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"Price Caps as Welfare-Enhancing Coopetition"

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Price Caps as Welfare-Enhancing Coopetition*

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Abstract: Assessing whether mergers, joint marketing, and other forms of coopetition harm consumers often requires information unavailable to antitrust authorities. Against this background, the paper analyzes the role of price caps. In the case of complements, price caps, like mergers, allow the firms to solve Cournot's multiple marginalization problem; but unlike mergers, they do not stifle price competition in case of substitutes. It demonstrates this first for non-repeated interaction and general demand and cost functions. Under more restrictive assumptions, it generalizes the results to repeated interaction; it also provides a new analysis of coordinated effects when goods are not necessarily substitutes.

Keywords: Price caps, information-light regulation, tacit collusion, complements and substitutes, mergers, joint marketing agreements, coopetition.

JEL Codes: D43, L24, L41, O34.

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Section 4.2 builds on an earlier Discussion Paper entitled "Cooperation vs. Collusion: How Essentiality Shapes Co-opetition".

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1 Introduction

Starting with the Sherman Act's Section 1 prohibition of any "contract, combination in the form of trust or otherwise, or conspiracy, in restraint of trade or commerce" (1890), the prevention of lessening of competition through agreements among potential competitors has been one of the two cornerstones of competition policy.[1] Major applications include the antitrust treatment of *mergers* and of *joint marketing*. [Under joint marketing, firms co-sell their goods or licenses either through a joint subsidiary – as in the case of patent pools, through which intellectual property owners sell licenses to bundles of their patents – or through an independent entity]. If improperly structured, such cooperative practices have the potential to lessen competition and harm consumers.

Assessing whether a cooperative agreement is likely to reduce competition and raise price is a notably difficult exercise. First, there is often a shortage of reliable price and demand data, leading to well-known difficulties in assessing the impact of, say, a merger.[2] For new technologies, there may even be no data at all, and yet antitrust authorities have to approve or block acquisitions of startups by incumbents or the formation of patent pools. Accordingly, authorities often have little information as to whether a merger will raise prices substantially; or even whether it will raise prices at all, that is, whether the merger is horizontal (involves substitutes) or not (involves complements)[3].

Second, the pattern of substitutability/complementarity may change over time, and a merger that is desirable at the date of the approval may no longer be so later on. Products evolve, as do usages. For instance, product B may be a complement to product A today, but later become a substitute. Such an allegation was made for instance in the Microsoft case,[4] in which the browser was definitely a complement to the operating system, but was alleged to have the potential to become an operating system itself through the writing of extra code. Similarly, molecules A and B may be jointly needed to cure disease D, but each may in the future suffice to curing disease D'. A proper merger assessment therefore may require not only past data, but also unavailable forward-looking ones.

Third, while economists and antitrust practitioners neatly distinguish between "substitutes" and "complements", in many industries products may exhibit dual patterns of complementarity and substitutability: They compete with each other for consumers having selected the technology or the platform to which the products are related; but they also have a joint interest in keeping prices low so as to make the technology or platform attractive against rival options (non-consumption or competing technologies and plat-

[1] Article 101 of the European Treaty provides a similar prohibition in the EU. The other cornestone is the monitoring of abuses of dominant positions (Section 2 of the Sherman Act, Article 102 of the European Treaty).

[2] See, e.g., https://en.wikipedia.org/wiki/Small_but_significant_and_non-transitory_inreases_in_price.

[3] Mergers of complements fall in the category of "conglomerate mergers" in antitrust circles.

[4] Technically, this was an abuse of dominant position case, but the same concerns would have emerged in a merger case.

forms). For instance, a technology built around multiple *patents* held by different owners becomes more attractive when licensing prices decrease, but these patents can also be substitutes in that they enable alternative implementations of given functionalities. *Contents* offered by a cable or satellite television operator compete among themselves for the attention of the operator's subscribers but are also complements to the extent that increased operator membership benefits all content providers. *Payment systems* using a common point-of-sale terminal or interface at merchant premises compete for cardholder clientele and usage while sharing a common interest in merchants' wide adoption of the terminals. *Health care providers* who are members of a health insurance network vie for patients insured by the network but also depend on rival providers for the attractiveness of the insurance network.[5] Thus hospitals can be both substitutes (because they compete with one another to attract insured patients) and complements (because they jointly determine the value of becoming insured by the network). *Supermarkets* offer competing brands for many product categories, but one-stop shop benefits create complementarities across categories.[6] Further illustrations include *music performance rights* (as, say, licensed by Pandora), *alcoholic beverages* (as in the Grand Met-Guinness merger), *retail outlets* (in department stores and commercial malls), *intermodal transportation*, *airline alliances*, or *books, tickets and hotel rooms* (on online platforms).

As we will see, products can be complements at low prices and substitutes at high prices, or the reverse. This means that local measurements of demand elasticities may mislead the observer as to the nature of competition. Existing data, even if available to the antitrust authority or the researcher, again may not tell the entire story.

The purpose of the paper is to add a regulatory instrument to the competition authorities' policy toolbox. The new cooperative arrangement would be an agreement among firms on price caps for their various products. Unlike in a merger or an old-style patent pool, firms would keep control – including pricing – over their products or licenses and would only be constrained to charge no more than the agreed-upon caps.

We argue that, when it is unclear whether products or services are substitutes or complements, and authorities feel hesitant on approving a merger or a joint-marketing alliance, they may well want to consider allowing instead price caps agreements. To make such a case, and motivated by the lack of data that plagues merger analysis, we analyze the general properties of price-cap cooperation. The intuition for why price caps can be attractive is that they allow producers of complements to cooperate and solve Cournot's double marginalization problem, but do not allow competitors to collude and raise prices of substitutes. This intuition requires scrutiny, though.

Section 2 first sets up the model, which allows for asymmetry among firms, for demand substitutability/complementarity, for strategic complementarity/substitutability, as well

[5] See Katz (2011).

[6] See Thomassen *et al.* (2017) for a recent empirical analysis of consumers' shopping patterns and supermarkets' pricing behavior, accounting for cross-category complementarities.

as for hybrid cases – it indeed provides two examples where these characteristics depend on price levels; in the first example (on technology adoption) complementary patents become substitutes as prices rise, whereas in the second example (the Hotelling model with network externalities) substitutes become complements at higher prices.

Section 3 first characterizes the set of prices that can be sustained by price caps in the absence of repeated interaction (Section 3.1). Section 3.2 then focuses on duopoly settings and shows that, under a reasonable assumption, price caps can only improve consumer welfare. Furthermore, letting firms negotiate price caps benefits them (and consumers as well, from the previous result) when goods are complements and have no impact when goods are substitute; we also demonstrate the benefits of price caps in hybrid environments. So, unlike mergers, price-cap are always socially beneficial. Finally, Section 3.3 extends these insights to symmetric oligopoly settings.

It might however be that price caps facilitate tacit collusion (called "coordinated effects" in merger analysis) by reducing the set of possible deviations on the equilibrium path or in punishment phases. For this reason, in Section 4 we extend our study to allow firms to coordinate tacitly through repeated interaction. Alas, even in the absence of price caps, the repeated-game literature has focused on the case of substitutes, and often on perfect substitutes. Before trying to assess the desirability of price caps, the paper must therefore start filling the gap and study tacit collusion with arbitrary degrees of substitutability or complementarity. It obtains two sets of results.

Section 4.1 focuses on symmetric stationary paths in symmetric oligopoly settings. In this context, the lessons of the static analysis are confirmed for both substitutes and complements, provided that substitutes are strategic complements (as is usually assumed in economic analysis). The intuition is as follows. What prevents firms from achieving perfect coordination through repeated interaction is their incentives to deviate so as to increase their short-run profits. In the case of substitutes, where collectively firms wish to collude and raise prices, the profitable individual deviations consist in undercutting the collusive prices, and price caps cannot be used to limit these deviations. By contrast, in the case of complements, firms instead want to cooperate on lowering their prices so as to eliminate the double marginalization; in this case, price caps actually inhibit deviations from such low prices, both on- and off-equilibrium.

Section 4.2 goes further in the study of tacit collusion for the above-mentioned technology adoption model in which individual users must select a) which licenses to purchase in the technological class and b) whether to adopt the technology at all. The first choice depends on the extent of patent substitutability within the class, while the second captures the complementarity dimension. Unlike the Hotelling-with-club-effects example, offerings are complements at low prices and substitutes at high ones. We measure the "essentiality" of offerings through the reduction in the value of the technology when users forego an offering – for the sake of tractability, users have the same preferences along this dimension, and only differ along another dimension: the cost of adopting the technology, or equivalently their opportunity cost of not adopting another technology. The model allows for a smooth transition between perfect substitutes and perfect complements.

Within this framework, we derive general results about the sustainability of "tacit collusion" (coordinated increase in price) or "tacit cooperation" (coordinated decrease in price), that is, about bad and good coordination through repeated interaction. We then note that price caps agreements are equivalent to setting a joint-marketing entity combining two features, individual licensing and unbundling, and that both features are needed to ensure that consumer welfare always (weakly) increases under the agreement. Interestingly, both features are embodied in European law on technology transfer agreements that rules on patent pools.[7]

Section 5 steps back and investigates how the prospect of price caps affects the incentive to innovate and introduce products. Because price caps do not affect profits under substitutes, they have no impact on entry either. By contrast, they boost profits in the case of complements, and thereby encourage entry, raising consumer welfare under mild assumptions: they both promote product variety and enable lower markups. Thus the "ex-ante picture" matches the ex-post one.

Finally, Section 6 concludes.

Related literature. Our paper contributes to three literatures: static oligopoly, tacit collusion in oligopoly, and an emerging literature that looks for information-free (or -light) regulatory tools.

The literature on static oligopoly, well-reviewed by Vives (1999), is large, but has not emphasized the themes of this paper. The emphasis on price-cap-constrained competition in particular is new. By contrast there is a large literature on the impact of mergers under non-repeated interaction. This line of research was initiated by the seminal paper of Farrell and Shapiro (1990), who consider Cournot, homogenous-goods competition, provide a necessary and sufficient condition for a merger to raise price, and warn against the hazards of using concentration indices.

Second, there is an extensive theoretical literature on repeated games, with and without observability of actions,[8] as well as a large theoretical and empirical literature on collusion in oligopoly.[9] Less attention has been devoted to the role of substitutability and complementarity, however, despite the importance of these factors in the antitrust treatment of mergers or marketing alliances. The exception is a literature which, following Deneckere (1983) and Wernerfelt (1989), studies the impact of product differentiation. The conventional view, pioneered by Stigler (1964), is that homogeneous cartels are more

[7]Since 2004 for independent licensing and 2014 for unbundling. Other jurisdictions only require that patent pools allow independent licensing.

[8]For a thorough study of this literature, see Mailath and Samuelson (2006).

[9]For a survey of this literature, see, for instance, Jacquemin and Slade (1998) and Marshall and Marx (2012).

stable than non-homogeneous ones (Jéhiel (1992) calls this the principle of minimum differentiation). In the context of symmetric horizontal differentiation, Ross (1992) shows however that stability does not increase monotonically with substitutability, because product differentiation both lowers the payoff from deviation and reduces the severity of punishments (if one restricts attention to Nash reversals; Häckner (1996) shows that Abreu's penal codes can be used to provide more discipline than Nash reversals, and finds that product differentiation facilitates collusion).[10] Building on these insights, Lambertini *et al.* (2002) argue that, by reducing product variety, joint ventures can actually destabilize collusion.

In a context of vertical differentiation, where increased product diversity also implies greater asymmetry among firms, Häckner (1994) finds that collusion is instead easier to sustain when goods are more similar (and thus firms are more symmetric). Building on this insight, Ecchia and Lambertini (1997) note that introducing or raising a quality standard can make collusion less sustainable.

Section 4 departs from the existing literature in several ways. First, it characterizes the scope for tacit coordination in settings with (varying degrees of) complementarity as well as substitutability. Second, it allows for explicit commercial cooperation, such as a price caps agreement or a patent pool, and studies its impact on the scope for tacit coordination. Finally, it derives the regulatory implications.

Third, the paper contributes to a small but growing literature searching for regulatory rules that require little or no information from regulators; information-free regulatory rules have been studied primarily in the context of intellectual property, including guidelines for joint marketing agreements, with and without market power and vertical integration, and for standard-setting bodies (see Lerner-Tirole (2004, 2015), Boutin (2016) and Reisinger-Tarantino (2017)).

2 Setting

2.1 General assumptions

We consider a classic oligopoly setting with $n \geq 2$ single-product firms, indexed by $i \in \mathcal{N} \equiv \{1, ..., n\}$. Let $C_i(q_i)$ denote the cost of producing a quantity q_i of good i, and $D_i(\mathbf{p})$ the demand for that good, as a function of the vector of prices $\mathbf{p} = (p_i)_{i \in \mathcal{N}} \in \mathbb{R}^n_+$. We will assume that, for $i \in \mathcal{N}$, $D_i(\cdot)$ and $C_i(\cdot)$ are both C^2 and:[11]

- $C_i(0) = 0$ and $C_i'(\cdot) \geq 0$;

[10] Raith (1996) emphasizes another feature of product differentiation, namely, the reduced market transparency that tends to hinder collusion.

[11] In what follows, $\partial_i F(\mathbf{p})$ denotes the first-order derivative of the function $F(\mathbf{p})$ with respect to the price p_i; likewise, $\partial^2_{ij} F(\mathbf{p})$ denotes the second-order derivative with respect to the prices p_i and p_j.

- $D_i(\cdot) > 0$, $\partial_i D_i(\cdot) < 0$ (individual demands are positive and downward sloping) and $\sum_{j \in \mathcal{N}} \partial_j D_i(\cdot) \leq 0$ (a uniform increase in all prices reduces individual demands)[12].

It will sometimes be useful to consider the following familiar environments:

(**S**) *Substitutes:* $\partial_j D_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$;

(**C**) *Complements:* $\partial_j D_i(\cdot) < 0$ for $j \neq i \in \mathcal{N}$.

Note that the goods are necessarily either substitutes or complements when demands are linear. With more general demands, however, the sign of $\partial_j D_i(\mathbf{p})$ may vary with $\mathbf{p}$ (see Section **2.2** for examples).

Throughout the paper, we will also maintain the following regularity conditions:

- for all $i \in \mathcal{N}$, the profit function

  $$\pi_i(\mathbf{p}) \equiv p_i D_i(\mathbf{p}) - C_i(D_i(\mathbf{p}))$$

  is strictly quasi-concave in p_i;

- for all $i \in \mathcal{N}$, the best-response function[13]

  $$R_i(\mathbf{p}_{-i}) \equiv \arg\max_{p_i} \pi_i(p_i, \mathbf{p}_{-i}),$$

  is well-defined, C^1, and bounded above.

