On the First Order Approximation of Counterfactual Price Effects in Oligopoly Models*

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Abstract

We develop how first order approximation can be used to make counterfactual price predictions in oligopoly models. We extend the theoretical results of Jaffe and Weyl (2013) on mergers to any counterfactual scenario involving perturbations to firms' first order conditions. We show that, for vertical shifts to firms' cost functions, first order approximation simplifies naturally and is exact for a class of demand systems. We then use Monte Carlo experiments to evaluate accuracy. We find that (i) first order approximation is more accurate than simulation in 91.7% of the mergers considered; (ii) it is more accurate than simulation in 98% of the cost shocks considered; and (iii) simple versions of approximation, of interest to antitrust practitioners, exist and systematically outperform merger simulation.

Keywords: first order approximation; cost pass-through; simulation; mergers
JEL classification: K21; L13; L41

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1 Introduction

This paper addresses the first order approximation of counterfactual price effects in oligopoly models. First order approximation may best be introduced in its relation to simulation, a methodology that is a staple in industrial organization and other fields of economics. The accuracy of simulation requires functional forms that characterize reasonably how economic relationships change away from the observed equilibria. First order approximation, by contrast, allows the researcher to remain largely agnostic about functional forms. Instead, the second-order properties of the relevant functions, in the neighborhood of the observed equilibria, are gleaned from pass-through and subsequently used to inform counterfactual predictions. The theoretical literature has long recognized that pass-through is connected to demand curvature (e.g., Bulow and Pfleiderer (1983)), and this has garnered more attention recently (e.g., Fabinger and Weyl (2012), Weyl and Fabinger (2013)). However, there is little prior research that explores the theoretical properties of first order approximation and none that investigates empirically the accuracy of its counterfactual predictions.

Our starting point is the theoretical work of Jaffe and Weyl (2013), which derives first order approximation in the context of mergers between horizontally differentiated competitors. We first extend the theory to any counterfactual scenario involving perturbations to firms' first order conditions, and focus especially on vertical shifts in the marginal cost and demand functions faced by firms. Such scenarios include, but are not limited to, pollution permits trading programs, production or sales taxes, exchange rate fluctuations, and some forms of innovation. Each involves the same fundamental issue: the extent to which firms transmit cost or demand shocks to consumers in the form of price adjustments. Predominately, papers in industrial organization use simulation to examine such scenarios – first order approximation provides an alternative methodology that potentially is more robust. We explain how the primitives required for implementation of first order approximation can be obtained from pass-through and show how the formulas can be manipulated to best make use of the available information.

We then present additional theoretical results for counterfactual scenarios involving vertical shifts in firms' marginal cost functions. First, we show that in such settings first order approximation simplifies and can be implemented by pre-multiplying the cost changes by the cost pass-through matrix. This result is both simple and powerful. The immediate implication is that reduced-form econometric estimates of pass-through can be used to make meaningful out-of-sample predictions, alleviating in some cases the need for structural estimation.[1] Second, we prove that first order approximation is exact in models characterized by constant pass-through, such as those that feature a class of demand functions identified in Bulow and Pfleiderer (1983).[2] Third, we show that the above results extend to scenarios involving "industry-wide" cost shocks that affect all firms equally. Knowledge of how firms respond to industry-wide shifts in the observed equilibria, either collectively or individually, is sufficient to support predictions based on first order approximation that are fully consistent with oligopoly interactions. This latter result is relevant to a large and growing literature in macroeconomics and international trade.

These theoretical results in hand, we use Monte Carlo experiments to evaluate the accuracy of first order approximation, both absolutely and relative to simulation. These experiments complement the theoretical results, which demonstrate the precision of approximation only for counter-factual scenarios involving arbitrarily small perturbations and in certain special cases, such as when firms have quadratic profit functions or for vertical cost and demand shifts with constant pass-through demand systems. The Monte Carlo experiments allow us to evaluate tractably the quality of counter-factual predictions in those settings that are most relevant for researchers and policy-makers.

We first parameterize the logit, almost ideal, linear and log-linear demand systems to reproduce each of 3,000 randomly drawn sets of data on market shares and margins. The data generating process is designed to cover a wide range of firm and industry conditions. Importantly, we calibrate the demand systems such that the demand elasticities are identical in each for a given draw of data. Marked differences in demand curvature and pass-through exist though and lead to differences in counterfactual predictions. We impose a number of counterfactual changes on each parameterized system, including (i) a merger between two firms; (ii) a firm-specific vertical shift in the marginal cost of one firm; and (iii) an industry-wide vertical shift in the marginal cost functions of all firms.[3]

[1]The result can be interpreted as provided external validity to reduced-form pass-through estimates because, provided consistent pass-through estimates are obtained (i.e., internal validity is achieved), the econometrician can extrapolate beyond the range of the data to model counter-factual scenarios based on the logic of first order approximation.

[2]Demand must induce firms to employ constant pass-through rates. Weyl and Fabinger (2013) provide versions of this result for the case of single-product firms. The applicability of the result in settings with multi-product firms is limited as there the only demand system with constant pass-through that also satisfies Slutsky symmetry is linear.

[3]The generated data confirm the Monte Carlo results of Crooke, Froeb, Tschantz, and Werden (1999) regarding the sensitivity of merger simulation to functional form assumptions. Our work thus has relevance to a burgeoning literature that compare merger simulation to direct *ex post* estimates of actual price effects (e.g., Nevo (2000); Peters (2006); Weinberg (2011); Weinberg and Hosken (2013); Bjornerstedt and Verboven (2012)), in that we highlight the potential importance of demand curvature assumptions in creating discrepancies between merger simulations and realized price effects.

We then compare the predictions of first order approximation, calculated using information on curvature and pass-through in the initial equilibrium, to the true price effect. We also compare first order approximation to the predictions of misspecified simulation, i.e., simulation conducted with correct elasticities but incorrect assumptions on the functional forms. Of course, simulation that is conducted with correct functional form obtains the true price effect. This empirical design yields results that are relevant to researchers who have estimated accurately the relevant elasticities, as they exist in the observed equilibria, but who have imperfect knowledge about how the elasticities change away from those equilibria.

In our data, first order approximation outperforms misspecified simulation systematically and substantially. Consider first the case of mergers. We find that the prediction errors that arise with first order approximation are tightly distributed around the true price effects, while with misspecified simulation the empirical distributions of prediction error exhibit bias and fatter tails. The median absolute prediction error that arises with first order approximation typically is an order of magnitude less than that of misspecified simulation, and the absolute prediction error with approximation is smaller than that of misspecified simulation in 91.7% of the merger scenarios considered. Further, when price effects are evaluated against a specific threshold (e.g., a 10% change), prediction based on first order approximation exhibits both few false positives and few false negatives, while prediction based on misspecified simulation typically exhibits either many false positives or many false negatives.

First order approximation is even more accurate for the counterfactual scenarios involving vertical shifts to the marginal cost functions. There our theoretical results establish exactness for the cases of linear and log-linear demand, for which optimal pass-through rates are constant. In the Monte Carlo experiments, we find that prediction error often is within rounding error of zero with logit or almost ideal demand, for which pass-through is not constant. First order approximation has smaller absolute prediction error than misspecified simulation in 98.4% of the firm-specific cost shock scenarios considered and in 99.8% of the industry-wide cost shock scenarios.

Finally, we use the Monte Carlo experiments to characterize the accuracy of two versions of first order approximation for mergers that we believe should be attractive to antitrust practitioners. These predictors build on the Farrell and Shapiro (2010a) logic that mergers create opportunity costs ("upward pricing pressure") because each merging firm, when making a sale, forgoes with some probability a sale by the other merging firm. The first predictor is calculated by multiplying upward pricing pressure by cost pass-through. We find that this predictor is substantially more accurate than misspecified merger simulation – for instance, it has smaller absolute prediction error than misspecified simulation in 89.6% of the mergers considered. The second predictor is calculated by multiplying upward pricing pressure by the identity matrix. We find that even this outperforms misspecified merger simulation in fully 77.9% of the mergers considered. These results indicate that upward pricing pressure is substantially more useful than initially conceptualized (e.g., see Schmalensee (2009); Carlton (2010); Farrell and Shapiro (2010a)), and underscores the value of the first order approach to merger analysis pioneered in Werden (1996) and advanced in Jaffe and Weyl (2013).

Most generally, our research has bearing on how economists in industrial organization approach making counterfactual predictions. We refer readers to Nevo and Whinston (2010) for an overview of simulation and to Werden and Froeb (2008) for an informed discussion of merger simulation specifically. First order approximation joins a recent literature exploring ways to construct counterfactuals that rely less on functional form assumptions. For instance, recent contributions demonstrate the random coefficients logit model is non-parametrically identified (Bajari, Fox, Kim, and Ryan (2012), Berry, Gandhi, and Haile (2013)). Provided the model is estimated appropriately, data from the observed equilibria can be allowed to determine the curvature of the relevant functional forms and inform counterfactual predictions.

This paper is also closely related to recent theoretical research studying the various uses of cost pass-through. Cost pass-through, due to its connection with demand curvature, can be used in a number of ways to inform modeling choices, estimation, and the generation of counterfactuals (e.g., Fabinger and Weyl (2012)). First order approximation is one option: our results show that, given knowledge of the demand elasticities, cost pass-through can be used to make robust counterfactual predictions. Miller, Remer, and Sheu (2013) show that cost pass-through also can be used to select an appropriate demand model for use in counterfactual simulation exercises and inform the demand elasticities given such a model. These findings leverage the observation that cost pass-through is connected theoretically to strategic complementarity, in the sense of Bulow, Geanakoplos, and Klemperer (1985), which in turn is connected to the degree to which consumers view products as substitutes.

Our results have special relevance to the substantial literature in macroeconomics and international trade that estimates the pass-through of industry-wide cost shocks by way of reduced-form regression. In particular, we show how the obtained pass-through rates enable out-of-sample predictions that are fully consistent with oligopoly theory – conveying a previously unrecognized sense of external validity. Those predictions are both simpler and likely more accurate than most simulation methodologies. While a full review of the literature is beyond the scope of our paper, we note that among the topics investigated are the sources of incomplete cost pass-through (e.g., Nakamura and Zerom (2010), Golberg and Hellerstein (2013)), the retail and wholesale components of cost pass-through (e.g., Nakamura (2008), Gopinath, Gourinchas, Hsieh, and Li (2011)) and how cost-pass through is affected by horizontal market structures (e.g., Atkeson and Burstein (2008), Berman, Martin, and Mayer (2011), Auer and Schoenle (2012), Hong and Li (2013)) and vertical market structures (e.g., Hellerstein and Villas-Boas (2010), Neiman (2010), Hong and Li (2013)).

The paper proceeds as follows. Section 2 introduces the theoretical underpinnings of first order approximation as a methodology for making counterfactual predictions. Special attention is given to counterfactual scenarios involving mergers and vertical shifts to firms' cost and demand functions. That section includes the new theoretical results and sketches the simplifications that could make first order approximation more palatable for antitrust practitioners. Section 3 pivots to the Monte Carlo experiments. There the data generating process is detailed, and summary statistics are presented on the pass-through that arises with logit, almost ideal, linear and log-linear demand. The sensitivity of simulation to functional form assumption is also explored. Section 4 compares the accuracy of first order approximation both against true price effects and vis-à-vis misspecified simulation. It addresses, in turn, counterfactual scenarios involving mergers, scenarios involving firm-specific and industry-wide cost shocks, and the simplified versions of approximation for antitrust practitioners. We conclude in Section 5 with a discussion of first order approximation and an overview of additional research opportunities.

2 The Theory of First Order Approximation

2.1 Definitions and model

We assume that there is a set of firms engaging in Bertrand-Nash competition, each facing a well-behaved, twice-differentiable demand function. The mathematics generalize to alternative equilibrium concepts if there is a single strategic variable per product (Jaffe and Weyl 2013) but we focus on the Bertrand-Nash case to ease exposition. Let each firm i produce some subset of the products available to consumers. The profit function of firm i is

$$\pi_i = P_i^T Q_i(P) - C_i(Q_i(P)),$$

where P_i is a vector of firm i's prices, Q_i is a vector of firm i's unit sales, P is a vector containing the prices of every product, and C_i is the cost function. The superscript T

denotes the vector/matrix transpose.

