs to get injunctions, while producing firms get injunctions only under special circumstances. For these reasons our baseline model assumes away the possibility of injunctions. However, since producing firms could potentially get a larger market share following an injunction, litigation incentives are affected by this assumption.

Suppose that asserting patents grants the producing firm with the largest portfolio an injunction with probability I. Then the payoff from patent assertion increases by $I(\pi_m - \pi)$ for the plaintiff and is reduced by $I\pi$ for the defendant. Hence, the total surplus from going to court is $I(\pi_m - 2\pi) - 2c$. Notice that in contrast with our baseline case the total surplus could *increase* by going to court. In that case, there are no transfers that stop litigation and firms in equilibrium will go to court. Even more, PAEs have weaker incentives to enforce patents because they cannot get injunctions. However if the expected gain from an injunction is small (for example when I is small), then the total surplus decreases by going to court. In this case litigation will not occur in equilibrium and firms will settle after negotiating transfers. Nonetheless, the willingness to sell patents to PAEs is affected by injunctions, which reduces the monetization effect of PAEs.

9 Conclusions

This paper provides a theoretical framework to understand the effect of Patent Assertion Entities (PAEs) on the incentives for litigation and innovation. In particular, we focus on the practice of "patent privateering," which describes the outsourcing of patent monetization to PAEs by producing firms.

Our main contribution is to identify different channels through which PAE privateers can affect the incentives to innovate. In our model, firms choose their level of R&D investment in anticipation of the continuation payoffs from patenting. PAEs change these payoffs because they increase patent monetization, destroy the value of defensive patent portfolios, and, in some cases, extract rents.

We find that, without PAEs, the fear of retaliation and the cost of litigation may preclude producing firms with similarly-sized patent portfolios from monetization. In these cases firms remain in a tacit "IP truce" equilibrium, meaning that they cannot license or enforce their patents even when they believe their rival infringes on them. When firms decide to invest in R&D, the incentives to obtain more patents decrease if those extra patents will not be monetized. Therefore transaction costs reduce the incentives to invest in R&D.

With PAEs, firms anticipate better continuation payoffs if they come out ahead after the R&D stage. The strategic advantage of PAEs is that they cannot be counter-sued, implying that their litigation incentives are stronger than those of an operating firm holding the same patents. PAEs are able to disrupt the "IP truce" equilibrium and to create incentives on the margin for firms to invest more in R&D, in order to capture that value which would otherwise not have been realized. In fact, when firms have large portfolios of similar size the payoffs they receive in the equilibrium with a PAE are identical to the payoffs they would receive in a world where litigation costs are zero. Hence in some cases PAEs allow firms to overcome transaction costs.

However, we find that PAEs also affect the continuation payoffs by reducing the value of defensive patent portfolios and by potentially extracting rents from the producing firms. These effects hinder ex-ante incentives to innovate. By avoiding counter-suits, PAEs destroy the value of patents that otherwise would be used defensively by counter-suing a producing firm. This reduces the incentives to obtain those patents in the first place. We show that this effect not only matters to the firm with the smallest portfolio, but also to the firm with the largest portfolio. Thus in our model PAEs reduce the marginal incentive to obtain one more patent when one of the firms owns a very large patent portfolio. Another negative effect of PAEs is their potential to extract rents from the market. When this happens firms have less incentives to invest in R&D in order to commercialize the final product earlier and bring the continuation payoff sooner.

Our main result characterizes exactly when each of these PAE effects dominates. Overall, we show that PAEs can increase R&D incentives even when: they lower total industry profits by extracting rents, they do not invest in R&D, they do not use the patents to produce products, and they do not have any cost advantage in litigation with respect to the producing firms. Perhaps surprisingly, by increasing litigation threats PAE privateers can facilitate rather than obstruct innovation. This is most likely to be the case when a large fraction of the firms' patent portfolios cannot be monetized–e.g. when legal costs are very significant and the number of components involved in the final product is large. Furthermore, we study the welfare and policy implications of our results. We show that the welfare effect of PAE activity may be to increase total welfare in cases where firms under-invest in R&D relative to the socially optimal level of investment. Our model is stylized because we want to isolate some of the channels through which PAE privateers can affect incentives. We discuss our results under various other assumptions about litigation costs and injunctions. Further extensions of our model could incorporate private information, reputational concerns, contracting issues, selection, alternative penalty structures, asymmetries, etc.

Finally, our paper highlights some interesting questions that could be explored empirically. For example, do producing firms try to negotiate licenses before they sell patents to the PAE? Our results show that PAEs may be able to extract rents when this is not the case. Thus it would be interesting to find out how and when PAE privateers earn rents. Another possible question is: do PAEs tend to approach firms with similarly-sized patent portfolios or firms with asymmetric portfolios? Our model shows that PAEs do not extract rents when firms have similar patent portfolios. A detailed examination of the sources of patents for PAEs could shed light on this issue.

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A Appendix A: Proofs

Proof of Lemma 1.

Proof. Consider Firm A with n patents, while its rival has m patents. By entering, Firm A gets at least $\pi - mV$ and at most nV by not. So entering is better iff $\pi - mV > nV \Leftrightarrow \pi > (n+m)V = NV$. □

Proof of Lemma 2.

Proof. Consider the bilateral bargain between Firm A and the PAE, taking the outcome of the negotiation between Firm B and the PAE as given. Using the change of variables $u = S_{PAE}(z, m') - p - S_{PAE}(0, m')$, the maximization problem (1) can be written as

$$\max_{z,u} u^{1-s}(J_{A,PAE}(z, m') - J_{A,PAE}(0, m') - u)^s.$$

The solution is $z^* \in \arg\max J_{A,PAE}(z, m')$ and $u^* = (1-s)[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')]$. which implies the transfer $p = s[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')] - [S_A(z^*, m') - S_A(0, m')]$. The agreement is incentive compatible as long as $J_{A,PAE}(z^*, m') \geq J_{A,PAE}(0, m')$. □

Proof of Proposition 1.

Proof. When a producing firm sells its entire portfolio to the PAE, the other producing Firm And the PAE achieve the same joint surplus at any allocation that monetizes all patents (when possible). Suppose without loss of generality that Firm A sold its entire portfolio to the PAE in their bilateral negotiation (that is, $n' = n$), and consider the negotiation between Firm B and the PAE.

Firm B, on its own, can use its entire portfolio and get $T(m, 0)$ from Firm A, because Firm A has no patents to use defensively. The PAE will use the patents acquired from Firm A against Firm B, to obtain $T(n, 0)$ in licenses. Hence, Firm B's outside option is $S_B(n, 0) = T(m, 0) - T(n, 0)$ and the PAE's outside option (from the bilateral negotiation with Firm B) is $S_{PAE}(n, 0) = T(n, 0)$.

The joint surplus between Firm B and the PAE without agreement is $J_{B,PAE}(n, 0) = T(m, 0)$. Any number $z > 0$ of patents allocated from Firm B to the PAE must generate a weakly lower surplus $J_{B,PAE}(n,z) \leq T(m,0)$, and it attains this level when $z = m$.[23] Therefore, an outcome of the bargain process between Firm B and the PAE is $z^* = m$ and $p(m) = T(m,0)$.

Thus, whenever Firm A has sold everything to the PAE, the joint surplus of Firm B and the PAE is maximized by selling all of B's patents to the PAE. Analogously, when Firm B is selling all its patents to the PAE, an outcome of the bilateral negotiation between Firm A and the PAE is to have Firm A sell all its portfolio to the PAE at price Vn. In each case, the PAE pays each Firm An amount exactly equal to its licensing revenue from the acquired patents, and so earns no profit. $\square$

Proof of Lemma 3.