When firms offer substitutable goods, it will be occasionally useful to focus on the case where prices are strategic complements:

(**SC**) *Strategic complementarity:* $\partial_j R_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$.

In our setting, this condition amounts to $\partial^2_{ij} \pi_i(\cdot) > 0$, and is implied by (**S**) when demand is linear.[14] More generally, under mild regularity conditions (and indeed, in all standard models of oligopoly price competition), prices are strategic complements when goods are substitutes.[15]

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[12] This condition is automatically satisfied if, for instance, consumers have unit demands for each good and idiosyncratic preferences $v(\#\mathcal{S})$ for any combination $\mathcal{S} \subseteq \mathcal{N}$ (with $v(0) = 0$). We then have:

$$D_i(\mathbf{p}) = \Pr[\max_{\{\mathcal{S} \subseteq \mathcal{N} | i \in \mathcal{S}\}} \{v(\#\mathcal{S}) - \sum_{j \in \mathcal{S}} p_j\} \geq \max_{\{\mathcal{S} \subseteq \mathcal{N} | i \notin \mathcal{S}\}} \{v(\#\mathcal{S}) - \sum_{j \in \mathcal{S}} p_j\}],$$

which decreases when all prices increase uniformly (as this can only induce consumers to switch to smaller baskets). When $n = 2$, the condition is satisfied for any preferences $v(S)$ such that $v(\varnothing) = 0$.

[13] As usual, it is sometimes convenient to express the price vector $\mathbf{p} = (p_1, ..., p_n)$ as $\mathbf{p} = (p_i, \mathbf{p}_{-i})$, where $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$ denotes the vector of all prices but p_i.

[14] We then have $\partial_j \pi_i(\mathbf{p}) = [p_i - C_i'(D_i(\mathbf{p}))] \partial_j D_i(\mathbf{p})$ and thus $\partial^2_{ij} \pi_i(\cdot) = [1 - C_i''(\cdot) \partial_i D_i] \partial_j D_i > 0$.

[15] Conversely, they are often (always, when demand is linear) strategic substitutes (i.e., $\partial_j R_i(\cdot) < 0$ for $j \neq i$) when goods are complements. See Vives (1999) for a detailed analysis.

Throughout the paper, we assume that there exists a unique Nash equilibrium, $\mathbf{p}^N = \left(p_i^N\right)_{i \in \mathcal{N}}$. To ensure that it is stable when prices are strategic complements, we further suppose that, for $j \in \mathcal{N}$:[16]

$$\sum_{i \in \mathcal{N} \setminus \{j\}} \partial_j R_i \left(\mathbf{p}_{-i}\right) < 1. \tag{1}$$

Finally, we assume that the industry profit

$$\Pi\left(\mathbf{p}\right) = \sum_{i \in \mathcal{N}} \pi_i \left(\mathbf{p}\right)$$

is strictly quasi-concave in $\mathbf{p}$ and achieves its maximum at $\mathbf{p}^M = \left(p_i^M\right)_{i \in \mathcal{N}}$; let $q_i^M \equiv D_i\left(\mathbf{p}^M\right)$ denote the monopoly output of good i.

2.2 Hybrid demands: two examples

We sketch here two environments exhibiting reversals of the complementarity/substitutability pattern. In the first, offerings are complements at low prices and substitutes at higher prices; the second exhibits the reverse pattern.

Technology adoption

Our first illustration of hybrid demands is a nested model in which (a unit mass of) users first select among licenses to patents covering a technology and then choose between this technology and selected licenses and an outside option (adopting a competing technology or no technology at all). For tractability we maintain a single-dimensional heterogeneity parameter: users differ in their opportunity cost or benefit of adopting the technology, but not in their preferences for the bundles of licenses within the technology. In the context of two symmetrical patents held by two patent owners, the description goes as follows. The users obtain value V from acquiring the two licenses, and $V - e$ from a single one. So $e \in [0, V]$ is an essentiality parameter: $e = 0$ for perfect substitutes and $e = V$ for perfect complements. An individual user is characterized by her technology adoption cost drawn from some cumulative distribution function $F\left(\cdot\right)$ with monotone hazard rate; users have quasi-linear preferences.

For low prices ($p_i < e$ for $i = 1, 2$), users secure both licenses and (assuming a zero marginal cost) firm i's profit is given by

$$p_i F\left(V - p_1 - p_2\right).$$

So offerings are local complements: each patent holder wishes that the other owner reduce her price. By contrast, for high prices ($p_i > e$ for $i = 1, 2$), users do not acquire

[16]A stronger version, namely, $\sum_{i \in \mathcal{N} \setminus \{j\}} \left|\partial_j R_i \left(\mathbf{p}_{-i}\right)\right| < 1$, suffices to guarantee the existence and uniqueness of the Nash equilibrium, and moreover ensures that it is stable under the standard tâtonnement process; see online Appendix A. However, our analysis does not rely on equilibrium stability in the case of strategic substitutes.

a second license and pick the lower price one if they adopt the technology at all. So offerings are local substitutes.

We will return to this technology adoption model in Section 4.2.

Hotelling with club effects

The online Appendix B studies the properties of the Hotelling model augmented with positive network externalities. The one difference with the familiar Hotelling model on a line is that the consumers' valuation is $v + \sigma(q_1 + q_2)$ where $\sigma > 0$ is the club-effects parameter. For low prices, the market is covered and there are no network externalities at the margin, as total demand is fixed and equal to the unit mass of consumers ($q_1 + q_2 = 1$). So offerings are imperfect substitutes. For high prices the market is not covered and the firms choose their prices as local monopolies. So each locally would like the other firm to lower its price and create more externalities. The reaction curves are represented in Figure 1, which illustrates some of the results obtained in the online Appendix: in particular, for v high enough, the offerings satisfy (**S**) and (**SC**), and the monopoly prices are greater than the Nash prices. By contrast, if v is small enough, then for low prices the offerings locally satisfy (**S**) and (**SC**), but around the Nash prices they satisfy (**C**) instead, and the monopoly prices are lower than the Nash prices.

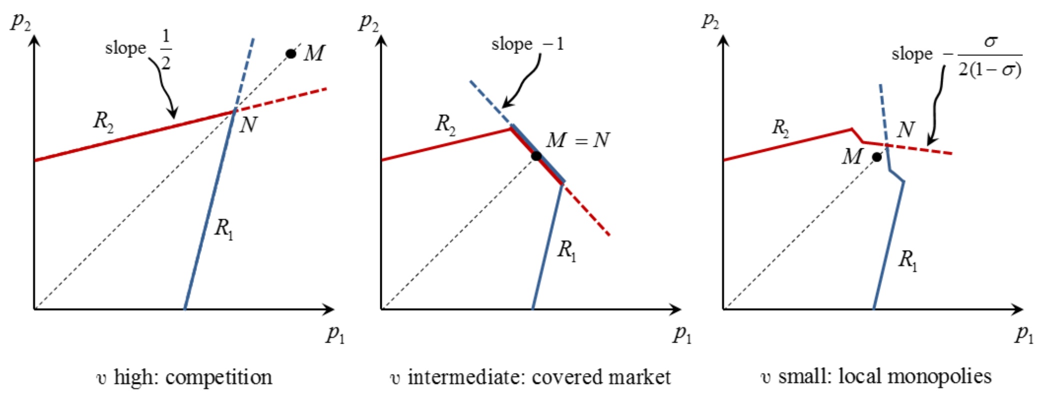

Figure 1: Hotelling with club effects

3 Impact of price caps on non-repeated interactions

We consider here a static, one-shot game in which firms simultaneously set their prices, and study the impact of price caps on the outcome of this game.

3.1 Preliminary analysis

The following lemmas provide useful properties of the monopoly and Nash outcomes. The first lemma shows that the monopoly outcome lies above firms' best-responses when goods are substitutes. When goods are complements instead, the monopoly outcome lies below at least one firm's best-response. It can be checked that this applies to all firms in the absence of cross-subsidization, that is, if all markups are non-negative. However, with complements, it may be optimal to subsidize some goods in order to boost the demand for others; the prices of these other goods may then lie above the best-responses.[17]

Lemma 1 (monopoly prices)

(*i*) $(\mathbf{S}) \Longrightarrow \forall i \in \mathcal{N}, p_i^M > C_i'\left(q_i^M\right)$ *and* $p_i^M > R_i\left(\mathbf{p}_{-i}^M\right)$.

(*ii*) $(\mathbf{C}) \Longrightarrow \exists (i,j) \in \mathcal{N}^2$ *such that* $p_i^M > C_i'\left(q_i^M\right)$ *and* $p_j^M < R_j\left(\mathbf{p}_{-j}^M\right)$*; furthermore, if* $n=2$*, then* $p_i^M > C_i'\left(q_i^M\right) \Longrightarrow p_j^M < R_j\left(p_i^M\right)$ *for* $j \neq i$.

Proof. See Appendix A. ∎

The next lemma shows that firms' best-responses always exceed their marginal costs:

Lemma 2 (best responses exceed marginal costs) *For any firm* $i \in \mathcal{N}$ *and any* $\mathbf{p}_{-i} \in \mathbb{R}_+^{n-1}$, $R_i\left(\mathbf{p}_{-i}\right) > C_i'\left(D_i\left(R_i\left(\mathbf{p}_{-i}\right), \mathbf{p}_{-i}\right)\right)$.

Proof. See Appendix B. ∎

Suppose now that, prior to setting their prices, the firms can agree on price caps, $\bar{p}_1, ..., \bar{p}_2$. They thus play a constrained game in which they simultaneously set their prices, subject to the agreed price caps. In the following, unless otherwise stated, "price caps" will mean "price caps without transfers". However, we will also consider "price caps with transfers", in which some firms may compensate the others in a lump-sum fashion for agreeing to price caps, an agreement intermediate between price caps and mergers (as a merged entity can control prices directly, and focus on joint profit).

The following proposition shows that price caps can be used to sustain any prices lying below firms' best responses, and only these prices:

Proposition 1 (price-cap implementable allocations)

(*i*) *The set of prices that can be sustained with price caps (with or without transfers) is:*

$$\mathcal{P} \equiv \left\{\mathbf{p} \in \mathbb{R}_+^n \mid p_i \leq R_i\left(\mathbf{p}_{-i}\right) \text{ for } i \in \mathcal{N}\right\}.$$

(*ii*) *In particular,* $\mathbf{p}^N \in \mathcal{P}$ *and, for any other price vector in* $\mathcal{P}$, $p_i < p_i^N$ *for some* $i \in \mathcal{N}$.

[17] For instance, consider the case of two goods, produced at the same constant unit cost $c > 0$, and with demands respectively given by $D_1(p_1, p_2) = D(p_1, p_2)$, with $\partial_2 D_1 < 0$, and $D_2(p_1, p_2) = \lambda D(p_2, p_1)$, with $\lambda \in (0,1)$. The monopoly prices are then asymmetric, and involve cross-subsidization (namely, $p_2^M < c < p_1^M$) for λ small enough (indeed, p_2^M tends to 0 as λ goes to 0). Furthermore, as by construction p_1^M satisfies $\partial_1 \pi_1\left(\mathbf{p}^M\right) = -\partial_1 \pi_2\left(\mathbf{p}^M\right) = -\left(p_2^M - c\right) \partial_1 D_2\left(\mathbf{p}^M\right) < 0$, it is such that $p_1^M > R_1\left(p_2^M\right)$.

Proof. (i) We first show that price caps can sustain only prices in $\mathcal{P}$. Consider a price vector $\hat{\mathbf{p}}$ that is sustainable with price caps $(\bar{p}_i)_{i\in\mathcal{N}}$. As $\pi_i(\mathbf{p})$ is strictly quasi-concave in p_i, we must have, for $i \in \mathcal{N}$:

$$\hat{p}_i = \arg\max_{p_i \le \bar{p}_i} \pi_i(p_i, \hat{\mathbf{p}}_{-i}) = \min\{R_i(\hat{\mathbf{p}}_{-i}), \bar{p}_i\} \le R_i(\hat{\mathbf{p}}_{-i}).$$

Hence, $\hat{\mathbf{p}} \in \mathcal{P}$.

Conversely, any price $\hat{\mathbf{p}} \in \mathcal{P}$ can be sustained with a vector price caps $\bar{\mathbf{p}} = \hat{\mathbf{p}}$. To see this, note that, for $i \in \mathcal{N}$: $\bar{p}_i = \hat{p}_i \le R_i(\hat{\mathbf{p}}_{-i})$; the strict quasi-concavity of $\pi_i(\mathbf{p})$ in p_i then yields the result.

(ii) See Appendix C. ■

This proposition already establishes that firms cannot use price caps to raise all prices above their Nash levels. More generally, as price caps can only prevent firms from charging high prices, they are intuitively unlikely to harm consumers. We explore this further in the next two subsections.

3.2 Duopoly

We focus here on the case of a duopoly (i.e., $\mathcal{N} = \{1, 2\}$). To confirm that price caps can only benefit consumers, we introduce the following regularity condition:

Assumption A: For any $i \neq j \in \{1, 2\}$ and any price $p_i \in [0, p_i^N)$, if $R_j(p_i) > p_i^N$, then:

$$R_j'(p_i) > -\frac{D_i(p_i, R_j(p_i))}{D_j(R_j(p_i), p_i)}.$$

Assumption A holds trivially when prices are strategic complements (as then $R_j'(\cdot) > 0$), as is the case with standard theoretical and empirical models of price competition, and it follows from the usual stability condition $|R_j'(p_i)| < 1$ when demand functions are "quasi-symmetric" in that $p_1^N = p_2^N$ and $p_i < p_j$ implies $D_i(p_i, p_j) \ge D_j(p_j, p_i)$;[18] it also holds for the hybrid demands considered in Section 2.2.[19] In online Appendix C, we provide a sufficient condition on demand ensuring that Assumption A holds.

We have:

Proposition 2 (duopoly: price caps benefit consumers) *Under Assumption A, any price vector $\mathbf{p} \neq \mathbf{p}^N$ that can be sustained with price caps (with or without transfers) yields a higher consumer surplus than $\mathbf{p}^N$.*

[18] Fix $p_i < p^N$. As $R_j'(p_i) < 1$, we then have:

$$R_j(p_i) - p_i = \int_{p^N}^{p_i} \left[R_j'(p) - 1\right] dp > 0.$$

Under "quasi-symmetry", we thus have $D_j(R_j(p_i), p_i) \le D_i(p_i, R_j(p_i))$, which, together with $R_j'(p_i) > -1$, implies that Assumption A is satisfied.