The first order conditions that characterize this firm's profit-maximizing prices can be expressed

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)}{\partial P_i}^T \right]^{-1} Q_i(P) - (P_i - MC_i(Q_i(P))) = 0, \tag{1}$$

where $MC_i = \partial C_i / \partial Q_i$ is a vector of firm i's marginal costs. This formulation is often convenient in the context of first order approximation, as we explain below, because marginal costs enter quasi-linearly with a coefficient of one. The first order conditions also can be written as

$$f_i^{alt}(P) \equiv Q_i(P) + \left[\frac{\partial Q_i(P)}{\partial P_i}^T \right] (P_i - MC_i) = 0, \tag{2}$$

which is simply a rearrangement of equation (1) with quantity sold entering quasi-linearly with a coefficient of one. This alternative formulation can be convenient in special cases.

Consider now an event – policy change or otherwise – that affects firms' profits through the cost function, the consumer demand schedule, or both. The first order conditions can be modified to account for resulting changes in firm incentives. Given the definition of $f_i(P)$ in equation (1), these "post-policy" first order conditions can be expressed

$$h_i(P) \equiv f_i(P) + g_i(P) = 0, \tag{3}$$

where $g_i(P)$ adjusts the pre-policy conditions $f_i(P)$ to account for the changes in incentives. Similarly, the alternative post-policy conditions are given by $h_i^{alt}(P) \equiv f_i^{alt}(P) + g_i^{alt}(P) = 0$.

First order approximation uses information on the pre-policy equilibrium expressed through the post-policy first order conditions just derived. Jaffe and Weyl (2013) provide the following theorem, which generalizes beyond their context of horizontal mergers:

Theorem 1: *Let P^0 be the pre-policy equilibrium price vector and let $h(P)$ be the post-policy first order conditions. If $h(P)$ is invertible then the price changes due to the policy, to a first approximation, are given by the vector*

$$\Delta P = - \left(\frac{\partial h(P)}{\partial P} \right)^{-1} \Bigg|_{P=P^0} h(P^0).$$

Proof: Let $h(P) = f(P) + g(P)$. Then $h(P^0) = g(P^0) \equiv r$ because $f(P^0) = 0$. Let P^1 denote the prices the characterize the post-policy equilibrium so that $h(P^1) = 0$. If h is invertible then

$$P^1 - P^0 \;=\; h^{-1}(0) - h^{-1}(r) = \left(\frac{\partial h^{-1}}{\partial h}(r)\right)(0-r) + O(\|r\|^2)$$
$$\approx\; -\left(\frac{\partial f}{\partial P}(P^0) + \frac{\partial g}{\partial P}(P^0)\right)^{-1} g(P^0) \quad \square$$

The formula provided in Theorem 1 approximates how firms transmit marginal cost and demand shocks to consumers in the form of price adjustments; if one replaces $h(P)$ with $h^{alt}(P)$ the formula applies to the alternative first order conditions.[4] Note that the shocks are represented in the formula by the vector $h(P^0)$, which is equivalent to $g(P^0)$ since by construction $f(P^0) = 0$. How these shocks are predicted to manifest as price changes is determined by the opposite inverse Jacobian of $h(P)$ with respect to P, evaluated at pre-policy prices, which we refer to as the *policy pass-through* matrix. As can be seen from equations (1) and (3), the policy pass-through matrix incorporates the first and second derivatives of the demand function. The matrix is closely related to the cost pass-through matrix that arises in the initial equilibrium, as we develop in Section 2.3.

First order approximation is exact when profit functions are quadratic, as they are with linear demand and constant marginal costs (Jaffe and Weyl (2013)). We extend in Section 2.4 the conditions under which exactness is obtained to counterfactual scenarios involving vertical marginal cost shifts and constant pass-through systems.

To build intuition on Theorem 1, we demonstrate first order approximation graphically. Figure 1 plots a hypothetical function $h(P)$ for a single-product monopolist. The intersection of $h(P)$ with the horizontal axis is the optimal post-policy price for the monopolist. The dashed line is tangent to $h(P)$ at the pre-policy price. A first order approximation to the optimal post-policy price is obtained by projecting this tangent to its point of intersection with the horizontal axis. This is equivalent to applying a single step of the Newton-Raphson method for finding the roots of a function. In this example, the convexity of $h(P)$ causes first order approximation to understate the optimal post-policy price of the monopolist. The convexity or concavity of $h(P)$ depends on higher-order properties of demand, meaning that in general, first order approximation could understate or overstate the price adjustments. Improved predictions could be obtained either by incorporating information on the third-order properties of demand,[5] yielding a second order approximation, or by applying multiple

[4] Indeed, first order approximation was initially proposed for merger evaluation in Froeb, Tschantz, and Werden (2005), based on the alternative first order conditions of equation (2).

[5] Equivalently, one could incorporate information on how pass-through rates change with level of costs.

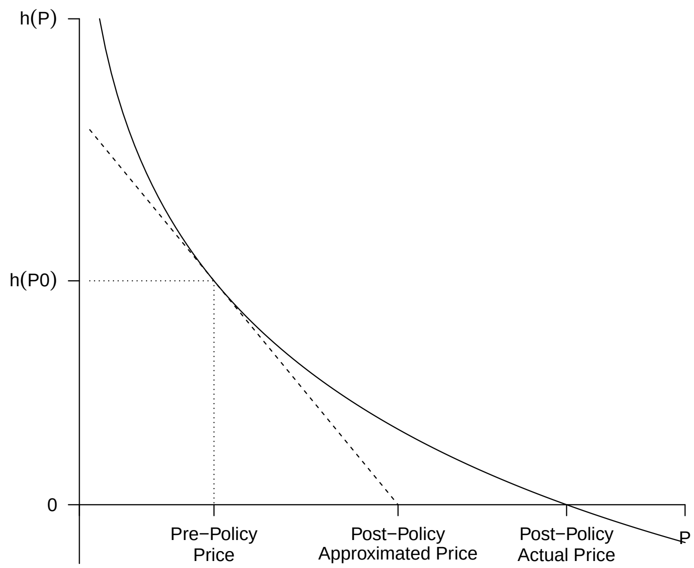

Figure 1: Graphical Illustration of First Order Approximation

steps of the Newton-Raphson method. Both prospects require information that is unavailable except in rare instances.

2.2 Policy changes and first order conditions

In order to provide some concrete examples of first order approximation, we examine three distinct cases: (i) policy changes that involve vertical shifts in the marginal cost schedules, (ii) policy changes that result in vertical shifts of the demand functions, and (iii) horizontal mergers between firms selling substitute products as in Jaffe and Weyl (2013). The first two cases allow for simple cost- and demand-side shocks, respectively, while the third incorporates interactions from both channels simultaneously.

The g function takes on a different form for each of these three policy types. For policies that vertically shift the marginal cost function, the g function is given by

$$g_i(P) = \Delta MC_i(Q_i(P)) \equiv MC_i^1(Q_i(P)) - MC_i(Q_i(P)), \tag{4}$$

where MC^1 is the post-policy marginal cost function. The pre- and post-policy first order conditions are equivalent aside from the substitution of MC^1 for MC. The tractability of this expression arises from the formulation of the $f_i(P)$, where costs enter quasi-linearly with a coefficient of one. For policies that vertically shift the demand function, holding fixed demand slope and curvature), the g function takes the form

$$g_i(P) = -\left[\frac{\partial Q_i(P)}{\partial P_i}^T\right]^{-1} \Delta Q_i(P) \equiv -\left[\frac{\partial Q_i(P)}{\partial P_i}^T\right]^{-1} (Q_i^1(P) - Q_i(P)), \tag{5}$$

which is the size of the vertical shift in the *inverse* demand curve that results from the policy. Again the pre- and post-policy first order conditions are equivalent aside from the substitution of $Q_i^1(P)$ for $Q_i(P)$. In the special case when the alternative first order conditions $h_i^{alt}(P)$ are being used, expression (5) reduces to $g_i^{alt}(P) = \Delta Q_i(P)$. The main benefit of the alternative first order conditions is that they simplify in these instances.

The g function that results from a horizontal merger is more complicated. A merger between substitutes can potentially have effects both on demand and on costs. Consider a merger between firms j and k. The g function that results is

$$g_j(P) = \underbrace{-\left(\frac{\partial Q_j(P)^T}{\partial P_j}\right)^{-1}\left(\frac{\partial Q_k(P)^T}{\partial P_j}\right)}_{\text{Matrix of Diversion from } j \text{ to } k} \underbrace{(P_k - MC_k^1)}_{\text{Markup of } k} - \underbrace{(MC_j - MC_j^1)}_{\text{Cost Efficiencies}}. \tag{6}$$

The form of $g_k(P)$ is analogous and $g_i(P) = 0$ for $i \neq j, k$. One can think of the first term in equation (6) as measuring a new opportunity cost that the firm internalizes after the merger. Each firm in the merger, when making a sale, forgoes with some probability a sale by the other firm. That is, when adjusting its prices, the merging firm can drive sales towards or away from its merging partner. The diversion matrix represents the fraction of sales lost by firm j's products that shift to firm k's products due to an increase in firm j's prices. When multiplied by the vector of firm k's markups, this yields the value of diverted sales; the more these sales are worth, the greater incentive a firm has to raise price following a merger. In turn, these incentives are counterbalanced by any marginal cost efficiencies created by the merger, which are captured in the second term of equation (6). Farrell and Shapiro (2010a) refer to $g_j(P^0)$ and $g_k(P^0)$ as the net *upward pricing pressure* created by the merger. The interpretation of upward pricing pressure as an opportunity cost is supported by the formulation of the first order conditions in equations (1) and (3), as both $g_i(P)$ and marginal costs enter the same way.[6]

[6] For an extended discussion on these issues, see for example Werden (1996); Weyl and Fabinger (2013);

2.3 Pass-through and identification

The formula for first order approximation requires as an input the Jacobian of $h(P)$. This incorporates the first and second derivatives of the demand function (equivalently, the demand elasticities and curvatures). We discuss in this section how pass-through can be exploited to obtain the second derivatives of demand, given the first derivatives. Let $\rho^c(P^0)$ be the initial cost pass-through matrix. Each element $\rho^c_{ij}(P^0)$ is the derivative of product i's equilibrium price with respect to product j's cost, evaluated at equilibrium prices. The off-diagonal elements are positive if and only if prices are strategic complements.[7] Jaffe and Weyl (2013) show that

$$\rho^c(P) \equiv \frac{\partial P}{\partial t} = -\left(\frac{\partial f(P)}{\partial P}\right)^{-1}, \tag{7}$$

where t is a vector of taxes that perturbs marginal costs. The matrices that appear above are of dimensionality $N \times N$, where N is the number of products. Thus, this expression provides N^2 equations, each of which matches an element of the cost pass-through matrix to a nonlinear function of the first and second derivatives of demand.

These equations can be used to recover the second derivatives, given certain identifying assumptions and knowledge of the first derivatives and cost pass-through. Assuming that demand satisfies Slutsky symmetry is sufficient for identification in the case of duopoly.[8] In other instances, second derivatives of the form $\partial^2 Q_i/(\partial P_j \partial P_k)$, for $i \neq j$, $i \neq k$ and $j \neq k$, are not identified from equation (7) even with Slutsky symmetry, without further restrictions. These second derivatives are plausibly small, however, and it may be reasonable to normalize

Farrell and Shapiro (2010a); Farrell and Shapiro (2010b); Kominers and Shapiro (2010); Jaffe and Weyl (2013); and Willig (2011). The 2010 Horizontal Merger Guidelines endorse upward pricing pressure as informative of the likely competitive effects of mergers. See Horizontal Merger Guidelines ∮6.1:

> "The value of sales diverted to a product is equal to the number of units diverted to that product multiplied by the margin between price and incremental cost on that product. In some cases, where sufficient information is available, the Agencies assess the value of diverted sales, which can serve as a diagnostic of the upward pricing pressure.... The Agencies rely much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated products."

[7] Among the four demand systems we consider in the Monte Carlo analysis, strategic complementarity exists for the linear, logit and almost ideal demand systems but not for the log-linear demand system, where equilibrium prices are unaffected by competitor costs. See Section 3 for numerical results or Miller, Remer, and Sheu (2013) for theoretical results.