Proof. Suppose by contradiction there is an equilibrium in which Firm B sells $m' < m$ and retains $k = m - m' > 0$. Given m', the strategy that maximizes the joint surplus between Firm A and the PAE is for Firm A to retain $\ell = \max\{0, k - \hat{c}\}$ and for the PAE to acquire $n - \ell$ patents from Firm A. To show this claim, we analyze two cases: $k \leq \hat{c}$ and $k > \hat{c}$.

When $k \leq \hat{c}$, Firm A does not face a direct litigation threat from Firm B. But if Firm A were to sue Firm B, those k patents would be used defensively. We distinguish the cases $m' > \hat{c}$ and $m' \leq \hat{c}$. When $m' > \hat{c}$, without an agreement between Firm A and the PAE, the PAE gets Vm' from threatening to sue Firm A with the patents acquired from Firm B, and Firm A gets $T(n,k)$ from Firm B. Thus, the bilateral joint surplus without agreement is $J_{A,PAE}(0,m') = Vm' + T(n,k) - Vm'$. When $m' \leq \hat{c}$, without an agreement between Firm A and the PAE, Firm A gets $T(n,k)$ by threatening to sue Firm B and the PAE gets 0. Thus, for any value of m', the bilateral joint surplus between Firm A and the PAE without agreement is $J_{A,PAE}(0,m') = T(n,k)$. By selling everything to the PAE, Firm A and the PAE generate their maximal joint surplus of $J_{A,PAE}(n,m') = Vn$, which is strictly larger than $J_{A,PAE}(0,m')$ for $m' < m$.

When $k > \hat{c}$ and Firm A sells everything to the PAE, Firm A leaves itself vulnerable to Firm B's litigation threat. However, Firm A can "cancel out" this litigation threat by holding on to some patents. The minimum number of retained patents that are sufficient to deter Firm B from litigation is $\ell = k - \hat{c}$. Again, we distinguish the cases $m' > \hat{c}$ and $m' \leq \hat{c}$. When $m' > \hat{c}$, without an agreement between Firm A and the PAE, the PAE gets Vm' from Firm A, using the patents acquired from Firm B, and Firm A gets $T(n,k)$ from Firm B. The bilateral joint

[23] When $z > 0$, the joint surplus between Firm B and the PAE is strictly less than $T(m,0)$ when $T(m,0) = Vm$, and $m - z < \hat{c}$ or $z < \hat{c}$, which means, respectively, that Firm B or the PAE cannot credible sue Firm A.

surplus without agreement is $J_{A,PAE}(0,m') = T(n,k)$. When $m' \leq \hat{c}$, without an agreement the PAE gets 0 and Firm A gets $T(n,k)$. Therefore, for any value of m', Firm A and the PAE get $T(n,k)$ as joint surplus without an agreement. Consider Firm A keeping $k - \hat{c}$ patents and selling $n' = n - k + \hat{c}$ to the PAE. Notice that $n > m \geq k$ implies that $n' > \hat{c}$, so the PAE can credibly monetize the patents acquired from Firm A. By keeping $k - \hat{c}$ patents, Firm A effectively deters Firm B from starting a lawsuit. Thus, the joint surplus between Firm A and the PAE in this case is $J_{A,PAE}(n', m') = Vn'$ which is strictly larger than $J_{A,PAE}(0, m')$. In fact, this is the largest joint surplus that Firm A and the PAE can achieve, because selling more than n' would imply that Firm B has a credible threat against Firm A (which lowers the bilateral joint surplus), and selling less than n' would imply that the PAE extracts less surplus from Firm B.

We have shown that Firm A and the PAE best respond to $m' < m$ by playing the strategy $n'(m') = n - \max\{0, k - \hat{c}\}$. But in this case, Firm B and the PAE do not maximize their joint surplus at $m' < m$, since $J_{B,PAE}(n'(m'), m') = T(m', 0) < Vm$. By selling everything to the PAE, Firm B and the PAE can guarantee a larger joint surplus of Vm. Therefore, selling $m' < m$ cannot be an equilibrium. □

Proof of Lemma 4.

Proof. Suppose there is an equilibrium in which Firm A sells $n' < n$ and retains $\ell = n - n' > 0$. If $\ell \leq \hat{c}$ Firm A is not monetizing those patents (Firm B cannot sue Firm A) so Firm A and the PAE could increase their joint surplus by transferring all of those patents to the PAE. Suppose, instead, that $\ell > \hat{c}$. Since the PAE cannot credibly sue Firm A using Firm B's patents, in this case, Firm B and the PAE best respond having Firm B retain all of its patents and counter-suing Firm A. But this cannot be an equilibrium, because when Firm B retains all of its patents, Firm A and the PAE maximize their joint surplus by allocating all the patents to the PAE and letting it monetize them. □

Proof of Proposition 2.

Proof. In this proof we find all the equilibria of the game.

1. When $n = m \leq \hat{c}$, any allocation is an equilibrium, since patents cannot be monetized. When $n = m > \hat{c}$, any equilibrium features firms either holding on to all their patents on selling them all. Suppose in equilibrium Firm A kept $0 < \ell < n$ patents. Firm B

can "cancel out" this threat by keeping $k = \ell - \hat{c}$ patents, and the PAE will be able to monetize the rest as long as $n > \ell$. Thus, Firm A will always lose the value of those ℓ patents, unless $\ell = n$. Therefore, in equilibrium firms either sell all or keep all.

$$\pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0.$$

2. When $m > \hat{c}$, by lemma 3, any equilibrium features Firm B selling everything to the PAE.

   Consider an equilibrium where Firm A keeps $\ell > 0$ patents. Because this is an equilibrium, Firm A and the PAE must get the highest joint surplus with this allocation of patents. If Firm A kept all of its patents, the joint surplus from the threat of litigation of Firm A and the PAE would equal Vn. Any other equilibrium allocation must have all the patents being monetized to achieve this maximal joint surplus Vn. This is the maximal joint surplus, because Firm B has sold all its patents to the PAE. Therefore, for any equilibrium with $\ell > 0$ we must have $\ell \in (\hat{c}, n - \hat{c})$. This implies that when Firm A deals with the PAE there is no increase in joint surplus between Firm A and the PAE. Thus, the PAE extracts no rents from Firm A.

   In addition, Firm B and the PAE must maximize their joint surplus when Firm B sells all of its patents to the PAE. Firm B and the PAE could increase their joint surplus by "cancelling out" Firm A's litigation threat. The minimum number of patents that Firm B must keep to prevent Firm A from starting a lawsuit is $k = \ell - \hat{c}$. If the PAE can still monetize the patents bought from Firm B, that is $m - k > \hat{c}$, and $k < m$, then Firm B and the PAE would have a profitable deviation since by cancelling out Firm A's threat, Firm B and the PAE can increase their joint surplus by $V\hat{c}$.[24] But this cannot be an equilibrium, since by lemma (3) Firm B sells all its patents in equilibrium. Hence, either Firm B does not have enough patents to cancel Firm A's lawsuit ($k > m$) or, by keeping some patents, the remaining patents cannot be monetized by the PAE or Firm B ($m - k < \hat{c}$). Suppose Firm B has enough to "cancel out" Firm A's litigation threat, but when doing it the remaining patents cannot be monetized by the PAE, which is equivalent to $m < \ell < m + \hat{c}$. In this case, by keeping all of its portfolio, Firm B and the PAE have a joint bilateral surplus of $-V(n - \ell)$ which is a profitable deviation. Thus, in equilibrium it must be the case that Firm B does not have enough patents to "cancel

[24]This is because by holding on to k patents Firm B has to pay $V(n - \ell)$ to the PAE, as the PAE monetizes the patents bought from Firm A, and also the PAE gets $V(m - k)$ from monetizing the patents bought from Firm B. When Firm B sells everything to the PAE, the joint surplus is $-V(n - m)$ as Firm B gets $-Vn$ from the litigation against Firm A and the PAE, and the PAE monetizes Firm B patents getting Vm.

out" Firm A's litigation threat. That is, $\ell > m + \hat{c}$. When Firm A keeps this large amount of patents, Firm B cannot avoid the litigation threat by holding on to some patents, and therefore Firm B's outside option is the same as if Firm A sold all of its patents to the PAE.