[19] In the technology adoption model, the reaction symmetric function exhibits a kink; still, Assumption A holds whenever the reaction function is differentiable, as well as for both right- and left-derivatives at the kink.

Proof. Consider a price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P} \backslash \{\mathbf{p}^N\}$. From Proposition 1, $\hat{p}_i < p_i^N$ for some $i \in \{1, 2\}$. If the price of the other firm, j, satisfies $\hat{p}_j \leq p_j^N$, then consumers clearly prefer $\hat{\mathbf{p}}$ to $\hat{\mathbf{p}}^N$. Suppose now that $\hat{p}_j > p_j^N$; from Proposition 1 we then have $R_j(\hat{p}_i) \geq \hat{p}_j > p_j^N$, let:[20]

$$\tilde{p}_i \equiv \inf \left\{ p_i \geq \hat{p}_i \mid R_j(p_i) \leq p_j^N \right\}.$$

By construction, $\tilde{p}_i \in (\hat{p}_i, p_i^N]$ and $R_j(\tilde{p}_i) = p_j^N$. Letting $S(p_i, p_j)$ denote total consumer surplus, we then have:

$$S(\hat{p}_i, \hat{p}_j) \geq S(\hat{p}_i, R_j(\hat{p}_i)) > S(\tilde{p}_i, R_j(\tilde{p}_i)) \geq S\left(p_i^N, p_j^N\right),$$

where the first inequality follows from Proposition 1, the last follows from $\tilde{p}_i \leq p_i^N$ and $R_j(\tilde{p}_i) = p_j^N$, and the strict one follows from $\tilde{p}_i > \hat{p}_i$ and Assumption A, which together imply:

$$S(\hat{p}_i, R_j(\hat{p}_i)) - S(\tilde{p}_i, R_j(\tilde{p}_i)) = \int_{\hat{p}_i}^{\tilde{p}_i} \left[D_i(p_i, R_j(p_i)) + D_j(R_j(p_i), p_i) R_j'(p_i) \right] dp_i > 0.$$

■

This Proposition shows that, under Assumption A, firms' use of price caps can only benefit consumers. Consider now firms' incentives to introduce price caps.

Intuitively, suppliers of substitutes wish to avoid competition and raise prices above the Nash level; in the light of Proposition 1, price caps are unlikely to help them. By contrast, suppliers of complements wish to avoid double marginalization, and price caps can enable them to achieve that. The following Proposition confirms this intuition:

Proposition 3 (duopoly: firms' incentives)

(i) *Under* (**S**), *firms cannot use price caps (with or without transfers) to increase both of their profits; if in addition* (**SC**) *holds, then firms cannot use price caps to increase their joint profit either.*

(ii) *Under* (**C**), *firms can use price caps to increase both profits (and thus, a fortiori, their joint profit); allowing price caps without transfers benefits consumers as well.*

Proof. We start with the observation that $\pi_i(R_i(p_j), p_j)$ increases (resp., decreases) with p_j under (**S**), (resp., under (**C**)); to see this, note that:

$$\begin{aligned} \frac{d}{dp_j} \{\pi_i(R_i(p_j), p_j)\} &= \partial_j \pi_i(R_i(p_j), p_j) \\ &= [R_i(p_j) - C_i'(D_i(R_i(p_j), p_j))] \, \partial_j D_i(R_i(p_j), p_j), \end{aligned}$$

[20] The reasoning that follows focuses on the range $[\hat{p}_i, \tilde{p}_i]$, because Assumption A is required to hold only for the prices $p_i < p_i^N$ that satisfy $R_j(p_i) > p_i^N$; if Assumption A holds for any $p_i < p_i^N$, then the reasoning applies to the entire range $\left[\hat{p}_i, p_i^N\right]$.

where the first equality follows from the envelope theorem. It follows from Lemma 2 that the last expression has the same sign as $\partial_j D_i(\cdot)$.

(i) Consider a price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P} \backslash \{\mathbf{p}^N\}$. From Proposition 1, $\hat{p}_j < p_j^N$ for some $j \in \{1, 2\}$. Under (**S**), we have, for $i \neq j \in \{1, 2\}$:

$$\pi_i(\hat{p}_i, \hat{p}_j) \leq \pi_i(R_i(\hat{p}_j), \hat{p}_j) < \pi_i\left(R_i\left(p_j^N\right), p_j^N\right) = \pi_i\left(p_i^N, p_j^N\right),$$

where the weak inequality stems from the definition of $R_i(\cdot)$ and the strict inequality follows from $\hat{p}_j < p_j^N$ and $\pi_i(R_i(p_j), p_j)$ being strictly increasing in p_j, as noted above. Therefore, firms cannot use price caps (with or without transfers) to increase both of their profits (and thus, *a fortiori*, their joint profit).

Furthermore, if prices are strategic complements, then any price vector $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2)$ in $\mathcal{P} \backslash \{\mathbf{p}^N\}$ is such that $\hat{p}_i < p_i^N$ for $i = 1, 2$.[21] The above argument then implies that both firms obtain strictly less profit than in the Nash equilibrium. Hence, in that case firms cannot use price caps (with or without transfers) to increase any of their profits.

(ii) By contrast, under (**C**), there exist prices in $\mathcal{P}$ that increase both firms' profits. To see this, note first that, from Lemma 2, both firms' margins are positive at the Nash equilibrium. It follows that, starting from the Nash equilibrium prices $\left(p_1^N, p_2^N\right)$, a small and uniform reduction in both prices increases both firms' profits, as reducing one firm's price has only a second-order effect on the profit of that firm, and a first-order, positive effect on the other firm's profit (as it increases that firm's demand). To conclude the argument, it suffices to note that these new prices (p_1, p_2) belong to $\mathcal{P}$: they satisfy $p_i < p_i^N$ for $i = 2$, and thus

$$R_j(p_i) - p_i = \int_{p_i^N}^{p_i} \left[R_j'(p) - 1\right] dp > 0.$$

Therefore, there are prices in $\mathcal{P}$ that give both firms more profit than the Nash equilibrium prices.

To conclude the proof, it suffices to note that increasing both firms' profits requires lowering prices below the Nash level. To see this, consider a price vector $\mathbf{p}$ that increases both firms' profits above their Nash levels; we then have, for $i \neq j \in \{1, 2\}$:

$$\pi_i(R_i(p_j), p_j) \geq \pi_i(\mathbf{p}) \geq \pi_i^N = \pi_i\left(R_i\left(p_j^N\right), p_j^N\right),$$

which, under (**C**), implies $p_j \leq p_j^N$. Hence, $\mathbf{p} \leq \mathbf{p}^N$. ■

Proposition 3 (which does not hinge on Assumption A) shows that: (i) price caps play no role when firms offer substitutable goods (case (**S**)), in which case firms' interests conflict with those of consumers – firms would like to raise prices, whereas price caps

[21] From Proposition 1, this has to be the case for at least one firm i; Proposition 1 and strategic complementarity then together imply that, for the other firm, j: $\hat{p}_j \leq R_j(\hat{p}_i) < R_j\left(p_i^N\right) = p_j^N$.

(which would benefit consumers) can only be used to lower prices; and (ii) price caps enable instead the firms to cooperate when they offer complements (case (**C**)), in which case firms' interests are somewhat aligned with those of consumers – they both long for lower prices. Furthermore, price caps benefit consumers whenever they enhance both firms' profits; which prices firms actually choose to sustain depends on factors such as firms' relative bargaining power or the feasibility of side transfers.

Finally, it is interesting to compare the use of price caps with the impact of a merger on firms' pricing policies (in the absence of merger-specific synergies). For the sake of exposition, it is useful to suppose that either:

($\mathbf{M_S}$) $p_i^M \geq p_i^N$ for $i = 1, 2$, with at least one strict inequality, and Assumption A holds; or

($\mathbf{M_C}$) $p_i^M \leq p_i^N$ for $i = 1, 2$, with at least one strict inequality, and $p_i^M \leq R_i\left(p_j^M\right)$ for $i \neq j \in \{1, 2\}$.

The first situation (case ($\mathbf{M_S}$)) always arises under (**S**) and (**SC**).[22] However, it can also arise even when (**S**) does not hold. This can be the case, for instance, when goods are substitutes for some prices but complements for other prices. For instance, neither (**S**) nor (**C**) holds in the two examples discussed in Section 2.2, and yet one of ($\mathbf{M_S}$) and ($\mathbf{M_C}$) is satisfied, depending on the specific values of some of the parameters (e.g., the degree of essentiality of the individual patents and/or the distribution of the adoption costs in the first model, or the value of the good and/or the magnitude of the club effects in the second model). Note also that, when firms are sufficiently asymmetric, then under (**C**) the monopoly price may lie above the Nash level for one firm, and below it for the other firm.[23] The online Appendix D provides an example in which, under monopoly pricing, applications running on a platform are sold at cost (and thus, below Nash prices), which allows the platform to be priced at a price exceeding the Nash level, as illustrated in Figure 2.

The analysis above suggests that, in the absence of information about the situation at stake, and in the absence of merger-specific efficiency gains, price caps constitute a socially safer alternative to mergers. Indeed, we have:

Proposition 4 (duopoly: price caps vs. mergers)

(*i*) *Under* ($\mathbf{M_S}$), *a merger harms consumers, whereas price caps (with or without transfers) can only benefit them.*

[22] From Lemma 1, under (**S**) the monopoly outcome lies above both firms' best-responses; under (**SC**), this in turn implies that monopoly prices strictly exceed Nash levels.

[23] Furthermore, as noted at the end of Subsection 3.1, with complements it may be useful to price one good below cost, in which case the price of the other good lies above the best-response.

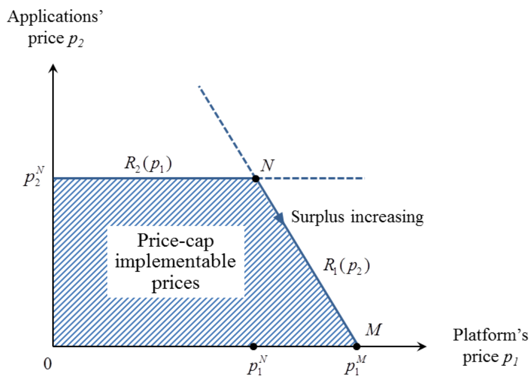

Figure 2: Platform & apps

(*ii*) *Under* ($\mathbf{M_C}$), *price caps* $\bar{p}_1 = p_1^M$ *and* $\bar{p}_2 = p_2^M$ *induce a unique continuation equilibrium, in which both firms charge monopoly prices; a merger and price caps with transfers thus both yield perfect cooperation, which benefits firms and consumers. Allowing price caps without transfers also benefits firms and consumers.*

Proof. (*i*) Under ($\mathbf{M_S}$), consumers prefer the Nash prices to the monopoly prices, and thus a merger harms them. By contrast, from Proposition 2, price caps can only benefit consumers.

(*ii*) Suppose now that the firms agree on price caps $\bar{p}_1 = p_1^M$ and $\bar{p}_2 = p_2^M$. Under ($\mathbf{M_C}$), we then have $R_i\left(p_j^M\right) \geq p_i^M = \bar{p}_i$ for $i \neq j \in \{1,2\}$; as firms' individual profits are strictly quasi-concave with respect to their own prices, charging the monopoly prices thus constitutes a continuation equilibrium.

Conversely, the strict quasi-concavity of the individual profits implies that any equilibrium price vector $(\hat{p}_1, \hat{p}_2)$ satisfies, for $j \neq i \in \{1,2\}$:

$$\hat{p}_i = \min\left\{R_i\left(\hat{p}_j\right), \bar{p}_i\right\} = \min\left\{R_i\left(\hat{p}_j\right), p_i^M\right\}. \tag{2}$$

Hence, $\hat{p}_i \leq p_i^M$ for $i = 1,2$. Let us show that $\hat{p}_j = p_j^M$ for at least one firm $j \in \{1,2\}$. Suppose instead that $\hat{p}_i < p_i^M$ for $i = 1,2$. From (2), we then have $\hat{p}_i = R_i\left(\hat{p}_j\right)$ for $i \neq j \in \{1,2\}$, and thus $\hat{\mathbf{p}} = \mathbf{p}^N$; but then, from ($\mathbf{M_C}$), $\hat{p}_i = p_i^N \geq p_i^M = \bar{p}_i$, a contradiction.

Hence, there exists $j \in \{1,2\}$ for which $\hat{p}_j = p_j^M$. But then, the price of firm $i \neq j \in \{1,2\}$ must satisfy $\hat{p}_i = p_i^M$, as $R_i\left(p_j^M\right) \geq p_i^M$ under ($\mathbf{M_C}$). Both firms charging the monopoly prices thus constitutes the unique continuation equilibrium. ■

3.3 Oligopoly

We extend here the analysis to an arbitrary number of firms. For the sake of tractability, we focus on (almost) symmetric firms and symmetric outcomes, in which all firms charge the same price. Specifically, we now assume that the $n \geq 2$ firms:

- face the same cost: $C_i(q_i) = C(q_i)$ for all $i \in \mathcal{N}$;
- face the same demand when the other firms charge the same price: $D_i(p_i, p, ..., p) \equiv D(p_i, p)$ for all $i \in \mathcal{N}$ and any $p_i, p \in \mathbb{R}_+$.

Let $\pi(p_i, p) \equiv p_i D(p_i, p) - C(D(p_i, p))$ denote firm i's profit when it charges p_i and all others charge the same price p, and

$$R(p) \equiv \arg\max_{p_i} \pi(p_i, p)$$

denote firm i's best-response to such prices. Condition (1) implies[24]

$$R'(\cdot) < 1.$$

Our general assumptions imply that the unique Nash equilibrium is symmetric:[25] $p_i^N = p^N$. We will assume that the monopoly prices, too, are symmetric: $p_i^M = p^M$;[26] let q^M denote the monopoly quantity.

Finally, it will be convenient to denote by $\pi^s(p)$ and $\Pi^s(p)$ the individual and industry profits achieved when all firms charge the same price p; that is, $\pi^s(p) \equiv \pi(p, p)$ and $\Pi^s(p) \equiv \Pi(p, ..., p) = n\pi^s(p)$.