[8] Slutsky symmetry implies $\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}$ and it follows that:

$$\frac{\partial^2 Q_i}{\partial^2 P_j} = \frac{\partial}{\partial P_j}\frac{\partial Q_i}{\partial P_j} = \frac{\partial}{\partial P_j}\frac{\partial Q_j}{\partial P_i} = \frac{\partial^2 Q_j}{\partial P_j \partial P_i}.$$

them to zero. Alternatively, Jaffe and Weyl (2013) suggest the following assumption on demand:

$$Q_i(P) = \psi\left(P_i + \sum_{j \neq i} \mu_j(P_j)\right), \tag{8}$$

for some $\psi : \mathbb{R} \to \mathbb{R}$ and $\mu : \mathbb{R} \to \mathbb{R}$, which is sufficient for full identification.[9] This is a slightly more restrictive version of the horizontality assumption employed in Weyl and Fabinger (2013). For ease of exposition we refer in later sections to equation (8) as defining the "horizontality" assumption. Once the additional identifying restrictions have been chosen, numerical optimization can be used to select the second derivatives that minimize the "distance" between the elements in the opposite inverse Jacobian of $f(P)$ and the elements in the observed pre-policy cost pass-through matrix. The selected second derivatives then can be used to calculate the Jacobian of $h(P)$. These steps require expressions of the Jacobians of $f(P)$ and $h(P)$, which we provide in Appendix A.

A similar strategy can be used if instead a measure of demand pass-through is available. Consider demand shocks that perturb the unit sales of each product (given prices), and denote the vector of demand shocks s. We show in Appendix B that this gives rise to the demand pass-through matrix,

$$\rho^d(P) \equiv \frac{\partial P}{\partial s} = -\left(\frac{\partial f^{alt}(P)}{\partial P}\right)^{-1}, \tag{9}$$

where entry ρ^d_{ij} is the effect of a demand shock for product j on i's price. This expression can be used to infer the second derivatives of demand that rationalize demand pass-through, following the logic outlined for cost pass-through.

2.4 Additional theoretical results

We develop in this section three results that, in turn, (i) simplify the calculation of calculation of first order approximation; (ii) expand the range of environments in which first order approximation provides exact prediction; and (iii) extend first order approximation to the industry-wide shifts that are relevant for macroeconomics and international trade.

[9] With horizontality, the needed second derivatives take the form

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k} = \frac{\partial^2 Q_i}{\partial^2 P_i} \frac{\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_k}}{\left(\frac{\partial Q_i}{\partial P_i}\right)^2}.$$

First, for scenarios involving vertical shifts in the marginal cost or demand functions, a first order approximation to the price changes can be obtained by multiplying the shift by the appropriate notion of cost or demand pass-through. Numerical optimization to obtain demand curvature from pass-through is unnecessary. Consider the case of a vertical shifts in the cost functions. Because the derivatives of the cost function are unchanged it follows that $\partial g(P)/\partial P = 0$. This implies that $\partial h(P)/\partial P = \partial f(P)/\partial P$ such that the policy pass-through matrix equals the cost pass-through matrix. Analogous logic applies to the case of vertical shifts in the demand functions. This again shows that manipulating the first order conditions can simplify dramatically the implementation of first order approximation. We formalize this result in the following corollary:

Corollary 1: *Let P^0 be the pre-policy equilibrium price vector and let $\rho^c(P)$ be the cost pass-through matrix. Further let $g(P) \equiv h(P) - f(P) = \Delta MC$, where ΔMC is a vertical shift in the marginal cost functions such that $\Delta MC \equiv MC^1(P) - MC(P)$. Then the price changes due to the marginal cost shifts, to a first order approximation, are given by the vector $\Delta P = \rho^c(P^0)\Delta MC$.*

This result is both simple and powerful. The immediate implication is that reduced-form econometric estimates of cost pass-through can be used to make meaningful out-of-sample predictions, alleviating in some cases the need for structural estimation. The result can be interpreted as provided external validity to reduced-form pass-through estimates because, provided consistent pass-through estimates are obtained (i.e., internal validity is achieved), the econometrician can extrapolate beyond the range of the data to model counterfactual scenarios in a way that is fully consistent with oligopoly interactions.

Second, also for scenarios involving vertical shifts in cost or demand functions, first order approximation provides exact predictions if pass-through is constant. This expands the range of economic environments in which first order approximation is exact beyond the case of quadratic profit functions considered by Jaffe and Weyl (2013). Constant pass-through arises for a class of demand functions identified in Bulow and Pfleiderer (1983) and commonly employed in the theoretical literature of industrial organization. With single-product firms, the class includes as special cases the linear, log-linear and constant markup demand systems.

Proposition 1: *Let P^0 be the pre-policy equilibrium price vector and let $\rho^c(P)$ be the cost pass-through matrix. Further let $g(P) \equiv h(P) - f(P) = \Delta MC$, where ΔMC is a vertical shift in the marginal cost functions such that $\Delta MC \equiv MC^1(P) - MC(P)$. If the underlying* *demand functions belong to the constant pass-through class then the first order approximation defined in Corollary 1 is exact.*

Proof: Let $P^*(MC(P))$ represent the equilibrium prices that arise with the cost functions $MC(P)$. Then the change in price due to the policy can be written

$$\int_0^{\Delta MC} \rho^c(P^*(MC(P^0) + x))dx.$$

Since pass-through is constant, $\rho^c(P) = \rho^c$ and the change in price simplifies to $\int_0^{\Delta MC} \rho^c dx = \rho^c \int_0^{\Delta MC} dx = \rho^c \Delta MC$, which is the first order approximation for vertical cost shifts. □

In research that predates this paper, Weyl and Fabinger (2013) provide versions of the above proposition for the case of single-product firms. The applicability of the proposition in settings with multi-product firms is limited as there the only demand system with constant pass-through that also satisfies Slutsky symmetry is linear.

Finally, we develop results for cases involving vertical shifts in cost or demand that affect all firms in an industry equally. Such "industry-wide" shifts are highly relevant to the macroeconomics and international trade literatures because they encompass a range of important policy topics including the connection between wages and prices, the effect of exchange rate fluctuations, and the implications of oil price shocks. The price effects of industry-wide shifts can be calculated, to a first order approximation, without knowledge of the full pass-through matrix. Instead, knowledge of how firms respond to industry-wide shifts, in the neighborhood of the initial equilibrium, and either individually or on average, is sufficient to support predictions that are fully consistent with oligopoly interaction. Furthermore, first order approximation in such settings is exact if the underlying demand functions belong to the constant pass-through class. These results, formalized below as a corollary to Theorem 1 and Proposition 1, can be interpreted as conveying an external validity to the substantial empirical literature on the pass-through of industry-wide shifts.

Corollary 2: *Let every product $i \in \{1, 2, \ldots, N\}$ face an identical shift in marginal cost, $\Delta MC_i = \Delta MC_j = \Delta \overline{MC}\ \forall i, j$. Define industry cost pass-through as the vector $\rho^I(P) = [(\sum_{j=1}^N \rho^c_{1,j}(P)), \ldots, (\sum_{j=1}^N \rho^c_{N,j}(P))]$, where $\rho^c_{ij}(P)$ is the cost pass-through of product i's price with respect to product j's cost. Further define average industry pass-through as the scalar $\overline{\rho}^I = \frac{1}{N}\sum_{i=1}^N \rho^I_i(P)$, where $\rho^I_i(P)$ is the i^{th} element of the industry pass-through vector. Then the following results apply:*

(i) *The product specific price changes, to a first approximation, are given by the vector*

$$\Delta P = \rho^I(P)|_{P=P^0}\Delta\overline{MC}.$$

(ii) *The average price change, to a first approximation, is given by the scalar*

$$\overline{\Delta P} = \overline{\rho}^I(P)|_{P=P^0}\Delta\overline{MC}$$

and moreover, if pass-through symmetry exists such that $\rho^c_{ij}(P) = \rho^c_{ji}(P)$ *and* $\rho^c_{ii}(P) = \rho^c_{jj}(P)$ *for all* i, j*, then the product-specific price changes equal the average price change.*

(iii) *If the underlying demand functions belong to the constant pass-through class then the first order approximations defined in (i) and (ii) are exact.*

2.5 Antitrust applications

First order approximation should be attractive to antitrust practitioners because it avoids the recognized sensitivity of merger simulation to functional form assumptions (e.g., Crooke, Froeb, Tschantz, and Werden (1999)) and because how the predictions are obtained – multiplying an opportunity cost by an appropriate notion of pass-through – likely can be conveyed transparently to non-economists. Here we outline two simplifications that could facilitate such an application.[10] First, practitioners could use cost pass-through as an imperfect proxy for policy pass-through. This obviates the need to obtain the second derivatives of demand via estimation or numerical optimization, and it also retains intuitive appeal because the prediction equals an opportunity cost multiplied by cost pass-through. Formally, we define "simple approximation" for mergers as

$$\Delta P = \rho^c(P^0)g(P^0), \tag{10}$$

where $\rho^c(P^0)$ is the cost pass-through matrix and $g(P^0)$ is in this instance net upward pricing pressure. This ought to provide conservative predictions of price increases relative to first order approximation because the elements in the policy pass-through matrix typically exceed those in the cost pass-through matrix, as we confirm this in our Monte Carlo experiments.

Second, practitioners could use upward pricing pressure as a predictor of price changes when information on cost pass-through is unavailable. This can be rationalized as an ap-

[10] These are not the only two simplifications available. For instance, Miller, Remer, Ryan, and Sheu (2012) and Jaffe and Weyl (2013) explore various options for dealing with imperfect information on pass-through.

proximation based on a policy pass-through matrix that equals the identity matrix:

$$\Delta P = g(P^0), \tag{11}$$

which is equivalent to supposing that (i) the upward pricing pressure on each firm is passed through to consumers at the rate of 100 percent and (ii) there are not feedback effects, via strategic complementarity or otherwise, that lead firms to pass-through the upward pricing pressure on other firms. While the policy pass-through matrix for mergers is never diagonal in reality, the accuracy of this calculation in predicting price changes is an empirical question that we explore in the Monte Carlo experiments.

3 Design of the Monte Carlo Experiments

3.1 Conceptual overview

We use Monte Carlo experiments to examine the accuracy of first order approximation. These experiments complement the preceding theoretical results, which demonstrate the precision of approximation only for counter-factual scenarios involving arbitrarily minute perturbations and in certain special cases, such as when firms have quadratic profit functions or for vertical cost/demand shifts with constant pass-through demand systems. The Monte Carlo experiments allow us to evaluate tractably the quality of counter-factual predictions in those settings that are most relevant for researchers and policy-makers.

We consider an array of economic environments. In each, we posit the demand and cost functions, including both the functional forms and the parameterizations. We first compute a "pre-policy" equilibrium under the assumption of Nash Bertrand competition and then compute "post-policy" equilibria for a number of counterfactual scenarios enumerated below. Comparisons of pre- and post-policy equilibria obtain the true price effects. These provide a baseline against which to measure both first order approximation and simulation conducted with incorrect assumptions on the relevant functional forms.

We focus on two types of counterfactual scenarios: (i) mergers between two horizontally differentiated firms and (ii) vertical shifts in the cost functions of either a single firm or all firms. We model mergers by assuming joint profit maximization on the part of the merging firms. Prices in the post-merger equilibria almost always dictate positive market share for both merging firms' products.[11] We anticipate that results obtained from the latter

[11] We constrain quantities to be non-negative in the post-merger equilibria. This binds in two of the 3,000

scenarios, involving vertical shifts to firms' cost functions, should generalize to scenarios involving vertical shifts in demand given the theoretical similarities outlined in Section 2.

In calculating first order approximation, we utilize the requisite information on the first-order characteristics (i.e., elasticities) and second-order characteristics (e.g., curvatures and pass-through) of the pre-policy equilibria. We derive the second-order characteristics from the posited functional forms, given parameterizations that we discuss below. In practice, such information likely would be obtained independently from econometric analyses of pass-through or other sources. In the merger scenarios, we calculate first order approximation alternately using the second derivatives of demand as inputs and using cost pass-through as an input with the horizontality assumption of equation (8). In the cost shift scenarios, these two methodologies are equivalent, from Corollary 1.

We also compute equilibrium using misspecified simulations. These are conducted with the correct first-order characteristics of the pre-policy equilibria – only the functional forms employed are incorrect. We implement by specifying four different demand systems, positing one as the true underlying demand system (as described above) and examining the predictions of misspecified simulation using the other demand systems. Because elasticities are identical across systems for a given pre-policy equilibrium, this isolates the role of functional forms in driving counter-factual predictions. Of course, simulation that is conducted with the correct functional forms obtains the true price effects. By comparing the predictions of misspecified simulations and those of first order approximation, we develop results that are useful to researchers who have estimated accurately the relevant economic relationships, as they exist in the pre-policy equilibrium, but who have imperfect knowledge about how these relationships change away from the pre-policy equilibrium.

