Outsourcing, Vertical Integration, and Cost Reduction*

Simon Loertscher† University of Melbourne

Michael H. Riordan‡ Columbia University

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Abstract

We study a buyer's incentives to source internally or externally in a stylized model of procurement. In stage one, suppliers invest in cost reductions. In stage two, the suppliers compete in prices. In stage three, the buyer selects a supplier. Vertical integration gives the buyer the option to source internally, which is advantageous for the buyer as it avoids a markup payment, but disadvantageous insofar as it discourages investments by independent suppliers. Just as suggested by Williamson's puzzle of selective intervention, the integrated firm can do exactly the same as the two stand alone entities, and can sometimes do better. But this ability to do better has detrimental incentive effects on cost reduction investments by the non-integrated suppliers. We derive conditions under which these detrimental effects outweigh the advantageous effects of vertical integration.

Keywords: outsourcing, vertical integration, puzzle of selective intervention.
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†Department of Economics, Level 4, FBE Building, University of Melbourne, 111 Barry St, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

‡Economics Department, 1038 International Affairs Building, 420 West 118th Street, New York, NY 10027 Email: mhr21@columbia.edu.

1 Introduction

A dramatic transformation of American manufacturing ocurred at the end of the twentieth century, away from vertical integration and toward outsourcing (Whitford, 2005). By the 1990's, outsourcing was widespread, to the point that even vertically integrated firms heavily relied on independent upstream suppliers (Atalay, Hortaçsu, and Syverson, 2014). This trend toward outsourcing went increasingly hand in hand with offshoring, until recently firms began re-evaluating the costs and benefits associated with offshoring. This recent empirical importance of outsourcing gives renewed salience to the puzzle of selective intervention posed by Williamson (1985): Why can't a merged firm do everything the two separate firms can do, and do strictly better by intervening selectively?

Our approach to the puzzle of selective intervention is to view the decision of whether to vertically integrate or to divest as occurring in a multilateral setting. The procurement environment we have in mind is motivated by Whitford (2005) description of new customer-supplier relationships that shifted and blurred the boundaries of firms, as original equipment manufacturers increasingly relied on independent suppliers for both the production and the design of specialized parts. A "customer" seeks to commercialize a new product or improve (or to expand the distribution of) an existing one in a downstream market for which the design of a specialized input process potentially has significant cost consequences. The customer has access to a group of qualified suppliers with different ideas and capabilities who invest in product and process design to prepare proposals for supplying the input. The customer selects the most attractive supply source, and a vertically integrated customer has the additional option to source internally if that is more cost effective.

This basic procurement problem capturers an array of applications. For example, the customer could be PepsiCo, which required a special sort of potatoes as input for expanding its potato chips business in China, and had the option of integrating with local producers or of sourcing externally from independent suppliers (Tap, Lu , and Loo, 2008). Alternatively, the customer could be AT&T which needed to procurement telecommunicaions equipment from an upstream industry including Ericsson, Nortel or Lucent, which it strategically divested in 1996 (Lazonick and March, 2011); or the customer could be Microsoft whose sources for the Window Phone 8 include the independent suppliers HTC, Huawei and Samsung, and Nokia, which it recently acquired.

Vertical integration has a tradeoff in such settings. On the one hand, there are rent-seeking and efficiency advantages from avoiding a markup when the input is sourced internally. Markup avoidance shifts rents away from lower cost independent suppliers by distorting the sourcing decision, and also increases efficiency because the project is pursued whenever its value exceeds the cost of internal sourcing. On the other hand, vertical integration has a disadvantageous "discouragement effect" on the investment incentives of the independent suppliers. Because the procurement process is tilted in favor of internal sourcing, independent suppliers are less inclined to make cost-reducing investments in the preparation of proposals. Furthermore, it is costly for the integrated firm to compensate by increasing its own *ex ante* investment, and, if the net investment discouragement effect outweighs the markup avoidance advantages, then the customer has reason to divest its internal division as a way to commit to a level playing field.

This is the tradeoff we study. The following is a sketch of the model we analyze. There is a buyer who procures a fixed input from a set of upstream suppliers via a tender. Prior to the tender, all suppliers make cost reducing investments that shift the support of the distribution from which costs are drawn. Absent integration, all suppliers then bid in a first-price reverse auction, and the buyer procures from the supplier with the lowest bid. For simplicity, the cost of investment is assumed to be quadratic, and the cost distribution is assumed to be exponential. Under vertical integration, the buyer owns one of the suppliers. The tender is still a first-price reverse auction, but now the integrated supplier has a preferred supplier status because he can produce whenever his realized cost is less than the lowest bid from any of the independent suppliers. If the buyer does not source internally, she buys from the independent supplier with the lowest bid.

Keeping investments fixed, vertical integration is always profitable in this model as it allows the buyer to captures rents from the independent suppliers by avoiding to pay the bid markup whenever she sources internally. Vertical integration is also detrimental to social welfare because the lowest cost supplier does not necessarily produce the input. (This is the case when an independent supplier draws the lowest cost but bids above the cost of the integrated supplier.) In contrast, without integration production is always efficient because the unique equilibrium of the first-price auction is symmetric and monotone. Moreover, the socially optimal investments, given that sourcing is efficient, are always an equilibrium outcome without integration. However, because it changes the buyer's make-or-buy decision, vertical integration also affects the incentives to invest in cost reductions away from the social optimum. In equilibrium, the integrated supplier overinvests while the non-integrated suppliers underinvest relative to the social optimum (and also relative to the second-best solution to the social planner's problem, which takes sourcing decisions as given and maximizes welfare over investments). Because investment costs are convex, the additional costs that accrue to the integrated firm from this excessive investment in equilibrium can be so large that they outweigh the benefits from integration.

As mentioned, in our model vertical integration effectively establishes a preferred supplier, which can be thought of as a supplier who submits his bid after all independent suppliers have submitted theirs. As in Burguet and Perry (2009), the preferred supplier limits the market power of the other suppliers. In our setup, the integrated firm avoids giving away rents by allocating production to its upstream division whenever its cost is below the low bid. These allocation distortions from a preferred suppler are similar to those analyzed by Burguet and Perry (2009). However, as a result the endogenous investments in cost reductions in our model, the preferred supplier has a more favorable cost distribution than the independent suppliers. This contrasts with the model of Burguet and Perry which assumes identical cost distributions.

Our emphasis on multilateral supply relationships, and in particular the argument that vertical is motivated partly by rent-seeking, is reminiscent of Bolton and Whinston (1993).[1] However, their model assumes efficient bargaining process under complete in-

[1]Integration also occurs in a multi-lateral setting in the models of Riordan (1998) and Loertscher and

formation to allocate scarce supplies. Vertical integration creates an "outside option" of the bargaining process that for given investments only influences the division of rents. In contrast, our model features incomplete information about costs, and for given investments vertical integration impacts the sourcing decision as well as the division of rents. Moreover, in our model the rent-seeking advantage of vertical integration leads to *ex post* sourcing distortions, which in turn create *ex ante* investment distortions relative to the first-best. In contrast, in the BW model the integrated downstream firm overinvests to create a more powerful outside option when bargaining with independent customers, but the *ex post* allocation decision is efficient conditional on investments. Consequently, the two models give rise to starkly different conclusions. For the case that corresponds to the unit model featured in our analysis, Bolton and Whinston find that outsourcing is never an equilibrium market structure although it is socially efficient.[2] The reason for this conclusion is that, as long as the outside option of sourcing from its own supplier is binding, the investment disincentives of the independent firm do not matter for the profits of the integrated firm. In contrast, non-integration can be privately advantageous in our setup because the investment disincentives of the independent sector matter for the profits of the integrated firm.

Most recent theories of vertical integration frame the problem in bilateral terms, focusing on how agency problems inside an integrated firm compare with contracting problems across separate firms. As Cremer (2010) explains, the key to these theories is that the "principal does not quit the stage" after vertical integration, meaning that contracts between the owner (principal) and managers unavoidably are incomplete. Thus, anticipating expropriation by an owner who is unable to commit, an employee-manager has low-powered incentives to invest in the relationship. The current theories are most compelling for evaluating incentive tradeoffs surrounding the vertical acquisition of an owner-managed firm. As observed by Williamson (1985), however, the explanation for vertical integration is more elusive when a separation of ownership and control prevails and diminishes incentives both upstream and downstream irrespective of the identity of the owner Our approach is to view the procurement problem in multilateral terms by embedding it in a broader market context while abstracting from agency problems inside the firm. Like in most contemporary theories, the principal does not quit the stage in our theory either. However, the problem with vertical integration in our model is not an inability of the owner to make commitments to managers, but rather an inability of the vertically integrated firm to make credible commitments to independent firms on whom it wants to rely for procurement.

If the vertically integrated firm simply replicated the way it procured before integrating, the profit of the integrated entity would just be equal to the joint profit of the two independent firms. However, just like Williamson (1985) argued, it can do strictly better than that because it can now avoid paying the markup for procuring from outside

Reisinger (2014). However, the setups in these papers are different as the upstream supply is perfectly competitive and vertical integration is a continuous variable, and their focus is on the competitive effects of vertical integration rather than on the incentives to integrate.

[2]See their Proposition 5.2, in which $\lambda = 1$ corresponds to the case with unit supply and demand in our model.

suppliers whenever the cost of production of the integrated supplier is below the lowest bid of the outside suppliers. In this sense, the vertically integrated firm's flexibility to change its behavior after integration is to its benefit. This seems to contrast sharply with the existing literature, where the vertically integrated firm's inability to commit may render integration unprofitable (Cremer 2010). But it raises the question why vertical integration would not always be profitable in our model. Essentially, the answer is that, because the integrated firm procures differently, the incentives for the outside suppliers to invest in cost-reduction decrease. This effect can be so strong that it dominates all the benefits from integration. It is exactly the ability of the vertically integrated firm to do better than it does without integration that ultimately may hinder it from so doing because this ability changes the investment behavior of the outside suppliers, which is outside the control of the integrated firm.

Our solution to the puzzle of selective intervention might be interpreted as joining the rent-seeking theory of the firm with the property-rights theory (Gibbons, 2005).[3] Our version of of the rent-seeking theory builds on Burguet and Perry (2009) to explains how a preferred integrated supplier creates a sourcing distortion, and hence changes the magnitude of the joint surplus of an upstream industry and a downstream customer. This theory is reminiscent of an older industrial organization literature that focused on how vertical integration matters for the exercise of market power.[4] Our version of the property-rights theory builds on Riordan (1990) to explain how inefficient *ex post* sourcing changes *ex ante* investments which also determine the joint surplus. The contemporary property-rights literature has focused on how vertical integration matters for relationship-specific investments, typically under the assumption of efficient bargaining, which of course implies efficient sourcing *ex post*.[5]

Lastly, the multilateral setting that is at the heart of our model permits a formaliza-

[3]Emphasizing the standard assumption of efficient *ex post* bargaining in the property-rights literature, Gibbons (2005, p. 205) summarizes the difference between the two as follows: *(I)n the property rights theory, the integration decision determines ex ante investments and hence total surplus, whereas in the rent-seeking theory, the integration decision determines ex post haggling and hence total surplus.*

[4]This literature, summarized by (Perry, 1989), has different strands. For example, backward vertical integration is motivated by the downstream firm's incentive to avoid paying above-cost prices to upstream suppliers of inputs . In the double markups strand, vertical integration of successive monopolists improves efficiency by reducing the markup to the single monopoly level. In the variable proportions strand, a non-integrated firm inefficiently substitutes away from a monopoly-provided input at the margin, and vertical integration corrects the resulting input distortion. (In our model, while alternative suppliers offer substitute inputs, there is no input distortion if upstream market power is symmetric.)

[5]Williamson (1985) argues that asset specificity, incomplete contracts, and opportunism conspire to undermine efficient investments. Grossman and Hart (1986) and Hart and Moore (1990) formalize the argument by modeling how asset specificity and incomplete contracting causes a holdup problem that diminishes the investment incentive of the party lacking control rights over productive assets, while Bolton and Whinston (1993) add that vertical integration may cause investment distortions motivated by the pursuit of a bargaining advantage. Riordan (1990) argued in a different vein, not assuming efficient bargaining, but still consistent with Cremer's interpretation of contemporary theories, that the changed information structure of a vertically integrated firm creates a holdup problem because the owner cannot commit to incentives for the employee-manager. The basic technological assumptions in our model extend those in Riordan (1990) to a multilateral setting while abstracting from the internal holdup problem.

tion of the notion that the extent of the market determines the division of labor which fares prominently in the work of Adam Smith (1776) and Stigler (1951). In our setup, as the extent of the market – measured by the number of suppliers – is small, there is a strong incentive for the customer to vertically integrate and to source internally. As the extent of the market increases, these incentives diminish, and reliance on outside supply and the division of labor increase. Moreover, our model adds to that the insight that, for a given number of suppliers, the division of labor – measured as the probability that the customer sources externally – decreases as the customer integrates with one of the suppliers, and so do the investments of the independent suppliers. Taking the independent suppliers' investment as a measure of the extent of the market, the extent of the market in our model is therefore also a function of the vertical structure of the customer.

The remainder of the paper is organized as follows. Section 2 lays out the model. In Section 3, we analyze bidding behavior, equilibrium with and without vertical integration, and derive a condition for outsourcing to be advantageous for the customer. We also perform a first-best and a second-best welfare analysis and develop intuition for the results. In Section 4, we analyze a natural bargaining game that allows us to determine the market structure endogenously. Section 5 extends the model in multiple ways and shows that our main results are robust by relaxing a number of assumptions imposed in Section 2. Section 6 concludes. Proofs are in the Appendix.

2 Model

There is one downstream firm, called the customer, who demands a fixed requirement of a specialized input for a project, and there are n upstream firms, called suppliers, capable of providing possibly different versions of the required input. Each of the suppliers makes a non-contractible investment in designing the input by exerting effort before making a proposal. *Ex ante*, that is, prior to the investment in effort, a supplier's cost of producing the input is uncertain. *Ex post*, that is after the investment, every supplier privately observes his cost realization. More effort shifts the supplier's cost distribution downward in the sense of first-order stochastic dominance, and, therefore, shifts average cost downward.

In the main model, we assume that the customer's demand is inelastic. Specifically, she buys the input from the cheapest supplier no matter what price. It captures in the extreme the idea that the likely value of the downstream good is very large relative to the likely cost of the input. This might be so for a highly valuable and differentiated downstream product. In Section 5, we extend the model to allow for an elastic demand function by assuming that the customer's value is a random draw.

There are two possible modes of vertical market organization. The customer either is independent of the n suppliers, which we refer to interchangeably as "non-integration" or "outsourcing", or is under common ownership with one of the suppliers, which is referred to as "integration". Restricting attention to limited partial integration serves to focus the analysis on vertical rather than horizontal market structure.[6]

Timing We study a two-stage game in which the vertical market structure – integration or non-integration – is given at the outset and common knowledge.

Stage 1: In stage 1, all suppliers i simultaneously make non-negative investments x_i, $i = 1, .., n$. The cost of investment x is $\Psi(x) = \frac{a}{2}x^2$, where $a > 0$ is a given parameter. The effect of investment x_i on costs is that it shifts the mean of the distribution $G(c_i; x_i)$ with support $[\underline{c}(x_i), \infty)$ from which i's cost of production c_i will be drawn in stage 2. Specifically, we assume that

$$G(c_i + x_i) = 1 - e^{-\mu(c+x_i-\beta)}$$

and

$$\underline{c}(x_i) = \beta - x_i,$$

where $\mu > 0$ and $\beta > 0$ are parameters of the exponential distribution.