When firms charge the same price p, they each obtain $\pi^s(p) = \Pi^s(p)/n$; their interests are thus aligned: intuitively, they wish to raise prices above the static Nash level when their goods are substitutes, and to lower prices when their goods are complements. Indeed, we have:

Lemma 3 (symmetric oligopoly: profitable prices)

(i) Under (**S**), $p^M > p^N$ *and* $\pi^s(p) \geq \pi^s(p^N) \implies p \geq p^N$*;*

(ii) Under (**C**), $p^M < p^N$ *and* $\pi^s(p) \geq \pi^s(p^N) \implies p \leq p^N$*.*

[24] Using symmetry and $R'(p) = \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j R_i(\mathbf{p})$, we have:

$$R'(p) = \frac{1}{n} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j R_i(\mathbf{p}) = \frac{1}{n} \sum_{j \in \mathcal{N}} \sum_{i \in \mathcal{N} \setminus \{j\}} \partial_j R_i(\mathbf{p}) < 1,$$

where the inequality stems from (1).

[25] The condition $R'(\cdot) < 1$, together with the boundedness of $R(\cdot)$, imply the existence of a unique symmetric Nash equilibrium.

[26] Given our previous assumptions, it suffices that the first-order condition:

$$D(p, p) + [p - C'(D(p, p))] \sum_j \partial_i D_j(\mathbf{p}) = 0$$

admits a solution for some $\mathbf{p} = (p, ..., p)$. As the left-hand side is positive for $p = 0$, it suffices in turn that it becomes negative for p large enough – a condition that is trivially satisfied in the case of complements, as the left-hand side boils down to $[p^N - C'(q^N)] \sum_{j \neq i} \partial_i D_j(\mathbf{p}^N) < 0$ for $p = p^N$.

Proof. See Appendix D. ■

We now study the impact of price caps on the set of symmetric equilibria.[27] Without loss of generality given our focus on symmetric outcomes, suppose that firms agree on a symmetric price cap, $\bar{p}$.[28]

From Proposition 1, firms cannot use price caps to raise their prices uniformly above the Nash level. We now show that, conversely, any uniform prices below the Nash level can be sustained. Let $\mathcal{P}^s$ denote the set of prices that can be sustained in a symmetric equilibrium outcome with price caps; that is, $\mathcal{P}^s$ is the set of prices $\hat{p}$ for which there exists $\bar{p}$ such that all firms charging $\hat{p}$ is an equilibrium when each firm $i \in \mathcal{N}$ faces the price cap $\bar{p}$. We have:

Proposition 5 (symmetric oligopoly: price caps benefit consumers)
$\mathcal{P}^s \equiv \left\{ \hat{p} \mid \hat{p} \leq p^N \right\}$.

Proof. See Appendix E. ■

This Proposition extends Proposition 2 in that, focussing on symmetric outcomes, price caps can only result in lower prices and thus benefit consumers. Using Lemma 3, it also implies that firms have no incentives to introduce a price cap under (**S**), and can instead use them to increase their profits under (**C**). Indeed, we have:

Proposition 6 (symmetric oligopoly: firms' incentives)

(*i*) *Under* (**S**), *firms cannot use price caps to sustain a more profitable symmetric outcome than that of the Nash equilibrium.*

(*ii*) *Under* (**C**), *firms can use price caps to sustain the monopoly outcome, which increases their profits and also benefits consumers, compared with the Nash outcome; furthermore, if firms face non-decreasing returns to scale, then the unique equilibrium under a price cap* $\bar{p} = p^M$ *is the monopoly outcome.*[29]

Proof. See Appendix F. ■

Proposition 6 extends Proposition 3: price caps play no role when firms offer substitutes, and enable instead the firms to achieve perfect cooperation when they offer complements, in which case this cooperation also benefits consumers.

Remark: price caps versus mergers. The above findings also extend the insight that, in the absence of merger-specific efficiency gains, price caps constitute a safer alternative

[27] Focussing on symmetric equilibria is natural given the assumed symmetric setting. Transfers are then not needed, as firms' interests are aligned.

[28] Clearly, if charging the same price p constitutes an equilibrium (with or without price caps), it also constitutes an equilibrium when all firms are subject to the same price cap $\bar{p} = p$.

[29] It can be checked that this result holds as long as costs are not too convex, namely, as long as $C'\left(D\left(p^M, 0\right)\right) < p^M$.

to mergers: (i) they both enable (perfect) socially desirable cooperation in case of complements; and (ii) in case of substitutes, price caps are innocuous whereas mergers harm consumers and social welfare.

4 Impact of price caps on repeated interactions

We now study the scope for tacit coordination through repeated interaction. We first assume away price-cap agreements, and consider an infinite horizon setting in which:

- in each period $t = 1, 2, ...$, firms set their prices $\{p_i^t\}_{i\in\mathcal{N}}$ for that period.
- each firm i maximizes the discounted sum of its per-period profits, $\sum_{t\geq 1} \delta^t \pi_i(\mathbf{p}^t)$, where δ is the discount factor, common to all firms.

To avoid technicalities, we focus on pure strategies and, in the case of substitutes, assume that prices are bounded above by an arbitrarily large bound: firms' strategies are of the form $\{p_i^t(\cdot)\}_{t=1,2,...}$, where $p_i^t(\cdot)$ is a mapping from H^t, the set of all possible histories at the beginning of period t, onto $[0, p^{\max}]$, where $p^{\max} > \max\{p^N, p^M\}$ (> 0).[30] We focus on the subgame-perfect equilibria of this game.

It is well-known that the repetition of the static Nash equilibrium constitutes a subgame-perfect equilibrium of this repeated game, and that multiple equilibria may exist when the discount factor is not too small.

We then introduce again a preliminary stage in which the firms can agree on price caps. Price caps can facilitate tacit coordination in two ways:

- price caps limit feasible deviations from the equilibrium path;
- price caps may also enlarge the set of feasible punishments following a deviation, by constraining the deviator's possible actions.

To study the overall impact of price caps on the scope for tacit coordination, we characterize and compare the equilibrium sets with and without price caps.

4.1 Symmetric oligopoly outcomes

We consider here the same setting as Section 3.3 and focus on stationary symmetric equilibrium paths in two scenarios: without price caps, and with a symmetric price cap $\bar{p}$.

Let $\mathcal{P}^+$ (resp., by $\mathcal{P}_c^+$) denote the set of prices p that are weakly more profitable than the static Nash equilibrium (i.e., $\pi^s(p) \geq \pi^s(p^N)$) and can be sustained in the

[30] This upper bound ensures the existence of a worst punishment, which we use for the case of substitutes.

absence of price caps (resp., with price caps). We first note that the static Nash outcome is sustainable, with or without price caps, and that price caps can only enhance the scope for tacit coordination:

Lemma 4 (price caps facilitate tacit coordination) $p^N \in \mathcal{P}^+ \subseteq \mathcal{P}_c^+$.

Proof. Any price in $\mathcal{P}^+$ remains sustainable when a "high-enough" price cap (e.g., $\bar{p} = p^{\max}$) is introduced; hence, $\mathcal{P}^+ \subseteq \mathcal{P}_c^+$. The conclusion then follows from the fact that, in the absence of price caps, the repetition of the static Nash outcome constitutes a subgame-perfect equilibrium of the repeated game. ■

From Lemma 4, the sets $\mathcal{P}^+$ and $\mathcal{P}_c^+$ are non-empty. We will denote by p^* and by p_c^* the most profitable prices in these sets: $p^* \equiv \arg\max_{p \in \mathcal{P}^+} \pi^s(p)$ and $p_c^* \equiv \arg\max_{p \in \mathcal{P}_c^+} \pi^s(p)$. From Lemma 3, p^* and p_c^* both lie above p^N when the two goods are substitutes, and below p^N when they are complements. Furthermore, if firms can achieve perfect coordination in the absence of price caps, they can do so as well with high enough price caps: $p_c^* = p^* \left(= p^M\right)$ when $p^M \in \mathcal{P}^+$. The more interesting case is therefore when, in the absence of price caps, firms cannot achieve perfect coordination through repeated interaction (i.e., $p^M \notin \mathcal{P}^+$). We have:

Proposition 7 (screening through price caps under tacit coordination)

(*i*) *Under* (**S**) *and* (**SC**), $\mathcal{P}_c^+ = \mathcal{P}^+$*; price caps* ~~*thus*~~ *have no impact on the scope for tacit collusion.*

(*ii*) *Under* (**C**), $p^M \notin \mathcal{P}^+ \Longrightarrow p_c^* = p^M < p^* \left(\leq p^N\right)$*; price caps* ~~*thus*~~ *enable the firms to achieve perfect cooperation, which also benefits consumers and society. Furthermore, under non-decreasing returns to scale, a price cap* $\bar{p} = p^M$ *yields a unique continuation equilibrium, in which firms repeatedly charge* p^M*.*

Proof. See Appendix G. ■

The intuition underlying Proposition 7 is simple. Under (**S**) and (**SC**), firms produce substitutes; they thus wish to avoid competition and raise prices. However, the targeted prices lie above firms' best-responses; as a result, price caps cannot be used to limit firms' deviations from these targeted prices and, when prices are strategic complements, they cannot limit deviations from any other feasible price either. Hence, price caps do not facilitate tacit collusion. By contrast, under (**C**), firms produce complements; they thus wish to lower their prices in order to eliminate double marginalization. In the absence of price caps, repeated interaction may not enable firms to achieve perfect coordination, in which case the most profitable sustainable price remains higher than the monopoly level. Price caps enable instead the firms to achieve perfect coordination as a unique equilibrium.

Discussion. If firms seize the coordination opportunities that arise, then prices are equal to p^* without price caps, and to p_c^* with price caps; it then follows from Proposition 7 that price caps are always socially beneficial. If instead firms do not always seize these coordination opportunities, a potential objection is that price caps may facilitate collusion by providing focal points. The field evidence on focal points is however difficult to interpret, due to the lack of appropriate counterfactual – for instance, the fact that firms adhere to the ceilings can simply reflect that these caps are effective in limiting double marginalization problems. Laboratory experiments have tried to circumvent this issue, but so far have failed to provide evidence of collusive, focal-point effects. See, e.g., Engelmann and Müller (2011) for an experiment designed to make collusion easier than in previous attempts, as well as a review of that literature.

4.2 The technology adoption model

To study more fully the impact of commercial cooperation on tacit coordination, let us return to the hybrid demand model sketched in Section 2.2. Unlike Section 4.1, we provide a complete characterization of sustainable prices, without and with price caps. We also provide extensions to asymmetric demands and to an arbitrary number of firms.

4.2.1 Setup and non-repeated interaction

There are two firms, $i = 1, 2$, and a mass 1 of users. Each firm owns a patent pertaining to the technology. While users can implement the technology by building on a single patent, it is more effective to combine both: users obtain a gross benefit V from the two patents, and only $V - e$ with either patent alone; the parameter $e \in [0, V]$ thus measures the *essentiality* of individual patents.

Adopting the technology involves an opportunity cost, θ, which varies across users and has full support $[0, V]$ and c.d.f. $F(\theta)$.[31] A user with cost θ adopts the complete technology, based on both patents, if and only if $V \geq \theta + P$, where P is the total licensing price; the demand is thus

$$D(P) \equiv F(V - P).$$

Similarly, the demand for the incomplete technology at price p is

$$D(p + e) = F(V - e - p).$$

That is, an incomplete technology sold at price p generates the same demand as the complete technology sold at price $p + e$; thus $p + e$ can be interpreted as the "quality-adjusted price."

[31] Conditionally on adopting the technology, users have here identical preferences over IP bundles, and thus select the same set of licenses; this also implies that a joint-marketing scheme need not bother with menus of offers (second-degree price discrimination).

Users obtain a net surplus $S(P)$ when they buy the complete technology at total price P, where $S(P) \equiv \int_0^{V-P} (V - P - \theta)\, dF(\theta) = \int_P^V D(\tilde{P})d\tilde{P}$, and a net surplus $S(p+e)$ from buying an incomplete technology at price p.

To ensure the concavity of the relevant profit functions, we will assume that the demand function is well-behaved:

Assumption B: $D(\cdot)$ is twice continuously differentiable and, for any $P \in [0, V]$, $D'(P) < 0$ and $D'(P) + PD''(P) < 0$.

If users buy the two licenses at unit price p, each firm obtains

$$\pi(p) \equiv pD(2p),$$

which is strictly concave under Assumption B; let $p^M \in [0, V]$ and π^M denote the per-patent monopoly price and profit:

$$p^M \equiv \arg\max_p \pi(p), \text{ and } \pi^M \equiv \pi\left(p^M\right) = p^M D(2p^M).$$

If instead users buy a single license at price p, industry profit is

$$\tilde{\pi}(p) \equiv pD(p+e),$$

which is also strictly concave under Assumption B; let $\tilde{p}^M(e)$ denote the monopoly price and $\tilde{\pi}^M(e)$ the total monopoly profit for the incomplete technology:

$$\tilde{p}^M(e) \equiv \arg\max_p pD(p+e), \text{ and } \tilde{\pi}^M(e) \equiv \tilde{\pi}\left(\tilde{p}^M(e)\right) = \tilde{p}^M(e)\, D(\tilde{p}^M(e) + e).$$

Like $\tilde{\pi}(p)$, $\tilde{\pi}^M(e)$ is decreasing in e: the profit derived from the incomplete technology decreases as each patent becomes more essential.

Consider the static game in which the two firms simultaneously set their prices. When a firm raises its price, either of two things can happen:

First, technology adopters may keep including the license in their basket, but because the technology has become more expensive, fewer users adopt it. In reaction to price p_j set by firm j, firm i sets price $r(p_j)$ given by:

$$r(p_j) \equiv \arg\max_{p_i} p_i D(p_i + p_j),$$

which under Assumption B satisfies $-1 < r'(p_j) < 0$ and has a unique fixed point, which we denote by $\hat{p}$:

$$\hat{p} = r(\hat{p}).$$

The two patents are then complements and their prices strategic substitutes (an increase in one price induces the other firm to lower its price). Furthermore, $\hat{p} > p^M$ due to double marginalization.[32]

Second, technology adopters may stop including the license in their basket; this occurs when the firm raises its price above e.