3.2 Data generating process

We generate data using four specific functional forms for demand: those of the logit, almost ideal, linear and log-linear demand systems. These four demand systems allow for a wide range of curvature and pass-through conditions and are commonly employed in antitrust analyses of mergers involving differentiated products (Werden, Froeb, and Scheffman (2004), Werden and Froeb (2008)). They also have been used in academic studies that examine the effect of demand curvature on the precision of counterfactual simulations and related topics (e.g., Crooke, Froeb, Tschantz, and Werden (1999), Huang, Rojas, and Bass (2008), Miller, Remer, and Sheu (2013)). With linear demand, there is no curvature and profit is a quadratic

linear demand cases considered.

function of prices if marginal costs are constant. By contrast, log-linear demand and the AIDS are quite convex and often generate pass-through in excess of unity. Logit demand is characterized by more moderate curvature and pass-through. Both linear and log-linear demand feature constant pass-through.

We select parameterizations that reproduce each of 3,000 randomly drawn sets of pre-policy data. This process, known widely as calibration, works well in our context because it generates realistic demand functions while remaining computationally tractable.[12] The pre-policy data include market shares, which we randomly allocate among four single-product firms and the outside good, and a margin for first firm, which we draw from a uniform distribution bounded by 0.20 and 0.80. We normalize all prices to one in the initial equilibrium, which does not limit generality and conveys the minor advantage that the price changes in the counterfactual scenarios are the same in levels and percentages.[13] We also limit attention to constant marginal cost functions. The presence of scale economies or diseconomies would affect pass-through and first order approximation, and it is unexamined in our experiments.

In the calibration process, we first use the pre-policy data and the Nash-Bertrand assumption to obtain the parameters of the logit model. These parameters imply a full set of own-price and cross-price demand elasticities, and we use those elasticities to calibrate the almost ideal, linear and log-linear demand systems. Because our starting point is logit demand, consumer substitution between products is proportional to share in the pre-policy equilibrium. This property is retained away from the pre-policy equilibrium only for logit demand. Importantly, the calibration process imposes that the demand elasticities are identical across the demand systems for a given draw of data in the pre-policy equilibria. Differences in policy responses therefore are driven by differences in curvature and pass-through. Once a counter-factual policy change is imposed, the elasticities that arise diverge across the demand systems based on the respective functional forms.[14] We defer to Appendix C the mathematical details of the calibration process.

The randomly drawn data sets produce some calibrations with extreme pass-through conditions and others with no post-policy equilibria. We exclude those calibrations from the analysis, treating as extreme a pass-through rate that is negative or exceeds ten. The pass-through criterion eliminates 74 AIDS calibrations and 164 log-linear calibrations. The existence of equilibrium criterion eliminates 268 AIDS calibrations and 359 log-linear cali-

[12] The primary methodological alternative, that of randomly drawing demand parameters directly, frequently produces parameter combinations that violate standard normalcy conditions.

[13] We have confirmed that this normalization does not affect results by checking alternative assumptions.

[14] Similar procedures have been employed elsewhere in the literature (e.g., Crooke, Froeb, Tschantz, and Werden (1999)).

brations with mergers and 209 AIDS calibrations with the cost shocks.

3.3 Summary statistics

In Table 1, we summarize the empirical distribution of selected variables as they arise in the 3,000 generated pre-policy equilibria. We use order statistics rather than empirical moments because many of the variables exhibit considerable skew. The market shares and margins of firm 1 are obtained from random draws. Because shares are allocated among the four products and the outside good, the distribution of firm 1's share is centered around 20 percent. The margin distribution reflects uniform draws with support over (0.20, 0.80). The own-price elasticity of demand, which equals the inverse margin, has a distribution centered around 2.08, and 90 percent of the elasticities fall between 1.32 and 4.38. These statistics are invariant to the posited demand system because the demand systems are calibrated to reproduce the first-order characteristics in the pre-policy equilibria.

By contrast, pass-through depends on demand curvature and varies across the four demand systems. The median *own-cost pass-through* of firm 1, by which we mean the derivative of firm 1's equilibrium price with respect to its cost, equals 0.80, 1.19, 0.53, and 1.87 for the logit, almost ideal, linear and log-linear demand systems, respectively. Own-cost pass-through exhibits considerable support for the almost ideal and log-linear demand systems but is more tightly distributed for the logit and (especially) the linear demand systems. The *cross-cost pass-through* statistics are based on the derivative of firm 1's equilibrium price with respect to firm 2's marginal cost.[15] Cross-cost pass-through reflects the degree of strategic complementarity in prices (Bulow, Geanakoplos, and Klemperer 1985). Median cross-cost pass-through is 0.04, 0.22, 0.09 and 0.00 for the logit, almost ideal, linear and log-linear demand systems, respectively. While the almost ideal and log-linear demand systems both tend to generate large own-cost pass-through, only the AIDS generates large cross-cost pass-through as prices are not strategic complements with log-linear demand.

In Table 2, we summarize the empirical distribution of price changes that arise under different counterfactual scenarios. The scenarios include (i) a merger of firms 1 and 2; (ii) cost increases applied to firm 1; and (iii) cost increases applied to all firms. Given the data generating process, the median merger induces a change in the Herfindahl-Hirschman Index (HHI) of 652 and creates upward price pressure of 0.11.[16] The interquartile range for these statistics is 313-1078 and 0.05-0.18, respectively. To create a comparable range in the cost

[15]Because the firms are treated identically in the data generating process, the distribution of cross-cost pass-through of firm i's cost to firm j's price is similar for any $i \neq j$.

[16]Upward pricing pressure is defined in equation (6).

Table 1: Summary Statistics on Pre-Policy Equilibria

	Median	5%.	10%	25%	75%	90%	95%
Characteristics Invariant to Demand Form							
Market share	0.21	0.03	0.06	0.13	0.28	0.35	0.40
Margin	0.48	0.23	0.26	0.34	0.62	0.72	0.76
Elasticity	2.08	1.32	1.38	1.60	2.94	3.91	4.38
Own-Cost Pass-Through							
Logit	0.80	0.63	0.67	0.73	0.88	0.94	0.97
AIDS	1.19	0.75	0.78	0.90	1.72	2.36	2.82
Linear	0.53	0.51	0.51	0.52	0.55	0.57	0.58
Log-Linear	1.87	1.29	1.34	1.50	2.52	3.39	3.98
Cross-Cost Pass-Through							
Logit	0.04	0.00	0.01	0.02	0.06	0.09	0.11
AIDS	0.22	0.03	0.06	0.12	0.39	0.70	0.98
Linear	0.09	0.01	0.02	0.05	0.12	0.15	0.17
Log-Linear	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Summary statistics are based on 3,000 randomly-drawn sets of data on the pre-policy equilibria. Statistics include the mean and standard deviation, as well as order statistics for the 10th, 25th, 50th, 75th and 90th percentiles. The market share, margin and elasticity are for the first firm. Market share and margin are drawn randomly in the data generating process while the elasticity is the own-price elasticity of demand and equals the inverse margin. Pass-through is calculated, following calibration, based on the curvature properties of the respective demand systems. Own-cost pass-through is the derivative of firm 1's equilibrium price with respect to its own marginal cost. The cross-cost pass-through statistics are based on the derivative of firm 1's equilibrium price with respect to firm 2's marginal cost. The statistics exclude calibrations with extreme pass-through rates that are negative or exceed ten.

Table 2: Summary Statistics on Price Changes

	Median	5%.	10%	25%	75%	90%	95%
Due to merger of firms 1 and 2							
Logit	0.09	0.01	0.02	0.05	0.16	0.24	0.30
AIDS	0.18	0.01	0.03	0.08	0.46	1.09	1.88
Linear	0.08	0.01	0.02	0.04	0.14	0.21	0.28
Log-Linear	0.30	0.02	0.05	0.12	0.77	2.08	4.11
Due to cost increases specific to firm 1							
Logit	0.05	0.01	0.02	0.02	0.10	0.13	0.14
AIDS	0.09	0.02	0.02	0.04	0.15	0.23	0.29
Linear	0.04	0.01	0.01	0.02	0.07	0.08	0.08
Log-linear	0.14	0.03	0.03	0.07	0.24	0.35	0.43
Due to industry-wise cost increases							
Logit	0.05	0.02	0.02	0.02	0.10	0.14	0.15
AIDS	0.14	0.03	0.03	0.07	0.25	0.41	0.55
Linear	0.05	0.01	0.02	0.03	0.10	0.12	0.13
Log-linear	0.14	0.03	0.03	0.07	0.24	0.35	0.43

Notes: Summary statistics for the change in firm 1's price, based on 3,000 randomly-drawn sets of data on the pre-policy equilibria. Statistics include the mean and standard deviation, as well as order statistics for the 10th, 25th, 50th, 75th and 90th percentiles. The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-policy equilibria.

increase scenarios, we apply to each set of pre-policy data firm-specific and industry-wide cost increases of \$0.02, \$0.05, \$0.10 and \$0.15. Without loss of generality, all variables shown in Table 2 pertain to the changes in firm 1's price.

Starting with the merger scenarios, the median changes in price are 0.09, 0.18, 0.08, and 0.30 for the logit, almost ideal, linear and log-linear demand systems, respectively. Because pre-policy prices are normalized to one, these statistics reflects both the mean level change and mean percentage change. There is considerable dispersion within each demand system, reflecting the range of upward pricing pressure as well as the variance in pass-through that exists within systems. The larger price increases with the AIDS and log-linear demand reflect more substantial pass-through of upward pricing pressure to consumers and is consistent with the higher cost pass-through rates that arise in these systems.

With the cost increase scenarios, the median changes in firm 1's price due to firm-specific cost increases are 0.05, 0.09, 0.04 and 0.14 for the logit, linear, almost ideal and log-linear demand systems, respectively. The differences across demand systems are similar to the differences in cost pass-through summarized previously. The median changes in firm 1's price due to an industry-wide increase in costs are 0.05, 0.14, 0.05, and 0.14 for the logit, linear, almost ideal and log-linear demand systems, respectively. That these price increases tend to exceed those that arise with a firm-specific cost increase is attributable to the strategic complementarity of prices or, equivalently, to positive cross-cost pass-through. The exception is log-linear demand, for which strategic complementarity does not exist and firm-specific and industry-wide cost increases have the same effect.

We provide a graphic illustration in Figure 2 of how functional form assumptions affect the predictions of simulation. The scatter plots characterize the accuracy of merger simulutions when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3) and log-linear (column 4). Merger simulations are conducted assuming demand is logit (row 1), almost ideal (row 2), linear (row 3) and log-linear (row 4). Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of simulation and its horizontal position is the true price effect. Dots that fall along the 45-degree line represent exact predictions while dots that fall above (below) the line represent over (under) predictions. Prediction error is zero when the functional form used in simulation matches that of the underlying demand system.[17]

As shown, logit merger simulation systematically and substantially under-predicts the price effects of mergers when the underlying demand system is almost ideal or log-linear.[18] When the underlying demand system instead is linear, logit merger simulation is roughly correct on average but frequently either over-predicts or under-predicts price effects. AIDS merger simulation substantially over-predicts prices increases when the underlying demand system is logit or linear but under-predicts when it is log-linear. The performance of linear merger simulation is analogous to that of logit merger simulation. Log-linear simulation substantially over-predicts when the underlying demand system is not log-linear. Since, in practice, counter-factual simulation tends to be conducted with little knowledge on the underlying functional forms, these results call into question the robustness of the methodology. We turn now to whether first order approximation obtains more robust predictions.

[17]Figure 2 is symmetric by construction. For example, the scatterplot for logit merger simulation when underlying demand is AIDS is the inverse of the scatterplot for AIDS merger simulation when underlying demand is logit.

[18]We provide order statistics on the prediction error that arises in these scenarios in Appendix Table D.1.