Therefore, for an equilibrium with $\ell > 0$ to exist, it must be the case that $m + \hat{c} < \ell < n - \hat{c}$. In this equilibrium, Firm B and the PAE's joint surplus is $-V(n-m)$, and the PAE does not increase their joint surplus either, so extracts no rents from Firm B.

This implies that the PAE extracts no surplus in this case.

3. When $m \leq \hat{c}$ and Firm B keeps $k = m - m'$ the outside option of Firm A is to use its portfolio to litigate when possible, which only happens when $n - k > \hat{c}$. In this case, Firm A obtains $T(n,k)$. By selling all its patents to the PAE, Firm A avoids the counterclaims brought by Firm B using the portfolio it withheld. Thus, the PAE monetizes all Firm A's patents and Firm A does not faces a threat of litigation from Firm B. Thus, the joint surplus between Firm A and the PAE is in this case Vn. The extra surplus from selling everything to the PAE is given by $Vn - T(n,k)$, which is split according to Firm A's bargain power s.

□

Proof of Lemma 5.

Proof. Consider random variables $\tau_i^A(x) \sim \exp(x)$, $i = 1, ..., N$ and $\tau_i^B(y) \sim \exp(y)$, for $i = 1, ..., N$ and assume they are all independent. Define the random variables

$$k = \sum_{i=1}^{N} 1(\tau_i^A < \tau_i^B), \text{ and } \tau = \max\{\min\{\tau_i^A, \tau_i^B\}\}.$$

Define also $Z_i(x,y) = \min\{\tau_i^A(x), \tau_i^B(y)\}$ and $W_i(x,y) = \tau_i^A(x) - \tau_i^B(y)$. A property of the exponential distribution is that Z_i and W_i are independent (see for example Ferguson (1964)). Therefore, the vectors of random variables $Z = (Z_1, ..., Z_N)$ and $W = (W_1, ..., W_N)$ are independent. Since k and τ are measurable functions of Z and W they are independent. □

Proof of Lemma 6

Proof. Given the investment of the rival firm, the problem with a PAEs is

$$\max_{x \geq 0} \mathbf{E}_{\tau,k}[e^{-r\tau} U_{PAE}(k)|x,y] - c_I(x),$$

which can be written as:

$$\max_{x\geq 0}\ \mathbf{E}_{\tau,k}[e^{-r\tau}U(k)|x,y]-c_I(x)+\mathbf{E}_{\tau,k}[e^{-r\tau}U_{PAE}(k)-U(k)|x,y].$$

□

Proof of Proposition 3.

Proof. We define $\delta(k)\equiv U_{PAE}(k)-U(k)$.

(a) For $\hat{c}\leq k\leq N-\hat{c}$, $U_{PAE}(k)=-U_{PAE}(N-k)$. For $k\leq\hat{c}$, $U_{PAE}(k)=-V(N-k)$. And for $k>N-\hat{c}$, $U_{PAE}(k)=Vk-(1-s)V\ell_k$, where ℓ_k is the number of patents retained by the firm with the smaller portfolio in equilibrium. Thus, for $k\geq N-\hat{c}$ we have that $U_{PAE}(k)+(1-s)V\ell_k=-U_{PAE}(N-k)$. In a symmetric equilibrium $p=\frac{1}{2}$ and by using the symmetry of the binomial coefficients we have:

$$RE\left(x^*_{PAE}\right)=\frac{1}{2^N}\sum_{k=0}^{N/2}\binom{N}{k}U_{PAE}(k)+\frac{1}{2^N}\sum_{k=N/2}^{N}\binom{N}{k}U_{PAE}(N-k).$$

Using the relation between $U_{PAE}(k)$ and $U_{PAE}(N-k)$ (for all k) we have:

$$\sum_{k=0}^{N}\binom{N}{k}U_{PAE}(k)=-(1-s)V\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\ell_k.$$

Defining $\eta(k;s)=-(1-s)V\binom{N}{k}\ell_k$ and noticing that $\eta\leq 0$, we have the result.

(b) We borrow algebra from the derivation of $\Pi_x(x,x)$ in Appendix C. Since $\delta(k)\geq 0$ for $k>\frac{N}{2}$ and non-positive otherwise, we have

$$RS(x^*_{PAE})=G(x^*_{PAE},x^*_{PAE})\frac{Vh'(x^*_{PAE})}{2^{N+2}h(x^*_{PAE})}\cdot\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)>0.$$

(c) To show the last part of the proposition, define

$$\kappa(x^*_{PAE})\equiv[WP(x^*_{PAE})+RE(x^*_{PAE})]\cdot\left[G(x^*_{PAE},x^*_{PAE})\frac{Vh'(x^*_{PAE})}{2^{N+1}h(x^*_{PAE})}\right]^{-1}.$$

We have that $\kappa(x^*_{PAE})>0$ if and only if

$$\sum_{k=0}^{N}\binom{N}{k}(2k-N)\Delta(k)+\frac{r\ln(N)}{2h(x^*_{PAE})}\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\eta(k;s)>0,$$

which implies that $\kappa(x^*_{PAE}) > 0$ if and only if

$$h(x^*_{PAE}) > \frac{r\ln(N)(1-s)V\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\ell_k}{2\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)}.$$

It is easy to see that

$$0 \le \frac{\sum_{k=N-\hat{c}}^{N}\binom{N}{k}(1-s)V\ell_k}{\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)} \le \frac{\sum_{k=0}^{\hat{c}}\binom{N}{k}Vk}{\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)} \le 1.$$

This implies the sufficient condition: $h(x^*_{PAE}) > \frac{r\ln(N)}{2}$.

□

Proof of Proposition 4.

Proof. Let x^* be the equilibrium without the PAE, so $foc(x^*) = 0$. By Proposition 3, we know that $\Delta_x(x) > 0$ for all $x \ge \tilde{x}_{PAE} = h^{-1}\left(\frac{r\ln N}{2}\right)$. Therefore, any x^*_{PAE} such that $foc(x^*_{PAE}) + PAE_x(x^*_{PAE}) = 0$, we have that $foc(x^*_{PAE}) < 0 = foc(x^*)$. But in the region where $h(x) > \frac{r\ln N}{2}$ we showed that $foc(x)$ is strictly decreasing and therefore we must have $x^* < x^*_{PAE}$. □

Proof of Lemma 7

Proof. An interior solution therefore implies the conditions

$$(2\pi + W)G_x(x,y) = c_I'(x) \quad \text{and} \quad (2\pi + W)G_y(x,y) = c_I'(y).$$

By concavity of h and convexity of c_I any interior solution is symmetric. The symmetric solution for the planner's problem is equivalent to the competitive Firm Best response when its rival invests zero, after relabeling the parameters values: $\pi \to 2\pi + W$ and $r \to \frac{r}{2}$. Therefore, under our assumptions in Appendix D, the planner solution $x_P > 0$ exists and is unique. □

Proof of Lemma 8.