It follows that the Nash equilibrium is unique and symmetric:[33] both firms charge

$$p^N \equiv \min\{e, \hat{p}\},$$

and face positive demand. We will denote the resulting profit by

$$\pi^N \equiv \pi\left(p^N\right).$$

The comparison of p^N with respect to p^M drives the nature of the interaction between the firms, and the coordination that they wish to pursue:[34]

- **Rivalry:** if $p^N < p^M$ (which arises when $e < p^M$, implying $p^N = e$), then the firms wish to *collude* by raising their prices above the static Nash level, which harms consumers and reduces social welfare. Note that charging a price above $p^N = e$ induces users to buy at most one license. We will assume that firms can share the resulting profit $\tilde{\pi}(p)$ as they wish.[35] In this incomplete-technology region, it is optimal for the firms to raise the price up to $\tilde{p}^M(e)$, if feasible, and share the resulting profit, $\tilde{\pi}^M(e)$.

- **Complementors:** if $p^N > p^M$ (which arises when $e > p^M$), then the firms wish to *cooperate* by lowering their prices below the static Nash level, which benefits users as well as firms. Ideally, the firms would reduce the per-patent price down to p^M, and so as to obtain per-firm profit π^M.

Along the lines of our previous results, it is easy to check that price caps can only be beneficial: in case of rivalry (i.e., if $p^N < p^M$), the equilibrium prices under a cap cannot exceed $e = p^N$ (each p_i is equal to the minimum of e and firm i's cap), and so letting firms agree on price caps has no effect on the outcome; if instead $p^N > p^M$, then allowing price caps lead to the lower-price monopoly outcome.

Interestingly, a pool offering the bundle at some pre-agreed price P, together with

[32] By revealed preference, $p^m D(2p^m) \geq \hat{p}D(2\hat{p}) \geq p^m D(\hat{p} + p^m)$ and thus $D(2p^m) \geq D(\hat{p} + p^m)$, implying $\hat{p} \geq p^m$. Assumption B moreover implies that this inequality is strict.

[33] See online Appendix E for a detailed exposition.

[34] It is tempting to refer to "substitutes" in case of rivalry and to "complements" in case of complementors. However, in this hybrid demand model, patents are always local complements for low prices, and local substitutes for high prices. For instance, in the case of "weak complementors" (namely, when $p^N = e > p^M$), patents are complements at prices below the Nash level (e.g., at monopoly prices), and local substitutes at higher prices.

[35] In our setting, they can do so by charging the same price $p > e$ and allocating market shares among themselves; more generally, introducing a small amount of heterogeneity in users' preferences would allow the firms to achieve arbitrary market shares by choosing their prices appropriately.

independent licensing, achieves here the same outcome.[36] Independent licensing means that the owners of the patents keep ownership of their patent and therefore can market it outside the pool. A price cap is equivalent to the combination of independent licensing and unbundling, where "unbundling" refers to the requirement that the pool sells individual licenses (at a total price below the bundle price) and not only the bundle; the pool's stand-alone prices then serve as price caps for the independent licensing pricing game. As we will see, independent licensing alone no longer provides a perfect screen in case of repeated interaction.

4.2.2 Repeated interaction

We now suppose that the firms play the game repeatedly, with discount factor $\delta \in (0,1)$. Let

$$v \equiv (1-\delta)\,\Sigma_{t\geq 0}\delta^t \frac{\pi_1^t + \pi_2^t}{2}$$

denote the average of firms' discounted profits over a pure-strategy equilibrium path, $\mathcal{V}^+$ denote the set of these equilibrium payoffs that are weakly more profitable than Nash (i.e., such that $v \geq \pi^N$), and v^* denote the maximal equilibrium payoff.[37] Tacit coordination raises profits only if $v^* > \pi^N$.

The location of e affects not only the nature of tacit coordination, but also the minmax profit:

Lemma 5 (minmax) *Let $\underline{\pi}$ denote the minmax profit.*

(i) If $e \leq \hat{p}$, the static Nash equilibrium (e,e) gives each firm the minmax profit: $\underline{\pi} = \pi^N = \pi(e)$.

(ii) If $e > \hat{p}$, the minmax profit is the incomplete-technology per-period monopoly profit: $\underline{\pi} = \tilde{\pi}^M(e) < \pi^N = \pi(\hat{p})$.

Proof. See online Appendix F. ■

Hence, when $e \leq \hat{p}$, the static Nash equilibrium (e,e) yields the minmax profit;[38] it thus constitutes the toughest punishment for both firms. When instead $e > \hat{p}$, each firm

[36]See Lerner and Tirole (2004). Current antitrust guidelines in Europe, Japan and the US require patent pools to allow independent licensing.

[37]This maximum is well defined, as the set $\mathcal{V}^+$ of Nash-dominating subgame perfect equilibrium payoffs is non-empty (it includes π^N) and compact (see Mailath and Samuelson (2006), chapter 2). Also, although we restrict attention to pure-strategy subgame perfect equilibria here, the analysis could be extended to public mixed strategies (where players condition their strategies on public signals) or, in the case of private mixed strategies, to perfect public equilibria (relying on strategies that do not condition future actions on private past history); see Mailath and Samuelson (2006), chapter 7.

[38]By charging e, a firm can ensure that its patent is included in the basket; by so doing, it obtains $eD(e+e) = \pi(e)$ if the other firm charges $p > e$, and $eD(e+p) \geq \pi(e)$ otherwise.

can guarantee itself the incomplete-technology monopoly profit $\tilde{\pi}^M(e)$,[39] which is then lower than the profit of the static Nash equilibrium $(\hat{p}, \hat{p})$; Abreu (1988)'s optimal penal codes can then be used to sustain the toughest punishment.

We now characterize the scope for tacit coordination in the case of rivalry and of complementors.

a) Rivalry: $p^N < p^M$ This case arises when $e < p^M$, implying $p^N = e$ and $\underline{\pi} = \pi^N = \pi(e)$; collusion then implies selling the incomplete technology, and the loss in demand due to partial consumption grows with essentiality. In particular, if e is close to p^M, the Nash equilibrium payoff $\pi(e)$ approaches the highest possible profit π^M, whereas pricing above e substantially reduces the demand for the patents; as each firm can guarantee itself $\pi(e)$, there is no collusion. Specifically, this occurs when patents are weak substitutes, namely, when $e \geq \underline{e}$, where $\underline{e}$ is the unique solution to

$$\tilde{\pi}^M(\underline{e}) = 2\pi(\underline{e}).$$

By contrast, for e close to 0, this loss in demand is small and the Nash profit is negligible; and so collusion, if feasible, is attractive for the firms. Because users then buy only one license, each firm can attract all users by slightly undercutting the collusive price. Like in standard Bertrand oligopolies, *maximal* collusion (on $\tilde{p}^M(e)$) is sustainable whenever *some* collusion is sustainable. As symmetric collusion is easier to sustain, and deviations are optimally punished by reverting to static Nash behavior, such collusion is indeed sustainable if:

$$\frac{\tilde{\pi}^M(e)}{2} \geq (1-\delta)\tilde{\pi}^M(e) + \delta\pi(e) \iff \delta \geq \delta^R(e) \equiv \frac{1}{2}\frac{1}{1-\frac{\pi(e)}{\tilde{\pi}^M(e)}}, \tag{3}$$

where $\delta^R(e)$ is increasing in e and exceeds 1 for $e \geq \underline{e}$. Building on these insights, we have:

Proposition 8 (rivalry) *When $e < \underline{e}$ and $\delta \geq \delta^R(e)$, $\mathcal{V}^+ = [\pi^N, v^*]$, and $v^* = \tilde{\pi}^M(e)/2$: tacit collusion is feasible and the most profitable collusion occurs at price $\tilde{p}^M(e)$; otherwise, the unique equilibrium is the repetition of the static Nash one.*

Proof. See online Appendix G. ■

Hence, greater essentiality hinders collusion, which is not feasible if $e \geq \underline{e}$; furthermore, as the threshold $\delta^R(e)$ increases with e for any given $\delta \in (1/2, 1)$ in the entire rivalry range $e \in [0, p^M]$, there exits a unique $\hat{e}(\delta) \in (0, \underline{e})$ such that collusion is feasible if

[39] As $r'(\cdot) < 0$, $e > \hat{p}$ implies $\tilde{p}^M(e) = r(e) < r(\hat{p}) = \hat{p} < e$. Hence, by charging $\tilde{p}^M(e)$, a firm can ensure that its patent is included in the basket; it thus obtains $\tilde{p}^M(e) D(\tilde{p}^M(e) + e) = \tilde{\pi}^M(e)$ if the other firm charges $p > e$, and $\tilde{p}^M(e) D(\tilde{p}^M(e) + p) \geq \tilde{\pi}^M$ otherwise.

and only if $e < \hat{e}(\delta)$. This is because the toughest punishment, given by the static Nash profit, becomes less effective as essentiality increases; although the gains from deviation also decrease, which facilitates collusion, this effect is always dominated.

b) Complementors: $p^M < \hat{p}$ This case arises when $e > p^M$. Like when $e \in [\underline{e}, p^M]$, selling the incomplete technology cannot be more profitable than the static Nash outcome.[40] Firms can however increase their profit by *lowering* their price below the Nash level. Furthermore, when demand is convex, it can be checked that cooperation on some total price $P < 2e$ is easiest when it is symmetric (i.e., when $p_i = P/2$). As $p^N = \min\{e, \hat{p}\}$, we can distinguish two cases:

• *Weak complementors:* $e < \hat{p}$, in which case $p^M < p^N = e$. The static Nash equilibrium $p^N = e$ still yields minmax profits and thus remains the toughest punishment in case of deviation. As $p_j \le e < \hat{p} = r(\hat{p}) < r(p_j)$, firm i's best deviation then consists in charging e. In particular, *perfect* cooperation on p^M is sustainable if and only if:

$$\pi^M \ge (1-\delta)\, eD\left(p^M + e\right) + \delta\pi(e), \tag{4}$$

which is satisfied for δ close enough to 1.

• *Strong complementors:* $e > \hat{p}$, in which case $p^M < p^N = \hat{p}$. Starting from a symmetric price $p \in \left[p^M, p^N\right]$, the best deviation profit is then given by $\max_{\tilde{p} \le e} \tilde{p}D(\tilde{p} + p)$. The static Nash equilibrium $(\hat{p}, \hat{p})$ however no longer yields the minmax payoff, equal here to the incomplete-technology monopoly profit: $\underline{\pi} = \tilde{\pi}^M(e)$; Abreu (1988)'s optimal penal codes then provide more severe punishments than the static Nash outcome. If firms are sufficiently patient, these punishments can be as severe as the minmax profits,[41] in which case perfect cooperation on p^M is sustainable if in addition:

$$\pi^M \ge (1-\delta) \max_{\tilde{p} \le e} \tilde{p}D(\tilde{p} + p) + \delta\tilde{\pi}^M(e).$$

Building on these insights leads to:

Proposition 9 (complementors) *When* $p^M < p^N$*:*

(i) There exists $\overline{\delta}^C(e) < 1$ *and* $\underline{\delta}^C(e) < \overline{\delta}^C(e)$*, where* $\overline{\delta}^C(e)$ *is decreasing for* e *close to* p^M*, close to* $\hat{p}$ *and close to* V*, and* $\underline{\delta}^C(e)$ *is decreasing in* e*, and equal to* 0 *for* $e = \hat{p}$*, such that*

– *perfect cooperation on price* p^M *is feasible (i.e.,* $v^* = \pi^M$*) whenever* $\delta \ge \overline{\delta}^C(e)$*;*

– *profitable cooperation is sustainable (i.e.,* $v^* > \pi^N$*) whenever* $\delta \ge \underline{\delta}^C(e)$*.*

[40] $\tilde{\pi}^M(e) = \tilde{p}^M(e)\, D\left(e + \tilde{p}^M(e)\right) < \left(e + \tilde{p}^M(e)\right) D\left(e + \tilde{p}^M(e)\right) \le 2eD(2e)$, where the first inequality stems from $e > 0$ and the second one from the fact that the aggregate profit $PD(P)$ is concave in P and maximal for $P^M = 2p^M < 2e \le e + \tilde{p}^M(e)$ (as $\tilde{p}^M(e) = r(e) \ge r(\hat{p}) = p \ge e$).

[41] See Lemma 7 in online Appendix H.

(*ii*) *Furthermore, if* $D'' \geq 0$*, then there exists* $v^*(e,\delta) \in (\pi^N, \pi^M]$*, which is (weakly) increasing in* δ*, such that the set of Nash-dominating sustainable payoffs is* $\mathcal{V}^+ = [\pi^N, v^*(e,\delta)]$.

Proof. See online Appendix H. ■

By contrast with the case of rivalry, where collusion inefficiently induces users to adopt the incomplete technology, avoiding double marginalization unambiguously raise profits here. It follows that some cooperation (and even perfect cooperation) is always sustainable, for any degree of essentiality, when firms are sufficiently patient; furthermore, in the case of strong complementors (i.e., $e > \hat{p}$), firms can always sustain some cooperation on a price $p < p^N = \hat{p}$, regardless of their discount factor: this is because starting from the static Nash price $\hat{p}$, a small reduction in the price then generates a first-order increase in profits, but only a second-order incentive to deviate.

c) Summary and welfare analysis Figure 3 summarizes the analysis so far.

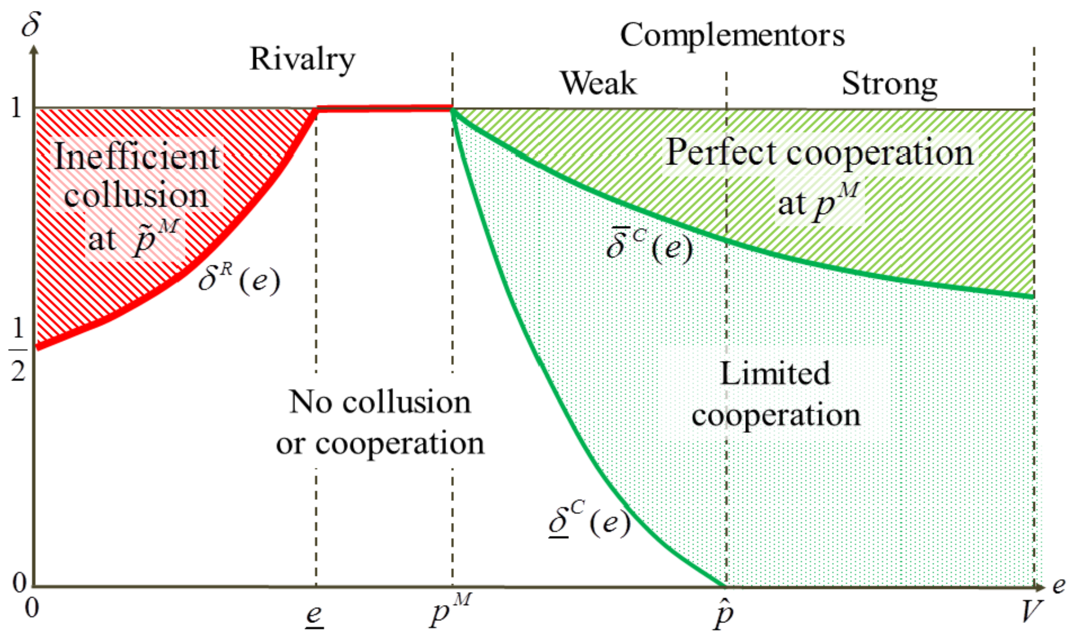

Figure 3: Tacit coordination without price caps in the technology model

Tacit coordination is easiest, and the gain from coordination highest, when the patents are close to being either perfect substitutes or perfect complements. Tacit coordination is impossible when patents are weak substitutes; raising price then leads users to adopt an incomplete version of the technology, and decreases overall profit. Collusion by contrast is feasible when patents are strong substitutes, and all the more so as they become closer substitutes. Likewise, cooperation is not always feasible when patents are weak complementors, but the scope for cooperation increases as patents become more essential; finally, some cooperation is always possible when patents are strong complementors.