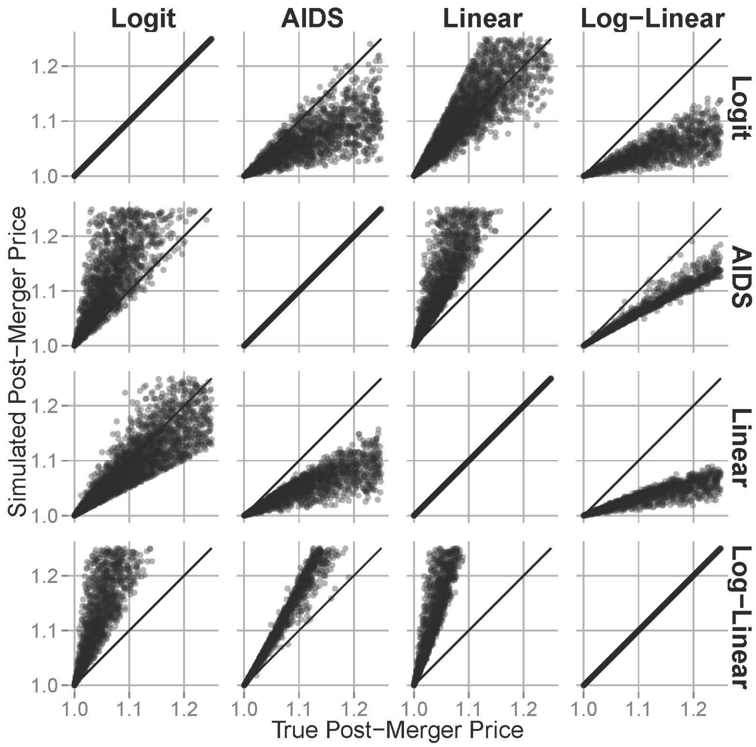

Figure 2: Prediction Error from Misspecified Merger Simulations

Notes: The scatter plots characterize the accuracy of merger simulutions when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3) and log-linear (column 4). Merger simulations are conducted assuming demand is logit (row 1), almost ideal (row 2), linear (row 3) and log-linear (row 4). Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of simulation and its horizontal position is the true price effect. Dots that fall along the 45° line represent exact predictions while dots that fall above (below) the line represent over (under) predictions. Prediction error is zero when the functional form used in simulation matches that of the underlying demand system.

4 Results of the Monte Carlo Experiments

4.1 First Order Approximation and Mergers

We first examine the accuracy of first order approximation in predicting the price effects that arise due to mergers, the application conceptualized in Jaffe and Weyl (2013). To start, in Figure 3 we use scatter plots to graph the prediction error that arises when the underlying demand system is logit (column 1), almost ideal (column 2) and log-linear (column 3). First order approximation is exact when demand is linear. We show the results of first order approximation calculated both using demand curvature as an input (row 1) and using cost pass-through as an input with the horizontality assumption (row 2). Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of simulation and its horizontal position is the true price effect. Dots that

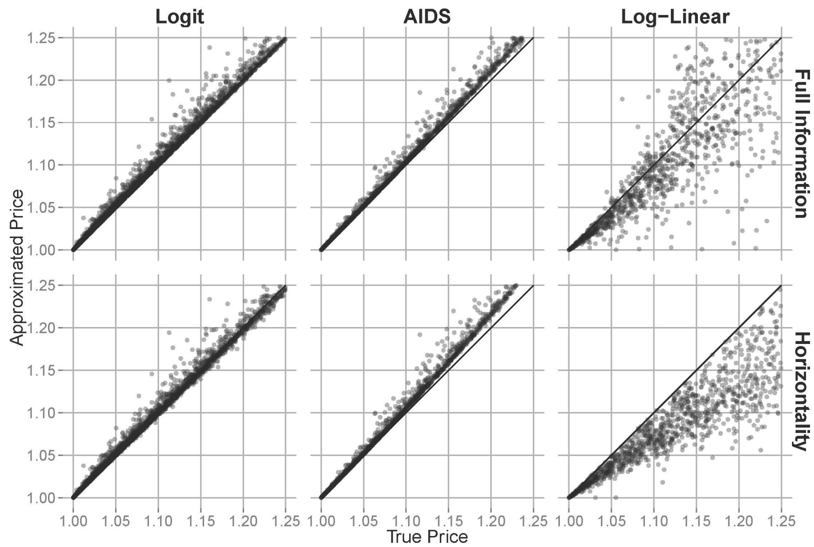

Figure 3: Prediction Error of First Order Approximation for Mergers

Notes: The scatter plots characterize the accuracy of first order approximation when the underlying demand system is logit (column 1), almost ideal (column 2) and log-linear (column 3). Approximation is exact when underlying demand is linear. First order approximation is calculated with demand curvature (row 1) and with cost pass-through (row 2). Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of first order approximation and its horizontal position is the true price effect. Dots that fall above (below) the 45° line represent over (under) predictions.

fall along the 45-degree line represent exact predictions while dots that fall above (below) the line represent over (under) predictions.

First order approximation yields accurate predictions when the underlying demand system is logit or almost ideal, as demonstrated the clustering of dots around the 45° line. Prediction error is somewhat larger with the log-linear demand system but even there it remains visibly smaller than the prediction error that arises with misspecified simulations and log-linear demand (see Figure 2). Clear and substantial bias arises only when underlying demand is log-linear and first order approximation is calculated with cost pass-through – there approximation systematically understates the true price effects.

Table 3 provides order statistics on the empirical distributions of prediction error. The median prediction error that arises with the logit, almost ideal and log-linear demand equals 0.001, 0.011 and −0.005, respectively, when first order approximation is calculated using curvature as an input. This can be evaluated against both the median true price effects of 0.09, 0.18 and 0.30, respectively, or against the median prediction that arises with misspecified simulation (Appendix Table D.1). Median prediction error is similar when first order approximation is calculated used cost pass-through as a input, with the exception of the previously mentioned log-linear case. The tightness of the empirical distribution of

Table 3: Empirical Distribution of Prediction Error for Mergers

Demand	Median	5%	10%	25%	75%	90%	95%
First order approximation calculated with demand curvature							
Logit	0.001	-0.001	-0.001	-0.000	0.005	0.011	0.018
AIDS	0.011	0.000	0.001	0.002	0.047	0.210	0.626
Log-Linear	-0.005	-3.375	-1.198	-0.072	0.087	0.463	1.129
First order approximation calculated with cost pass-through							
Logit	0.000	-0.007	-0.004	-0.001	0.003	0.007	0.013
AIDS	0.015	0.000	0.001	0.003	0.082	0.416	1.256
Log-Linear	-0.101	-3.554	-1.443	-0.389	-0.028	-0.008	-0.003

Notes: The table summarizes the empirical distribution of prediction errors for firm 1's price change that arise with first order approximation when the true underlying demand system is logit, almost ideal and log-linear, respectively. Separate statistics are shown for first order approximation calculated using the second derivatives of demand ("demand curvature") and using cost pass-through with the horizontality assumption ("cost pass-through"). The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-merger equilibria.

prediction error varies with the underlying demand system but, as we develop below, in each case the distribution for first order approximation is more tightly centered around the true price effect than the empirical distributions that arise with misspecified simulation.

Table 4 furthers the comparison of first order approximation and misspecified simulation. For brevity results are shown only for calculations that use demand curvature as an input; the results are similar for calculations based on pass-through and the horizontality assumption. Panel A provides the fraction of mergers examined for which first order approximation has smaller absolute prediction error than misspecified simulation. When underlying demand is logit, approximation is more accurate than AIDS, linear and log-linear simulation in 99.8%, 85.5% and 100% of the mergers, respectively. When underlying demand is almost ideal, approximation is more accurate than logit, linear and log-linear simulation in 95.1%, 96.5% and 99.3% of the mergers, respectively. With linear demand approximation is exact and so outperforms misspecified simulations in every instance. Finally, when underlying demand is log-linear, approximation is more accurate than logit, AIDS and linear simulation in 76.2%, 64.7% and 77.7% of the mergers, respectively. Aggregating across these scenarios, first order approximation is more accurate than misspecified simulation in 91.7% of mergers considered. Panel B shows the median absolute prediction errors that arise with first order approximation and misspecified simulations. As shown, the median absolute prediction error of misspecified simulation often is an order of magnitude larger than that of approximation.

Table 4: First Order Approximation versus Merger Simulation

Panel A: Frequency with which FOA is More Accurate

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	.	95.1%	100%	76.2%
AIDS Simulation	99.8%	.	100%	64.7%
Linear Simulation	85.3%	96.5%	.	77.7%
Log-Linear Simulation	100%	99.3%	100%	.

Panel B: Median Absolute Prediction Error

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	0.002	0.011	0.000	0.080
Logit Simulation	.	0.088	0.016	0.207
AIDS Simulation	0.090	.	0.103	0.122
Linear Simulation	0.016	0.102	.	0.220
Log-Linear Simulation	0.215	0.122	0.228	.

Notes: Panel A shows the fraction of data draws for which first order approximation has a smaller absolute prediction error than merger simulation in predicting firm 1's price change. Panel B shows the median absolute prediction error of first order approximation and misspecified merger simulations in predicting firm 1's price change. First order approximation is calculated using the second derivatives of demand ("demand curvature"). The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-merger equilibria.

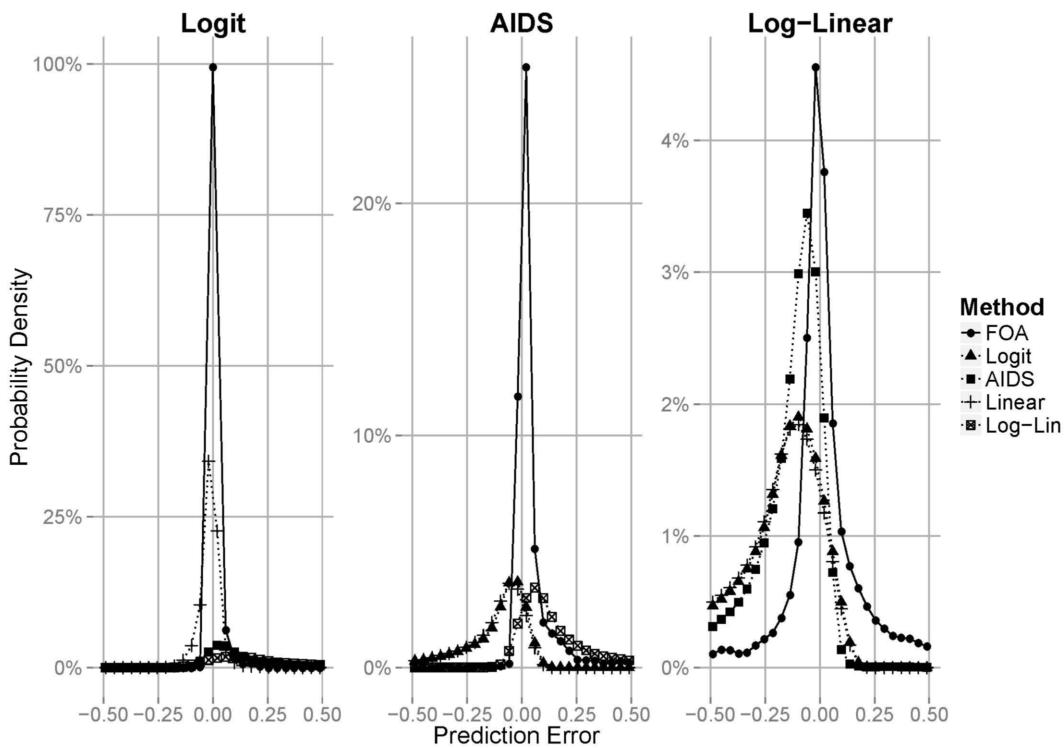

Figure 4: Kernel Density Estimation of Prediction Error Distributions for Mergers
Notes: The panels plot kernel denstity estimates of the distributions of prediction errors. Separate panels are provided for when the underlying demand system is logit, almost ideal and log-linear. First order approximation is exact when underlying demand is linear. Kernel density estimates are provided for first order approximation and for misspecified merger simulations. First order approximation is calculated using the second derivatives of demand ("demand curvature").

There are other ways to make the comparison. In Figure 4, we plot kernel density estimates of the prediction error distributions. Separate panels are provided for when the underlying demand system is logit, almost ideal and log-linear. The estimated densities for approximation appear as a solid black line while the estimated densities for misspecified simulations appear as dotted lines. In each case, the prediction error distribution for approximation is centered around zero. Those distributions are tight for the almost ideal and (especially) the logit demand systems. This is in stark relief to the prediction error distributions for misspecified simulations, which typically exhibit clear bias and fat tails. With log-linear demand, the prediction error distribution for approximation has fatter tails but nonetheless remains tighter and more closely centered around zero than the prediction error distributions of misspecified simulations.