The distribution of the minimum cost with n suppliers with a vector of investments $\mathbf{x} = (x_1, ...x_n)$ satisfying $x_1 \geq x_2... \geq x_n$ is for $c \geq \beta - x_n$

$$L(c; \mathbf{x}) = 1 - \prod_{i=1}^{n} [1 - G(c + x_i)] = 1 - e^{-n\mu(c-\beta)-\mu\sum_{i=1}^{n} x_i} \tag{1}$$

and for $c \in [\beta - x_j, \beta - x_{j+1}]$ with $j \geq 1$ it is

$$L(c; \mathbf{x}) = 1 - e^{-j\mu(c-\beta)-\mu\sum_{i=1}^{j} x_i}. \tag{2}$$

If the investments are symmetric, that is $x_i = x$ for all i, then

$$L(c + x, n) \equiv 1 - [1 - G(c + x)]^n = 1 - e^{-n\mu(c+x-\beta)}.$$

All the distributions are defined on an extended support, so that, for example, $G(c+x) = 0$ and $L(c+x, n) = 0$ for all $c \leq \beta - x$. The investment x_i and the cost realization c_i are private information of supplier i. The mean-shifting investments in our model are the same as in the Laffont and Tirole (1993) model of procurement. In contrast to the typical Laffont-Tirole model, however, supplier heterogeneity is realized after investments and the realized cost is the private information of the supplier.

Stage 2: In stage 2, the customer solicits bids from the suppliers in a reverse auction. For now, we assume that there is no reserve price, which can be justified on the ground that the precise input specifications are non-contractible *ex ante*, and the buyer cannot commit to reject a profitable offer. All suppliers i simultaneously make an *ex ante* effort choice x_i and then privately observe their *ex post* costs c_i, where $i = 1, .., n$.

[6] A more ambitious analysis could also consider horizontal consolidations that bring addition suppliers under common ownership, but a thorough analysis along such lines needs a richer model of downstream market structure to consider adequately the antitrust issues. If the upstream industry were diversified horizontally into supplying other downstream firms in the same market or into other markets entirely, then a horizontal consolidation of the industry would attract antitrust scrutiny.

Under non-integration, each supplier bids a price b_i in a first-price auction. The bids $\mathbf{b} = (b_1, .., b_n)$ are simultaneous. The customer selects the low-bid supplier. Under integration, supplier 1 is owned by the customer. The remaining $n - 1$ independent suppliers simultaneously each submit a bid b_i. The customer sources internally if $c_1 \leq \min\{\mathbf{b}_{-1}\}$, purchases from the low-bid independent supplier if $\min\{\mathbf{b}_{-1}\} \leq \min\{c_1\}$. In Section 4, we endogenize the market structure by analyzing a bargaining model by adding a stage 0 in which the buyer makes take-it-or-leave-it offers for acquiring or divesting the supply unit.

Section 5 considers robustness to a number of extensions: non-quadratic cost of investment, different parametric cost distributions, downward sloping demand, reserve prices, and agency problems inside the firm. Some our of preliminary results and, more importantly, the general nature of the tradeoffs between outsourcing and vertical integration depend neither on exponential cost distributions nor on quadratic investment costs. However, the comparison of the benefits and costs of alternative organizations of procurement requires parametric functional forms, and the quadratic-exponential specification is particularly convenient. What it means exactly to put the customer and supplier 1 under common ownership is a matter of interpretation. In the spirit of the property-rights theory of the firm, one can think of the customer as having control rights over a downstream production process, and vertical integration as the acquisition of those control rights by one of the suppliers, who thus gains the ability to exclude rivals from supplying the customer. Admittedly, under the assumption of inelastic demand it is awkward to imagine control rights with infinite value, but the awkwardness is removed by allowing for downward-sloping demand. Alternatively, one can think of of the customer as acquiring the assets of an upstream supplier. This interpretation seems deficient because it abstracts from from the problem of motivating the integrated supplier to invest, but the apparent deficiency can be remedied by interpreting the cost of investment to include agency costs.

3 Analysis

We now turn to the analysis of our model. We first derive the equilibrium bidding function of the independent suppliers, which is independent of the vertical market structure. Then we derive in turn the equilibrium under outsourcing and vertical integration, respectively. In Section 3.4, we compare the benefits and costs of vertical integration relative outsourcing from the perspective of the customer and the integrated supplier. Section 3.5 studies the planner's investment problem under first- and second-best scenarios, and Section 3.6 develops intuition for the results.

3.1 Bidding

Bidding under Outsourcing The equilibrium bidding function $b_O(c)$ under outsourcing when all n independent suppliers invest the same amount x is well known from auction theory. The auction being a first-price procurement auction, $b_O(c)$ is equal to the expected value of the lowest cost of any of the $n - 1$ competitors, conditional on this cost being larger than c. That is

$$b_O(c) = \frac{\int_c^\infty y dL(y+x, n-1)}{1 - L(c+x, ; n-1)} = c + \frac{1}{\mu(n-1)}.$$

The constant hazard rate of the exponential results in constant markup bidding.

Given that we confine attention to symmetric equilibria, the focus on symmetric investments x for the equilibrium bidding function is without loss of generality: supplier i's deviation to some $x_i \neq x$ will not be observed by any of its competitors, and any bidder i's equilibrium bid does not depend on its own distribution, only on its own cost realization. Consequently, if i deviates to some $x_i < x$, it will optimally bid according to $b_O(c)$ for any possible cost realization. On the other hand, if $x_i > x$, i's optimal bid will simply be $b_O(\beta - x)$ for all $c \in [\beta - x_i, \beta - x]$ and $b_O(c)$ for all $c > \beta - x$.

Bidding under Vertical Integration Vertical integration effectively establishes a preferred supplier, who serves to limit the market power of non-integrated suppliers as in Burguet and Perry (2009). Let x_1 be the equilibrium investment level of the integrated supplier and x_2 be the symmetric investment level of all non-integrated suppliers.

The equilibrium bidding function $b_I(c)$ of the non-integrated suppliers is then such that

$$c = \arg\max_z \left\{ [b_I(z) - c] \left[1 - G(b_I(z) + x_1)\right] [1 - G(z + x_2)]^{n-2} \right\}.$$

As G is exponential and assuming $x_1 \geq x_2$, $b_I(c)$ is such that

$$c = \arg\max_z \left\{ [b_I(z) - c] \, e^{-\mu(b_I(z) + (n-2)z)} k \right\},$$

where $k = e^{-\mu(x_1 + (n-2)x_2 - (n-1)\beta)}$ is a constant (that is, independent of z and $b_I(z)$). The first-order condition, evaluated at $z = c$, is

$$\left[b_I'(c) - \mu(b_I(c) - c)(b_I'(c) + n - 2)\right] \left[e^{-\mu(b_I(z) + (n-2)z)} k\right] = 0.$$

Imposing the boundary condition $\lim_{c \to \infty} b_I(c) - c = 0$, this differential equation has the unique solution

$$b_I(c) = c + \frac{1}{\mu(n-1)}. \tag{3}$$

Observe that $b_I(c) = b_O(c)$. That is, provided $x_1 \geq x_2$, equilibrium bidding by the non-integrated suppliers is independent of the form of the vertical market structure. Notice also that $b_I^{-1}(c) = c - \frac{1}{\mu(n-1)}$.

Below we will show that there is an equilibrium satisfying $x_1 \geq x_2$. Showing that $x_1 \geq x_2$ in equilibrium is straightforward as unilateral deviations from a prescribed equilibrium level x_1 will not be observed by the non-integrated suppliers and will thus not affect equilibrium bidding off the equilibrium path. As with outsourcing, downwards deviations $x_i < x_2$ by $i = 2, .., n$ will never induce i to bid differently from what $b_I(c)$ prescribes. If the independent supplier i invested more than x_2, he will, obviously, bid according to $b_I(c_i)$ for all $c_i \geq \beta - x_2$ for nothing changes in his optimization problem at the bidding stage compared to the case where $x_i = x_2$. If $x_i > x_2$, cost realizations $c_i < \beta - x_2$ occur with positive probability. For these realizations, the optimal bidding for i is as described in Lemma

Lemma 1 *For cost realizations $c_i < \beta - x_2$, bidder i's optimal bid $b(c_i)$ satisfies*

$$b(c_i) = \begin{cases} \beta - x_2 + \frac{1}{\mu(n-1)} & \text{if} \quad \beta - x_2 \geq c_i \geq \beta - x_2 - \frac{1}{\mu}\frac{n-2}{n-1} \\ c_i + \frac{1}{\mu} & \text{if} \quad \beta - x_2 - \frac{1}{\mu}\frac{n-2}{n-1} \geq c_i \geq \beta - x_1 - \frac{1}{\mu} \\ \beta - x_1 & \text{otherwise} \end{cases}$$

if all other independent suppliers invest x_2 and the integrated supplier invests x_1 with $x_1 \geq x_2$.

The bidding $b(c_i)$ is useful for analyzing deviations from a candidate equilibrium in which suppliers invest symmetrically.

3.2 Outsourcing

The expected profit at the investment stage of supplier i when investing x_i while each of the $n-1$ competitors invests x, anticipating that he will bid according to $b_O(c_i)$ when his cost is c_i with $c_i \geq \beta - x$ and $b_O(\beta - x)$ whenever $c_i < \beta - x$ is

$$\Pi_O(x_i, x) = \int_{\beta-x}^{\infty} [b_O(c) - c][1 - G(c+x)]^{n-1} dG(c+x_i) + \int_{\beta-x_i}^{\beta-x} [b_O(\beta-x) - c] dG(c+x_i) - \frac{a}{2}x_i^2.$$

For $x_i < x$, we have

$$\Pi_O(x_i, x) = \int_{\beta-x_i}^{\infty} [b_O(c) - c][1 - G(c+x)]^{n-1} dG(c+x_i) - \frac{a}{2}x_i^2.$$

Plugging in the expressions for the exponential distributions and integrating, we get for $x_i \geq x$

$$\Pi_O(x_i, x) = x_i - x - \frac{1}{\mu}\frac{n-2}{n-1} + \frac{n-1}{\mu n} e^{-\mu(x_i - x)} - \frac{a}{2}x_i^2$$

and for $x_i < x$

$$\Pi_O(x_i, x) = \frac{1}{\mu n(n-1)} e^{-\mu(n-1)(x-x_i)} - \frac{a}{2}x_i^2$$

The first-order condition for a symmetric equilibrium with $x_i = x^*$ is thus

$$\frac{\partial \Pi_O(x^*, x^*)}{\partial x_i} = \frac{1}{n} - ax^* = 0,$$

yielding $x^* = \frac{1}{an}$ as investment levels in any candidate symmetric equilibrium. That is, in equilibrium marginal costs of investment are equal to expected market shares.[7]

[7]This result – that is, that marginal costs of investment are equal to market shares – holds much more generally than for the exponential distribution and quadratic cost functions we assume here. By the envelope theorem, it holds for any symmetric equilibrium in a model with mean shifting investments.

The equilibrium expected procurement cost to the customer under outsourcing equals the expected low bid. Given symmetric investment levels x, the formula for the equilibrium expected procurement cost is

$$PC_O(x) = \int_{\beta-x}^{\infty} b(c)dL(c+x,n) = \beta - x + \frac{1}{\mu n} + \frac{1}{\mu(n-1)},$$

where $\beta - x + \frac{1}{\mu n}$ is the expected cost production cost given investments x and $\frac{1}{\mu(n-1)}$ is the markup.

Evaluating at the equilibrium value under outsourcing, that is at $x = x^O := \frac{1}{an}$, we thus get the equilibrium value of expected procurement cost $PC_O^* \equiv PC_O(x^O)$ of the customer and the expected profit of a representative supplier $\Pi_O^* \equiv \Pi_O(x^O, x^O)$ as follows:

Lemma 2 *In a symmetric equilibrium under outsourcing, the expected procurement cost PC_O^* of the customer is*

$$PC_O^* = \beta - \frac{1}{an} + \frac{1}{\mu n} + \frac{1}{\mu(n-1)},$$

and the expected profit of a representative supplier is

$$\Pi_O^* = \frac{1}{\mu n(n-1)} - \frac{1}{2an^2}.$$

Such an equilibrium exists if and only if $\frac{\mu}{a} < \frac{n}{n-1}$. In this equilibrium, the procurement cost PC_O^* and the suppliers' equilibrium profit Π_O^* decrease in n.

These formulas have very intuitive interpretations. Expected procurement costs PC_O^* are equal to the sum of the expected cost of production given equilibrium investments x^O, which is $\beta - \frac{1}{an} + \frac{1}{\mu n}$, and the constant bid markup $\frac{1}{\mu(n-1)}$. A supplier's expected equilibrium profit Π_O^* is equal to this markup, times the probability of winning, which is $\frac{1}{n}$, minus the investment costs when investment equals x^O.

3.3 Vertical Integration

We now turn to the equilibrium analysis when the customer is vertically integrated with supplier 1. The integrated firm's maximization problem is now to choose its investment x_1 to minimize the sum of expected procurement costs, denoted $PC_I(x_1, x_2)$, and investment costs $\frac{a}{2}x_1^2$, anticipating that the $n-2$ independent suppliers invest x_2 and bid according to $b_I(c)$ and that it will source externally if and only if the lowest bid of the independent suppliers is below its own cost realization c_1. The expected procurement cost given

$x_1 \geq x_2$ is[8]

$$PC_I(x_1, x_2) = \beta - x_1 + \frac{1}{\mu} - \frac{1}{\mu}\frac{n-1}{n}e^{-\mu(x_1-x_2)-\frac{1}{n-1}}.$$

Consider next a representative non-integrated supplier. Given investments x_1 and x_2 by the integrated supplier and the $n-2$ competing independent suppliers, the expected profit $\Pi_I(x_i, x_1, x_2)$ of an independent supplier i when investing $x_i \leq x_2$ is

$$\Pi_I(x_i, x_1, x_2) = \frac{1}{\mu n(n-1)}e^{-\mu(x_1-x_2)-\frac{1}{n-1}+\mu(n-1)(x_i-x_2)} - \frac{a}{2}x_i^2. \tag{4}$$

A necessary condition for a symmetric equilibrium (symmetric in the investment level x_2 of the independent suppliers) with $x_1 \geq x_2$ is therefore

$$\frac{\partial \Pi_I(x_i, x_1, x_2)}{\partial x_i}|_{x_i=x_2} = \frac{1}{n}e^{-\mu(x_1-x_2)-\frac{1}{n-1}} - ax_2 = 0. \tag{5}$$

Letting Δ be the unique non-negative solution[9] to the

$$a\Delta^I = 1 - e^{-\mu\Delta^I-\frac{1}{n-1}}, \tag{6}$$

the equilibrium values for x_1 and x_2, given by the first-order conditions (8) and (5), can be succinctly expressed as

$$x_1 = \frac{1}{an} + \frac{n-1}{n}\Delta^I \quad \text{and} \quad x_2 = \frac{1}{an} - \frac{1}{n}\Delta^I \tag{7}$$

with $\Delta^I = x_1 - x_2$. Evaluating $PC_I(x_1, x_2)$ and $\Pi_I(x_1, x_2)$ at the equilibrium investment levels, we get that the expected equilibrium procurement cost $PC_I^* \equiv PC_I(x_1, x_2)$ and the expected equilibrium profit $\Pi_I^* = \Pi_I(x_1, x_2)$ of an independent supplier are as follows:

Lemma 3 *The expected cost of procurement of the integrated firm is*

$$PC_I^* = \beta + \frac{a-\mu}{\mu}x_1$$

while the expected of profit of a non-integrated supplier is

$$\Pi_I^* = \frac{1}{\mu(n-1)}ax_2 - \frac{a}{2}x_2^2,$$

where x_1 and x_2 are given by (7).

---

[8] A necessary condition for $PC_I(x_1, x_2) + \frac{a}{2}x_1^2$ to be minimized over x_1 is therefore that

$$-1 + \frac{n-1}{n}e^{-\mu(x_1-x_2)-\frac{1}{n-1}} + ax_1 = 0.$$

Notice that the second order condition for a minimum is $-\mu\frac{n-1}{n}e^{-\mu(x_1-x_2)-\frac{1}{n-1}} + a \geq 0$. Since $e^{-\mu(x_1-x_2)-\frac{1}{n-1}} \leq 1$, a sufficient condition for this to be the case is $\frac{\mu}{a} \leq \frac{n}{n-1}$.

[9] To see that a non-negative solution exists and is unique, observe that both sides of the equation are increasing in Δ. The lefthand side is linear in Δ and equal to 0 at $\Delta = 0$ while the righthand side is concave and positive for any finite n at $\Delta = 0$. Therefore, a non-negative solution exists and is unique.