We now consider the impact of tacit coordination on users and society. To perform a welfare analysis we assume that, whenever equilibria exist that are more profitable than the static Nash outcome, then firms coordinate on one – anyone – of those equilibria.[42]

Proposition 10 (welfare) *Whenever firms coordinate on an equilibrium that is more profitable than the static Nash benchmark, such tacit coordination:*

(i) harms users and reduces total welfare under rivalry ($e < p^M$).

(ii) benefits users and increases total welfare for complementors ($e > p^M$).

Proof. See online Appendix I. ■

4.2.3 Impact of price caps on repeated interaction

Let us now investigate tacit coordination under price caps. Let $\mathcal{V}_c^+$ denote the set of equilibrium payoffs that are weakly more profitable than Nash when price caps can be introduced, and v_c^* denote the maximal payoff in this set. We have:

Proposition 11 (benefits of price caps) *Price caps:*

(i) have no impact on profitable collusion in case of rivalry: if $e < p^M$, then $\mathcal{V}_c^+ = \mathcal{V}^+$;

(ii) enable perfect cooperation, which benefits consumers as well, in case of complementors: if $e \geq p^M$, then $v_c^* = \pi^M$; in particular, introducing a price cap $\bar{p} = p^M$ yields a unique continuation equilibrium, in which firms repeatedly charge p^M.

Proof. See online Appendix J. ■

Within the context of technology adoption, Proposition 11 extends Proposition 7 in that it considers the entire set of Nash-dominating equilibria (stationary or not, symmetric or not) that can be sustained with and without price caps. The findings can be illustrated by comparing Figure 3 with Figure 4: price caps can only benefit consumers when firms use them to increase their profits; they do not allow for any additional undesired collusion in case of rivalry, and allow instead for perfect, desirable cooperation in case of complementors.

[42] We remain agnostic about equilibrium selection, as the conclusions hold for *any* profitable coordination.

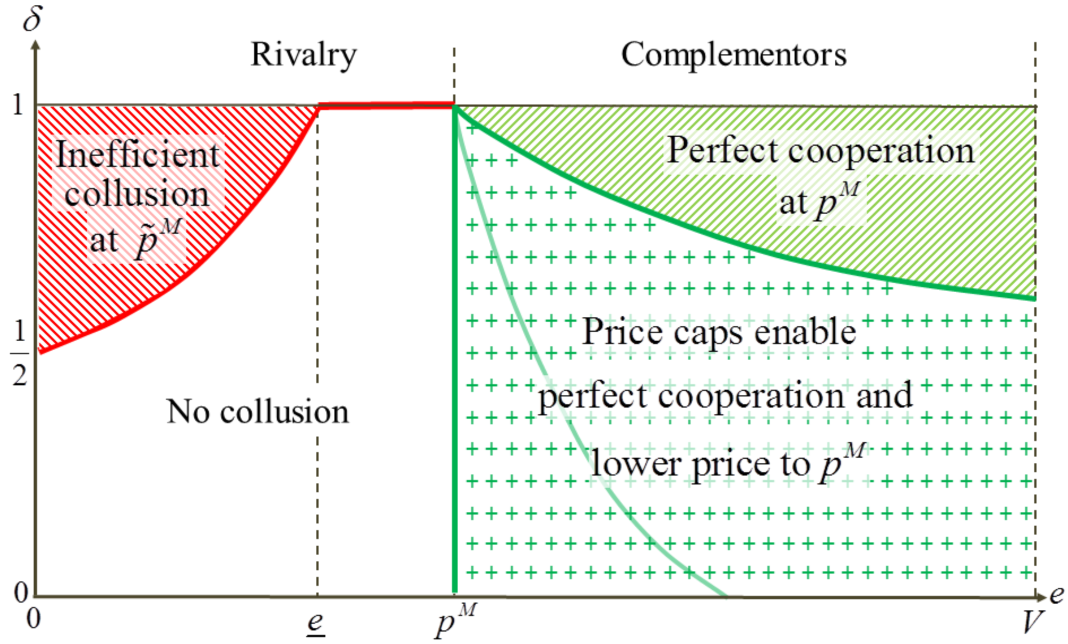

Figure 4: Tacit coordination under price caps in the technology model

4.2.4 Independent licensing is not a perfect screen under repeated interaction

We saw that independent licensing provides a perfect screen under non-repeated interaction: it prevent pools from sustaining any collusion in case of rivalry, and does not prevent pool to achieve perfect cooperation in case of complementors. Alas, this is no longer so under repeated interaction. A pool subject to independent licensing still improves cooperation and lowers price for complementors $\left(e > p^M\right)$, and it also benefits consumers when (inefficient) collusion would already arise in the absence of a pool, by allowing them to consume both offerings; however, a pool, even subject to independent licensing, may harm consumers by enabling collusion in case of weak rivalry.

To study this, let us introduce a pool subject to independent licensing in the repeated game considered in Section 4.2.2. The pool sets the price of the bundle[43] and specifies a sharing rule for its dividends: some fraction $\alpha_i \geq 0$ (with $\alpha_1 + \alpha_2 = 1$) goes to firm i. In addition, each pool member can offer licenses on a stand-alone basis if it chooses to. The game thus operates as follows:

1. At date 0, the firms form a pool and fix a pool price P for the bundle, as well as the dividend sharing rule.

2. Then at dates $t = 1, 2, ...$, the firms non-cooperatively set prices p_i^t for their individual licenses; the profits of the pool are then shared according to the agreed rule.

[43] It can be checked that the firms cannot gain from asking the pool to offer unbundled prices as well.

Online Appendix K characterizes the set of equilibria that can be sustained with a pool subject to independent licensing; comparing it to the equilibria sustained without a pool, or with a pool not subject to independent licensing, leads to the following proposition:

Proposition 12 (screening through independent licensing) *Independent licensing provides a useful but imperfect screen:*

(*i*) *Appending independent licensing to a pool is always welfare-enhancing.*

Relative to the absence of a pool:

(*ii*) *In case of complementors, a pool with independent licensing enables the firms to achieve perfect cooperation, which is welfare-enhancing.*

(*iii*) *In case of rivalry, if some collusion is already sustainable without a pool, then a pool with independent licensing enables the firms to collude more efficiently, which results in lower prices and is thus welfare-enhancing; however, there exists* $\underline{\delta}^R(e)$*, which increases from* $\delta^R(0) = 1/2$ *to* 1 *as* e *increases from* 0 *to* p^M*, and lies strictly below* $\delta^R(e)$ *for* $e \in (0, p^M)$*, such that, for* $\delta \in [\underline{\delta}^R(e), \delta^R(e))$*, the pool raises price by enabling the firms to collude.*

Proof. See online Appendix K. ■

These insights are illustrated in Figure 5.

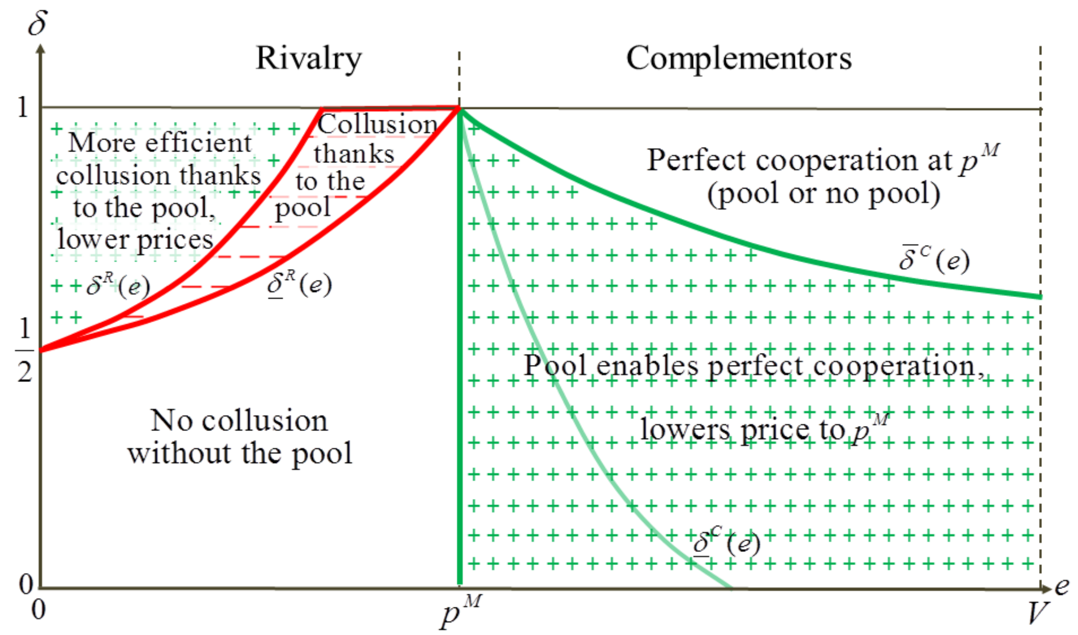

Figure 5: Impact of a pool with Independent Licensing (relative to no pool)

Without independent licensing, a pool would enable the firms to sustain the monopoly outcome which also benefits consumers in case of complementors but harms them in case of rivalry. Appending independent licensing does not prevent the pool from achieving the desired cooperation in case of complementors, and in case of rivalry, it enables the firms to collude more efficiently, which again benefits consumers when firms could already collude without the pool. However, the pool can also enable the firms to collude where they could not otherwise, in which case it hurts consumers. This is because, by eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive.

Thus, by relying on independent licensing alone, authorities run the risk of generating some welfare loss by approving a pool of weak substitutes.[44] By contrast, price caps (which, as already noted, amount to appending unbundling to independent licensing) provide a perfect screen.

4.2.5 Asymmetric offerings

Suppose now that essentiality differs across firms: The technology has value $V - e_i$ if the user buys only patent j (for $i \neq j \in \{1,2\}$); without loss of generality, suppose that $e_1 \geq e_2$. The following proposition shows that price caps still provide a perfect screen. As in Section 4.2.3, let $\mathcal{V}_c^+$ denote the set of pure-strategy equilibrium payoffs that are weakly more profitable than Nash when price caps can be introduced, and v_c^* denote the maximal per firm payoff in this set; we have:

Proposition 13 (asymmetric offerings) *Price caps:*

(i) *do not affect the scope for profitable collusion in case of rivalry: if* $e_1 + e_2 < P^M$*, then* $\mathcal{V}_c^+ = \mathcal{V}^+$*;*

(ii) *enable consumer-welfare-augmenting perfect cooperation in case of complementors: if* $e_1 + e_2 \geq P^M$*, then* $v_c^* = \pi^M$*; in particular, any vector of price caps* $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2)$ *satisfying* $\bar{p}_1 + \bar{p}_2 = P^M$ *and* $\bar{p}_i \leq e_i$ *induces* $\mathbf{p} = \bar{\mathbf{p}}$ *in every period as unique continuation equilibrium.*

Proof. See online Appendix J. ■

4.2.6 Oligopoly

Suppose now that there are $n \geq 2$ symmetric firms: The technology has value $V(m)$ if the user buys $m \leq n$ licenses, with $0 = V(0) \leq V(1) \leq ... \leq V(n)$ and $V(n) > 0$. The demand for the bundle of n patents at total price P becomes

$$D(P) \equiv F(V(n) - P),$$

[44] This occurs only when no collusion is sustainable in the absence of a pool (i.e., $\delta < \delta^R$) *and* the pool enables some collusion (e.g., $\delta \geq \underline{\delta}^R$); the pool is instead beneficial when inefficient collusion was already sustainable (i.e., $\delta \geq \delta^R$) and is neutral when collusion remains unsustainable (i.e., for δ low enough).

where the c.d.f. $F(\cdot)$ satisfies the same regularity conditions as before (that is, Assumption B holds). Lerner and Tirole (2004) show that, in the unique symmetric static Nash outcome, users buy patents at price

$$p^N \equiv \min\{\tilde{p}, \hat{p}\},$$

where $\tilde{p}$ is the unique price p satisfying

$$V(n) - np = \max_{m<n} \{V(m) - mp\},$$

and where $\hat{p}$ is now defined as:

$$\hat{p} \equiv \arg\max_p \{pD(p + (n-1)\hat{p})\}.$$

As in a duopoly, multiple marginalization implies $\hat{p} > p^M \equiv \arg\max_p npD(np)$, leading to three relevant regimes:

- Rivalry when $\tilde{p} < p^M$, implying $p^M > p^N = \tilde{p}$.
- Weak complementors when $p^M < \tilde{p} < \hat{p}$, implying $p^M < p^N = \tilde{p}$.
- Strong complementors when $\tilde{p} \geq \hat{p}$, implying $p^M < p^N = \hat{p}$.