Finally, we examine the propensity of first order approximation and misspecified simulation to produce "false positives" and "false negatives." We define a false positive as an instance in which the true price effect is less than ten percent but the prediction exceeds ten

Table 5: Type I and II Prediction Error for Mergers

Panel A: Frequency of False Positives (Type I)

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	1.6%	1.5%	0.0%	0.9%
Logit Simulation	.	0.3%	9.6%	0.0%
AIDS Simulation	23.2%	.	30.4%	0.0%
Linear Simulation	2.2%	0.0%	.	0.0%
Log-Lin Simulation	34.4%	12.6%	41.8%	.

Panel B: Frequency of False Negatives (Type II)

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	0.0%	1.0%	0.0%	14.5%
Logit Simulation	.	25.0%	2.2%	39.1%
AIDS Simulation	0.2%	.	0.0%	13.3%
Linear Simulation	9.6%	32.4%	.	46.8%
Log-Lin Simulation	0.0%	0.0%	0.0%	.

Notes: Panel A shows the fraction of data draws for which the true price change in firm 1's price is less than 10 percent but the prediction exceeds 10 percent. Panel B the fraction of data draws for which the true price change exceeds 10 percent but the prediction is less than 10 percent. First order approximation is calculated using the second derivatives of demand ("demand curvature"). The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-merger equilibria.

percent. Analogously, we define a false negative as an instance in which the true price effect exceeds ten percent but the prediction is less than ten percent. We select ten percent purely based on the empirical distribution of true prices changes: in each demand system, many mergers produce true price effects both above and below this threshold. We have examined other thresholds and the qualitative results are unaffected.

Overall, prediction based on first order approximation exhibits both few false positive and few false negatives while prediction based on misspecified merger simulation typically exhibits either many false positive or many false negatives. When the underlying demand system is logit or linear, first order approximation dominates misspecified merger simulation, in the sense that approximation produces fewer false positives and at least as few false negatives. When demand is almost ideal, first order approximation produces both low rates of false positives – many fewer than log-linear simulation – and also low rates of false negatives – many fewer than logit and linear simulation. With log-linear demand, first order approximation has a low rate of false positives that nonetheless exceeds the false positive rates of misspecified simulations. This reflects the much larger price effects that arise with log-linear demand. Its rate of false negatives is comparable to that of AIDS simulation and much lower than that of logit or linear simulation.

4.2 First Order Approximation and Cost Shifts

We now examine the predictions of first order approximation and simulation for counterfactual scenarios involving vertical shifts to the marginal cost functions. We impose cost increases of \$0.02, \$0.05, \$0.10 and \$0.15, first on the marginal cost of the first firm (a "firm-specific cost shock") and then on all four firms (an "industry-wide cost shock"). Recall from Proposition 1 that first order approximation is exact in such settings if cost pass-through is constant, as it is with linear and log-linear demand. Also, from Corollary 1, the predictions of first order approximation are unchanged whether they are calculated using second derivatives of demand, cost pass-through with horizontality, or cost pass-through as a proxy for policy pass-through.

Figure 5 provides scatter plots of the predicted and true price effects when the underlying demand system is logit (column 1) and almost ideal (column 2), and for the firm-specific cost shocks (row 1) and the industry-wide cost shocks (row 2). Each dots falls close to the 45° line, indicating that the predictions of first order approximation are nearly exact in these settings. For firm-specific cost shocks, the median prediction error that arises when underlying demand is logit and almost ideal, respectively, is -0.001 and 0.002, relative to median price effects of 0.05 and 0.09. In the case of industry-wide cost shocks, the median prediction errors are within rounding error of zero. More order statistics are presented in Appendix Tables D.2 and D.3 to flesh out the empirical distribution of prediction error.

Table 6 compares the accuracy of first order approximation to that misspecified simulation for the case of the firm-specific cost shocks. Panel A provides the fraction of cost shocks examined for which first order approximation has smaller absolute prediction error than misspecified simulation. When underlying demand is logit, approximation is more accurate than AIDS, linear and log-linear simulation in 96.3%, 99.3% and 100% of the mergers, respectively. When underlying demand is almost ideal, approximation is more accurate than logit, linear and log-linear simulation in 95.9%, 100% and 99.9% of the mergers, respectively. With linear and log-linear demand, first order approximation is exact and so outperforms misspecified simulations in every instance. Panel B shows the median absolute prediction

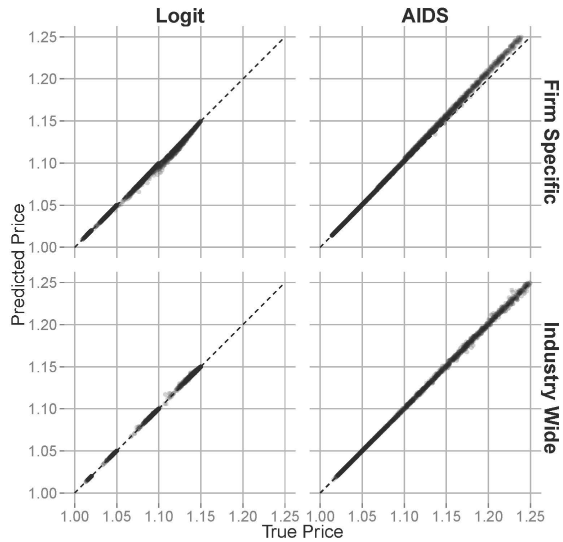

Figure 5: Prediction Error of First Order Approximation for Cost Shocks

Notes: The scatter plots characterize the accuracy of first order approximation when the underlying demand system is logit (column 1) and almost ideal (column 2). First order approximation is exact for vertical marginal cost shifts when underlying demand is linear or log-linear. Firm-specific cost shocks are represented in the top row and industry-wide cost shocks are represented in the bottom row. Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of first order approximation and its horizontal position is the true price effect. Dots that fall above (below) the dotted 45° line represent over (under) predictions.

errors that arise with first order approximation and misspecified simulations. These are near zero for first order approximation but tend to range between 0.02 and 0.10 for misspecified simulations. Appendix Table D.2 provides further information on the empirical distribution of prediction error for misspecified simulations for firm-specific cost shocks.

The same comparisons can be made for the case of the industry-wide cost shocks. Indeed we find that first order approximation produces smaller absolute prediction error than misspecified simulation more than 99% of the time for industry-wide cost shocks, regardless of the underlying demand system or the misspecified simulation technique employed. The median absolute prediction error that arises with first order approximation are either 0.001 (when the underlying demand system is almost ideal) or within rounding error of zero. The median absolute prediction errors that arise with misspecified simulation are larger, typically between 0.01 and 0.08. Appendix Table D.3 provides further information on the empirical distribution of prediction error for misspecified simulations for industry-wide cost shocks.

Table 6: FOA versus Simulation for Firm-Specific Cost Shocks

Panel A: Frequency with which FOA is More Accurate

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	.	95.9%	100%	100%
AIDS Simulation	96.3%	.	100%	100%
Linear Simulation	99.3%	100%	.	100%
Log-Linear Simulation	100%	99.9%	100%	.

Panel B: Median Absolute Prediction Error

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	0.001	0.002	0.000	0.000
Logit Simulation	.	0.022	0.018	0.071
AIDS Simulation	0.022	.	0.041	0.058
Linear Simulation	0.018	0.039	.	0.097
Log-Linear Simulation	0.074	0.058	0.101	.

Notes: Panel A shows the fraction of data draws for which first order approximation has a smaller absolute prediction error than simulation in predicting firm 1's change in price, for counter-factuals involving a firm-specific marginal cost shock. Panel B shows the median absolute prediction error of first order approximation and misspecified merger simulations in predicting firm 1's change in price. The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-policy equilibria.

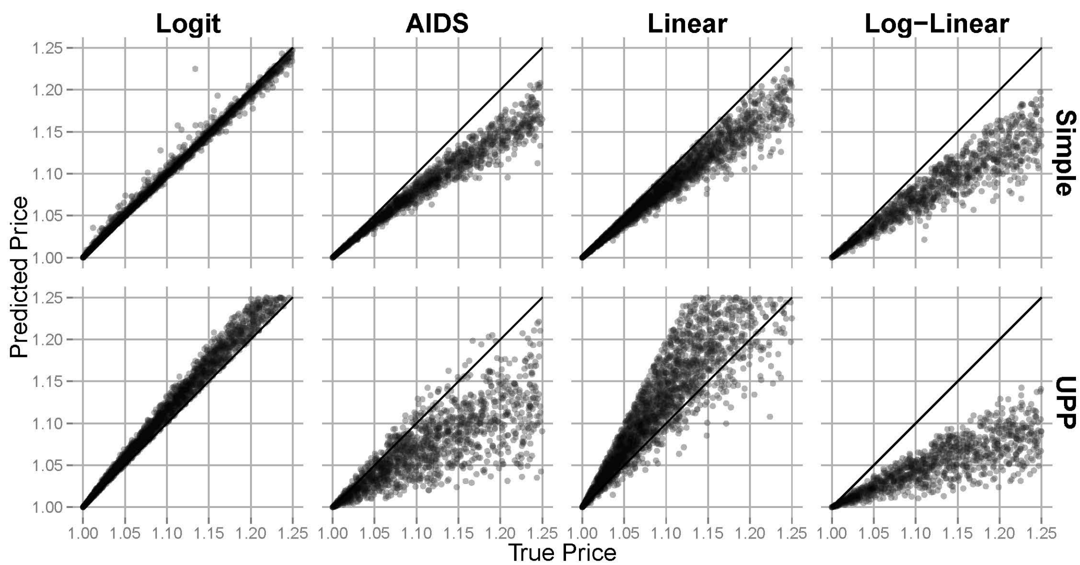

Figure 6: Prediction Error of Simplified First Order Approximation for Mergers
Notes: The scatter plots characterize the accuracy of first order approximation when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3) and log-linear (column 4). Simple approximation is calculated using cost pass-through as a proxy for policy pass-through (row 1), whereas UPP is calculated using the identity matrix as a proxy for policy pass-through (row 2). Each dot represents the predicted and true changes in firm 1's price for a given draw of data. Its vertical position is the prediction of first order approximation and its horizontal position is the true price effect. Dots that fall above (below) the dotted 45° line represent over (under) predictions.

4.3 Simplified predictors for mergers

Lastly, we examine the accuracy of two versions of first order approximation that should be useful to antitrust practitioners: simple approximation, for which cost pass-through is used as a proxy for policy pass-through, and upward pricing pressure (UPP), for which an identity matrix is used in the place of policy pass-through. To start, Figure 6 provides scatter plots that display the predicted price effects of simple approximation (row 1) and upward pricing pressure (row 2) against the the true price effects that arise when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3), and log-linear (column 4). Unlike the first order approximation, these versions of approximation are not exact when demand is linear.

The simple approximation yields accurate predictions when the underlying demand system is logit, as demonstrated by the tight clustering of dots along the 45-degree line, but

Table 7: MAPEs with Simplified Approaches for Mergers

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Simple	0.002	0.049	0.010	0.123
UPP	0.011	0.077	0.023	0.197

Notes: MAPE is median absolute prediction error as it relates to predicting firm 1's change in price. Separate statistics are shown for the cases in which the underlying demand system is logit, almost ideal, linear and log-linear. Simple approximation is calculated using cost pass-through as a proxy for policy pass-through, whereas UPP is calculated using the identity matrix as a proxy for policy pass-through. The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-merger equilibria.

otherwise provides predictions that systematically understate the true price effects. This property arises because the elements of the policy pass-through matrix typically exceed those of the cost pass-through matrix. In some instances, the property also may be desirable as the conservative predictions of simple approximation lessen the frequency of false positives. Upward pricing pressure is less consistent and tends to exceed the true price effects when underlying demand is logit or linear but understate the true price effects with almost ideal or log-linear demand.

Table 7 shows the median absolute prediction error that arises with simple approximation is 0.002, 0.049, 0.010 and 0.123 with logit, almost ideal, linear and log-linear demand, respectively. Analogously, the median absolute prediction error that arises with upward pricing pressure approximation is 0.011, 0.077, 0.023 and 0.197. Thus some precision is lost relative to first order approximation but these versions of approximation nonetheless are more accurate than misspecified merger simulation for most demand systems (see Table 4).[19] We provide details on the empirical distribution of prediction error in Appendix Tables D.1.