3.4 Comparison

Vertical divesture, or outsourcing, is mutually profitable for the customer and an integrated supplier if $PC_I^* + \frac{a}{2}x_1^2 + \Pi_O^* > PC_O^*$. The supplier profit under outsourcing Π_O^* can be thought of as part of the opportunity cost of vertically integrated procurement. This amounts to assuming that the integrated firm can sell its supply unit to an independent outside supplier, thereby increasing the number of non-integrated suppliers from $n-1$ to n.

Proposition 4 *Divesture of vertically integrated supplier is jointly profitable if and only if*

$$\Phi(n,\mu) := \frac{a-\mu}{\mu}x_1 + \frac{a}{2}x_1^2 - \frac{2}{\mu n} + \frac{1}{an} - \frac{1}{2an^2} > 0, \tag{8}$$

where x_1 and x_2 are determined by (6) and (7).

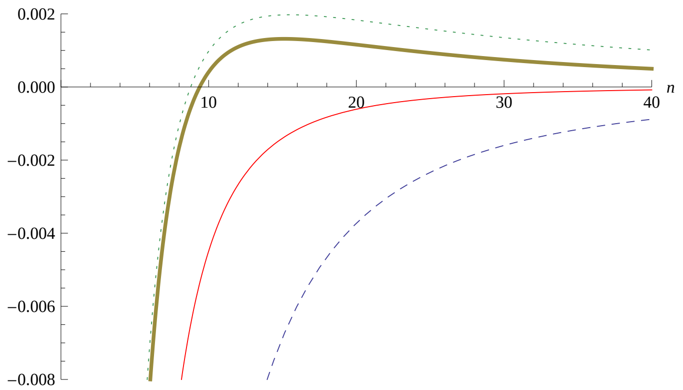


Figure 1: The function $\Phi(n,\mu)$ evaluated at $\mu \in \{0.25, 0.5, 0.75, 1\}$ as a function of n.

The relevant range of parameters satisfy $\mu \leq a\frac{n}{n-1}$; otherwise a symmetric equilibrium under outsourcing does not exist. Figure 1 shows the net benefits of divestiture are positive for sufficiently high values of μ and n within the relevant range of parameters. $\Phi(n,\mu)$ is negative μ is sufficiently small, but may be positive for higher values of μ if n is sufficiently large.

To appreciate this result, it is important to understand the powerful advantages of vertical integration. With inelastic demand and quadratic effort cost, the aggregate investment in effort is the same under non-integration and integration. This follows because the equilibrium marginal costs of effort are equal to market shares which sum to one. Furthermore, since the exponential distribution has a constant hazard rate, the distribution of minimum production cost is more favorable under vertical integration. The support of minimum cost distribution is the union of the supports of the cost distributions of the integrated and independent suppliers, and depends only on aggregate investment on the support of an independent firm. Because the additional investment of the integrated firm shifts its support downward, however, the minimum cost distribution shifts to the left. On top of that advantage of vertical integration, the integrated firm self-sources in some instances, thereby avoiding paying a markup and further reducing its procurement cost compared to non-integration.

From this perspective, the downside to vertical integration might seem more modest. Because the cost of effort is convex, the total effort cost increases as the same total investment is redistributed from independent suppliers to the independent supplier. In other words, even though the vertically integrated firm fully compensates for the investment discouragement of the independent suppliers, it does so at a higher cost. The proposition shows that the higher investment costs can be enough to substantially offset and even outweigh the benefits of vertical integration.

Notice that a "revealed preference argument" that the customer can do no worse by changing its conduct under vertical integration does not apply to this situation because of the response of the independent suppliers. Even though the integrated firm could keep its investment at the pre-integration level but chooses not to, and the integrated firm could source its requirements the same as under non-integration but chooses not to, the other firms nevertheless reduce their investments in equilibrium. All we can conclude from revealed preference is that, given that the other firms reduce their investments, the integrated buyer prefers slightly more to less investment, but this does not allow us to conclude that it is better off with integration.

3.5 Planner's Problem

First-Best It is instructive to compare equilibrium outcomes with those that would obtain if a social planner made the investment and sourcing decisions. Obviously, the planner would always choose to have the supplier with the lowest realized cost produce. The planner's problem is then to choose investments $\mathbf{x} = (x_1, .., x_n)$ to minimize the sum of expected production costs and investment costs $\frac{a}{2}\sum_{i=1}^{n} x_i^2$. Without loss of generality, let $x_i \geq x_{i+1}$. Defining $X_j := \sum_{i=1}^{j} x_i$ and $x_{n+1} := -\infty$, the expected cost of production $EC(\mathbf{x})$ is then given as

$$EC(\mathbf{x}) = \int_{\underline{c}(\mathbf{x})}^{\infty} cdLc; \mathbf{x}).$$

Let $TC(\mathbf{x}) := EC(\mathbf{x}) + \frac{a}{2}\sum_{i=1}^{n} x_i^2$. The planner's problem is then to choose $\mathbf{x}$ to

$$\min_{\mathbf{x}} TC(\mathbf{x}). \tag{9}$$

Proposition 5 *The solution to the planner's problem (9) is symmetric and satisfies* $x_i^* = \frac{1}{an}$ *for all* $i = 1, .., n$ *if and only if* $\mu \leq a$*. For* $\mu \geq a$*, the socially optimal investments are asymmetric and satisfy* $x_1^* = \frac{1}{an} + \frac{n-1}{n}\Delta^*$ *and* $x_i^* = \frac{1}{an} - \frac{1}{n}\Delta^*$ *for* $i = 2, .., n$*, where* Δ^* *is the unique positive number satisfying*

$$a\Delta^* = 1 - e^{-\mu\Delta^*}.$$

Observe that the planner's problem has a unique solution. Notice also that $\Delta^* = 0$ at $\mu = a$ and that Δ^* increases in μ for $\mu \geq a$.[10]

[10]To see that $\Delta^* = 0$ at $\mu = a$, notice that in this case the equality $a\Delta = 1 - e^{-\mu\Delta}$ can be written as $z = 1 - e^{-z}$ with $z = a\Delta$, which only holds if $z = 0$. An easy way to see that Δ^* increases in μ for $\mu \geq a$

The symmetric solution corresponds to the symmetric equilibrium investments under outsourcing, which exists for $\frac{\mu}{a} < \frac{n}{n-1}$. Therefore, the symmetric equilibrium under outsourcing exists even for a parameter range – for $1 < \frac{\mu}{a} < \frac{n}{n-1}$ – for which it is not socially optimal. In contrast, the asymmetric solution differs from the equilibrium investment levels under vertical integration in that in equilibrium the difference between investments is larger than would be socially optimal, that is $\Delta > \Delta^*$ holds. This difference is driven by the sourcing distortion under vertical integration.

Second-Best Likewise, it is of interest to look at the second-best scenario, according to which the planner can choose the investment level x_1 for the integrated supplier and the investment levels x_2 for the $n-1$ independent suppliers, taking as given that there is a sourcing distortion given by the size of the constant markup $\frac{1}{\mu(n-1)}$. Denote by x_1^S and x_2^S the solution values to the planner's second-best problem and let $\Delta^S \equiv x_1^S - x_2^S$.

Lemma 6 *The solution to the planner's second-best problem Δ^S is given by the unique positive number satisfying*

$$1 - \frac{n}{n-1} e^{-\mu\Delta^S - \frac{1}{n-1}} = \Delta^S$$

and satisfies $\Delta^S < \Delta^I$.

That is, in equilibrium there is excessive investment by the integrated supplier and too little little investment by the independent suppliers even relative to the second-best solutions. However, because $\Delta^S > 0$, the symmetric equilibrium investment levels under outsourcing are not socially optimal if there is a sourcing distortion because the buyer has a preferred supplier.

3.6 Discussion

The unique solution to the social planner's investment problem coincides with the symmetric equilibrium outcome under outsourcing when supplier heterogeneity is sufficiently great, that is, when μ is small. Despite the social undesirability of vertical integration, the buyer has the incentive to rely exclusively on outsourced supply only when heterogeneity in the upstream industry is not too great and when the upstream market is not too concentrated. The general intuition for this result is that, by creating a preferred supplier, vertical integration squeezes the profits of the upstream sector by avoiding paying markups, and this benefit dominates the higher production costs that result from sourcing distortions and the discouragement of independent suppliers investments in cost reduction. More precisely there is a positive incentive for vertical integration if the reduction in rents paid to the independent firms exceeds the increase in the total cost of production.

To deepen this intuition, re-consider the second-best planning problem, in which supplier 1 is a preferred supplier of the sort studied by Burguet and Perry (2009), and suppose that the planner is able to reallocate investments away from the independent

is to observe that the function $a\Delta$ is trivially independent of μ while the function $1 - e^{-\mu\Delta}$ increases in μ. Thus, the fixed point Δ^* must increase in μ.

sector, toward the preferred supplier. The total amount of investment is one, and $\Delta = 0$ and corresponds to symmetric investments equal to $x_i = \frac{1}{n}$ for $i = 1, ...n$, while $1 \geq \Delta > 0$ corresponds to an investment of $x_1 = \frac{1}{n} + \frac{n-1}{n}\Delta$ for preferred supplier and $x_2 = \frac{1}{n} - \frac{1}{n}\Delta$ for each of the independent suppliers. We restrict attention to those circumstances in which symmetric investments are first-best, i.e. $0 < \mu \leq 1$, and focus primarily on the boundary case $\mu = 1$. In the boundary case, any lesser degree of supplier heterogeneity – that is, any larger values of μ – would lead the social planner to an asymmetric solution under first-best. That is, the planner would designate one of the suppliers to invest more in cost reduction than the others. For this boundary case, Figure 2 illustrates the costs and benefits of establishing a preferred supplier. It also illustrates the further consequences of asymmetric cost distributions by increasing the investment of the preferred supplier case in which upstream industry is highly concentrated, specifically for $n = 4$.

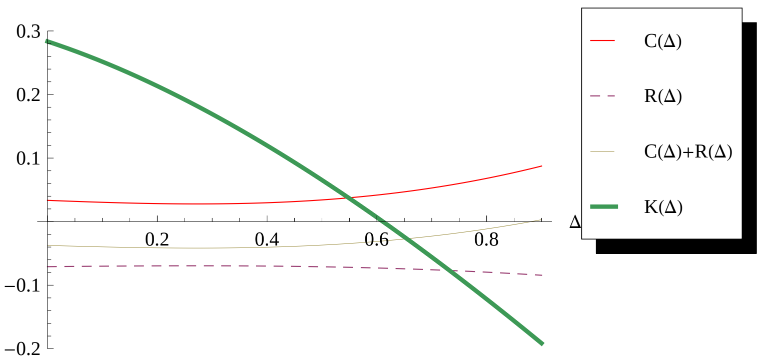


Figure 2: Profitability of Vertical Integration for $\mu = 1$ and $n = 4$.

First, observe that $\Delta = 0$ corresponds to the Burguet and Perry (2009) model in which the preferred supplier has the same cost distribution as independent suppliers. Given the sourcing distortion, the planner has an incentive to reallocate investments toward the preferred supplier, resulting in asymmetric cost distributions. Holding the sourcing distortion constant, the planner's incentive to reallocate is shown by the downward sloping concave curve in Figure 2, which has the functional form

$$K(\Delta) = 1 - e^{-\mu\Delta - \frac{1}{n-1}} - \Delta.$$

This curve graphs the difference between the marginal return to investment by the preferred supplier and the marginal return to investment of an independent supplier at a given allocation Δ. We interpret $K(\Delta)$ to measure the "efficiency effect" of an investment allocation, that is, the marginal reduction in expected total production cost given the market shares of the preferred supplier and the independent firms.

Second, notice that $K(\Delta)$ also indicates the difference in private incentives for investment under vertical integration. If $K(\Delta) > 0$, then a vertically-integrated supplier has a unilateral incentive to invest more, and an independent supplier has a unilateral incentive to invest less, whereas if $K(\Delta) < 0$ the opposite is true. The equilibrium difference in investment levels Δ^I occurs precisely at the point such that $K(\Delta^I) = 0$. In other words, equilibrium under vertical integration is equivalent to establishing a preferred supplier and reallocating investments such that the efficiency effect is zero.

Third, consider how investment reallocations impact expected total production cost if market shares are not held constant. A sourcing distortion in favor of a preferred supplier raises total production cost by shifting production to the sometimes less efficient preferred provider. By granting the preferred supplier a more favorable cost distribution, an investment reallocation has an adverse "sourcing effect" as the buyer to allocate even more market share to preferred provider. The overall consequences of an investment reallocation on production cost depends on the magnitudes of the efficiency effect and sourcing effect. The tradeoff between the two effects is demonstrated in Figure 2 with the convex curve labeled $C(\Delta)$, which graphs the increased total cost that results from creating a preferred supplier and reallocating investment so that the preferred supplier invests Δ more each of the others. This cost distortion relative to the first-best has the following functional form:

$$C(\Delta) = \frac{1}{\mu}\left(1 - e^{-\mu\Delta - \frac{1}{n-1}}\right) + \frac{n-1}{2n}(\Delta - 1)^2 - \frac{1}{\mu n} - \frac{n-1}{2n}.$$

For $\Delta < \Delta^I$, the eficiency effect and sourcing effect have opposite signs. The efficiency effect dominates for sufficiently small Δ, and $C(\Delta)$ declines to its minimum at $\Delta = \Delta^S$ where the two effect exactly balance, while for $\Delta^I > \Delta > \Delta^S$ the adverse sourcing effect overcomes the beneficial efficiency effect to push up total cost. For $\Delta > \Delta^I$, both effects are negative. Therefore, Δ^S solves the second-best planning problem.

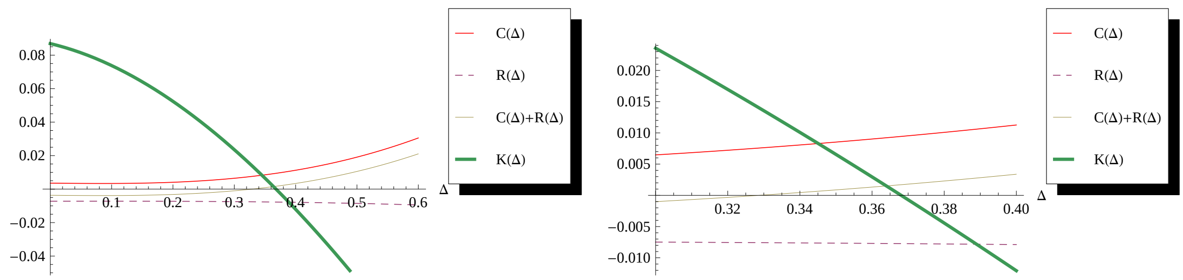
Foruth, consider the extent to which the creation of a preferred supplier squeezes the profits of its competitors. The sourcing distortion reduces rents paid to non-preferred suppliers by avoiding a markup whenever the cost of the preferred supplier is below the lowest bid. Furthermore, the profits are squeezed further as investment is reallocated toward the preferred supplier, as illustrated by the curve labeled $R(\Delta)$. The functional form for the boundary case yields a relatively flat curve:

$$R(\Delta) = -\frac{1}{\mu n}\left(1 - e^{-\mu\Delta - \frac{1}{n-1}}\right) - \frac{n-1}{2n^2}(\Delta - 1)^2 + \frac{n-1}{2n^2}.$$

In other words, the creation of a preferred supplier has a significant profit squeezing effect, but the magnitude of the effect is not very sensitive to an investment reallocation. Observe that $R(\Delta) = -\frac{1}{n}C(\Delta) - \frac{1}{\mu n}$.

Finally, consider the incentive for vertical integration versus outsourcing. Integration is profitable for the buyer and supplier 1 if and only if $C(\Delta)+R(\Delta) \geq 0$, which occurs for values of Δ below a critical value $\hat{\Delta}$. Establishing a pure preferred supplier in an industry with symmetric investments, and therefore symmetric cost distributions, is always profitable, i.e. $C(0) < -R(0)$, as shown by Burguet and Perry (2009). Asymmetric cost distributions resulting from increasingly reallocating investments toward the preferred supplier, however, eventually turns the tide against vertical integration, because the cost distortion rises much faster than rents are reduced. The net cost $C(\Delta)+R(\Delta)$ intersects the horizontal axis at a critical investment allocation $\hat{\Delta}$ above which the advantages of creating a preferred supplier with a superior cost distribution are outweighed by higher investment costs. The investments rise with reallocation because of the convexity of the quadratic investment cost function. Therefore, the profitability of vertical integration compared to outsourcing depends on whether the equilibrium point (Δ^I) occurs to the right or to the left of $\hat{\Delta}$. Figure 2 illustrates a particular upstream market structure in which the equilibrium intersection occurs to the left of $\hat{\Delta}$, and so vertical integration is profitable.

