Optimal opaqueness: Multi-product monopoly pricing without contractual restrictions*

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Abstract

This paper studies multi-product monopoly problems on the Hotelling line with a continuum of uniformly distributed consumers who are privately informed about their locations and have linear transportation costs. The setup gives rise to new challenges for *market definition* and to merger synergies without technological returns to scale and contractual restrictions. If the consumers' gross valuation exceeds one half, the multi-product monopoly optimally sells some consumers opaquely priced products, leaving them in the dark as to what they obtain until after the purchase. In contrast, independent sellers serve their customers at a uniform price without competing with each other if the gross valuation is less than one. With optimally placed products, every product is also optimally opaquely priced. The popular focus on uniform (or market-clearing) prices in multi-product monopoly pricing problems is therefore without loss of generality if and only if the multi-product monopoly optimally sells the goods in the same way as independent sellers do. Extensions analyze vertical differentiation, optimally placing and selling many goods and quadratic transportation costs.

Keywords: multi-product monopoly, private information, lotteries, mechanism design
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1 Introduction

The study of multi-product monopoly (MPM) pricing has a long tradition in economics and considerable real-world relevance as firms typically sell multiple products, with single-product sellers being the exception rather than the rule. The literature on MPMs either assumes that the firm sells each product at a market-clearing (or uniform) price or imposes other contractual restrictions to prevent the vacuous full surplus extraction and efficiency that arises under perfect price discrimination. However, ad-hoc restrictions of this form, even when empirically validated, are problematic because it is unclear, and difficult to evaluate, whether restrictions that may be satisfied before a policy change will continue to hold afterwards. For example, consider two single-product sellers of horizontally differentiated products who use uniform pricing. If the two sellers merge will, it be optimal for the merged entity to continue to set uniform prices? To answer these questions and to avoid contractual restrictions, this paper takes a mechanism design approach with incomplete information to MPM pricing. Specifically, we study variants of the Hotelling model with linear transportation costs, uniformly distributed locations and assume that each buyer is privately informed about its location on the unit interval. These assumptions imply that the firm faces a mechanism design problem with one-dimensional private information, which makes the analysis amenable to the methodology in the tradition of Myerson (1981).

Assuming the firm has zero marginal costs for producing the two products, we use this framework to revisit classic problems. We show that much of perceived wisdom is either misleading or incomplete. If the firm's products are at fixed locations, focusing on market-clearing pricing by a MPM is without loss of generality if and only if the MPM prices in the same way as two single-product firms. If the firm places its products optimally, it never relies entirely on market-clearing prices. Rather, it optimally offers an additional, opaquely priced product—formally, a fifty-fifty lottery between the left and the right product—that leaves consumers in the dark as to what they bought until after the purchase. The opaque product is inefficient because it represents (deliberate) random allocation that leads to excessive transportation costs. It is profitable for the firm because it reduces the consumers' information rent, which allows it to extract more revenue from buyers with location closer to the two products.[1] Among other things, the model thus provides a simple rationale for inefficient but profitable *product proliferation* (see, for example, Schmalensee, 1978). The optimal placement problem also brings to light subtle and interesting forces. If the buyers' gross valuation is large enough that every buyer is always optimally served, the firm opti-

[1]Technically, the optimality of opaqueness arises from the countervailing incentives inherent to the Hotelling model, which, loosely speaking, mean that buyers with small (large) locations face incentive compatibility constraints that are upward (downward) binding; see Loertscher and Muir (2024a) for details.

mally places its products at 3/16 and 13/16 rather than at 1/4 and 3/4, where the latter is the placements that minimize consumers' transportation costs. The inefficiency of the optimal selling mechanism induces the firm to choose inefficient placements.[2]

The analysis also sheds new light on pertinent issues in antitrust and regulation. Normalizing the maximum transportation cost to one and assuming that the products are exogenously given at 0 and 1, two independent sellers would optimally sell their products at a market-clearing price if the buyers' gross valuation is between one half and one. As their optimal prices are half of the buyers' gross valuation, the two sellers' market coverage does not overlap. An outside observer such as an antitrust authority faced with a proposed merger by the two sellers might therefore conclude that, because there is no overlap, the two markets are independent and no harm to consumers will arise from a merger. However, a MPM sells opaque products as soon as the buyers' gross valuation exceeds one half and prices the "pure" products at the gross value minus a quarter. Thus, the merger that would lead to a MPM and would be deemed neutral based on traditional measures of market definition would reduce consumer surplus—it raises all pure prices and gives zero utility to buyers who obtain the opaque product—while being profitable and potentially increasing social surplus.[3] With regard to regulation, we show that a Ramsey regulator whose aim it is to maximize a convex combination of the multi-product firm's profit and social surplus can achieve its objective by imposing an appropriately chosen price ceiling. This is true whether or not the firm is free to choose the placement of its products. It merely assumes that the firm respects the price constraints imposed by the regulator. Otherwise, the firm is free to choose the optimal selling mechanism (and placements). This sufficiency of price ceilings contrasts with problems with production in which randomization arises because of a non-regularity in the agents' type distribution (see e.g. Loertscher and Muir, 2024b).

Extensions allow for quadratic transportation costs, vertical differentiation, and many products.

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[2]The underlying forces are subtle and do not lack a certain irony: Given an efficient selling mechanism (that is, market-clearing pricing), the firm would optimally choose efficient placements, and given efficient placements, its optimal selling mechanism would be efficient. Yet, the benefit of opaqueness is large enough to make it choose inefficient placements and an inefficient selling mechanism. On a superficial level, this inefficiency contrasts with Spence (1976). However, his analysis assumes perfect price discrimination, which with private information is not possible due to consumers' incentive compatibility constraints. The optimality of inefficient placements arises because we assume consumers (and products) are located on a line. If in contrast they were located on a circle as in Salop (1979), then the firm would choose efficient placements for the products and then use the inefficient selling mechanism involving opaquely priced products.

[3]The latter occurs if the buyers' gross valuation is not bigger than 5/6.

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