Table 8 furthers this comparison with misspecified simulation. Panel A provides the fraction of mergers examined for which simple approximation has smaller absolute prediction error than misspecified simulation. When underlying demand is logit, approximation is more accurate than AIDS, linear and log-linear simulation in 99.5%, 91.3% and 100% of the mergers, respectively. When underlying demand is almost ideal, simple approximation

[19]Logit simulation is more accurate than upward pricing pressure when demand is linear and AIDS simulation is more accurate than both simple approximation and upward pricing pressure when demand is log-linear.

Table 8: Simplified Approaches versus Simulation for Mergers

Panel A: Frequency Simple Approx. is More Accurate

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	.	94.4%	52.7%	100%
AIDS Simulation	99.5%	.	99.8%	43.5%
Linear Simulation	91.3%	97.3%	.	99.2%
Log-Linear Simulation	100%	97.5%	100%	.

Panel B: Frequency with which UPP is More Accurate

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	.	95.3%	20.7%	100%
AIDS Simulation	97.1%	.	94.3%	7.0%
Linear Simulation	58.1%	87.7%	.	89.1%
Log-Linear Simulation	100%	86.0%	100%	.

Notes: Panels A and B show the fraction of data draws for which simplified FOA and UPP, respectively, have a smaller absolute prediction error than merger simulation in predicting firm 1's change in price. Simplified FOA is calculated using cost pass-through as a proxy for policy pass-through, whereas UPP is calculated using the identity matrix as a proxy for policy pass-through.The statistics exclude calibrations with (i) extreme pass-through rates that are negative or exceed ten, or (ii) no post-merger equilibria.

is more accurate than logit, linear and log-linear simulation in 94.4%, 97.3% and 97.5% of the mergers, respectively. When underlying demand is linear, simple approximation is more accurate than logit, almost ideal and log-linear simulation in 52.7%, 99.8% and 100% of the mergers, respectively. And finally, when underlying demand is log-linear, simple approximation is more accurate than logit, almost ideal, and linear simulation in 100%, 43.5%, and 99.2% of the mergers, respectively. Aggregating across these scenarios, simple approximation outperforms misspecified simulation in 89.6% of the mergers examined.

Panel B offers the same comparison for upward pricing pressure. When underlying demand is logit, UPP is more accurate than AIDS, linear and log-linear simulation in 97.1%, 58.1% and 100% of the mergers, respectively. When underlying demand is almost ideal, UPP is more accurate than logit, linear and log-linear simulation in 95.3%, 87.7% and 86.0% of the mergers, respectively. When underlying demand is linear, UPP is more accurate than logit, almost ideal and log-linear simulation in 20.7%, 94.3% and 100% of the mergers, respectively. And finally, when underlying demand is log-linear, UPP is more accurate than logit, almost ideal, and linear simulation in 100%, 7.0%, and 89.1% of the mergers, respectively. Aggregating across these scenarios, UPP outperforms misspecified simulation in 77.9% of the mergers examined.

5 Concluding Remarks

Our results demonstrate that first order approximation can be a powerful tool with which to make counterfactual predictions while remaining agnostic about the functional forms of the underlying economic model. While grounded in the oligopoly theory of industrial organization, its usefulness extends into other fields of economics including macroeconomics and international trade. A full application of the methodology requires the researcher to bring more information to bear – namely information on pass-through or demand curvature in the observed equilibria – than do most simulation-based prediction methodologies. This points to a potentially valuable research agenda that we sketch here.

First, the prospect of first order approximation accentuates the value of empirical research that examines the pass-through behavior of firms. Recent progress has been made on that front. For instance, in addition to the research cited herein, Marion and Muehlegger (2011) and Fabra and Reguant (2013) explore pass-through in the gasoline retail markets and wholesale electricity markets, respectively. Related is the econometric question whether, and under which conditions, reduced-form regressions can obtain consistent estimates of pass-through when pass-through is non-constant or when costs are partially observed.

Second, placing weight on estimated pass-through begs the question of whether the derived theoretical relationship between local demand curvature and cost pass-through extends to real-world settings, or whether menu costs and rule-of-thumb pricing obstruct the connection. Such issues may create a relevant distinction between long run and short run pass-through rates. This distinction is emphasized in the sizable literature on asymmetric pass-through (e.g., Borenstein, Cameron, and Gilbert (1997), Peltzman (2000)) and increasingly is modeled explicitly (e.g., Nakamura and Zerom (2010), Neiman (2010), Neiman (2011), Golberg and Hellerstein (2013)), but more work on this subject would be valuable.

Lastly, first order approximation is not the only way to make reasonable counterfactual predictions without imposing functional form assumptions on an economic model. Recent contributions demonstrate the random coefficients logit model is non-parametrically identified (Bajari, Fox, Kim, and Ryan (2012), Berry, Gandhi, and Haile (2013)) and therefore capable of providing estimates of demand curvature that are independent of the elasticity estimates. Research that ascertains the empirical variation required in practice to identify the second order properties of the model would have value, as would research that examines the accuracy of simulations based on flexibly-estimated random coefficients logit model when the true underlying model is not logit.

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Appendix

A Derivation of Policy Pass-Through

In this appendix, we provide an expression for the Jacobian of $h(P)$, which can be used to construct the policy pass-through matrix as used in Theorem 1. Using the definition $h(P) \equiv f(P) + g(P)$, we have

$$\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}. \tag{A.12}$$

The Jacobian of $f(P)$ can be written as:

$$\frac{\partial f(P)}{\partial P} = \begin{bmatrix} \frac{\partial f_1(P)}{\partial p_1} & \cdots & \frac{\partial f_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(P)}{\partial p_1} & \cdots & \frac{\partial f_J(P)}{\partial p_N} \end{bmatrix}, \tag{A.13}$$

where N is the total number of products and J is the number of firms. The vector P includes all prices; we use lower case to refer to the prices of individual products, so that p_n represents the price of product n. In the case that product n is sold by firm i,

$$\frac{\partial f_i(P)}{\partial p_n} = - \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right], \tag{A.14}$$

where Q_i and P_i are vectors representing the quantities and prices respectively of the products owned by firm i, and the initial vector of constants has a 1 in the firm-specific index of the product n. For example, if product 5 is the third product of firm 2, then the 1 will be in the 3^{rd} index position when calculating $\partial f_2(P)/\partial p_5$. If product n is not sold by firm i, the vector of constants is $\vec{0}$, and thus

$$\frac{\partial f_i(P)}{\partial p_n} = \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right]. \tag{A.15}$$

The matrix $\partial g(P)/\partial P$ can be decomposed in a similar manner:

$$\frac{\partial g(P)}{\partial P} = \begin{bmatrix} \frac{\partial g_1(P)}{\partial p_1} & \cdots & \frac{\partial g_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_K(P)}{\partial p_1} & \cdots & \frac{\partial g_K(P)}{\partial p_N} \\ & & \\ 0 & \cdots & 0 \\ \downarrow & & \downarrow \end{bmatrix}, \tag{A.16}$$

where N is the number of products and K is the number of merging firms. Notice that $\partial g(P)/\partial P$ includes zeros for non-merging firms, because the merger does not create opportunity costs for these firms. In the case that product n is sold by a firm merging with firm i (this does not include firm i itself),

$$\begin{aligned} \frac{\partial g_i(P)}{\partial p_n} &= -\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right]\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \\ &+ \left(\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right]\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right] - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T\right]\right)(P_j - C_j), \end{aligned} \tag{A.17}$$

where Q_j, P_j, and C_j are vectors of the quantities, prices, and marginal costs respectively of products sold by firms merging with firm i, and the vector of 1s and 0s has a 1 in the merging firm's firm-specific index of the product n. For example, if product 5 is the third product of firm 2, and firm 2 is merging with firm 1, then the 1 will be in the 3^{rd} index position when calculating $\partial g_1(P)/\partial p_5$. It is an important distinction that – supposing there are more than two merging parties – the index j refers to all of the merging parties' products, excluding firm i's products. If product n is not sold by any firm merging with firm i (including a product sold by firm i),

$$\frac{\partial g_i(P)}{\partial p_n} = \left(\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right]\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right] - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T\right]\right)(P_j - C_j). \tag{A.18}$$

B Derivation of Demand Pass-Through

Consider demand shocks that perturb the unit sales of each product (given prices), and denote the vector of demand shocks s. Since unit sales enter quasi-linearly into the alternative first order conditions of each firm with a coefficient of one, the post-shock first order conditions can be expressed

$$f^{alt}(P) + s = 0. \tag{B.1}$$

Differentiating with respect to s obtains

$$\frac{\partial P}{\partial s}\frac{\partial f^{alt}(P)}{\partial P} + I = 0, \tag{B.2}$$

and algebraic manipulations then yield the demand pass-through matrix:

$$\rho^{\mathrm{d}} \equiv \frac{\partial P}{\partial s} = -\left(\frac{\partial f^{alt}(P)}{\partial P}\right)^{-1}. \tag{B.3}$$

C Mathematical Details of the Calibration Process

We provide mathematical details on the calibration process in this appendix. To distinguish the notation from that of Section 2, we move to lower cases and let, for example, s_i and p_i be the market share and price of firm i's product, respectively.[20] Recall that in the data generating process we randomly assign market shares among the four single-product firms and the outside good, draw the price-cost margin of the first firm's product from a uniform distribution with support over (0.2, 0.8), and normalize all prices to unity. The calibration process then obtains parameters for the logit, almost ideal, linear and log-linear demand systems that reproduce these draws of data.

Calibration starts with multinomial logit demand, the basic workhorse model of the discrete choice literature. The system is defined by the share equation

$$s_i = \frac{e^{(\delta_i - \alpha p_i)}}{1 + \sum_{j=1}^{N} e^{(\delta_j - \alpha p_j)}}. \tag{C.1}$$

The parameters to be calibrated include the price coefficient α and the product-specific quality terms δ_i. We recover the price coefficient by combining the data with the first order

[20] We define market share $s_i = q_i / \sum_{j=1}^{N} q_j$, where q_i represents unit sales.

conditions of the first firm. Under the assumption of Nash-Bertrand competition this yields:

$$\alpha = \frac{1}{m_1 p_1 (1 - s_1)} \tag{C.2}$$

where m_1 is the price-cost margin of firm 1. We then identify the quality terms that reproduce the market shares:

$$\delta_i = \log(s_i) - \log(s_0) + \alpha p_i, \tag{C.3}$$

for $i = 1 \ldots N$. We follow convention with the normalization $\delta_0 = 0$. Occasionally, a set of randomly-drawn data cannot be rationalized with logit demand and we replace it with a set that can be rationalized. This tends to occurs when the first firm has both an unusually small market share and an unusually high price-cost margin.

The logit demand system often is criticized for its inflexible demand elasticities. Here, the restrictions on substitution are advantageous and allow us to obtain a full matrix of elasticities with a tractable amount of randomly drawn data. The derivatives of demand with respect to prices, as is well known, take the form

$$\frac{\partial q_i}{\partial p_j} = \begin{cases} \alpha s_i (1 - s_i) & \text{if } i = j \\ -\alpha s_i s_j & \text{if } i \neq j. \end{cases} \tag{C.4}$$

We use the logit derivatives to calibrate the more flexible almost ideal, linear and log-linear demand systems. This ensures that each demand system has the same first order properties in the pre-policy equilibrium, for a given draw of data.

The AIDS is written in terms of expenditure shares instead of quantity shares (Deaton and Muellbauer 1980). The exenditure share of product i takes the form

$$w_i = \alpha_i + \sum_{j=0}^{N} \gamma_{ij} \log p_j + \beta_i \log(x/P), \tag{C.5}$$

where x is total expenditure and P is a price index. We incorporate the outside good as product $i = 0$ and normalize its price to one; this reduces to N^2 the number of price coefficients in the system that must be identified (i.e., γ_{ij} for $i, j \neq 0$). We further set $\beta_i = 0$ for all i, a restriction that imposes in income elasticity of unity. Under this restriction, total expenditures are given by

$$\log(x) = (\widetilde{\alpha} + u\widetilde{\beta}) + \sum_{k=1}^{N} \alpha_k \log(p_k) + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \gamma_{kj} \log(p_k) \log(p_j) \tag{C.6}$$

for some utility u. We identify the sum $\widetilde{\alpha} + u\widetilde{\beta}$ rather than $\widetilde{\alpha}$, u and $\widetilde{\beta}$ individually.[21]

Given this structure, product i's unit sales are given by $q_i = xw_i/p_i$ and the first derivatives of demand take the form

$$\frac{\partial q_i}{\partial p_j} = \begin{cases} \frac{x}{p_i^2}(\gamma_{ii} - w_i + w_i^2) & \text{if } i = j \\ \frac{x}{p_i p_j}(\gamma_{ij} + w_i w_j) & \text{if } i \neq j. \end{cases} \tag{C.7}$$

The calibration process for the AIDS then takes the following four steps:

1. Calculate x and w_i from the randomly drawn data on market shares, using a market size of one to translate market shares into quantities.