Figure 2 is drawn for a concentrated upstream industry ($n = 4$) in which high markups make the returns from reducing rents very high relative to the cost penalty resulting from sourcing distortions. As the number of suppliers increases, the $C(\Delta) + R(\Delta)$-curve and the $K(\Delta)$-curve both shift upward, but the latter more so. Eventually the equilibrium value of Δ, Δ^I, moves to the right of $\hat{\Delta}$, and outsourcing becomes the preferred vertical structure. The reason for this is that, while there is not much in the way rents to be squeezed in an unconcentrated industry, there nevertheless is a relatively large cost penalty from vertical integration because of a still significant sourcing distortion and resulting investment reallocation. This point is illustrated in Figure 3 for $n = 12$.



(a) $n = 12, \mu = 1$ (b) Enlargement

Figure 3: Profitability of Outsourcing for $\mu = 1$ and $n = 12$

In fact, there is a threshold value $\hat{n}$ such that vertical integration is preferred for $n > \hat{n}$, and outsourcing is preferred for $n < \hat{n}$. The threshold value $\hat{n}$ is computed as follows. Let $\hat{\Delta}(n, \mu)$ be the value of Δ for which $C(\Delta) + R(\Delta) = 0$ and substitute $\hat{\Delta}(n, \mu)$ into the function $K(\Delta)$. The value $\hat{n}$ is then defined as the number (if it exists) satisfying $K(\hat{\Delta}(\hat{n}, \mu)) = 0$.

The function $K(\hat{\Delta}(n, \mu))$ is illustrated in Figure 4 for three different value of μ. For $\mu = 1$, $\hat{n} \approx 8.78$, so outsourcing is preferred when there are 9 or more upstream suppliers. For $\mu = 1/3$, $K(\hat{\Delta}(n, \mu)) < 0$ for all $n \geq 2$, implying that vertical integration is always the buyer's preferred market structure. This is another way to state the result in Proposition 4.

4 Endogenous Market Structure

We now analyze a bargaining game in which the market structure is determined endogenously. We first analyze an acquisition game.

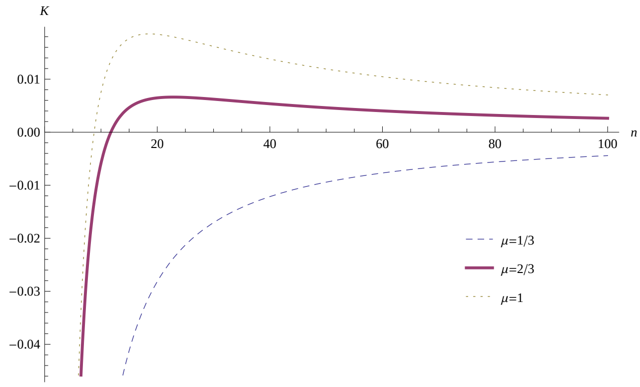


Figure 4: The function $K(\hat{\Delta}(n,\mu))$ for $\mu \in \{1/3, 2/3, 1\}$.

Acquisition Game The starting assumption is that the underlying parameters are as above and common knowledge. At the outset, the market structure is non-integration. The customer then makes sequential take-it-or-leave-it offers t_i to the independent suppliers $i = 1,..,n$. The sequence in which offers are made is pre-determined. Without loss of generality, we assume that supplier i receives the i-th offer. If i accepts, the acquisition game ends and the game with vertical integration analyzed above ensues. If firm $i < n$ rejects, the customer makes the offer t_{i+1} to firm $i+1$. If supplier n receives an offer but rejects it, the game with outsourcing analyzed above ensues.

The equilibrium behavior is readily determined. Suppose first that $\Phi(n,\mu) < 0$. That is, vertical integration is jointly profitable. Then the subgame perfect equilibrium offers are $t_i = \Pi_I^*$ for $i < n$ and $t_n = \Pi_O^*$. On and off the equilibrium path, every offer is accepted. Notice that in order for suppler to accept the offer he receives, he must be offered $t_n \geq \Pi_O^*$ because the alternative to his rejecting is that the game with the non-integrated market structure ensues, in which case he nets Π_O^*. Anticipating that the last supplier would accept the offer if and only if he is offered Π_O^*, the alternative for any supplier $i < n$ when rejecting is that the ensuing market structure will be non-integration if $\Phi < 0$ and integration, with i as an independent supplier netting Π_I^* otherwise. Therefore, it suffices to offer $t_i = \Pi_I^*$ to i with $i = 1,..,n-1$, provided $t_n = \Pi_O^*$. But as the latter is only a credible threat if $\Phi(n,\mu) \leq 0$, it follows that vertical integration is more profitable than the necessary (and sufficient) condition for it to be an equilibrium outcome suggests: $\Phi(n,\mu) \leq 0$ must be the case for integration to occur on the equilibrium path, but if $\Phi(n,\mu) \leq 0$, the profit of integration to the customer is actually strictly larger than $\Phi(n,\mu)$ because she has to pay less than Π_O^* on the equilibrium path.

Lastly, if $\Phi(n,\mu) > 0$, vertical integration is not jointly profitable and the customer will only make offers that will be rejected (e.g. $t_i \leq 0$ for all i would be a sequence of such offers).

Divesture Game Suppose now that the initial market structure is vertical integration and that the customer would be better off with outsourcing (i.e. $\Phi(n,\mu) > 0$). Assuming the customer can make an offer to an outsider who is willing to pay any price that allows him to break even, the customer can sell her supply unit at the price Π_O^*.

Bargaining with Externalities The acquisition process involves bargaining with externalities: A supplier's reservation price for selling is different when he is assured that if he does not sell no other supplier will sell, and if he has no such assurance. This reservation price is given by the profit under outsourcing, that is, when the supplier does not sell, minus the reduction in profits when another supplier sells. In our acquisition game with sequential take-it-or-leave-it offers, this is reflected by the higher offer the last supplier receives (off the equilibrium path), for whom the reduction in profits is zero if he does not accept the offer because no other offer will be made subsequently. The equilibrium in this acquisition game is unique because of the sequential nature of moves and the power of subgame perfection. For the same reason, the equilibrium outcome remains unique when $\Phi(n, \mu) > 0$ even though equilibrium no longer is simply because any sequence of offers that will be rejected are part of an equilibrium. Notice also that in our acquisition and divesture games, the equilibrium conditions are such that whenever there is an incentive to integrate, there is no incentive to divest, and conversely.

Of course, alternative bargaining procedures are conceivable. For example, following Jehiel and Moldovanu (1999) one could consider a second-price auction in which all suppliers simultaneously submit bids, and the bidder with lowest bid wins and is paid the second-lowest bid. Suppose first that the buyer has the right to reject all offers but does not set a reserve. If $\Phi(n, \mu) < 0$, the unique equilibrium outcome is such that the buyer acquires a supply unit at the price Π_I^* essentially because of the standard Bertrand (or second-price auction) arguments. In contrast, when $\Phi(n, \mu) > 0 > \hat{\Phi}(n, \mu) := PC_I^* + \frac{a}{2} x_I^* + \Pi_I^* - PC_O^*$ the equilibrium outcome is no longer unique. Observe that $-\hat{\Phi}(n, \mu)$ is the profit from vertical integration accruing to the buyer when he only has to pay the price Π_I^* instead of Π_O^* to acquire the supply unit.[11] This game now has two equilibrium outcomes. In every equilibrium leading to the first one, every supplier submits such a high bid that it will be rejected by the buyer, and no acquisition occurs just like in the acquisition game with sequential take-it-or-leave-it offers. However, there are also equilibria in which two or more suppliers submit bids equal to Π_I^* and the buyer selects one of these lowest price bidders at random. For suppliers, however, these equilibria are Pareto dominated by any equilibrium in which no acquisition occurs. Suppose now that the buyer can commit to a reserve price R with the usual meaning that the buyer is committed to buy from (one of the lowest) bidders whenever the lowest bid is at or below R but not otherwise. For $\hat{\Phi}(n, \mu) < 0$, the buyer always acquires a unit at the price Π_I^*. By setting the reserve $R = \Pi_O^*$, the buyer induces the suppliers to bid very aggressively. Notice that under the condition $\hat{\Phi}(n, \mu) < 0 < \Phi(n, \mu)$, this requires the buyer to set a reserve above her willingness to pay.[12] Third, interpreting integration as forward integration by a supplier, it is natural to assume that the sellers bid for the right

[11] It can be shown that $\hat{\Phi}(n, \mu)$ can be negative or positive as a function of μ and n. In fact, the emerging picture is very similar to Figure 1 except that all curve are shifted downwards a bit.

[12] This reflects the insight of Jehiel and Moldovanu (1999) that with negative externalities in a sale auction, the seller may optimally set a reserve below his value.

to acquire the downstream unit, with the buyer selling to the supplier with the highest bid. The equilibrium conditions for acquisition to occur, and the scope for multiplicity of equilibrium outcomes, are the same as in the second-price auction without a reserve just described.[13]

5 Extensions

In this section, we study a number of extensions that appear natural and demonstrate that the main insights derived from the model with inelastic demand, quadratic costs of effort, and with no reserves are not limited to the confines of this model.

5.1 Investment Cost Functions

A quadratic cost of investment function is convenient for our analysis because it implies the adding-up condition. Normalizing $a = 1$ for simplicity, equilibrium investments in cost reduction sum to 1 under both outsourcing and integration. This result follows because, in equilibrium, each supplier equates the marginal cost of its investment to its expected market share (i.e. probability of production). Since the marginal cost is equal to the level of investment under the quadratic specification, and since market shares sum to 1 under inelastic demand, it follows immediately that total investment must equal 1. Consequently the equilibrium effect of vertical on investments is only to reallocate a fixed amount of investment from the independent supply sector to the integrated supplier, while holding total investment constant.

Now consider the shifting support model with a more general marginal cost of investment $\psi(x)$. Symmetric equilibrium investment under outsourcing satisfies

$$\psi(x) = \frac{1}{n}$$

while the equilibrium investment conditions under vertical integration become

$$\begin{aligned}
\psi(x_2) &= \frac{1}{n-1}\int_{-\infty}^{\infty}[1 - F(G(c) + x_1)]dL(c + x_2, n-1), \\
\psi(x_1) &= \int_{-\infty}^{\infty} G(b(c) + x_1)dL(c + x_2, n-1)
\end{aligned}$$

and

$$(n-1)\psi(x_2) + \psi(x_1) = 1. \tag{10}$$

Thus equilibrium aggregate investment depends on the shape of the effort cost function.

[13] Somewhat intriguingly, whenever acquisitions occur in equilibrium for a broader range of parameter value than divestures occur, there is a potential Ponzi scheme inherent in the model. For example, with a reserve price and a second-price auction, the buyer could buy at the price Π_I^* and would be willing and able to sell at the price Π_O^* under the condition $\hat{\Phi}(n, \mu) < 0 < \Phi(n, \mu)$, suggesting that with such bargaining procedures the model would need to be extended to rule out the existence of money-pumps.

Proposition 7 *In the shifting support model, aggregate effort under vertical integration is the same, higher or lower than without vertical integration if, for all $x \geq 0$, $\psi''(x) = 0$, $\psi''(x) < 0$ or $\psi''(x) > 0$.*

Expected production costs are minimized under non-integration. Outcomes under vertical integration depart from this benchmark in three important ways. First, if $\psi''(x) \neq 0$, then equilibrium aggregate effort is either too high or too low under vertical integration. Second, even assuming $\psi''(x) = 0$ so that aggregate effort is fixed, vertical integration equilibrium inefficiently deploys the aggregate effort to the integrated supplier. This misallocation not only increases expected production cost, but also the cost of effort because the marginal cost of effort is increasing. Third, the sourcing decision is distorted in favor of the vertically integrated firm. This sourcing bias increases expected production cost, even though the integrated firm is motivated to reduce procurement cost.

Returning the exponential model, allowing an invertible marginal cost of investment function, the equilibrium difference in investments $\Delta = x_1 - x_2$ under vertical integration solves

$$\Delta = \psi^{-1}(1 - \frac{n-1}{n} e^{-\mu\Delta - \frac{1}{n-1}}) - \psi^{-1}(\frac{1}{n} e^{-\mu\Delta - \frac{1}{n-1}})$$

and equilibrium investments are

$$x_1 = \psi^{-1}(1 - \frac{n-1}{n} e^{-\mu\Delta - \frac{1}{n-1}}) \quad \text{and} \quad x_2 = \psi^{-1}(\frac{1}{n} e^{-\mu\Delta - \frac{1}{n-1}}).$$

To illustrate how the tradeoffs between vertical integration and outsourcing change with the shape of the cost investnment function, consider

$$\psi(x) = \begin{cases} x & \text{for} \quad x \leq \frac{1}{n} \\ x + \gamma(x - \frac{1}{n})^2 & \text{for} \quad x > \frac{1}{n} \end{cases}$$

This marginal cost function adds a quadratic component to the linear marginal cost function for investment levels above equilibrium investment under outsourcing, $\frac{1}{n}$. The exponential-quadratic model corresponds to $\gamma = 0$. In that model, if $\mu = 1$, vertical integration raises procurement costs for $n \geq 8$. If $\gamma = 1$, however, outsourcing is preferred for $n > 6$. Thus, a more steeply rising marginal cost above the efficient level of investment reduces the attractiveness of vertical integration, because the equilibrium cost-reduction by the integrated firm fails even more to compensate for the discouragement effect of the sourcing distortion on the investments of the independent sector.

5.2 Reserve Prices

A simple first-price auction models a standard pattern of commercial negotiations that requires minimal commitments. Suppliers make offers and the customer accepts the best offer. Such a transparent procurement process also is consonant with our motivation that suppliers compete ideas as well prices, i.e. suppliers innovate on the design of the input in order to reduce costs. In such a setting, our analysis demonstrates a tradeoff between extracting rents and motivating investments of independent suppliers.

If the required input were more standardized, so that acceptable designs were contractible, then the customer plausibly could exercise monopsony power by committing to a reserve price. For the case of inelastic demand, a positive reserve price is suboptimal under outsourcing, because the risk of failing to procure the input is disastrous. A reserve price is valuable under vertical integration, however, because the monopsonist is able to fall back on internal sourcing if independent suppliers cannot beat the reserve price. Thus, the ability to set a credible reserve price option clearly favors vertical integration under inelastic demand. Nevertheless, as we show below, a similar benefit-cost trade-off emerges, albeit with more stringent conditions for the superiority of outsourcing.

We perform the analysis of the effect of reserve prices within our baseline model with inelastic demand, exponentially distributed costs, and a quadratic cost of effort function. Suppose that the vertically integrated customer commits to a reserve price r after learning the cost of internal supply c_1. Given the symmetric equilibrium investment of independent firms x_2, the optimal reserve price satisfies

$$c_1 = r + \frac{G(r + x_2)}{g(r + x_2)} \equiv \Gamma_{x_2}(r)$$

while the symmetric bidding function $b(c, r)$ depends on the reserve price r according to[14]

$$b(c, r) = c + \frac{1}{n-1}\left[1 - e^{-\mu(n-1)(r-c)}\right].$$

In equilibrium, the vertically integrated firm chooses its own investmentx_1 to minimize expected procurement cost given x_2, and each independent suppliers invests to maximize expected profit given x_1 and x_2. The optimal reserve given $c_1 \geq \beta - x_2$ then satisfies

$$r(c_1) := \Gamma_{x_2}^{-1}(c_1). \tag{11}$$

Total equilibrium procurement cost (net of investment cost) is equal to the expected cost of internal supply minus the expected cost savings from outsourcing:

$$\beta - x_1 + \frac{1}{\mu} - \int_{\beta-x_2}^{\infty} \int_{\beta-x_2}^{\Gamma_{x_2}^{-1}(c)} [c_1 - b(c, \Gamma_{x_2}^{-1}(c_1)] dL(c + x_2, n-1) dG(c_1 + x_1).$$

Assuming $x_1 > x_2$, the expected profit of a representative independent firm choosing x in the neighborhood of x_2 is equal to the expected value of the markup times the probability of winning the auction:

$$\int_{\beta-x_2}^{\infty} \int_{\beta-x}^{\Gamma_{x_2}^{-1}(c_1)} [b(c, \Gamma_{x_2}^{-1}(c_1)) - c][1 - L(c + x_2, n-2)] dG(c + x) dG(c_1 + x_1)$$

In equilibrium each independent supplier chooses $x_2 = x$.[15]

[14] In the exponential case, the virtual cost function $\Gamma_{x_2}(r)$ is strictly increasing in r for given x_2, and therefore invertible. We denote its inverse by $\Gamma_{x_2}^{-1}(c_1)$ The bid function $b(c, r)$ solves the usual necessary differential equation for optimal bidding with the boundary condition $b(r, r) = r$.