Proof. We show in Appendix C that

$$G_x(x,x) = G(x,x)\frac{r\ln(N)}{4h(x)^2}h'(x) \quad \text{and} \quad \Pi_x(x,x) = \frac{h'(x)}{h(x)}\frac{V}{2^{N+1}}\sum_{k:\,|2k-N|\ge\hat{c}}\binom{N}{k}(2k-N)^2.$$

Substituting these in and rearranging, we obtain the condition

$$(\pi+W)G_x(x,x) > G(x,x)\Pi_x(x,x) \quad \Leftrightarrow \quad h(x) < \frac{2^{N-1}(\pi+W)r\ln(N)}{V\cdot\sum_{k:\,|2k-N|\geq\hat{c}}\binom{N}{k}(2k-N)^2} \equiv h(x_M).$$

□

B Appendix B: continuation payoffs with a PAE for $N < 3\hat{c}$

In this section we complete the analysis of the continuation payoffs for the cases $N < 3\hat{c}$. First of all, when $N \leq \hat{c}$ the PAE has no effect in continuation payoffs as the firms will never monetize their portfolios. Here, again we break the indifference by assuming that the firm with the smallest portfolio holds on to all its patents.

Case $N \in [2\hat{c}, 3\hat{c})$**:**

This case is qualitatively similar to the case presented in the body of the paper $N > 3\hat{c}$. From Proposition 2 we obtained the payoffs for the different cases. If $n \in \left[0, \frac{N-\hat{c}}{2}\right]$, Firm A gets $\pi + (2-\mu)Vn - VN$. If $n \in \left[\frac{N-\hat{c}}{2}, \hat{c}\right]$, Firm A gets $\pi + \mu V(n-N)$. For $n \in [\hat{c}, N-\hat{c}]$ Firm A gets $\pi + V(2n-N)$. For $n \in \left[N-\hat{c}, \frac{N+\hat{c}}{2}\right]$, Firm A's payoff is $\pi + \mu Vn$. Finally, if $n \in \left[\frac{N+\hat{c}}{2}, N\right]$, Firm A's payoff is $\pi + (2-\mu)Vn - (1-\mu)VN$.

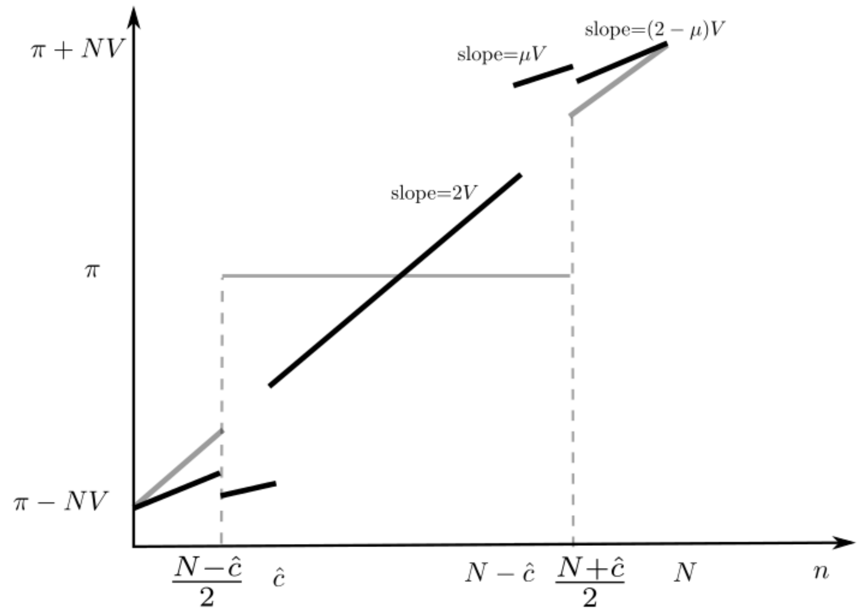

Figure 8: Consider a producing firm which discovers n components, while its rival discovers $N-n$, and $N \in [2\hat{c}, 3\hat{c})$. The firm's payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE.

Case $N \in [\hat{c}, 2\hat{c})$**:**

From Proposition 2 we obtained the payoffs for the different cases. If $n \in \left[0, \frac{N-\hat{c}}{2}\right]$, Firm A gets $\pi - V(N-n)$. If $n \in \left[\frac{N-\hat{c}}{2}, N-\hat{c}\right]$, similar to the previous case, Firm A gets $\pi + \mu V(n-N)$. If $n \in [N-\hat{c}, \hat{c}]$, Firm A gets π, since both n and m are less than $\hat{c}$. If $n \in \left[\hat{c}, \frac{N+\hat{c}}{2}\right]$, the PAE is able to monetize Firm A's portfolio, while Firm A on its own cannot. Thus, Firm A's payoff is $\pi + sVn$. Finally, if $n \in \left[\frac{N+\hat{c}}{2}, N\right]$, Firm A could monetize its portfolio, although Firm B would counter-sue. The PAE allows Firm A to avoid the counter-suing, and thus Firm A's payoff is $\pi + V(2n-N) + sV(N-n)$.

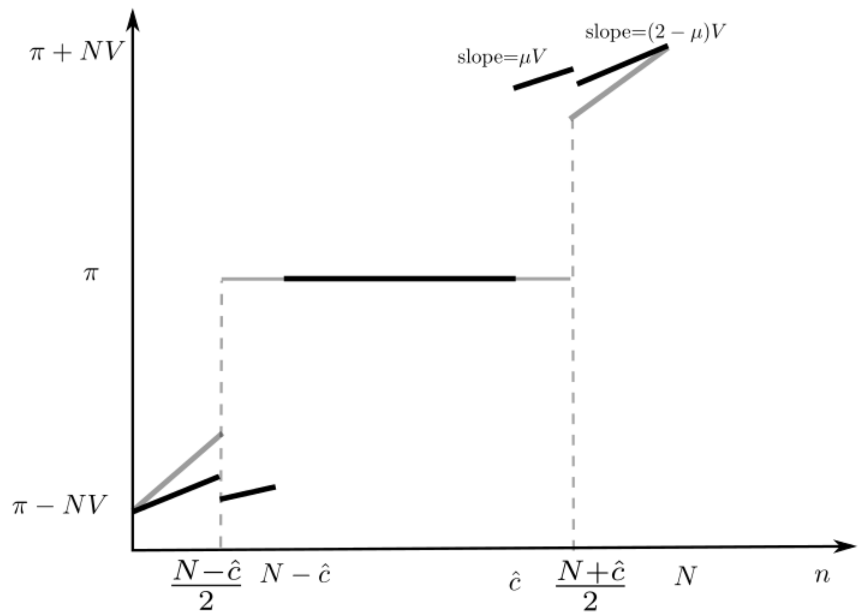

Figure 9: Consider a producing firm which discovers n components, while its rival discovers $N-n$, and $N \in [\hat{c}, 2\hat{c})$. The firm's payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE.

Notice that this case is qualitatively different to the case presented in the paper, because the PAE is not able to break the "IP-truce" when the patent portfolios are of similar size. Therefore, the effect of the PAE is potentially negative in this case.

C Appendix C: Derivation of $G(x, y)$ and $\Pi(x, y)$

We derive an explicit formula for $G(x, y)$ and $\Pi(x, y)$ and study their properties.

Derivation of $G(x, y)$

Explicit formula

By symmetry, let's call $\lambda = h(x)+h(y)$. Since each component has an independent exponential arrivals we have by definition

$$G(x, y) = g(\lambda) = \int_0^\infty e^{-rt} N(1-e^{-\lambda t})^{N-1} \lambda e^{-\lambda t} dt.$$

After standard mathematical manipulations (available upon request) we obtain

$$g(\lambda) = \sum_{k=0}^{N} \binom{N}{k} \frac{(-1)^k r}{r + \lambda k}.$$

If can also be shown that

$$g(\lambda) = \frac{\Gamma(N+1)}{\Gamma\left(N+1+\frac{r}{\lambda}\right)}.$$

where $\Gamma(z)$ is the Gamma function, which is increasing and convex for $z > 1$. It's easy to see that for $N > 1$ we have $g(0) = 0$, $g'(0) = 0$, $\lim_{\lambda\to\infty} g(\lambda) = 1$ and $\lim_{\lambda\to\infty} g'(\lambda) = 0$. Also, we can show that g is increasing and S-shaped. It is convex first and then concave.