2. Recover the price coefficients γ_{ij} for $i, j \neq 0$ that equate the AIDS derivatives given in equation (C.7) and the logit derivatives given in equation (C.4). Symmetry is satisfied because consumer substitution is proportional to share in the logit model. The outside good price coefficients, γ_{i0} and γ_{0i} for all i, are not identified and do not affect outcomes under the normalization the $p_0 = 1$. Nonetheless, they can be conceptualized as taking values such that the adding up restrictions $\sum_{i=0}^{N} \gamma_{ij} = 0$ hold for all j.

3. Recover the expenditure share intercepts α_i from equation (C.5), leveraging the normalization that $\beta_i = 0$. The outside good intercept α_0 is not identified and does not affect outcomes, but can be conceptualized as taking a value such that the adding up restriction $\sum_{i=0}^{N} \alpha_i = 1$ holds.

4. Recover the composite term $(\widetilde{\alpha} + u\widetilde{\beta})$ from equation (C.6).

This process creates an AIDS that, for any given set of data, has quantities and elasticities that are identical in the pre-policy equilibrium to those that arise under logit demand. The system possess all the desirable properties defined in Deaton and Muellbauer (1980). Our approach to calibration differs from Epstein and Rubinfeld (2001), which does not model the price index as a function of the parameters, and from Crooke, Froeb, Tschantz, and Werden (1999), which assumes total expenditures are fixed.

We turn now to the linear and log-linear demand systems. The first of these takes the form

$$q_i = \alpha_i + \sum_j \beta_{ij} p_j, . \tag{C.8}$$

[21] The price index P is defined implicitly by equation (C.6) as the combination of prices that obtains utility u given expenditure x. A formulation is provided in Deaton and Muellbauer (1980).

The parameters to be calibrated include the firm specific intercepts α_i and the price coefficients β_{ij}. We recover the price coefficients directly from the logit derivatives in equation (C.4). We then recover the intercepts to equate the implied quantities in equation (C.8) with the randomly drawn market shares, again using a market size of one. Of similar form is the log-linear demand system:

$$\log(q_i) = \gamma_i + \sum_j \epsilon_{ij} \log p_j, \tag{C.9}$$

where the parameters to be calibrated are the intercepts γ_i and the price coefficients ϵ_{ij}. Again we recover the price coefficients from the logit derivatives (converting first the derivatives into elasticities). We then recover the intercepts to equate the implied quantities with the market share data. This process creates linear and log-linear demand systems that, for any given set of data, has quantities and elasticities that are identical to those of the calibrated logit and almost ideal demand systems, in the pre-policy equilibrium.

D Additional Results

We include here three tables that provide order statistics, summarizing the empirical distributions of prediction error, for each of the four demand systems and each of the prediction methodologies. Table D.1 considers the merger counterfactual scenarios, Table D.2 considers the firm-specific cost shock scenarios, and Table D.3 considers the industry-wide cost shock scenarios. We refer the reader to Section 3 for details on the data generating processes.

Table D.1: Empirical Distribution of Prediction Errors for Mergers

Demand	Median	5%	10%	25%	75%	90%	95%
Underlying Demand System is Logit							
FOA with Full Info	0.001	-0.001	-0.001	-0.000	0.005	0.011	0.018
FOA with Pass-through	0.000	-0.007	-0.004	-0.001	0.003	0.007	0.013
Simplified FOA	-0.001	-0.012	-0.008	-0.003	0.001	0.003	0.005
UPP	0.011	0.001	0.002	0.005	0.020	0.031	0.038
Logit Simulation	.	.	.	.	.	.	.
AIDS Simulation	0.090	0.001	0.004	0.023	0.319	0.906	1.608
Linear Simulation	-0.006	-0.087	-0.063	-0.029	0.003	0.022	0.047
Log-Linear Simulation	0.215	0.012	0.026	0.073	0.658	1.975	4.244
Underlying Demand System is Almost Ideal							
FOA with Full Info	0.011	0.000	0.001	0.002	0.047	0.210	0.626
FOA with Pass-through	0.015	0.000	0.001	0.003	0.082	0.416	1.256
Simplified FOA	-0.049	-1.324	-0.651	-0.189	-0.013	-0.003	-0.001
UPP	-0.077	-1.580	-0.886	-0.297	-0.017	-0.002	0.001
Logit Simulation	-0.088	-1.617	-0.907	-0.315	-0.023	-0.004	-0.001
AIDS Simulation	.	.	.	.	.	.	.
Linear Simulation	-0.102	-1.631	-0.922	-0.334	-0.033	-0.012	-0.006
Log-Linear Simulation	0.121	0.007	0.015	0.045	0.355	1.135	2.610
Underlying Demand System is Linear							
FOA with Full Info	0.000	0.000	0.000	0.000	0.000	0.000	0.000
FOA with Pass-through	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Simplified FOA	-0.010	-0.077	-0.051	-0.025	-0.004	-0.001	-0.000
UPP	0.019	-0.019	-0.003	0.005	0.044	0.078	0.100
Logit Simulation	0.006	-0.047	-0.022	-0.003	0.029	0.063	0.087
AIDS Simulation	0.103	0.006	0.012	0.033	0.340	0.918	1.619
Linear Simulation	.	.	.	.	.	.	.
Log-Linear Simulation	0.228	0.014	0.030	0.083	0.671	2.025	4.237
Underlying Demand System is Log-Linear							
FOA with Full Info	-0.005	-3.375	-1.198	-0.072	0.087	0.463	1.129
FOA with Pass-through	-0.101	-3.554	-1.443	-0.389	-0.028	-0.008	-0.003
Simplified FOA	-0.123	-3.544	-1.568	-0.455	-0.035	-0.010	-0.004
UPP	-0.197	-3.930	-1.883	-0.617	-0.064	-0.022	-0.010
Logit Simulation	-0.207	-3.962	-1.904	-0.631	-0.070	-0.026	-0.012
AIDS Simulation	-0.122	-2.587	-1.132	-0.358	-0.046	-0.016	-0.007
Linear Simulation	-0.220	-3.958	-1.921	-0.643	-0.081	-0.029	-0.014
Log-Linear Simulation	.	.	.	.	.	.	.

Notes: The table summarizes the empirical distribution of prediction errors that arise with eight different methodologies when the true underlying demand system is logit, almost ideal, linear and log-linear, respectively. FOA with Full Information is first order approximation calculated using the second derivatives of demand. FOA with Pass-through is first order approximation calculated using cost pass-through and the horizontality assumption. Simplified FOA is calculated using cost pass-through as a proxy for policy pass-through. UPP is calculated using the identity matrix as a proxy for policy pass-through.

Table D.2: Empirical Distribution of Prediction Errors for Firm-Specific Cost Shocks

Demand	Median	5%	10%	25%	75%	90%	95%
Underlying Demand System is Logit							
FOA	-0.001	-0.005	-0.004	-0.002	-0.000	-0.000	-0.000
UPP	0.011	0.001	0.002	0.005	0.021	0.033	0.039
Logit Simulation	.	.	.	.	.	.	.
AIDS Simulation	0.020	-0.012	-0.005	0.003	0.061	0.133	0.196
Linear Simulation	-0.018	-0.059	-0.049	-0.035	-0.008	-0.004	-0.003
Log-Linear Simulation	0.074	0.013	0.018	0.035	0.147	0.261	0.350
Underlying Demand System is Almost Ideal							
FOA	0.002	0.000	0.000	0.000	0.006	0.010	0.014
UPP	-0.008	-0.159	-0.104	-0.041	0.005	0.021	0.029
Logit Simulation	-0.020	-0.187	-0.126	-0.058	-0.003	0.005	0.013
AIDS Simulation	.	.	.	.	.	.	.
Linear Simulation	-0.039	-0.218	-0.158	-0.085	-0.019	-0.009	-0.006
Log-Linear Simulation	0.058	0.012	0.013	0.028	0.094	0.127	0.155
Underlying Demand System is Linear							
FOA	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UPP	0.030	0.009	0.009	0.016	0.051	0.071	0.072
Logit Simulation	0.018	0.003	0.004	0.008	0.035	0.049	0.059
AIDS Simulation	0.041	0.006	0.010	0.019	0.088	0.164	0.229
Linear Simulation	.	.	.	.	.	.	.
Log-Linear Simulation	0.101	0.018	0.024	0.046	0.179	0.296	0.384
Underlying Demand System is Log-Linear							
FOA	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UPP	-0.056	-0.300	-0.216	-0.119	-0.027	-0.013	-0.009
Logit Simulation	-0.071	-0.329	-0.243	-0.140	-0.034	-0.017	-0.013
AIDS Simulation	-0.058	-0.156	-0.127	-0.094	-0.028	-0.013	-0.012
Linear Simulation	-0.097	-0.360	-0.278	-0.171	-0.045	-0.023	-0.018
Log-Linear Simulation	.	.	.	.	.	.	.

Notes: The table summarizes the empirical distribution of prediction errors that arise with eight different methodologies when the true underlying demand system is logit, almost ideal, linear and log-linear, respectively. FOA is first order approximation. With vertical shifts to the marginal cost function, FOA is identical whether calculated using second derivatives, cost pass-through with horizontality, or cost pass-through as a proxy for policy pass-through. UPP is calculated using the identity matrix as a proxy for policy pass-through.

Table D.3: Empirical Distribution of Prediction Errors for Industry-Wide Cost Shocks

Demand	Median	5%	10%	25%	75%	90%	95%
Underlying Demand System is Logit							
FOA	0.000	-0.000	0.000	0.000	0.000	0.001	0.001
UPP	0.003	0.000	0.000	0.001	0.007	0.013	0.017
Logit Simulation	.	.	.	.	.	.	.
AIDS Simulation	0.060	0.007	0.011	0.024	0.148	0.308	0.453
Linear Simulation	-0.007	-0.030	-0.024	-0.015	-0.003	-0.001	-0.001
Log-Linear Simulation	0.065	0.010	0.015	0.031	0.135	0.244	0.332
Underlying Demand System is Almost Ideal							
FOA	0.000	-0.015	-0.005	-0.000	0.001	0.002	0.003
UPP	-0.054	-0.422	-0.290	-0.136	-0.020	-0.007	-0.004
Logit Simulation	-0.059	-0.431	-0.297	-0.144	-0.024	-0.011	-0.006
AIDS Simulation	.	.	.	.	.	.	.
Linear Simulation	-0.070	-0.447	-0.310	-0.155	-0.032	-0.015	-0.010
Log-Linear Simulation	0.001	-0.148	-0.075	-0.018	0.011	0.027	0.039
Underlying Demand System is Linear							
FOA	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UPP	0.012	0.001	0.003	0.005	0.024	0.035	0.041
Logit Simulation	0.007	0.001	0.001	0.003	0.015	0.024	0.030
AIDS Simulation	0.071	0.010	0.015	0.032	0.161	0.325	0.471
Linear Simulation	.	.	.	.	.	.	.
Log-Linear Simulation	0.076	0.013	0.018	0.036	0.149	0.261	0.349
Underlying Demand System is Log-Linear							
FOA	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UPP	-0.056	-0.300	-0.216	-0.119	-0.027	-0.013	-0.009
Logit Simulation	-0.062	-0.310	-0.226	-0.126	-0.029	-0.015	-0.010
AIDS Simulation	-0.002	-0.039	-0.027	-0.011	0.014	0.060	0.111
Linear Simulation	0.073	-0.327	-0.242	-0.141	-0.034	-0.018	-0.013
Log-Linear Simulation	.	.	.	.	.	.	.

Notes: The table summarizes the empirical distribution of prediction errors that arise with eight different methodologies when the true underlying demand system is logit, almost ideal, linear and log-linear, respectively. FOA is first order approximation. With vertical shifts to the marginal cost function, FOA is identical whether calculated using second derivatives, cost pass-through with horizontality, or cost pass-through as a proxy for policy pass-through. UPP is calculated using the identity matrix as a proxy for policy pass-through.