[15] We computes the equilibrium investments levels (x_1, x) solving the necessary first-order conditions, presuming the appropriate second-order conditions are satisfied.

The condition for outsourcing to be preferred to vertical integration is similar to before. Expected procurement cost plus investment cost under vertical integration must exceed expected procurement cost plus the profit of a representative supplier under outsourcing. The difference between expected procurement costs under vertical integration and under outsourcing must not exceed the profit must be less than expected supplier profit under outsourcing. Figures 5 graphs the difference as function of n for $\mu = 1$ and compares it to the case without reserves, depicted in Figure 1. The curve is shifted to the right compared to the base model in which there is no reserve price. Although an optimal reserve price does lower procurement costs under vertical integration, outsourcing nevertheless is preferred for n sufficiently large.

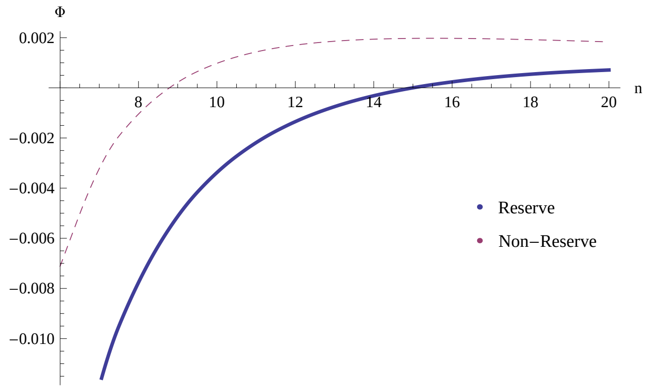


Figure 5: The function Φ with and without reserves for $\mu = 1$.

5.3 Elastic Demand

While inelastic demand is a useful simplifying assumption that helps illuminate the main tradeoffs between outsourcing and integration, it is of course more realistic for the buyer to abandon the project entirely if costs are prohibitively high. Fortunately, it is reasonably straightforward to generalize the analysis to allow for a downward sloping demand curve.

Setup We now assume that the customer has value v for the input, drawn from an exponential probability distribution $F(v) = 1 - e^{-\lambda(v-\alpha)}$ with support $[\alpha, \infty)$. The mean of the exponential distribution is $\alpha + \frac{1}{\lambda}$ and can be interpreted to indicate the expected profitability of the downstream market. The variance, which is $\frac{1}{\lambda}$, can be interpreted to indicate uncertainty about product differentiation. This model converges to the inelastic case as $\lambda \to 0$. The customer learns the realization of v before making the purchase (or production) decision.

Under vertical integration, however, the investment x_1 in cost reductions is made before the customers learns the realized v. Independent suppliers know F but v. All other assumptions regarding timing and investment costs are as in Sections 2 and 3. In particular, the cost of exerting effort x is $\frac{a}{2}x^2$ and given investment x_i supplier i's cost is drawn from the exponentiation distribution $1 - e^{\mu(c+x_i-\beta)}$ with support $[\beta - x_i, \infty)$ for all $i = 1, .., n$ and with $\mu \leq a$. To simplify the equilibrium analysis, we impose the parameter restriction

$$\beta - \alpha \geq \frac{1}{\lambda + (n-1)\mu} - \frac{1}{\lambda + \mu}. \tag{12}$$

Conditions (12) makes sure that under outsourcing the lowest equilibrium bid is always larger than the lowest possible draw of v. Observe that the righthand side in (12) is negative, so that $\beta \geq \alpha$ is sufficient for (12) to be satisfied. Our analysis can be extended beyond the specific parameterization satisfying (12) and beyond the case where v is drawn from an exponential distribution. However, these generalizations come at the costs of added complexity, which do not appear to be outweighed by sufficient benefits of additional insights. To explicitly account for the lower bounds of their supports, we denote by

$$\overline{F}(v) := \max\{F(v), 0\} = 1 - e^{-\lambda \max\{v-\alpha, 0\}}$$

and

$$\overline{G}(c_1 + x_1) := \max\{G(c_1 + x_1), 0\}\} = 1 - e^{-\mu \max\{c_1+x_1-\beta, 0\}},$$

respectively, the distributions of the buyer's valuation and the integrated firm's cost when its investment is x_1, where the second equalities hold for the exponential distributions.

Bidding The bidding function with elastic is denoted as $b_E(c)$ and given by

$$b_E(c) = c + \frac{1}{\lambda + \mu(n-1)} \tag{13}$$

for $c \geq \alpha - \frac{1}{\lambda+(n-1)\mu}$. Coincidentally, just like in the inelastic demand case without reserves, the bidding function is the same with or without vertical integration with elastic demand and no reserves.[16]

Profits Consider first the case under **outsourcing** when the investments of the independent suppliers are x. The profit $\Pi^B_{EO}(x)$ accruing to the buyer under outsourcing given symmetric investments x is

$$\Pi^B_{EO}(x) = n \int_{b_E(\beta-x)}^{\infty} \int_{\beta-x}^{y(v)} [v - b_E(c)][1 - G(c+x)]^{n-1} dG(c+x) dF(v),$$

where $y(b) = b - \frac{1}{\lambda+\mu(n-1)}$ denotes the inverse of the bidding function $b_E(c)$.

[16]To see that $b_E(c)$ is also the bidding function under integration, notice that the customer will buy from the independent suppliers if and only if the lowest submitted bid b is less than $\hat{v} = \min\{v, c_1\}$, where v is the customer's realized value and c_1 the cost draw of the integrated supplier. The distribution of $\hat{v}$ is $1-(1-\overline{F}(\hat{v}))(1-\overline{G}(\hat{v}+x_1))$. For our exponential specifications, the probability that $b \leq \hat{v}$ is thus $1 - e^{-(\mu \max\{\hat{v}+x_1-\beta, 0\} + \lambda \max\{\hat{v}-\alpha, 0\})}$. Arguments that are analogous to those that led to the expression (3) can then be invoked to conclude that $b_E(c)$ is also the bidding function under integration.

The expected profit $\Pi_{EO}(x_i, x)$ of an independent supplier under outsourcing who invests x_i while each of the other suppliers is expected to invest x with $x \geq x_i$ is

$$\Pi_{EO}(x_i, x) = \int_{b_E(\beta - x_i)}^{\infty} \int_{\beta - x}^{y(v)} [b_E(c) - c][1 - G(c + x)]^{n-1} dG(c + x_i) dF(v) - \frac{a}{2} x_i^2.$$

With **integration**, the buyer's profit is

$$\begin{aligned} \Pi_{EI}^B(x_1, x_2) &= \int_{\alpha}^{\infty} \int_{\beta - x_1}^{\max\{v, \beta - x_1\}} [v - c_1] dG(c_1 + x_1) dF(v) \\ &+ \int_{\beta - x_1}^{\infty} (1 - \overline{F}(c_1)) \int_{\beta - x_2}^{\max\{y(c_1), \beta - x_2\}} [c_1 - b_E(c_2)] dL(c_2 + x_2, n - 1) dG(c_1) \\ &+ \int_{\alpha}^{\infty} (1 - \overline{G}(v)) \int_{\beta - x_2}^{\max\{y(v), \beta - x_2\}} [v - b_E(c_2)] dL(c_2 + x_2, n - 1) dF(v) - \frac{a}{2} x_1^2. \end{aligned}$$

This profit is computed by deriving the expected profit from self-sourcing, which is done in the first line in the above expression, by then adding the cost savings from sourcing from the independent supplier with the lowest bid, which is captured in the second line, and by finally adding in the third line the expansion effect (relative to having no outside suppliers) that arises whenever $c_1 > v$ and $b_E(\min\{c_j\}) < v$ with $j \neq 1$.

Given its own investment x_i, investments $x_2 \geq x_i$ by all other non-integrated suppliers and x_1 by the integrated supplier, the expected profit $\Pi_{EI}(x_i, x_1, x_2)$ of an independent supplier under vertical integration is $\Pi_{EI}(x_i, x_1, x_2) =$

$$\int_{\beta - x_i}^{\infty} [b_E(c) - c][1 - F(b_E(c))][1 - G(b_E(c) + x_1)][1 - G(c + x_2)]^{n-2} dG(c + x_i) - \frac{a}{2} x_i^2.$$

Equilibrium Investments Under outsourcing, the necessary first-order conditions for the symmetric equilibrium investment x is

$$x = \frac{1}{a} \frac{\mu}{\lambda + n\mu} e^{-\lambda\left[\frac{1}{\lambda + (n-1)\mu} + \beta - \alpha - x\right]}. \tag{14}$$

With vertical integration, the vertically integrated supplier invests x_1 and all $n - 1$ independent suppliers invest x_2 satisfying

$$x_1 = x_2 + \frac{1}{a} \frac{\mu}{\lambda + \mu} e^{-\mu(x_1 - x_2)} \left[e^{\mu(\beta - \alpha - x_2)} - e^{-\lambda(\beta - \alpha - x_2) - \frac{\lambda + \mu}{\lambda + (n-1)\mu}} \right] \tag{15}$$

and

$$x_2 = \frac{1}{a} \frac{\mu}{\lambda + n\mu} e^{-\lambda(\beta - \alpha - x_2) - \mu(x_1 - x_2) - \frac{\lambda + \mu}{\lambda + (n-1)\mu}} \tag{16}$$

according to the necessary first-order conditions for equilibrium. As in our analysis of reserves with inelastic demand, we presume that the second-order conditions are satisfied rather than belabor the details.

Profitability of Outsourcing Evaluating (14), (15) and (16) numerically we can determine the buyer's and the independent suppliers' equilibrium profits under outsourcing and vertical integration. Denoting these equilibrium payoffs with a star "*", the analogue for the case of elastic demand to the function $\Phi(n, \mu)$ defined in (8) is

$$\Phi_E(n, \mu, \alpha, \lambda, \beta) := \Pi^{B*}_{EO} + \Pi^{*}_{EO} - \Pi^{B*}_{EI}.$$

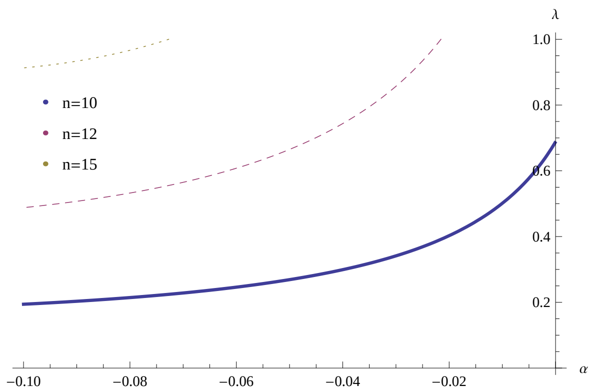


Figure 6: $\Phi_E(n, \mu, \alpha, \lambda) = 0$ for $n = 10$ and $n = 15$.

Figure 6 contains contour sets of $\Phi_E(n, \mu, \alpha, \lambda) = 0$ for different values of n in (α, λ)-space with $\mu = 1$ and $\beta = 0$. Outsourcing is profitable for a given n for values of α and λ below the corresponding curve.

Elastic Demand with a Reserve The analysis with elastic demand can also be extended to account for optimal reserves. Under outsourcing, the optimal reserve will be a function of the realized value v and will be given by the function $r(.)$ defined in (11). With vertical integration, the optimal reserve will be given by the same function $r(.)$, which is now evaluated at $\hat{v} := \min\{c_1, v\}$. Because of continuity, it is intuitive that with elastic demand and optimal reserves outsourcing will be profitable in the neighborhood of the parameter region for which it is profitable with perfectly inelastic demand and a reserve, that is, for values of λ close to zero. This intuition is corroborated by numerical analysis. Figure 7 plots the buyer's gain from outsourcing with reserves, denoted Φ_{ER}, and her gain from outsourcing without reserves, Φ_E, as a function of λ for $n = 16$ and $\alpha = \beta = 0$.

5.4 Agency Cost

So far we have ignored agency problems inside the firm. A conceptually straightforward way to introduce agency costs into the model is to assume a separation of ownership and control for suppliers. More specifically, suppose that the owner of an upstream firm is taken to be a risk-neutral principal who delegates the choice of effort to a risk-averse agent. The utility function of the agent is $U(w) - \Psi(x)$, where $U(w)$ is strictly-increasing

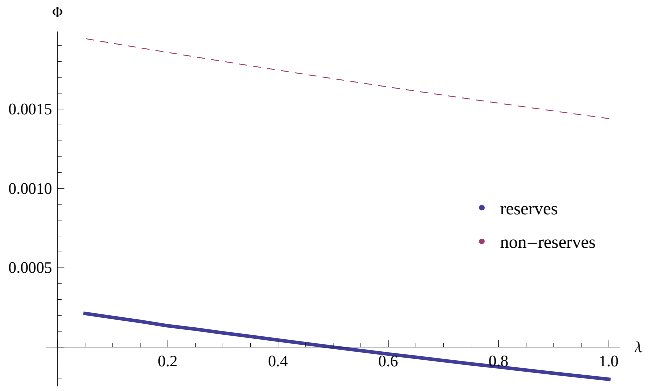


Figure 7: Φ_{ER} and Φ_E as function of λ.

concave function of the wage w and $\Psi(x)$ is the cost of effort. The principal sets the wage as a function of the realized cost, i.e. $w = w(c)$. To implement a particular effort x, the principal chooses a wage function to minimize the expected wage subject to an incentive constraint and a participation constraint. The solution to this problem determines an expected cost, $W(x)$, that incorporates agency cost. The key point is that the same agency problem exists under non-integration and vertical integration. Thus the comparative organization analysis can proceed along the same lines as before, replacing $\Psi(x)$ by $W(x)$.[17]

[17]The exponential cost distribution puts considerable structure on agency problem. One can view the principal as seeking implement a particular x at minimum expected cost cost subject to a participation constraint and and incentive constraints, as in Grossman and Hart (1983). The binding participation constraint is

$$\int_{\beta-x}^{\infty} w(c)f(c+x)dc = \Psi(x)$$

and the necessary first-order condition for the agent's problem is

$$U(w(\beta-x))f(\beta) + \int_{\beta-x}^{\infty} w(c)f'(c+x)dc = \psi(x).$$

The exponential distribution implies $f(\beta) = \lambda$ and $f'(c+x) = \lambda f(c+x)$, so the local incentive constraint becomes

$$\lambda\{U(w(\beta-x)) + \int_{\beta-x}^{\infty} w(c)f'(c+x)dc\} = \psi(x).$$

Substituting the participation constraint into the local incentive constraint and rearranging gives

$$\lambda\{U(w(\beta-x)) + \Psi(x)\} = \psi(x)$$

or

$$w(\beta-x) = U^{-1}(\frac{1}{\lambda}\{\psi(x) - \Psi(x)\})$$

and additionally imposing a quadratic cost of effort function gives

$$w(\beta-x) = U^{-1}(\frac{1}{\lambda}\{x - \frac{1}{2}x^2)\}).$$

5.5 Alternative Cost Distribution

The exponential cost distribution is convenient because it allows a closed form solution of the bid function under vertical integration. More generally, consider a cost distribution of the form $G(c+x)$ with support $[\beta - x, \infty)$ and density $f(z+x)$, satisfying $\lim_{c\to\infty} cf(c+x) = 0$. The shifting support model has the convenient property that $\frac{\partial G(z+x)}{\partial x} = g(z+x)$. Thus, cost-reducing investment maintains the shape of the cost distribution while shifting its support downward.