Notice that $\ln(g(\lambda)) = \ln(N!) - \ln(\Gamma(h_N(\lambda)))$, where $h_N(\lambda) = N + 1 + \frac{r}{\lambda}$. Thus,

$$g'(\lambda) = \frac{g(\lambda) r}{\lambda^2} \frac{\Gamma'(h_N(\lambda))}{\Gamma(h_N(\lambda))} > 0, \text{ and } g''(\lambda) = \frac{g(\lambda)}{\Gamma^2} \left\{ [\Gamma']^2 - [\Gamma]''\Gamma - \frac{r}{\lambda^2}[\Gamma'\Gamma] \right\} |_{h_N(\lambda)}.$$

The convex region of g is where $\left\{ [\Gamma']^2 - [\Gamma]''\Gamma - \frac{r}{\lambda^2}[\Gamma'\Gamma] \right\} |_{h_N(\lambda)} > 0$.

Approximation

In order to get more tractable properties of $G(x, y)$ we will use the Stirling approximation, which is a highly precise approximation for the Gamma function, even for small values of N:

$$\frac{\Gamma(x+1+\beta)}{\Gamma(x+1+\alpha)} \approx x^{\beta-\alpha}.$$

Assuming that N is larger than 6 (which gives a highly precise approximation), a good approximation for $g(\lambda)$ is:

$$\hat{g}(\lambda) = N^{-\frac{r}{\lambda}} = e^{-r\frac{\ln(N)}{\lambda}}$$

The properties if g are easily derived using this approximation:

$$\begin{aligned} \hat{g}'(\lambda) &= \hat{g}(\lambda)\frac{r\ln(N)}{\lambda^2} \\ \hat{g}''(\lambda) &= \frac{\hat{g}(\lambda) r \ln(N)}{\lambda^4}(r\ln(N) - 2\lambda) \end{aligned}$$

It is easy to see that $\hat{g}$ is increasing and S-shaped: it is initially convex for $\lambda < \bar{\lambda} = \frac{r\ln(N)}{2}$, and for $\lambda > \bar{\lambda}$ it is always concave.

Derivation of $\Pi_x(x, y)$

Let $p(x,y) = \frac{h(x)}{h(x)+h(y)}$. We have that $\frac{\partial}{\partial x} P(k; x, y) = \binom{N}{k} \frac{p'}{p(1-p)} p^k (1-p)^{N-k}(k - Np)$. Therefore, the marginal change in the expected payoff is given by:

$$\Pi_x(x,y) = \frac{p_x}{p(1-p)} \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np) \cdot U(k).$$

Since $p_x = \dfrac{h'(x)}{h(x)} p(1-p)$ and using the symmetry of $U(k)$ we obtain

$$\Pi_x(x,y) = \frac{Vh'(x)}{h(x)} \sum_{k\colon |2k-N| \geq \hat{c}} \binom{N}{k} p^k (1-p)^{N-k}(k-Np)(2k-N).$$

In a symmetric equilibrium, $p^* = \frac{1}{2}$ and therefore the expression above equals:

$$\Pi_x(x,x) = \frac{Vh'(x)}{2^{N+1}h(x)} \cdot \sum_{k\colon |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2 \equiv \frac{h'(x)}{h(x)} \Psi.$$

Notice that Ψ measures the intensity of rent seeking incentives, and it is decreasing in $\hat{c}$, and increasing in V.

D Appendix D: Existence and Uniqueness of Equilibrium

Existence

We cannot apply the standard results of existence of equilibria, despite the game being symmetric, the payoffs are continuous, and the actions are chosen from a convex and compact set.[25] Consider Firm A's problem

$$\max_{x \in [0,M]} u_A(x,y) \equiv G(x,y)\Pi(x,y) - c(x)$$

When $\pi > NV$ both firms always enter. We first impose a participation condition: even when Firm B does not invest in R&D, Firm A still wants to invest in R&D. Equivalently, if we denote A's best response to B by $x^*(y)$, we can write this as $x^*(0) > 0$, where:

$$x^*(0) \in \arg \max_{x \in [0,M]} u_A(x,0) \equiv G(x,0)\Pi(x,0) - c(x).$$

[25] Clearly the game is symmetric game, and the payoffs are continuous. Since $G(x,y) \leq 1$ and $\Pi(x,y) \leq \pi + NV$, no firm will choose an investment level above $M = c^{-1}(\pi + NV)$. Thus, strategies are chosen from the interval $[0, M]$.

As long as π is large enough we have that $x^*(0) > 0$, since $\Pi(x,0) = \pi + NV$ for all $x > 0$, so we have $u_A(x,0) = G(x,0)(\pi + NV) - c(x)$. Clearly there exists π large enough such that the optimum is strictly positive.

Next, notice that $u_A(0,0) = 0$ and $u_A(\infty,0) = -\infty$. Under general conditions it can be shown that $u_A(x,0)$ has at most two zeros in $(0,\infty)$.[26] The next figure depicts the typical shape of $u_A(x,0)$, showing that in fact it is not quasi-concave for small values of x.

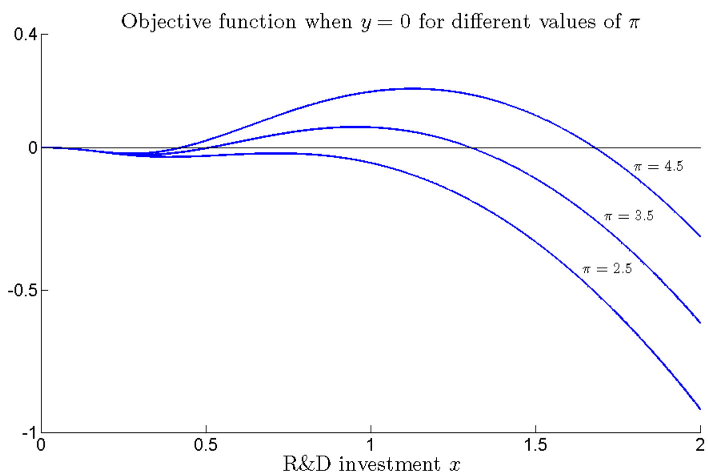

Figure 10: Simulation of the objective function for parameter values $N = 8$, $\pi = 2.5, 3.5, 4.5$, $R = 0.167$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $r = 1$, $c(x) = \frac{1}{2}x^2$, and $\hat{c} = 1.5$.

The shape of the objective function is not surprising. Intuitively, when Firm B invests 0, Firm A trades off the investment cost against the benefit of an earlier arrival of the continuation payoff, $\pi + NV$. When x is small, Firm A pays the investment cost and receives almost no benefit, since discoveries arrive far in the future. This is why the payoff decreases below zero for some small x. When x is larger than some threshold, the time at which all the components are discovered is significantly reduced, and the continuation payoff becomes significant and not so heavily discounted, so the firm has incentives to invest. Finally, for relatively large values of x, increasing x even more will not improve firm's profits because the gains from discovering faster are relatively smaller than the cost of investment.

[26] $u_A(x,0) = 0$ is equivalent to $\pi + NV = e^{\frac{r\ln(N)}{h(x)}}c(x) \equiv K(x)$, and $K(\cdot)$ is always positive and its derivative is zero whenever $\frac{c'(x)h^2(x)}{c(x)h'(x)} = \frac{r\ln(N)}{2}$. If there a unique x (or none) that satisfies this equation, then by continuity we can have at most two solutions. This is the case, for example, when $h(x) = x^\alpha$ and $c(x) = x^\beta$ with $\alpha < 1 < \beta$.