Our previous insights readily extend to any number of patents in the case of complementors. Likewise, in the case of rivalry, we show in Online Appendix L raising total profit above the static Nash level requires again selling an incomplete bundle. To go further, we focus for simplicity on the stationary symmetric outcomes that can be sustained by reversal to Nash; let $\hat{\mathcal{P}}^+$ (resp., $\hat{\mathcal{P}}_c^+$) denote the set of prices that are weakly more profitable than the static Nash equilibrium and can be sustained in the absence of price caps (resp., with price caps)[45]. We have:

Proposition 14 (oligopoly) *Price caps:*

(*i*) *do not affect the set of profitable prices that can be sustained by reversal to Nash in case of rivalry: if $p^N < p^M$, then $\hat{\mathcal{P}}_c^+ = \hat{\mathcal{P}}^+$;*

(*ii*) *enable perfect cooperation, which benefits consumers as well, in case of complementors: if $p^N \geq p^M$, then introducing a price cap $\bar{p} = p^M$ yields a unique continuation equilibrium, in which firms repeatedly charge p^M.*

Proof. See online Appendix L. ■

In the rivalry case, in order to increase profits firms must raise prices, which induces users to buy only a subset of patents. But then, a firm cannot profitably deviate by raising further its price, as it would exclude itself from the basket. Price caps thus have no bite on

[45] We allow for asymmetric price caps; however, given the symmetry of the environment, symmetric price caps are as effective as asymmetric ones.

profitable deviations, and so cannot enhance the scope for collusion. By contrast, in the case of complementors, price caps enable the firms to increase their profits by reducing their prices down to the monopoly level, and preventing any profitable deviation towards higher prices.

5 Investment incentives

This section analyses the impact of price caps on investment and entry incentives. For the sake of exposition, we focus on symmetric cost and demand conditions and assume that, in any n-firm oligopoly, there is a unique static Nash equilibrium, in which each firm obtains a profit π_n^N, and a unique monopoly outcome, in which each firm obtains a profit $\pi_n^M > \pi_n^N$, and consumers obtain a total surplus S_n^M.

We will focus on the cases $(\mathbf{S})-(\mathbf{SC})$ (i.e., all products are substitutes, and prices are strategic complements) and $(\mathbf{C})$ (i.e., all products are complements). Intuitively, entry is socially more desirable from in the latter case, as adding one product then enhances the benefits that consumers derive from the other products. To ensure that these additional benefits are shared between the firms and the consumers, we introduce the following regularity assumption:

Assumption C: Under $(\mathbf{C})$, S_n^M increases with n, and $C''(\cdot) \leq 0$.

In online Appendix M, we show that consumer net surplus under monopoly pricing increases with the number of products when demand is linear, and provide a couple of other illustrating examples. In online Appendix N, we show that the second part of the assumption (i.e., non-decreasing returns to scale) guarantees that π_n^M increases with n.

Interestingly, price caps do foster entry in case of complements, whereas they have no impact in the other case. From the previous analysis, price caps have no impact on the profits that they can achieve under $(\mathbf{S})-(\mathbf{SC})$, and thus do not affect investment or entry decisions. By contrast, under $(\mathbf{C})$, price caps enable the firms to cooperate perfectly. The following proposition shows that, as is intuitive, price caps boost investment and entry incentives.

To capture this insight, we consider two scenarios.

- *Single entrant scenario:* n incumbents are already present in the market, and a potential competitor can invest I to enter the market.
- *Free-entry scenario:* a large number of potential competitors, with entry costs $I_1 \leq I_2 \leq ...$, simultaneously decide whether to invest to enter the market.

To characterize investment incentives, we need to select among the continuation equilibria in case of repeated interactions. We assume that, in both scenarios, firms select the most profitable continuation equilibrium, given the number of firms present in the market. We have:

Proposition 15 (investment incentives) *In the absence of repeated interactions, or if firms select the most profitable continuation (post-entry) equilibrium:*

(*i*) *Under* (**S**) *and* (**SC**)*, price caps have no impact on investment / entry decisions.*

(*ii*) *Under* (**C**)*, price caps foster entry: in the single entrant scenario, they allow entry for a broader range of the investment cost* I*, which benefits the incumbents as well as the entrant; in the free-entry scenario, they (weakly) increase the number of firms entering the market. In both scenarios, price caps thus benefit consumers by expanding the range of products offered, as well as by reducing prices for any given product range.*

Proof. See online Appendix N. ■

6 Concluding remarks

The reviewing of mergers and joint marketing agreements can be hindered by poor information associated with patchy or non-existent price and demand data as well as time- and price-dependent patterns of complementarity/substitutability. This suggests enriching the antitrust toolbox with new and less information-intensive regulatory instruments. This paper is an attempt at meeting the challenge. It investigates price caps as a possible alternative to mergers and patent pools. The only function of joint marketing arrangements such as patent pools would then be the enabling of one-stop shopping: users could buy individual products or licenses at the price caps agreed upon by individual firms, which however would keep ownership of products or patents.

We saw that price caps, if used by the firms, raise consumer welfare – for quite general demand and cost functions in the case of non-repeated interactions and for the more specific repeated-interactions environments we were able to analyze. We thereby provided a novel analysis, of independent interest, of coordinated effects in an industry in which goods are not necessarily substitutes, let alone perfect substitutes.

This paper is only a first step in an extensive research agenda. We conclude with five important lines for future research. First, the theory of repeated interaction with arbitrary degrees of substitutability and complementarity should be developed for general cost and demand functions. Second, while we have assumed that firms take advantage of existing opportunities to coordinate tacitly, we know little about whether discussions such as those on price caps could enhance market transparency,[46] or create a focal point that

[46] Information exchanges about cost and demand conditions, or about past behavior, are reputed to facilitate tacit collusion – see, e.g., Green and Porter (1984) for a formal analysis and Ivaldi *et al.* (2003), Vives (2007) and Whinston (2006) for general discussions; rules restricting information exchanges, as they already exist for merger negotiations, could then be useful. In a recent paper, Sugaya and Wolitzky (2017) however challenge this common wisdom and show that maintaining privacy may help firms collude by refraining to compete in each other's markets.

would help the firms to indeed achieve tacit collusion. Further experimental work could inform us on this question. Third, we should think beyond mergers and pools. Other joint marketing arrangements, such as alliances for instance, ought to be considered; similarly, "indirect" joint marketing through platforms should receive more attention. Fourth, we could extend the analysis to competitive non-linear pricing and other forms of price discrimination, so as to unveil the proper counterparts of price caps in such settings.[47] Finally, the analysis should incorporate cost synergies and look at agreements that might increase our confidence in their competitiveness. We leave these and other fascinating aspects of coopetition to future research.

[47] For instance, China Eastern Airlines and Qantas (which are substitutes on the Shanghai-Sydney route and complements on connecting flights) submitted to the Australian competition agency (the ACCC) a coordination agreement covering schedules, frequencies and connection times, but also new fare products and frequent flyer programs – the airline industry is highly prone to yield management and loyalty programs. To accept the agreement, the ACCC imposed minimal quantity requirements (expressed in terms of seat capacity between Shanghai and Sydney, and of aggregate seat capacity between Shanghai and Australia). See ACCC decision N° A91470, available at http://www.accc.gov.au. We thank Graeme Woodbridge for drawing our attention to this case.

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Appendix

A Proof of Lemma 1

We first show that monopoly prices exceed marginal costs for at least one firm. Suppose instead that $p_i^M \leq C_i'\left(q_i^M\right)$ for all $i \in \mathcal{N}$, and consider a small and uniform increase in prices: $dp_i = dp > 0$ for $i \in \mathcal{N}$. We then have $dq_j = \sum_{i \in \mathcal{N}} \partial_i D_j\left(\mathbf{p}^M\right) dp \leq 0$ for all $j \in \mathcal{N}$, and thus:

$$d\Pi = \sum_{j \in \mathcal{N}} \left[p_j^M - C_j'\left(q_j^M\right)\right] dq_j + \sum_{j \in \mathcal{N}} q_j^M dp > 0,$$

a contradiction. Therefore, $p_i^M > C_i'\left(q_i^M\right)$ for some $i \in \mathcal{N}$.

We now show that, under (**S**), $p_i^M > C_i'\left(q_i^M\right)$ for every $i \in \mathcal{N}$. To see this, suppose that there exists a non-empty subset of $\mathcal{N}$, $\mathcal{N}^-$, such $p_j^M \leq C_j'\left(q_j^M\right)$ for every $j \in \mathcal{N}^-$, and consider a small and uniform increase in these prices: $dp_j = dp > 0$ for $j \in \mathcal{N}^-$. Under (**S**), we then have:

- for $i \in \mathcal{N} \setminus \mathcal{N}^-$, $dq_i = \sum_{j \in \mathcal{N}^-} \partial_j D_i\left(\mathbf{p}^M\right) dp > 0$, as $\partial_j D_i\left(\mathbf{p}^M\right) > 0$ for $j \neq i$.
- for $i \in \mathcal{N}^-$, $dq_i = \sum_{j \in \mathcal{N}^-} \partial_j D_i\left(\mathbf{p}^M\right) dp \leq \sum_{j \in \mathcal{N}} \partial_j D_i\left(\mathbf{p}^M\right) dp < 0$.

Therefore:

$$d\Pi = \sum_{j \in \mathcal{N} \setminus \mathcal{N}^-} \underbrace{\left[p_j^M - C_j'\left(q_j^M\right)\right]}_{>0} \underbrace{dq_j}_{>0} + \sum_{j \in \mathcal{N}^-} \underbrace{q_j^M}_{>0} \underbrace{dp}_{>0} + \sum_{j \in \mathcal{N}^-} \underbrace{\left[p_j^M - C_j'\left(q_j^M\right)\right]}_{\leq 0} \underbrace{dq_j}_{<0} > 0,$$

a contradiction. Therefore, under (**S**), $p_i^M > C_i'\left(q_i^M\right)$ for every $i \in \mathcal{N}$.

We now compare monopoly prices to firms' best-responses. The monopoly prices satisfy, for $i \in \mathcal{N}$:

$$0 = \partial_i \Pi\left(\mathbf{p}^M\right) = \partial_i \pi_i\left(\mathbf{p}^M\right) + \sum_{j \in \mathcal{N} \setminus \{i\}} \partial_i \pi_j\left(\mathbf{p}^M\right),$$

and thus:

$$\partial_i \pi_i\left(\mathbf{p}^M\right) = -\sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j \pi_i\left(\mathbf{p}^M\right) = -\sum_{j \in \mathcal{N} \setminus \{i\}} \left\{\left[p_j^M - C_j'\left(q_j^M\right)\right] \partial_i D_j\left(\mathbf{p}^M\right)\right\}. \qquad (5)$$

Therefore:

(i) Under (**S**), the right-hand side of (5) is negative, as $p_j^M > C_j'\left(q_j^M\right)$, from the first part of the lemma, and $\partial_j D_i(\cdot) > 0$ for $j \neq i \in \mathcal{N}$; hence, for $i \in \mathcal{N}$, we have $\partial_i \pi_i\left(\mathbf{p}^M\right) < 0$, which, together with the quasi-concavity of π_i with respect to p_i, implies $p_i^M > R_i\left(\mathbf{p}_{-i}^M\right)$.

(ii) Suppose that for all $j \in \mathcal{N}$, $p_j^M \geq R_j\left(\mathbf{p}_{-j}^M\right)$, implying $\partial_j \pi_j\left(\mathbf{p}^M\right) \leq 0$. We then have, for $j \in \mathcal{N}$:

$$0 \geq \partial_j \pi_j\left(\mathbf{p}^M\right) = D_j\left(\mathbf{p}^M\right) + \left[p_j^M - C_j'\left(q_j^M\right)\right] \partial_j D_j\left(\mathbf{p}^M\right),$$

and thus, under (**C**), $p_j^M > C_j'\left(q_j^M\right)$ for every $j \in \mathcal{N}$. But then, as $\partial_j D_i(\cdot) < 0$ for $j \neq i \in \mathcal{N}$ under (**C**), (5) implies $\partial_i \pi_i\left(\mathbf{p}^M\right) > 0$, a contradiction. Hence, the monopoly outcome satisfies $p_j^M < R_j\left(\mathbf{p}_{-j}^M\right)$ for some firm j.

Finally, when $n = 2$, (5) implies, for $j \neq i \in \{1, 2\}$:

$$\partial_j \pi_j\left(\mathbf{p}^M\right) = -\left[p_i^M - C_i'\left(q_i^M\right)\right] \partial_j D_i\left(\mathbf{p}^M\right).$$

Under (**C**), $\partial_j D_i(\cdot) < 0$ and thus $p_i^M > C_i'\left(q_i^M\right)$ implies $p_j^M < R_j\left(p_j^M\right)$.

B Proof of Lemma 2

Starting from $\mathbf{p} = \left.(p_i, \mathbf{p}_{-i})\right|_{p_i = R_i(\mathbf{p}_{-i})}$, the impact of a slight increase in p_i on firm i's profit is given by:

$$\left[p_i - C_i'\left(D_i(\mathbf{p})\right)\right] \partial_i D_i(\mathbf{p}) + D_i(\mathbf{p}).$$

If firm i's margin were non-positive, this impact would be positive (as $\partial_i D_i(\cdot) < 0 < D_i(\cdot)$), a contradiction. Hence, $R_i(\mathbf{p}_{-i}) > C_i'\left(D_i\left(R_i(\mathbf{p}_{-i}), \mathbf{p}_{-i}\right)\right)$.

C Proof of Proposition 1(ii)

By construction, $\mathbf{p}^N$ lies on firms' best-responses, and thus belongs to $\mathcal{P}$. Consider now a price vector $\hat{\mathbf{p}}$ in $\mathcal{P} \setminus \left\{\mathbf{p}^N\right\}$, and suppose that $\hat{p}_i \geq p_i^N$ for all $i \in \mathcal{N}$. For every $i \in \mathcal{N}$, we then have:

$$\begin{aligned}
\hat{p}_i - p_i^N &\leq R_i\left(\hat{\mathbf{p}}_{-i}\right) - p_i^N, \\
&= R_i\left(\hat{\mathbf{p}}_{-i}\right) - R_i\left(\mathbf{p}_{-i}^N\right) \\
&= \int_0^1 \frac{d}{d\lambda}\left\{R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\mathbf{p}_{-i}^N\right)\right\} d\lambda \\
&= \int_0^1 \{\sum_{j \in \mathcal{N} \setminus \{i\}} \partial_j R_i\left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\mathbf{p}_{-i}^N\right)\left(\hat{p}_j - p_j^N\right)\} d\lambda.
\end{aligned}$$

Summing-up these inequalities for $i \in \mathcal{N}$ and using (1) yields:

$$
\begin{aligned}
\sum_{i\in\mathcal{N}} \left(\hat{p}_i - p_i^N\right) &\leq \sum_{i\in\mathcal{N}} \int_0^1 \{ \sum_{j\in\mathcal{N}\backslash\{i\}} \partial_j R_i \left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\, \mathbf{p}_{-i}^N\right) \left(\hat{p}_j - p_j^N\right) \} d\lambda \\
&= \sum_{j\in\mathcal{N}} \left(\hat{p}_j - p_j^N\right) \int_0^1 \{[\sum_{i\in\mathcal{N}\backslash\{j\}} \partial_j R_i \left(\lambda \hat{\mathbf{p}}_{-i} + (1-\lambda)\, \mathbf{p}_{-i}^N\right)]\} d\lambda \\
&< \sum_{j\in\mathcal{N}} \left(\hat{p}_j - p_j^N\right),
\end{aligned}
$$

a contradiction. Therefore, $\hat{p}_i < p_i^N$ for some $i \in \mathcal{N}$.