Equilibrium bidding under outsourcing with symmetric suppliers is well understood from auction theory. Suppose n suppliers have the same cost distribution $G(c+x)$, and consider a representative firm with cost realization c when rival bidders use an invertible bid strategy $b(c)$. A representative firm chooses β to maximize $(\beta - c)[1 - G(b^{-1}(\beta))]^{n-1}$. Therefore, a symmetric equilibrium bidding strategy $b_O(c)$ satisfies

$$c = \arg\max_z \left\{ [b_O(z) - c] \left[1 - G(c+x)\right]^{n-1} \right\},$$

or

$$b_O(c) = c + \frac{\int_c^\infty \left[1 - G(z+x)\right]^{n-1} dz}{\left[1 - G(c+x)\right]^{n-1}}.$$

imposing the boundary condition

$$\lim_{c\to\infty} [b_O(c) - c] = 0$$

Note that $b(c)$ is an increasing function and is indeed invertible on the support of $G(\cdot)$. Assuming that first-order conditions are necessary and sufficient for equilibrium investments, it is straightforward that each independent supplier invests $x = \frac{1}{n}$ in equilibrium (normalizing $a = 1$).

Under vertical integration with investment x_1 by the integrated supplier and x_2 by each independent supplier, the equilibrium bidding function $b_I(c)$ satisfies

$$b_I(c) = c + \frac{\int_c^\infty (1 - G(t+x_2))^{n-2}(1 - G(b_I(t) + x_1))dt}{(1 - G(c+x_2))^{n-2}(1 - G(b_I(c) + x_1))}$$

Letting $L(c; x, n) \equiv 1 - [1 - G(c+x)]^n$ denote the distribution of the minimum order statistic with n independent suppliers, equilibrium investments satisfy[18]

$$x_2 = \frac{1}{n-1} \int_{-\infty}^{\infty} [1 - G(b_I(c) + x_1)] dL(c, x_2, n-1)$$

Unfortunately, the usual first-order approach to fully solving the principal's problem (Rogerson, 1985) does not apply directly to the exponential-quadratic case, and an analysis of a full solution to the principal's problem is outside the scope of this paper. It seems straightforward, however, to characterize $W(x)$, for example, by assuming that the principal restricts attention to linear $w(c)$, or, alternatively, to paying a fixed wage plus a bonus if realized c is sufficiently close to $\beta - x$.

[18] The equilibrium condition for integrated firm uses the fact that the marginal return to cost reduction when b is the minimum bid of the independent sector is $G(b + x_1)$.

and

$$x_1 = \int_{-\infty}^{\infty} G(b_I(c) + x_1) dL(c, x_2, n-1).$$

Thus, the shifting support model retains the "adding-up condition" $(n-1)x_2 + x_1 = 1$ of the exponential model (see also the proof of Proposition 5).

Difficulties with this more general formulation arise because the bidding function under vertical integration does not in general admit a closed form solution, which makes it challenging to characterize procurement costs under vertical integration. The exponential case is exceptional because it yields a constant markup bidding function for any $n \geq 2$.

A model in which G is a uniform distribution and n is equal to 2 is another special case that admits a closed form solution for b_I. That is, suppose that given investment x_i supplier i's costs are uniformly distributed on $[\mu - x_i, 1 + \mu - x_i]$. Facing a competing supplier who invests $x \geq x_i$, the independent bidder bids according to

$$b(c) = \frac{c + 1 + \mu - x}{2}$$

for $c \in [\mu - x, 1 + \mu - x]$ and submits an arbitrary bid $b > 1 + \mu - x$ for $c > 1 + \mu - x$ with and without integration.

For this specification, a numerical analysis demonstrates that vertical integration reduces procurement costs over the relevant range of μ. A numerical analysis of the uniform case for larger values of n requires nesting a numerical solution for the bidding function, which has not closed form solution.

6 Conclusion

We offer a new comparative theory of outsourcing and vertical integration that features a key tradeoff between markup avoidance and investment discouragement. In our simple stylized model of procurement, upstream suppliers make relationship-specific investments in cost reduction before bidding to supply an input requirement to a downstream customer. Since neither the investment nor the cost realization are observable, independent suppliers exercise some degree market power by bidding above-cost markups. By unifying the customer and one of suppliers under common ownership, vertical integration improves their joint profits because it enables the customer to avoid the markup by sourcing internally, keeping investments fixed. Moreover, if the procurer's demand is elastic, integration increases efficiency and further increases profits, keeping investments fixed, because the markup avoidance also leads to an output expansion. Therefore, just like in Williamson (1985)'s famous puzzle of selective intervention, an integrated firm can do the same as the separate entities do, and sometimes it can do strictly better. This would seemingly lead to the conclusion that vertical integration is inevitably profitable. This prediction is puzzling because it is at odds with the empirical observations, which include the recent trend towards outsourcing.

In our model, however, vertical integration is not always profitable because it changes the incentives to invest for the suppliers, making equilibrium investment levels smaller for non-integrated suppliers and larger for the integrated supplier. Thus vertical integration effectively reallocates investment away from independent suppliers and toward the integrated supplier. Such a reallocation raises total investment costs because the marginal cost of investment is increasing. The discouragement effect on cost-reducing investments of independent suppliers can be so costly for the integrated firm that it outweighs the aforementioned benefits from vertical integration. Not only does vertical integration change the behavior of the integrated entity in the way suggested by Williamson, but, exactly because it does so, it also changes the behavior of the non-integrated firms. Put differently, vertical integration occurs within a competitive procurement environment, and depending on how this environment's behavior is affected by vertical integration, vertical integration or outsourcing may be the procurer's preferred organizational structure.

The usual statement of Williamson's puzzle interprets the vertical integration decision in a narrow bilateral context, implicitly holding constant the conduct of outside parties. All that seems to matter for the decision are the incentives of the manager of the supply division and the ability of the integrated firm to adapt to the external environment. Accounting for the investment response of independent suppliers, however, creates a tradeoff between the advantages of markup avoidance on the one hand, and the cost disadvantage of realigned investment incentives on the other. In this multilateral setting, the puzzle vanishes. The tradeoff favors vertical integration in some circumstances, and vertical divestiture and outsourcing in others.

Our procurement model is motivated by the idea that specialized suppliers make non-contractible investments in cost-reducing product and process design, consistent with Whitford (2005)'s description of the type of customer-supplier relationships that emerged in manufacturing at the end of the 20th century. Whitford (2005) calls the new organizational form "contested collaboration", colorfully describing it as a"waltz" whereby customer-supplier pairs cooperate gracefully on cost-reducing design innovations, but contest awkwardly over price. The investment stage of our model captures in a stylized way that an original equipment manufacturer outsources cost-reducing design innovations, while bidding in a procurement auction against a preferred supplier captures in a stylized way that supply negotiations do not always proceed efficiently. From this perspective, vertical divestiture is a commitment to a level playing field that encourages independent suppliers to invest in cost reduction.

Our theory also helps explain a trend toward outsourcing in an increasingly global economy marked by faster technological change and shorter product cycles. It predicts that vertical divesture is under certain circumstances an attractive strategy to encourage cost-reducing investments by independent suppliers as it shifts rents their direction. The conditions favoring vertical divestiture include a moderate cost variance across a greater number of potential suppliers, and greater demand uncertainty. These conditions contributes to reducing supplier markups, thus weakening the markup avoidance advantages of vertical integration. By increasing the size of markets both upstream and downstream, globalization promotes more offshoring opportunities for procurement and also improves incentives for downstream product improvements, consistent with the conditions of our theory. Recent increases in labor costs in China and elsewhere can be interpreted as decreases in competition, favoring vertical integration going forward.

Our theory also helps explain the documented prevalence of external sourcing in American manufacturing even by vertically integrated firms. In our model, a vertically integrated firm chooses to source externally whenever doing so can meet its input requirements less expensively than self supply. A high variance of costs across potential upstream suppliers with differing design and process approaches is consistent with substantial external sourcing by downstream manufacturers, including those who have the option to source internally.

Appendix

Proof of Lemma 1: The optimal bid when competing only with the integrated supplier – that is, if the optimal bid b is on the interval $[\beta - x_1, \beta - x_2 + \frac{1}{\mu(n-1)}]$ - is given by the solution to $\arg\max_b (b - c_i)(1 - G_0(b + x_1))$. This solution is $b^*(c_i) = c_i + \frac{1}{\mu}$ if $c_i + \frac{1}{\mu} \geq \beta - x_1$, which is equivalent to the lower bound of the second condition stated in the lemma, i.e. $c_i \geq \beta - x_1 - \frac{1}{\mu}$, and $b^*(c_i) = \beta - x_1$ otherwise. For $b^*(c_i)$ to be on the interval $[\beta - x_1, \beta - x_2 + \frac{1}{\mu(n-1)}]$, it further has to be the case that $c_i + \frac{1}{\mu} \leq \beta - x_2 + \frac{1}{\mu(n-1)}$, which is equivalent to the upper bound of the second condition stated in the lemma, i.e. $c_i \leq \beta - x_2 - \frac{n-2}{\mu(n-1)}$. ■

Proof of Lemma 2: The necessary conditions have been derived in the main text. We are thus left to verify the conditions under which the second-order conditions for an equilibrium are satisfied. For $x_i \geq x$, the first derivative of $\Pi_O(x_i, x)$ with respect to x_i is

$$\frac{\partial \Pi_O(x_i, x)}{\partial x_i} = 1 - \frac{n-1}{n} e^{-\mu(x_i - x)} - a x_i$$

Evaluated at $x_i = x$, the second derivative with respect to x_i is therefore

$$\frac{\partial^2 \Pi_O(x_i, x)}{\partial x_i^2}\Big|_{x_i = x} = \frac{\mu(n-1)}{n} - a.$$

For $x_i < x$, the first partial of $\Pi_O(x_i, x)$ with respect to x_i is

$$\frac{\partial \Pi_O(x_i, x)}{\partial x_i} = \frac{1}{\mu n(n-1)} e^{-\mu(x_i - x)} - a x_i.$$

Evaluated at $x_i = x$, the second partial is thus

$$\frac{\partial^2 \Pi_O(x_i, x)}{\partial x_i^2}\Big|_{x_i = x} = \frac{\mu n}{n-1} - a.$$

The second-order condition is thus satisfied if and only if $\frac{\mu}{a} < \frac{n}{n-1}$.

To see that PC_O^* decreases in n, observe that

$$\frac{\partial PC_O^*}{\partial n} = \frac{(\mu - a)(n-1)^2 - an^2}{\mu a n^2 (n-1)^2},$$

which is negative if and only if $\frac{\mu}{a} < 1 + \frac{n^2}{(n-1)^2}$. The derivative of Π_O^* with respect to n is

$$\frac{\partial \Pi_O^*}{\partial n} = \frac{\mu(n-1)^2 - an(2n-1)}{\mu a n^2 (n-1)^2},$$

which has the same sign as $\mu(n-1) - an\left(1 + \frac{n}{n-1}\right)$. This is negative if and only if $\frac{\mu}{a} < \left(1 + \frac{n}{n-1}\right)$. Both inequalities are satisfies if the necessary and sufficient condition for the existence of such an equilibrium is satisfied. ■

Proof of Lemma 3: The arguments in the main text imply that PC_I^* and Π_I^* are the equilibrium payoffs of the integrated firm and the independent suppliers with Δ given by (6) and x_1 and x_2 given by (7). We are thus left to derive the conditions under which such an equilibrium exists.

Case 1: $x_i < x_2$ We first look at a downward deviation $x_i < x_2$ by a non-integrated supplier. The first and second partials of $\Pi_I(x_i, x_1, x_2)$ with respect to x_i are

$$\frac{\partial \Pi_I(x_i, x_1, x_2)}{\partial x_i} = \frac{1}{n} e^{-\mu\Delta - \frac{1}{n-1} + \mu(n-1)(x_i - x_2)} - a x_i$$

and

$$\frac{\partial^2 \Pi_I(x_i, x_1, x_2)}{\partial x_i^2} = \frac{\mu(n-1)}{n} e^{-\mu\Delta - \frac{1}{n-1} + \mu(n-1)(x_i - x_2)} - a.$$

The profit function is thus concave on $[0, x_2]$ if and only if $\frac{\mu(n-1)}{n} e^{-\mu\Delta - \frac{1}{n-1} + \mu(n-1)(x_i - x_2)} - a \leq 0$. As the term $\frac{\mu(n-1)}{n} e^{-\mu\Delta - \frac{1}{n-1} + \mu(n-1)(x_i - x_2)}$ increases in x_i, this second-order condition is thus satisfied if and only if

$$\frac{\mu}{a} \leq \frac{n}{(n-1)(1 - a\Delta)}.$$

Since $a\Delta < 1$, this second-order condition is always satisfied if the second-order condition under outsourcing is.

Let $\hat{x} = x_2 + \frac{n-2}{\mu(n-1)}$.

Case 2: $x_i \in (x_2, \hat{x}]$ Next we consider deviations by i such that $c_i \in \left[\beta - x_2 - \frac{1}{\mu}\frac{n-2}{n-1}, \beta - x_2\right]$ occur with positive probability, and no lower c_i can occur. From Lemma 1 we know that for cost realizations in this interval, the optimal bid by i will be the constant bid $\beta - x_2 + \frac{1}{\mu(n-1)}$.

For $x_i \in [x_2, \hat{x}]$ the profit function for the deviating supplier i is

$$\begin{aligned} \Pi_I(x_i, x_1, x_2) &= \frac{1}{n-1} \int_{\beta - x_2}^{\infty} e^{-\mu[n(c_i - \beta) + x_1 + (n-2)x_2 + x_i + \frac{1}{\mu(n-1)}]} dc_i \\ &+ \int_{\beta - x_i}^{\beta - x_2} \mu \left(\beta - x_2 + \frac{1}{\mu(n-1)} - c_i\right) e^{-\mu\Delta - \frac{1}{n-1} - \mu(c_i + x_i - \beta)} - \frac{a}{2} x_i^2. \end{aligned}$$

Integrating yields

$$\Pi_I(x_i, x_1, x_2) = e^{-\mu\Delta - \frac{1}{n-1}} \left[x_i - x_2 - \frac{n-2}{\mu(n-1)} + e^{-\mu(x_i - x_2)} \frac{n-1}{\mu n}\right] - \frac{a}{2} x_i^2.$$

The first and second partial derivatives are

$$\begin{aligned} \frac{\partial \Pi_I(x_i, x_1, x_2)}{\partial x_i} &= e^{-\mu\Delta - \frac{1}{n-1}} \left[1 - \frac{n-1}{n} e^{-\mu(x_i - x_2)}\right] - a x_i \\ \frac{\partial^2 \Pi_I(x_i, x_1, x_2)}{\partial x_i^2} &= e^{-\mu\Delta - \frac{1}{n-1}} \left[\mu \frac{n-1}{n} e^{-\mu(x_i - x_2)}\right] - a. \end{aligned}$$

Therefore, on $[x_2, \hat{x}]$, the deviator's profit function is concave in x_i, and maximized at $x_i = x_2$ if and only if

$$\frac{\mu}{a} < \frac{n}{(n-1)(1-a\Delta)}.$$

Case 3: $x_i \in [\hat{x}, x_1 + \frac{1}{\mu}]$ We next consider investments $x_i \in [\hat{x}, x_1 + \frac{1}{\mu}]$. The expected profit of the deviating supplier is

$$\begin{aligned}
\Pi_I(x_i, x_1, x_2) &= \frac{1}{n-1} \int_{\beta - x_2}^{\infty} e^{-\mu[n(c_i - \beta) + x_1 + (n-2)x_2 + x_i + \frac{1}{\mu(n-1)}]} dc_i \\
&+ \int_{\beta - x_2 - \frac{n-2}{\mu(n-1)}}^{\beta - x_2} \mu \left(\beta - x_2 + \frac{1}{\mu(n-1)} - c_i \right) e^{-\mu\Delta - \frac{1}{n-1} - \mu(c_i + x_i - \beta)} \\
&+ \int_{\beta - x_i}^{\beta - x_2 - \frac{n-2}{\mu(n-1)}} e^{-\mu[2(c-\beta) + x_1 + x_i] - 1} dc_i - \frac{a}{2} x_i^2.
\end{aligned}$$

Integrating one gets

$$\Pi_I(x_i, x_1, x_2) = e^{-\mu\Delta - \frac{1}{n-1} - \mu(x_i - x_2)} \frac{1}{\mu} \frac{n-1}{n} - \frac{1}{2\mu} e^{-\mu\Delta - \frac{1}{n-1}} \left[e^{-\mu(x_i - x_2) + \frac{n-2}{n-1}} - e^{\mu(x_i - x_2) - \frac{n-2}{n-1}} \right]$$

as the profit function for a deviating independent supplier choosing investment $x_i \in [\hat{x}, x_1 + \frac{1}{\mu}]$. Using the facts that $1 - a\Delta = e^{-\mu\Delta - \frac{1}{n-1}}$ and $x_2 = \frac{1}{an}(1 - a\Delta)$ and defining $y := \mu(x_i - x_2) - \frac{n-2}{n-1}$, we can express the deviator's profit equivalently as

$$\begin{aligned}
\hat{\Pi}_I(y, x_1, x_2) &= \frac{1 - a\Delta}{\mu} \left[e^{-y - \frac{n-2}{n-1}} \frac{1}{\mu} \frac{n-1}{n} \right. \\
&+ \left. \frac{1}{2} \left[e^y - e^{-y} \right] \right] - \frac{a}{2} \left(\frac{1}{\mu} \left[y + \frac{n-2}{n-1} \right] + \frac{1}{an}(1 - a\Delta) \right)^2,
\end{aligned}$$

for $y \in [0, \mu\Delta + \frac{1}{n-1}]$.