Next consider the general problem when $y > 0$. Firm A now also faces a rent-seeking effect, as firms compete to get more components than their rivals. Moving in the opposite direction is the "free riding" benefit that Firm A gets when Firm B invests more, because the continuation payoff arrives earlier.

To understand how $x^*(y)$ changes with y, we can compute the cross partial derivative

$$\frac{\partial^2 u_A(x,y)}{\partial x \partial y} = G_{x,y}(x,y)\Pi(x,y) + G_x(x,y)\Pi_y(x,y) + G_y(x,y)\Pi_x(x,y) + G(x,y)\Pi_{x,y}(x,y)$$

Re-arranging terms, we can show that:

$$G_{x,y}(x,y) = G(x,y) r \ln(N) \frac{h'(x)h'(y)}{(h(x)+h(y))^4}[r\ln(N) - 2h(x) - 2h(y)],$$

$$G_x(x,y)\Pi_y(x,y)+G_y(x,y)\Pi_x(x,y) = \frac{G(x,y) r \ln(N) h'(x) h'(y)}{(h(x)+h(y))^2 h(x) h(y)} \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} (k-Np) U(k)[h(y)-h(x)]$$

and

$$\Pi_{x,y}(x,y) = -\frac{h'(x)h'(y)}{h(x)h(y)} \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} [(k-Np)^2 - Np(1-p)] U(k).$$

Next we can combine these expressions to sign $\frac{\partial^2 u_A(x,y)}{\partial x \partial y}$. In particular, since we are interested in a symmetric equilibrium we can study the local behavior when $x = y$. In a symmetric equilibrium we have:

$$\begin{aligned}
G_{x,y}(x,x) &= G(x,x) r \ln(N) \frac{h'(x)^2}{(2h(x))^4}[r\ln(N) - 4h(x)] \quad < \quad 0 \quad \text{iff} \quad h(x) > \frac{r\ln(N)}{4} \\
G_x(x,x)\Pi_y(x,x) + G_y(x,x)\Pi_x(x,x) &= \frac{G(x,x) r \ln(N) h'(x)^2}{4h(x)^4} \Psi [h(x) - h(x)] \quad = \quad 0 \\
\Pi_{x,y}(x,x) &= -\frac{h'(x)^2}{h(x)^2} \underbrace{\sum_{k=0}^{N} \binom{N}{k} \frac{1}{2^N} \left[\left(k - \frac{N}{2}\right)^2 - \frac{N}{4} \right] U(k)}_{=0} \quad = \quad 0
\end{aligned}$$

Combining the 3 terms above, in a symmetric equilibrium we have

$$\frac{\partial^2 u_A(x,x)}{\partial x \partial y} = G_{x,y}(x,x)\Pi(x,x)$$

and hence

$$\text{sign } \frac{\partial^2 u_A(x,x)}{\partial x \partial y} = \text{sign } (r\ln(N) - 4h(x))$$

Therefore as long as the intersection of the best response and the 45 degree line occurs at x such that $h(x) > \frac{r\ln(N)}{4}$, the local behavior in the symmetric equilibrium is such that

$\frac{\partial^2 u_A(x,x)}{\partial x \partial y} < 0$. Since we have a symmetric game, this condition implies that locally around the region where $x = y$ the investments x and y are strategic substitutes: when y increases, the best response $x^*(y)$ decreases.

The figure below depicts the typical shape of the best responses when the parameter π is large enough so the condition $\hat{\pi} > N > 3\hat{c}$ is satisfied.

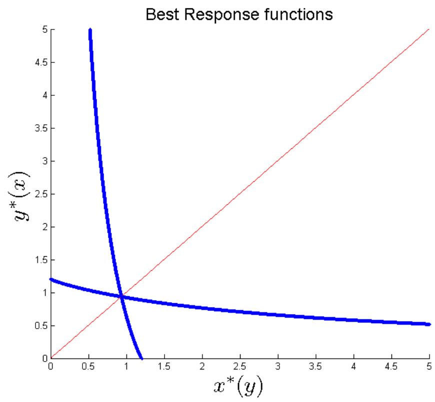

Figure 11: Simulation of the best responses for parameter values $r = 1$, $N = 8$, $\pi = 5$, $R = 0.167$, $\beta = 0.8$, $h(x) = x^{\alpha}$, $\alpha = 0.8$,$c(x) = \frac{1}{2}x^2$, $\hat{c} = 1.5$.

Notice there is a unique symmetric equilibrium where firms exert a positive level of R&D. Next, we show that whenever the symmetric equilibrium involves a level of investment large enough, there is a unique symmetric equilibrium.

Condition for uniqueness

We show that when there exists a symmetric equilibrium with positive level of investment x^* satisfying $h(x^*) \geq \frac{r \ln(N)}{2}$, then it is unique. We know that any interior symmetric equilibrium x must satisfy the condition:

$$G_x(x, x)\pi + G(x, x)\Pi_x(x, x) = c'(x).$$

Define $foc(x) = G_x(x, x)\pi + G(x, x)\Pi_x(x, x) - c'(x)$. Using the approximation for $G(x, x)$ and the computation of $\Pi_x(x, x)$ (derived in Appendix C) we have

$$foc(x) = e^{\frac{-r \ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[\frac{\pi r \ln(N)}{2h(x)} + \Psi \right] - c'(x).$$

Taking derivatives we obtain:

$$foc'(x) = e^{\frac{-r\ln(N)}{2h(x)}}\frac{h'(x)}{h(x)}\left[\frac{r\ln(N)h'(x)}{2h^2(x)}A(x) - \frac{\pi r\ln(N)}{2h(x)} + \frac{h(x)}{h'(x)}\left(\frac{h''(x)}{h(x)} - \frac{(h')^2(x)}{h^2(x)}\right)A(x)\right] - c''(x),$$

where $A(x) = \dfrac{\pi r\ln(N)}{2h(x)} + \Psi$. Rearranging terms we obtain:

$$foc'(x) = e^{\frac{-r\ln(N)}{2h(x)}}\frac{h'(x)}{h(x)}\left[\left(\frac{r\ln(N)}{2h(x)} - 1\right)\left(\frac{r\ln(N)h'(x)}{2h^2(x)}\pi + \Psi\right) - \frac{\pi r\ln(N)}{2h(x)} + \frac{h''(x)}{h'(x)}A(x)\right] - c''(x),$$

Notice that for any x such that $\frac{r\ln(N)}{2h(x)} < 1$ we have that $f'(x) < 0$. Let $\bar{x}$ be such that

$$h(\bar{x}) > \frac{r\ln N}{2}.$$

Hence, there can be only one symmetric equilibrium with a level of investment such that $h(\bar{x}) > \frac{r\ln N}{2}$.

Notice that $foc(x) = 0$ does not imply that x is an equilbrium. For example, $foc(0) = 0$ and we showed that $x^* = 0$ is not an equilibrium for π large enough. Depending on the parameters there might be other candidates for equilibrium. For any x such that $foc(x) = 0$ and $foc'(x) > 0$, x cannot be an equilibrium, since locally around x there are profitable deviations, as the condition $foc'(x) > 0$ implies that x is a local minimum of $G(x, y)\Pi(x, y) - c(x)$ at $y = x$. Generally there are at most two positive levels of investment such that $foc(x) = 0$, as depicted in the figure below: one where $foc'(x) > 0$ and one where $foc'(x) < 0$. In this case the unique equilibrium with positive level of investment is where $foc(x) = 0$ and $foc'(x) < 0$.