We are now going to show that $\hat{\Pi}_I(y, x_1, x_2)$ is decreasing and concave in y for all $y \in [0, \mu\Delta + \frac{1}{n-1}]$. We do so by first establishing that $\frac{\partial \hat{\Pi}_I(y, x_1, x_2)}{\partial y}|_{y=0} < 0$. Second, we show that the third derivative with respect to y is positive. This implies that the second derivative is largest over this interval at $y = \mu\Delta + \frac{1}{n-1}$. The final step in the argument is then to show that $\frac{\partial^2 \hat{\Pi}_I(y, x_1, x_2)}{\partial y^2}|_{y = \mu\Delta + \frac{1}{n-1}} < 0$, which then implies that $\hat{\Pi}_I(y, x_1, x_2)$ is concave over the interval in question.

Step 1:

$$\frac{\partial \hat{\Pi}_I(y, x_1, x_2)}{\partial y} = \frac{1 - a\Delta}{\mu} \left[-e^{-y - \frac{n-2}{n-1}} \frac{n-1}{n} + \frac{1}{2} \left(e^y + e^{-y} \right) \right] - \frac{a}{\mu} \left[\frac{1}{\mu}(y + \frac{n-2}{n-1}) + \frac{1}{an}(1 - a\Delta) \right].$$

At $y = 0$, we get

$$\frac{\partial \hat{\Pi}_I(y, x_1, x_2)}{\partial y}|_{y=0} - \frac{n-1}{n}(1 - a\Delta) - \frac{a}{\mu} \left[\frac{1}{\mu} \frac{n-2}{n-1} + \frac{1}{an}(1 - a\Delta) \right] < 0.$$

Step 2: Differentiating further we get

$$\begin{aligned}\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2} &= \frac{1-a\Delta}{\mu}\left[e^{-y-\frac{n-2}{n-1}}\frac{n-1}{n}+\frac{1}{2}\left(e^y-e^{-y}\right)\right]-\frac{a}{\mu^2}\\ &= \frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^y+\left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}}-\frac{1}{2}\right)e^{-y}\right]-\frac{a}{\mu^2},\end{aligned}$$

where $\frac{n-1}{n}e^{-\frac{n-2}{n-1}}-\frac{1}{2}\leq 0$ for all $n\geq 2$ with strict inequality for $n>2$ (at $n=2$, it is equal to 0; differentiating with respect to n yields $-\frac{e^{-\frac{n-2}{n-1}}}{n^2(n-1)}$, which is negative), and

$$\frac{\partial^3\hat{\Pi}_I(y,x_1,x_2)}{\partial y^3} = \frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^y-\left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}}-\frac{1}{2}\right)e^{-y}\right]>0.$$

Thus, $\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}$ is an increasing function of y and hence largest at $y=\mu\Delta+\frac{1}{n-1}$.

Step 3: Evaluating $\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}$ at $y=\mu\Delta+\frac{1}{n-1}$ one gets

$$\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}}=\frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^{\mu\Delta+\frac{1}{n-1}}+\left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}}-\frac{1}{2}\right)e^{-\mu\Delta-\frac{1}{n-1}}\right]-\frac{a}{\mu^2}.$$

Replacing $e^{-\mu\Delta-\frac{1}{n-1}}$ by $1-a\Delta$ and $e^{\mu\Delta+\frac{1}{n-1}}$ by $\frac{1}{1-a\Delta}$ and collecting terms yields

$$\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}}=\frac{\mu-2a}{\mu^2}+\left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}}-\frac{1}{2}\right)\frac{(1-a\Delta)^2}{\mu}.$$

As just noticed the last expression is not positive. Therefore, $\frac{\partial^2\hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}}<0$ if $\frac{\mu}{a}<2$, which is certainly the case if $\frac{\mu}{a}<\frac{n}{n-1}$, which is the necessary and sufficient condition for the existence of a symmetric equilibrium under outsourcing.

Case 4: $x_i>x_1+\frac{1}{\mu}$ Finally, consider investments $x_i>x_1+\frac{1}{\mu}$. For such investments, the expected profit of a deviating non-integrated supplier is

$$\begin{aligned}\Pi_I(x_i,x_1,x_2) &= \frac{1}{n-1}\int_{\beta-x_2}^{\infty}e^{-\mu[n(c_i-\beta)+x_1+(n-2)x_2+x_i+\frac{1}{\mu(n-1)}]}dc_i\\ &+\int_{\beta-x_2-\frac{n-2}{\mu(n-1)}}^{\beta-x_2}\mu\left(\beta-x_2+\frac{1}{\mu(n-1)}-c_i\right)e^{-\mu\Delta-\frac{1}{n-1}-\mu(c_i+x_i-\beta)}\\ &+\int_{\beta-x_1-\frac{1}{\mu}}^{\beta-x_2-\frac{n-2}{\mu(n-1)}}e^{-\mu[2(c-\beta)+x_1+x_i]-1}dc_i\\ &+\int_{\beta-x_i}^{\beta-x_1-\frac{1}{\mu}}(\beta-x_1-c_i)\mu e^{-\mu(c_i+x_i-\beta}dc_i-\frac{a}{2}x_i^2.\end{aligned}$$

Integrating yields

$$\begin{aligned}\Pi_I(x_i,x_1,x_2) &= \frac{1}{\mu}\frac{n-1}{n}e^{-\mu\Delta-\frac{1}{n-1}-\mu(x_i-x_2)}\\ &+ \frac{1}{2\mu}e^{-\mu(x_i-x_i)+1}\left[1-e^{-2(\mu\Delta-\frac{1}{n-1})}\right]\\ &+ x_i-x_1-\frac{1}{\mu}-\frac{a}{2}x_i^2.\end{aligned}$$

The key observation is that the terms in the fist and second line decrease in x_i. The derivative of the last line with respect to x_i is $1-ax_i$. Since $x_i \geq x_1+\frac{1}{\mu} \geq \frac{1}{an}+\frac{1}{\mu}$, we have

$$1-ax_i \leq \frac{n-1}{n}-\frac{a}{\mu} \leq 0,$$

where the inequality follows because it is equivalent to $\frac{\mu}{a} \leq \frac{n}{n-1}$. ■

Proof of Proposition 4: Inserting the expressions obtained in Lemmas 2 and 3, one gets

$$\beta+\frac{a-\mu}{\mu}x_1+\frac{a}{2}x_1^2+\frac{1}{n}\left[\frac{1}{\mu(n-1)}-\frac{1}{2an}\right]$$

for $PC_I^*+\frac{a}{2}x_1^2+\Pi_O^*$. As $PC_O^*=\beta-\frac{1}{an}+\frac{1}{\mu}\frac{2n-1}{n(n-1)}$, vertical divesture is thus jointly profitable if and only if

$$\beta+\frac{a-\mu}{\mu}x_1+\frac{a}{2}x_1^2+\frac{1}{n}\left[\frac{1}{\mu(n-1)}-\frac{1}{2an}\right] > \beta-\frac{1}{an}+\frac{1}{\mu}\frac{2n-1}{n(n-1)},$$

which is equivalent to the inequality in the proposition. ■

Proof of Proposition 5: Substituting the expressions for the exponential gives us the following expression for the expected production cost:

$$EC(\mathbf{x})=\mu\sum_{j=1}^{n} je^{-\mu X_j}\int_{\beta-x_j}^{\beta-x_{j+1}} ce^{-j\mu(c-\beta)}dc,$$

where, as in the main text, $X_j := \sum_{i=1}^{j} x_i$ and $x_{n+1} := -\infty$.

Integrating $\int_{\beta-x_j}^{\beta-x_{j+1}} ce^{-j\mu(c-\beta)}dc$ by parts we get

$$\int_{\beta-x_j}^{\beta-x_{j+1}} ce^{-j\mu(c-\beta)}dc = \frac{1}{j\mu}e^{j\mu x_j}\left[\beta-x_j+\frac{1}{j\mu}-\left(\beta-x_{j+1}+\frac{1}{j\mu}\right)e^{-j\mu(x_j-x_{j+1})}\right].$$

Therefore, the j-th summand, denoted $S_j=j\mu e^{-\mu X_j}\int_{\beta-x_j}^{\beta-x_{j+1}} ce^{-j\mu(c-\beta)}dc$, is given as

$$S_j=e^{-\mu(X_j-jx_j)}\left[\beta-x_j+\frac{1}{j\mu}-\left(\beta-x_{j+1}+\frac{1}{j\mu}\right)e^{-j\mu(x_j-x_{j+1})}\right].$$

Observe that $S_n = e^{-\mu(X_n - nx_n)}\left[\beta - x_n + \frac{1}{n\mu}\right]$.

Consequently, $EC(\mathbf{x})$ can be written as

$$EC(\mathbf{x}) = \sum_{j=1}^{n} S_j = \sum_{j=1}^{n} e^{-\mu(X_j - jx_j)}\left[\beta - x_j + \frac{1}{j\mu} - \left(\beta - x_{j+1} + \frac{1}{j\mu}\right)e^{-j\mu(x_j - x_{j+1})}\right].$$

For j such that $1 < j \leq n$, we have

$$\begin{aligned}
\frac{\partial S_{j-1}}{\partial x_j} &= -(j-1)\mu e^{-\mu(X_j - jx_j)}(\beta - x_j) \\
\frac{\partial S_j}{\partial x_j} &= (j-1)\mu e^{-\mu(X_j - jx_j)}\left[\beta - x_j + \frac{1}{j\mu} - \left(\beta - x_{j+1} + \frac{1}{j\mu}\right)e^{-j\mu(x_j - x_{j+1})}\right] \\
&- e^{-\mu(X_j - jx_j)} + j\mu e^{-\mu(X_{j+1} - (j+1)x_{j+1})}\left(\beta - x_{j+1} + \frac{1}{j\mu}\right).
\end{aligned}$$

Noticing that

$$j\mu e^{-\mu(X_{j+1} - (j+1)x_{j+1})}\left(\beta - x_{j+1} + \frac{1}{j\mu}\right) = j\mu\left[e^{-\mu(X_j - jx_j)}\left(\beta - x_j + \frac{1}{j\mu}\right) - S_j\right]$$

and

$$e^{-\mu(X_j - jx_j)}\left[\beta - x_j + \frac{1}{j\mu} - \left(\beta - x_{j+1} + \frac{1}{j\mu}\right)e^{-j\mu(x_j - x_{j+1})}\right] = S_j,$$

we can that write

$$\frac{\partial S_j}{\partial x_j} = -\mu S_j + j\mu e^{-\mu(X_j - jx_j)}(\beta - x_j)$$

Moreover, for all $i > j$ we have

$$\frac{\partial S_i}{\partial x_j} = -\mu S_i.$$

Letting $S_0 := 0$ and $\sum_{i=n+1}^{n} \frac{\partial S_i}{\partial x_n} = 0$, we thus have for all $j = 1, .., n$

$$\frac{\partial EC(\mathbf{x})}{\partial x_j} = \mu e^{-\mu(X_j - jx_j)}(\beta - x_j) - \mu\sum_{i=j}^{n} S_i$$

and for all $j < n$,

$$\frac{\partial EC(\mathbf{x})}{\partial x_j} - \frac{\partial EC(\mathbf{x})}{\partial x_{j+1}} = -\frac{1}{j}e^{-\mu(X_j - jx_j)}(-1 + e^{-\mu(x_j - x_{j+1})}).$$

For $j = n$ we have

$$\frac{\partial S_n}{\partial x_n} = \mu(n-1)e^{-\mu(X_n - nx_n)}\left(\beta - x_n + \frac{1}{n\mu}\right).$$

Consequently,

$$\frac{\partial EC(\mathbf{x})}{\partial x_n} = -\frac{1}{n} e^{-\mu(X_n - n x_n)}.$$

Using the first-order condition, we get the boundary condition

$$\frac{1}{n} e^{-\mu(X_n - n x_n)} = a x_n. \tag{17}$$

We now analyze the second-order conditions for a cost minimum. The derivative of $EC(\mathbf{x})$ with respect to x_n is

$$\frac{\partial EC(\mathbf{x})}{\partial x_n} = \frac{\partial S_n}{\partial x_n} + \frac{\partial S_{n-1}}{\partial x_n} = -\frac{1}{n} e^{-\mu(X_n - n x_n)}.$$

At symmetry, i.e. with $x_i = x$ for all i, the second partials are therefore

$$\frac{\partial^2 EC(\mathbf{x})}{\partial x_n^2} = -\mu \frac{n-1}{n} \quad \text{and} \quad \frac{\partial^2 EC(\mathbf{x})}{\partial x_n \partial x_{n-1}} = \frac{\mu}{n}.$$

Therefore, the second partials of the total cost $TC(\mathbf{x}) := EC(\mathbf{x}) + \frac{a}{2} \sum_{i=1}^{n} x_i^2$ are

$$\frac{\partial^2 TC(\mathbf{x})}{\partial x_n^2} = -\mu \frac{n-1}{n} + a \quad \text{and} \quad \frac{\partial^2 TC(\mathbf{x})}{\partial x_n \partial x_{n-1}} = \frac{\mu}{n}.$$

Thus, at symmetry the Hessian matrix has $a - \mu \frac{n-1}{n}$ on the main diagonal and $\frac{\mu}{n}$ everywhere else. Thus, it is positive semi-definite if and only if $a - \mu \frac{n-1}{n} \geq \frac{\mu}{n}$, which is equivalent to $a \geq \mu$. This proves the the symmetric solution is a local minimum if and only if $\frac{\mu}{a} \leq 1$. We next show that the symmetric solution is also a global minimum whenever it is a local minimum.

Subtracting $\frac{\partial TC(\mathbf{x})}{\partial x_i}$ from $\frac{\partial TC(\mathbf{x})}{\partial x_{i+1}}$ and simplifying yields for $i = 1, .., n-2$ with $n > 2$

$$\frac{\partial TC(\mathbf{x})}{\partial x_{i+1}} - \frac{\partial TC(\mathbf{x})}{\partial x_i} = \frac{1}{i} e^{-\mu X_i} \left[e^{i\mu x_{i+1}} - e^{i\mu x_i} \right] = a(x_{i+1} - x_i). \tag{18}$$

We thus have a system of first-order difference equations

$$\frac{1}{i} e^{-\mu X_i} \left[e^{i\mu x_{i+1}} - e^{i\mu x_i} \right] = a(x_{i+1} - x_i) \tag{19}$$

with the boundary condition (17) and the constraints $x_i \geq x_{i+1}$. Notice that the symmetric solution $x_i = \frac{1}{an}$ for all $i = 1, .., n$ is always a solution of this system. We are now going to show that for $a \geq \mu$ it is the unique solution.