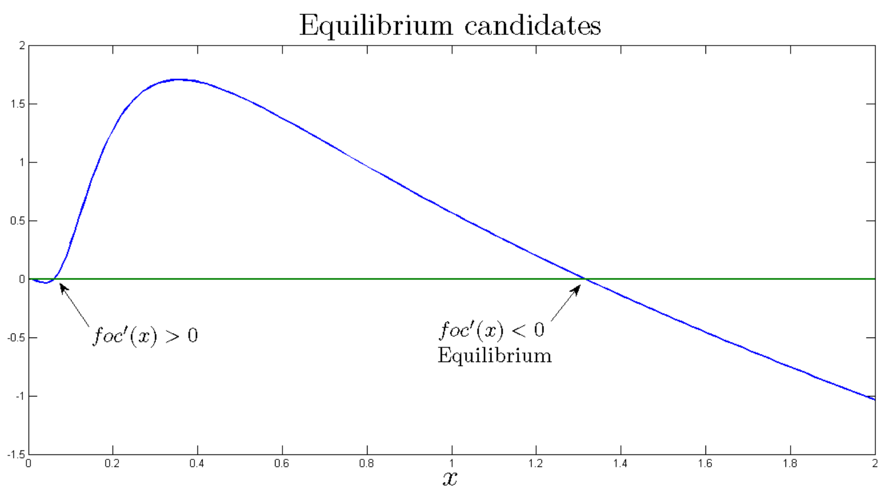

Figure 12: Simulation of the best response functions for parameter values $N = 6$, $\pi = 2$, $R = 1$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $r = 1$, $c(x) = \frac{1}{2}x^2$, and $\hat{c} = 1.5$.

For a large set of parameter values the conditions presented in the previous discussion are satisfied. Moreover, for those parameters we find that there is a unique equilibrium and it is symmetric. In the figure below we show comparative statics on the parameters π and r. When π increases the symmetric equilibrium features larger levels of investment, as expected. When r changes we obtain a non-monotonic behavior in the equilibrium level of R&D. The intuition is simple: when r is very small firms are very patient and they do not heavily discount profits. Hence the effect that dominates is the winner premium effect. As r starts to increase discounting provides an extra incentive for firms to invest. However, when r is very large the present discounted value of entering the market is small, even when a firm wins the race for every component.

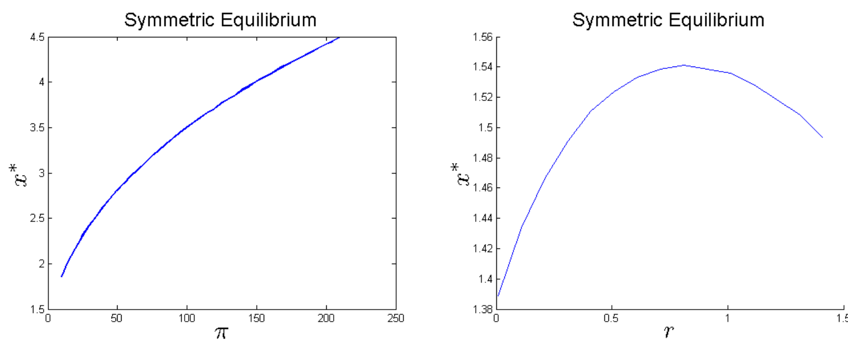

Figure 13: Comparative statics on π and r. Parameter values $N = 6$, $R = 1$, $\beta = 0.8$, $h(x) = x^{\alpha}$, $\alpha = 0.8$, $c(x) = \frac{1}{2}x^2$, $\hat{c} = 1.5$, with $r = 1$ (on the left) and $\pi = 10$ (on the right).

Non existence of equilibrium with positive investment

As is standard in rent seeking games, we cannot guarantee that the investments are strategic substitutes for all parameter values. For example, when r is high or when π is low the best responses might be strategic complements everywhere, or in some region.

In the first example we have increased the value of the discount rate r. Firms discount so heavily the payoff after entry that a firm finds it worth investing only when its rival has also invested sufficient resources in R&D. Moreover, in this case R&D investments are strategic complements, and when Firm B increases its R&D investment, Firm A will slowly increase its own R&D investment. In fact, the best response looks like the best response of a firm that simply maximizes $G(x, y)\pi - c(x)$. That is, firms are more concerned with their effect on bringing the payoff sooner than with competing for rents. In the existence section we ruled out this case by assuming that r is small and π is large, since for those parameters we know the best response to $y = 0$ is $x^*(0) > 0$ and not $x^*(0) = 0$. When r is large, so the payoff from entry is still heavily discounted, but the competition effect is significant, investments are both strategic substitutes and complements in different portions of the best response function. In these cases there is no equilibrium with positive level of investment.

For intermediate values of the parameter r we can find cases in which we have a discontinuity in the best response (since we cannot guarantee that the objective function is pseudo-concave with respect to the firm's own decision variable). This is because the rent seeking effect discontinuously becomes relevant after the rival has invested a small amount in R&D. Thus the other firm can still 'catch up' and compete against its rival.

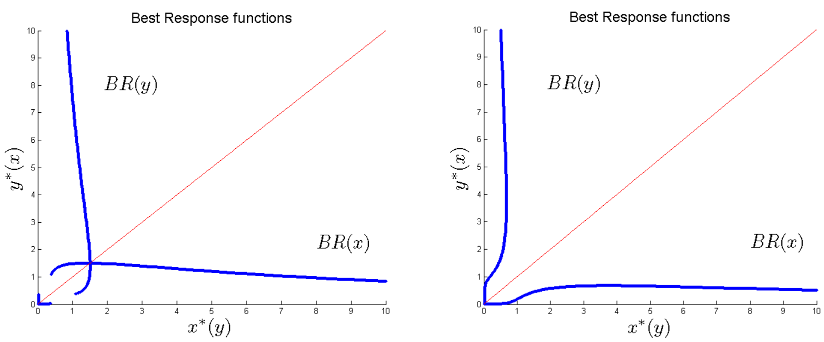

Figure 14: Simulation of the best response functions for $r = 2.5$ (on the left) and $r = 5$ (on the right), with parameter values $N = 6$, $\pi = 10$, $R = 2$, $\beta = 0.8$, $h(x) = x^{\alpha}$, $\alpha = 0.8$, $c(x) = \frac{1}{2}x^2$, and $\hat{c} > 2$.

E Appendix E: What if firms can bargain over licenses before trading with the PAE?

An important effect of PAEs in our baseline model is that they may decrease the incentives to invest in R&D because they may extract some of the total industry surplus as bargaining rents. However, if producing firms anticipate this possibility and are able to negotiate licenses before selling any patents to a PAE, they can avoid the possibility that the PAE extracts rents. Instead, firms will bargain over licenses under the threat of selling patents to a PAE. In this case the existence of the PAE will still affect the firms' incentives to invest in R&D, but the producing firms will capture all of the total industry surplus, which would eliminate one of the negative effects of patent monetization by PAEs. In this section we re-examine the payoffs from the licensing and litigation subgames when firm can negotiate licenses before selling patents to a PAE, and then re-consider how this affects our results in the R&D investment stage. As before, we proceed by backward induction.

When both firms own more than $\hat{c}$ patents after the R&D stage, the PAE does not extract rents from the bilateral negotiations. In those cases firms are indifferent between bargaining licenses among them or selling patents to the PAE in bilateral negotiations, because the PAE does not reduce the joint surplus of the producing firms. In other words, when the continuation game has an efficient equilibrium firms are indifferent between trading patents before or after engaging with the PAE.

In patent allocations such that one of the firms owns fewer than $\hat{c}$ patents and the other firm has more than $\hat{c}$ patents after the R&D stage, firms will jointly lose surplus if they do not negotiate licenses before bargaining with the PAE. In this case Proposition 2 shows that the PAE would be able to extract $(1-s)[Vn-T(n,m-m')]$ from the bilateral negotiation stage, where m' is the number of patents sold by firm B to the PAE.

By negotiating, firms can avoid this loss in suplus due to the PAE. Firms will bargaining over licenses under the threat of bilaterally bargaining with the PAE. We assume without loss of generality that $n > \hat{c} \geq m$ patents. In the bilateral negotiation with the PAE, Firm A splits surplus with the PAE, which depends on the number of patents that Firm B sold to the PAE. Therefore, it is optimal for Firm B to threaten to sell nothing to the PAE if the licensing negotiation with Firm A fails. This strategy maximizes the surplus that is bargained over between Firm A and Firm B. Let b be the bargaining power of Firm A in this negotiation with Firm B. Using the results in Proposition 2, we can show that firms trade licenses and the payoffs are given by the Nash bargaining solution:

$$\begin{aligned}\pi_A =& \pi + T(n,m) + s[Vn - T(n,m)] + b(1-s)[Vn - T(n,m)],\\ \pi_B =& \pi - Vn + (1-b)(1-s)[Vn - T(n,m)].\end{aligned}$$

Defining $\mu = s(1-b) + b$ the payoffs can be written as:

$$\pi_A = \pi + T(n,m) + \mu[Vn - T(n,m)], \quad \pi_B = \pi - Vn + (1-\mu)[Vn - T(n,m)].$$

Combining these payoffs with those in Proposition 2 for the rest of the patent allocations, we can find the continuation payoffs of the firms after the R&D stage.[27]

Proposition 5. *When firms bargain over licenses under the threat of bilateral trade with the*

[27]The proof is omitted because it follows analogously to that of Proposition 2.

PAE, the equilibrium payoffs for Firm A are:

$$\pi_A = \pi + \begin{cases} V(n-m) & n > \hat{c} \;\; \textit{and} \;\; m > \hat{c} \\ T(n,m) + \mu[Vn - T(n,m)] & n > \hat{c} \;\; \textit{and} \;\; m \leq \hat{c} \\ -Vm + (1-\mu)[Vm - T(m,n)] & n \leq \hat{c} \;\; \textit{and} \;\; m > \hat{c} \\ 0 & n \leq \hat{c} \;\; \textit{and} \;\; m \leq \hat{c} \end{cases}$$

Firm B's payoffs are symmetric to Firm A's.

In particular, in the case $N > 3\hat{c}$, we have that $T(n,m) = V(n-m)$ for any $m \leq \hat{c}$, and the following payoffs:

$$U^A_{PAE}(n) \equiv= \pi_A(n, N-n) = \pi + \begin{cases} (2-\mu)Vn - VN & n \leq \hat{c} \\ V(2n-N) & n > \hat{c} \;\; \text{and} \;\; m > \hat{c} \\ (2-\mu)Vn - (1-\mu)VN & n \geq N - \hat{c} \end{cases}.$$

Figure 15 depicts the continuation payoff for Firm A in the case $N > 3\hat{c}$ below.

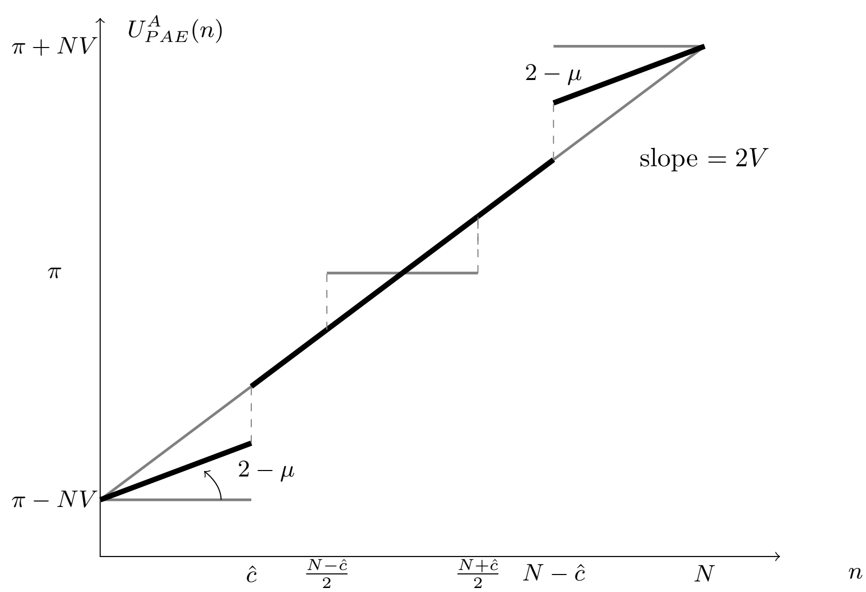

Figure 15: Consider a producing firm which discovers n components, while its rival discovers $N-n$, and $N > 3\hat{c}$. The firm's payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE. Notice that $\mu = s(1-b) + b \leq 1$.

This figure shows the effects of the PAE on continuation payoffs, compared to the case without PAEs. We see two effects that are also present in the main version of the model. First, the payoff function is steeper in the middle region, because in the game without a PAE patents would not be monetized, while in the game where firms can negotiate licenses under the threat of selling patents to a PAE, they can credibly monetize the value of their patents in this region. Second, at the extremes, the threat of selling patents to a PAE destroys the defensive value of a small patent portfolio. In this case the payoff function becomes flatter, as a function of the firm's portfolio, but the levels of payoffs become more extreme.

However, the fact that firms can negotiate licenses instead of selling patents to a PAE means that the PAE extracts no surplus and total profits of the producing firms do not decrease. I.e. the area under the payoff function is the same in the games with and without a PAE. This eliminates one of the channels through which PAE activity decreases the incentives to invest in R&D. Firms will split the surplus that the PAE would extract if their negotiation fails, and that implies the slopes at the extremes are $2-\mu$. The parameter $\mu = s + b(1-s)$ measures how the surplus that the PAE can potentially extract from the firms is divided between Firm A and Firm B.

Next, consider the R&D investment stage in the case where firms anticipate the payoffs from negotiating licenses under the threat of selling patents to a PAE. The analysis in this case precisely follows that in Section 6, except we have to replace Propositions 3 and 4 with the following two propositions.[28]

Proposition 6. *When $N > 3\hat{c}$, for symmetric R&D investments $x = y = x^*_{PAE}$ we have:*

- *The rent extraction effect (RE) is always zero.*
- *The winner premium (WP) effect is strictly positive and equal to*

$$WP(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{h'(x^*_{PAE})}{2^{N+1} h(x^*_{PAE})} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)\Delta(k) > 0.$$

- *The marginal effect of PAEs is always positive.*

Most importantly, this proposition differs from the one in the text because the rent extraction effect is always 0 when the PAE cannot capture bargaining rents. We can now also see how the threat of selling patents to a PAE affects the equilibrium R&D investments relative to the game where a PAE does not exist, analogously to Proposition 4.

[28]The proofs are again omitted because they closely follow those of the corresponding propositions in the text.

Proposition 7. *Suppose symmetric equilibria exist in the games with and without the PAE, and the equilibrium investments are such that* $h(x) > \frac{r \ln N}{2}$. *Total R&D investments are larger in the equilibrium with the PAE than in the equilibrium without the PAE.*

Similarly to our main result, this proposition shows that the possibility of monetizing patents through a PAE can *increase* the equilibrium level of R&D investments. Hence our results on welfare continue to hold in the case where firms can negotiate licenses before trading with a PAE, and are in fact stronger in this case.