Notice first that the right-hand side of (19) is, trivially, linear in x_{i+1} with slope a. The left-hand side of (19) is increasing and convex in x_{i+1} with slope μ at symmetry. Fix then an arbitrary x_1. Provided $\mu \leq a$, $x_2 = x_1$ is the unique solution to (19). Iterating the argument, we get that $x_i = x_1$ is the unique solution to (19) for all $i = 1, .., n-1$. Notice then that the left-hand side of (17) is convex and increasing in x_n with slope $\mu \frac{n-1}{n}$ at symmetry. Since $\mu \leq a$ implies $\mu \frac{n-1}{n} < a$, where a is the slope of the right-hand side of (17), it follows that symmetry, i.e. $x_n = x_1$, is the unique solution to (17). But at symmetry, (17) implies $x_n = \frac{1}{an}$. Thus, for $\mu \leq a$, $x_i = \frac{1}{an}$ for all $i = 1, .., n$ is the unique solution.

We now characterize the planner's solution for the case $\mu > a$. We first prove a result that is of some independent interest. We say that investments shift the mean if the cumulative probability of a cost draw not bigger than c is $G(c)$ when the investment is 0 and $G(c + x)$ when the investment is x. Let $\underline{c}(\mathbf{x})$ be the lower bound in the common support when the investments are $\mathbf{x}$ and define $G(c + x_i)$ on an extended support (i.e. $G(c_i+x_i) = 0$ for all $c_i < \underline{c}(x_i)$, where $\underline{c}(x_i) = \beta - x_i$ is the lower bound in i's support given x_i). Observe that this model nests our investment model with exponential distributions. The distribution $L(c; \mathbf{x})$ of the minimum cost c given investments $\mathbf{x}$ is then given as

$$L(c; \mathbf{x}) = 1 - \prod_{i=1}^{n} (1 - G(c + x_i))$$

with support $[\underline{c}(\mathbf{x}), \infty)$; see also (1). The expected cost of production is $EC(\mathbf{x}) = \int_{\underline{c}(\mathbf{x})}^{\infty} c dL(c; \mathbf{x})$ and total cost is $TC(\mathbf{x}) = EC(\mathbf{x}) + \frac{a}{2} \sum_{i=1}^{n} x_i$.

Integrating $\int_{\underline{c}(\mathbf{x})}^{\infty} c dL(c; \mathbf{x})$ by parts we get $EC(\mathbf{x}) = \underline{c}(\mathbf{x}) + \int_{\underline{c}(\mathbf{x})}^{\infty} [1 - L(c; \mathbf{x})] dc$. Taking the derivative of $EC(\mathbf{x})$ with respect to x_i gives

$$\frac{\partial EC(\mathbf{x})}{\partial x_i} = -\int_{\underline{c}(\mathbf{x})}^{\infty} \frac{\partial L(c; \mathbf{x})}{\partial x_i} dc.$$

This is so regardless of whether x_i affects $\underline{c}(\mathbf{x})$ because if it does the two effects cancel. Notice that $\frac{\partial L(c;\mathbf{x})}{\partial x_i} = g(c + x_i) \prod_{j \neq i}^{n} (1 - G(c + x_j))$. Consequently,

$$\frac{\partial EC(\mathbf{x})}{\partial x_i} = -\int_{\underline{c}(\mathbf{x})}^{\infty} g(c + x_i) \prod_{j \neq i}^{n} (1 - G(c + x_j)) dc.$$

At an optimum, we have $\frac{\partial EC(\mathbf{x})}{\partial x_i} + a x_i = 0$ for all i. Adding up over all i yields

$$-\int_{\underline{c}(\mathbf{x})}^{\infty} \sum_{i=1}^{n} g(c + x_i) \prod_{j \neq i}^{n} (1 - G(c + x_j)) dc + a \sum_{i=1}^{n} x_i = 0. \tag{20}$$

Observe then that $dL(c; \mathbf{x}) = \sum_{i=1}^{n} g(c + x_i) \prod_{j \neq i}^{n} (1 - G(c + x_j)) dc$. Thus, (20) can be written as

$$-\int_{\underline{c}(\mathbf{x})}^{\infty} dL(c; \mathbf{x}) + a \sum_{i=1}^{n} x_i = 0.$$

Since $\int_{\underline{c}(\mathbf{x})}^{\infty} dL(c; \mathbf{x}) = 1$, this implies that at an optimum investments always add up to a constant, that is

$$\sum_{i=1}^{n} x_i = \frac{1}{a}. \tag{21}$$

Next we show that the planner's asymmetric solution consists of two different investment levels only. Let $k_1 = \max\{i|x_i = x_1\}$. The difference equation (19) then implies

$$ak_1(x_{k_1+1} - x_1) = e^{-\mu k_1 x_1}\left(e^{\mu k_1 x_{k_1+1}} - e^{\mu k_1 x_1}\right) = e^{\mu k_1 (x_{k_1+1} - x_1)} - 1.$$

Letting $\Delta_1 = k_1(x_1 - x_{k_1+1})$ this is the same as

$$a\Delta_1 = 1 - e^{-\mu\Delta_1} > 0. \tag{22}$$

Next let $k_2 = \max\{i|x_i = x_{k_1+1}\}$ and let $\Delta_2 = k_2(x_{k_1} - x_{k_2})$. Then (19) implies

$$a\Delta_2 = e^{-\mu\Delta_1}\left(e^{\mu\Delta_2} - 1\right).$$

This equation has two possible solutions: (i) $\Delta_2 = 0$ and (ii) $\Delta_2 > 0$. Solution (i) implies $k_2 = n$. The adding-up constraint (21) implies $\Delta_1 = \frac{1}{a} - nx_n$ and the boundary condition (17) and (22) imply $anx_n = e^{-\mu(1/a - nx_n)}$. Thus, the solution with $\Delta_2 = 0$ is admissible. Next we show that the other solution, i.e. (ii), is not.

To see this, observe that equalities (17) and (22) and the adding-up constraint (21) imply

$$1 \geq a(nx_n + \Delta_1) = 1 - e^{-\mu\Delta_1} + e^{-\mu(1/a - nx_n)},$$

where the inequality is strict if $\Delta_2 > 0$. But this implies

$$e^{-\mu\Delta_1} \geq e^{-\mu(1/a - nx_n)},$$

which in turn implies $1/a - nx_n \leq \Delta_1$. Taken together, this implies

$$\Delta_1 + nx_n = \frac{1}{a}.$$

That is, Δ_1 and nx_n add up to $1/a$. Thus, $\Delta_2 > 0$ would violate the adding-up constraint. Hence, we conclude that $\Delta_2 = 0$ and thus $k_2 = n$.

The final step shows that $k_1 = 1$. We show that by assuming to the contrary that there are $k > 1$ suppliers who invest x_1 and then showing the reallocating $\varepsilon/(k-1) > 0$ from each of them to supplier 1 decreases total costs. Using a change of variables $y = \beta - c$, the part of total costs affected by this reallocation of investment can be shown to be

$$\begin{aligned} &\mu e^{-\mu(x_1+\varepsilon)} \int_{-x_1}^{-x_2+\frac{\varepsilon}{k-1}} ye^{-\mu y}dy + (k-1)\mu e^{-\mu(k-1)x_2+x_1} \int_{-x_2+\frac{\varepsilon}{k-1}}^{-x_3} ye^{-\mu y}dy \\ + \quad &\frac{a}{2}\left[(x_1+\varepsilon)^2 + (k-1)(x_2 + \varepsilon/(k-1))^2\right]. \end{aligned}$$

Taking the derivative with respect to ε, evaluated at $x_1 = x_2 = x$ and $\varepsilon = 0$, gives

$$\mu x e^{-\mu x}\left[e^{-\mu k x} - \frac{k}{k-1}e^{-\mu x}\right] < 0.$$

Thus, this reallocation of investments decreases total costs. This is a contradiction to $k > 1$ being optimal. ■

Proof of Lemma 6: Taking the sourcing distortion $\frac{1}{\mu(n-1)}$ as given, the expected cost of production given investment levels x_1 and x_2 under the second-best scenario, denoted $EC^S(x_1, x_2)$, is

$$\begin{aligned} EC^S(x_1, x_2) &= \mu \int_{\beta - x_1}^{\beta - x_2 + \frac{1}{\mu(n-1)}} ce^{-\mu(c+x_1-\beta)} dc + \mu \int_{\beta - x_2 + \frac{1}{\mu(n-1)}}^{\infty} ce^{-\mu(c+x_1-\beta)} e^{-\mu(n-1)(c - \frac{1}{\mu(n-1)} + x_2 - \beta)} dc \\ &+ \mu(n-1) \int_{\beta - x_2}^{\infty} ce^{-\mu(c + \frac{1}{\mu(n-1)} + x_1 - \beta)} e^{-\mu(n-1)(c+x_2-\beta)} dc. \end{aligned}$$

The first integral captures those cost realizations of the integrated supplier for which this supplier produces with probability 1. The second integral represents the instances in which the integrated supplier produces when the lowest cost draw of the independent suppliers is sufficiently high but not otherwise. The last integral covers those cost realizations for which the independent supplier with the lowest cost draw produces. Integrating and simplifying yields

$$EC^S(x_1, x_2) = \beta - x_1 + \frac{1}{\mu} - \frac{1}{\mu} e^{-\mu(x_1 - x_2) - \frac{1}{n-1}}.$$

At an optimum,

$$\frac{\partial EC^S(x_1, x_2)}{\partial x_1} + ax_1 = -1 + e^{-\mu(x_1 - x_2) - \frac{1}{n-1}} + ax_1 = 0$$

and

$$\frac{\partial EC^S(x_1, x_2)}{\partial x_1} + (n-1)ax_2 = -e^{-\mu(x_1 - x_2) - \frac{1}{n-1}} + (n-1)ax_2 = 0.$$

Taking the difference $a(x_1 - x_2)$ then gives

$$a(x_1 - x_2) = 1 - \frac{n}{n-1} e^{-\mu(x_1 - x_2) - \frac{1}{n-1}}.$$

The solution Δ^S to the equation $a\Delta = 1 - \frac{n}{n-1} e^{-\mu\Delta - \frac{1}{n-1}}$ cannot be 0 because $\frac{n}{n-1} e^{-\frac{1}{n-1}} < 1$ for any finite $n \geq 2$.

To see that $\Delta^S < \Delta^I$, recall that Δ^I is the positive solution to $a\Delta = 1 - e{-}\mu\Delta - \frac{1}{n-1}$. The left-hand side of both equations being the same (and increasing in Δ) and the right-hand side of either equation being decreasing in Δ but being strictly smaller for the equation that determines Δ^S, the result follows. ■

Proof of Proposition 7: Under nonintegration, equilibrium effort is given by $\psi(x^*) = \frac{1}{n}$. On the other hand, rewriting the consolidated equilibrium condition with vertical integration, (10), as $\frac{n-1}{n}\psi(x_2) + \frac{1}{n}\psi(x_1) = \frac{1}{n}$, it follows from Jensen's inequality that $(n-1)x_2 + x_1 = nx^*$ if $\psi'' = 0$ and $(n-1)x_2 + x_1 > nx^*$ if $\psi'' < 0$ and $(n-1)x_2 + x_1 < nx^*$ if $\psi'' > 0$. ■

Derivation of the bidding function $b_E(c)$ in (13) We now show that with elastic demand the function in (13) is the equilibrium bidding function with and without integration, beginning with the latter. Given symmetric investments x, a symmetric equilibrium bidding strategy $b(c)$ is such that

$$c = \arg\max_z \left\{[b(z) - c]\,[1 - F(b(z))]\,[1 - G(c + x)]^{n-1}\right\}.$$

For F and G exponential, a representative supplier's problem becomes

$$\max_z (b(z) - c)e^{-\lambda \max\{b(z)-\alpha,0\}-\mu(n-1))(z+x-\beta)}.$$

Taking the derivatives with respect to z separately for the cases $b(z) \leq \alpha$ and $b(z) > \alpha$, evaluated at $z = c$, gives, respectively, the first-order conditions

$$b'(c) - (n-1)\mu[b(c) - c)] = 0$$

and

$$b'(c) - [\lambda b'(c) + (n-1)\mu][b(c) - c)] = 0.$$

Imposing the boundary constraint $b(\hat{c}) = \hat{c}$ for some arbitrary $\hat{c}$ for the former gives the unique solution

$$b_i(c) = c + \frac{1}{(n-1)\mu} + ke^{(n-1)\mu c}$$

for $c \leq \hat{c}$ for some constant k (that remains to be determined) while imposing the boundary condition $\lim_{c\to\infty}(b(c) - c)/c = 0$ on the latter yields the unique solution

$$b_{ii}(c) = c + \frac{1}{\lambda + (n-1)\mu}.$$

Setting $b_{ii}(\hat{c}) = \alpha$ gives us $\hat{c}$ as stated in the lemma. Standard arguments imply that the equilibrium bidding function has to be continuous everywhere. As the functions b_i and b_{ii} are continuous for $c \leq \hat{c}$ and $c \geq \hat{c}$, we are left to insure that $b_i(\hat{c}) = b_{ii}(\hat{c})$, which will then determine k. Imposing this equality gives us

$$\frac{1}{\lambda + (n-1)\mu} = \frac{1}{(n-1)\mu} + ke^{(n-1)\mu\hat{c}},$$

implying

$$k = -\left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu\hat{c}}.$$

Plugging k back into $b_i(c)$, we get

$$b_i(c) = c + \frac{1}{(n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu(\hat{c}-c)}.$$

To see that we can write the bidding function $b_E(c)$ as stated in in (13), observe that for $c \geq \alpha - \frac{1}{\lambda+\mu(n-1)}$.

For the case with integration, notice that in a first-price auction, $b(c) \geq c$ will always hold. Therefore, there are a priori four cases to consider:

i) $b(\beta - x_2) > \max\{\alpha, \beta - x_1\}$
ii) $\alpha > b(\beta - x_2) > \beta - x_1$
iii) $\beta - x_1 > b(\beta - x_2) > \alpha$
iv) $\min\{\alpha, \beta - x_1\} > b(\beta - x_2)$
However, assuming $x_1 \geq x_2$, we are left we cases i) and ii).

The proof proceeds along similar lines as the one for the case without integration. For F and G exponential and $x_1 \geq x_2$, a representative non-integrated supplier's problem is

$$\max_z (b(z) - c)e^{-\lambda \max\{b(z)-\alpha,0\}-\mu(b(z)+x_1-\beta)-\mu(n-2))(z+x_2-\beta)}.$$

Taking the derivatives with respect to z separately for the cases $b(z) \leq \alpha$ and $b(z) > \alpha$, evaluated at $z = c$, gives, respectively, the first-order conditions

$$b'(c) - [b'(c)\mu + (n-2)\mu][b(c) - c)] = 0$$

and

$$b'(c) - [(\lambda + \mu)b'(c) + (n-2)\mu][b(c) - c)] = 0.$$

Imposing the boundary constraint $b(\hat{c}) = \hat{c}$ for some arbitrary $\hat{c}$ for the former gives the unique solution

$$b_I(c) = c + \frac{1}{(n-1)\mu} + ke^{(n-1)\mu c}$$

for $c \leq \hat{c}$ for some constant k (that remains to be determined) while imposing the boundary condition $\lim_{c\to\infty}(b(c) - c)/c = 0$ on the latter yields the unique solution

$$b_{II}(c) = c + \frac{1}{\lambda + (n-1)\mu}.$$

Setting $b_{II}(\hat{c}) = \alpha$ gives us $\hat{c} = \alpha - \frac{1}{\lambda+(n-1)\mu}$. Standard arguments imply that the equilibrium bidding function has to be continuous everywhere. As the functions b_I and b_{II} are continuous for $c \leq \hat{c}$ and $c \geq \hat{c}$, we are left to insure that $b_I(\hat{c}) = b_{II}(\hat{c})$, which will then determine k. Imposing this equality gives us

$$\frac{1}{\lambda + (n-1)\mu} = \frac{1}{(n-1)\mu} + ke^{(n-2)\mu\hat{c}},$$

implying

$$k = \left[\frac{1}{\lambda + (n-1)\mu} - \frac{1}{(n-1)\mu}\right] e^{-(n-1)\mu\hat{c}}.$$

Plugging k back into $b_I(c)$, we get

$$b_I(c) = c + \frac{1}{(n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu(\hat{c}-c)}.$$

■

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