D Proof of Lemma 3

We have:

$$
p^M - R\left(p^M\right) = \int_{p^N}^{p^M} \left[1 - R'(p)\right] dp,
$$

where the integrand of the right-hand side is positive. Hence, the sign of $p^M - p^N$ is the same as that of $p^M - R\left(p^M\right)$. Therefore, using Lemma 1, $p^M > p^N$ under (**S**) and $p^M < p^N$ under (**C**). The implication for profitable prices follows from the strict quasi-concavity of the industry profit: $\Pi^s(\hat{p}) \geq \Pi^s\left(p^N\right)$ then implies $\hat{p} \geq p^N$ under (**S**), and $\hat{p} \leq p^N$ under (**C**).

E Proof of Proposition 5

From Proposition 1, $p^N \in \mathcal{P}^s$ and any other price $\hat{p}$ in $\mathcal{P}^s$ is such that $\hat{p} < p^N$. Conversely, any price $\hat{p} \leq p^N$ can be sustained with a symmetric price cap $\bar{p} = \hat{p}$. To see this, it suffices to note that $\hat{p} \leq p^N$ implies $\hat{p} \leq R(\hat{p})$ for any $i \in \mathcal{N}$: as $R\left(p^N\right) = p^N$, we have:

$$
R(\hat{p}) - \hat{p} = \int_{p^N}^{\hat{p}} \left[R'(p) - 1\right] dp \geq 0,
$$

where the inequality stems from $\hat{p} \leq p^N$ and $R'(\cdot) < 1$. Therefore,$R(\hat{p}) \geq \hat{p} = \bar{p}$; the strict quasi-concavity of $\pi_i(\mathbf{p})$ in p_i then yields the result.

F Proof of Proposition 6

(i) Focussing on symmetric prices, from Proposition 5 price caps can only sustain prices below the Nash level; it thus follows from Lemma 3 that, under (**S**), firms cannot use price caps to sustain more profitable symmetric outcomes.

(ii) Suppose that, under (**C**), the firms agree on a price cap $\bar{p} = p^M$. As $R\left(p^M\right) >$

$p^M = \bar{p}$ from Lemma 1, and firms' profits are quasi-concave in their own prices, all firms charging p^M constitutes a continuation equilibrium. Furthermore, if $C''(\cdot) \leq 0$, then by charging $p_i = p^M$ each firm $i \in \mathcal{N}$ can obtain

$$\pi_i\left(p^M, \mathbf{p}_{-i}\right) = \pi^s\left(p^M\right) + \int_{q^M}^{D_i\left(p^M, \mathbf{p}_{-i}\right)} \left[p^M - C'\left(q_i\right)\right] dq_i \geq \pi^s\left(p^M\right),$$

where the inequality stems from the fact that (a) $p_j \leq \bar{p} = p^M$ for every $j \in \mathcal{N}$ implies $D_i\left(p^M, \mathbf{p}_{-i}\right) \geq D_i\left(p^M, ..., p^M\right)$, as goods are complements, and (b) using Lemma 1 and $C''(\cdot) \leq 0$, $p^M > C'\left(q^M\right) \geq C'(q_i)$ for $q_i \geq q^M$; hence, $\pi_i\left(p^M, \mathbf{p}_{-i}\right) \geq \pi^s\left(p^M\right)$. As each firm can secure $\pi^s\left(p^M\right)$, it follows that the monopoly outcome constitutes the unique continuation equilibrium.

G Proof of Proposition 7

(i) We already noted that $\mathcal{P}^+ \subseteq \mathcal{P}_c^+$. Conversely, fix a price $\hat{p} \in \mathcal{P}_c^+$ and an equilibrium sustaining this price $\hat{p}$ thanks to a price cap $\bar{p}$. Letting $\hat{\underline{V}}_i$ denote the lowest sustainable continuation value for firm $i \in \mathcal{N}$, consider the alternative "bang-bang" strategies: (i) along the equilibrium path, firms stick to $\hat{p}$; and (ii) any deviation by firm $i \in \mathcal{N}$ (from the equilibrium path, or for any other history) is punished with the continuation value $\hat{\underline{V}}_i$. These alternative strategies still sustain $\hat{p}$, as any deviation (including from off-equilibrium) is punished at least as severely with the alternative strategies.

Thus, without loss of generality, consider an equilibrium sustaining $\hat{p}$ thanks to the price cap $\bar{p}$, in which any deviation by any firm $i \in \mathcal{N}$ is punished with the continuation value $\hat{\underline{V}}_i$. At any point in time, all deviations from the prescribed continuation path are punished in the same way; hence, the best deviation is the "myopic" deviation that maximizes the current profit. As firms' individual profits are moreover strictly quasi-concave with respect to their own prices, it follows that, for any $t = 1, 2, ...$, and any $h^t \in H^t$, firm i's best deviation from the prescribed price vector $\mathbf{p}^t\left(h^t\right)$ consists in charging:

$$\bar{R}_i\left(\mathbf{p}_{-i}^t\left(h^t\right); \bar{p}\right) \equiv \min\left\{R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right), \bar{p}\right\}.$$

By construction, $\bar{p} \geq \hat{p}$; furthermore, as by assumption $\hat{p}$ is at least as profitable as p^N, and the goods are substitutes, $\hat{p} \geq p^N$. We thus have $\bar{p} \geq p^N$, which, using $R'(\cdot) < 1$ and $R\left(p^N\right) = p^N$, yields:

$$\bar{p} - R(\bar{p}) = \int_{p^N}^{\bar{p}} \left[1 - R'(p)\right] dp \geq 0.$$

We thus have $\bar{p} \geq R(\bar{p})$ for any firm $i \in \mathcal{N}$. But by construction, any prescribed prices

$\mathbf{p}^t(h^t)$ must satisfy $p_j^t(h^t) \leq \bar{p}$ for all $j \in \mathcal{N}$. Under (**SC**), we thus have:

$$R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right) \leq R(\bar{p}) \leq \bar{p}.$$

It follows that the price cap $\bar{p}$ never limits firms' deviations: in any period $t = 1, 2, ...,$ and for any history $h^t \in H^t$, firm i's best deviation from the prescribed price vector $\mathbf{p}^t(h^t)$, for any $i \in N$, is the same as in the absence of any price caps:

$$\bar{R}_i\left(\mathbf{p}_{-i}^t\left(h^t\right);\bar{p}\right) = \min\left\{R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right), \bar{p}\right\} = R_i\left(\mathbf{p}_{-i}^t\left(h^t\right)\right).$$

Hence, the same strategies constitute an equilibrium in the absence of price caps, and thus $\hat{p} \in \mathcal{P}^+$.

(ii) We first establish that $p^M \notin \mathcal{P}^+$ implies $p^* > p^M$. Suppose instead that the firms can sustain a price $p^* < p^M$ without price caps. To prevent a deviation by firm $i \in \mathcal{N}$, there must exist a continuation payoff $\underline{V}_i$ such that:

$$\pi^s(p^*) \geq (1-\delta)\,\pi(R(p^*), p^*) + \delta \underline{V}_i. \tag{6}$$

However, note that, for $p < p^M$:

$$\frac{d}{dp}\left\{\pi^s(p) - (1-\delta)\,\pi(R(p), p)\right\} = (\pi^s)'(p) - (1-\delta)\frac{d}{dp}\left\{\pi(R(p), p)\right\} > 0,$$

where the inequality stems from the fact that $\pi^s(p) = \Pi^s(p)/n$ increases with p for $p < p^M$, and that, under (**C**), $\pi_i^s(R(p), p) = \max_{p_i} \pi_i^s(p_i, p)$ decreases as p increases when the goods are complements:

$$\frac{d}{dp}\left\{\pi(R(p), p)\right\} = \left[R(p) - C'(D(R(p), p))\right] \sum_{j \in \mathcal{N}\setminus\{i\}} \partial_j D_i(R(p), p, ..., p) < 0,$$

where the equality follows from the envelope theorem and the inequality from the margins being positive, from Lemma 2, together with $\partial_j D_i(\cdot) < 0$ under (**C**).

As $p^* < p^M$ by assumption, it follows from (6) that

$$\pi^s\left(p^M\right) \geq (1-\delta)\,\pi\left(R\left(p^M\right), p^M\right) + \delta \underline{V}_i.$$

Hence, p^M can be sustained as well, a contradiction.

The last part of the proof follows that of Proposition 6. If the firms agree on a price cap $\bar{p} = p^M$, then, as $R\left(p^M\right) > p^M = \bar{p}$ from Lemma 1, and firms' profits are quasi-concave in their own prices, charging p^M in every period constitutes a continuation equilibrium. Furthermore, as no other firm can charge more than p^M, if $C'' \leq 0$, then by charging $p_i = p^M$ in every period, each firm $i \in \mathcal{N}$ can secure at least $\pi^s\left(p^M\right)/(1-\delta)$; hence, the monopoly outcome constitutes the unique continuation equilibrium.

Price Caps as Welfare-Enhancing Coopetition

Online Appendix (Not for publication)

Patrick Rey and Jean Tirole

14 April 2017

A Nash equilibrium

We establish here the existence of a unique Nash equilibrium in the setting considered in Section 2:

Lemma 6 (Nash equilibrium) *In the setting considered in Section 2, if in addition*

$$\forall j \in \mathcal{N}, \sum_{i \in \mathcal{N} \setminus \{j\}} \left| \partial_j R_i \left(\mathbf{p} \right) \right| < 1,$$

then there exists a unique static Nash equilibrium, which is moreover "stable" under the standard tâtonnement process.

Proof. As it is never optimal for firm i to charge a negative price, and $R_i(\cdot)$ is bounded above by some finite B_i (which obviously must satisfy $B_i > 0$), we have, for $\mathbf{p}_{-i} \in \mathbb{R}_+^{n-1}$:

$$R_i \left(\mathbf{p}_{-i} \right) \in \mathcal{C}_i \equiv \left[0, B_i \right],$$

where $\mathcal{C}_i$ is a non-empty compact interval of $\mathbb{R}_+$. Note that, by construction, any Nash equilibrium price vector $\mathbf{p}^N = \left(p_i^N \right)_{i \in \mathcal{N}}$ is such that $p_i^N \in \mathcal{C}_i$.

Next, define $\boldsymbol{\phi}(\mathbf{p}) \equiv \left(\phi_i(\mathbf{p}) \right)_{i \in \mathcal{N}}$, where $\phi_i(\mathbf{p}) = R_i(\mathbf{p}_{-i})$. $\boldsymbol{\phi}$ is a contraction mapping from $\mathcal{C} \equiv \mathcal{C}_1 \times ... \times \mathcal{C}_n$ to $\mathcal{C}$, endowed with the ℓ_1 norm: for any $\mathbf{p} \in \mathcal{C}$, $\boldsymbol{\phi}(\mathbf{p}) \in \mathcal{C}$ and, in addition, for any $\mathbf{p}' \in \mathcal{C}$:

$$
\begin{aligned}
\left\|\boldsymbol{\phi}\left(\mathbf{p}'\right)-\boldsymbol{\phi}\left(\mathbf{p}\right)\right\| &= \sum_{i\in\mathcal{N}}\left|\phi_i\left(\mathbf{p}'\right)-\phi_i\left(\mathbf{p}\right)\right| \\
&= \sum_{i\in\mathcal{N}}\left|R_i\left(\mathbf{p}'_{-i}\right)-R_i\left(\mathbf{p}_{-i}\right)\right| \\
&= \sum_{i\in\mathcal{N}}\left|\int_0^1 \frac{d}{d\lambda}\left\{R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right\}d\lambda\right| \\
&\leq \sum_{i\in\mathcal{N}}\int_0^1\{\sum_{j\in\mathcal{N}\setminus\{i\}}\left|\partial_j R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right|\left|p'_j-p_j\right|\}d\lambda \\
&= \int_0^1\{\sum_{j\in\mathcal{N}}[\sum_{i\in\mathcal{N}\setminus\{j\}}\left|\partial_j R_i\left(\lambda\mathbf{p}'_{-i}+(1-\lambda)\,\mathbf{p}_{-i}\right)\right|]\left|p'_j-p_j\right|\}d\lambda \\
&\leq \sum_{j\in\mathcal{N}} k\left|p'_j-p_j\right| \\
&= k\left\|p'-p\right\|,
\end{aligned}
$$

where:

$$
k=\max_{\mathbf{p}\in\mathcal{C},j\in\mathcal{N}}\sum_{i\in\mathcal{N}\setminus\{j\}}\left|\partial_j R_i\left(\mathbf{p}\right)\right|<1.
$$

It follows from the Banach fixed point theorem that $\boldsymbol{\phi}(\mathbf{p})$ has a unique fixed point in $\mathcal{C}$, $\mathbf{p}^N$, and that any sequence $\{\mathbf{p}_n\}_{n\in\mathbb{N}}$ satisfying $\mathbf{p}_{n+1}=\boldsymbol{\phi}(\mathbf{x}_n)$ converges to this fixed point. Hence, $\mathbf{p}^N$ is the unique Nash equilibrium of the static game, and it is stable under the standard tâtonnement process. ■

B Hotelling with club effects

B.1 Model

Consider the following symmetric duopoly setting, in which for notational simplicity costs are zero (i.e., $C_i(q_i)=0$ for $i=1,2$) and:

- As in the standard Hotelling model, a unit mass of consumers is uniformly distributed along a unit-length segment; the two firms are located at the two ends of the segment, and consumers face a constant transportation cost per unit of distance, which is here normalized to 1.

- Unlike in the Hotelling model, however, consumers enjoy club effects: their gross surplus is $v+\sigma Q$, where $v>0$, $\sigma\in(0,1/2)$ reflects the magnitude of these positive externalities and $Q=q_1+q_2$ denotes the total number of consumers.

For low enough prices, the entire market is covered (i.e., $Q = 1$), and as long as prices are not too asymmetric, each firm faces the classic Hotelling demand given by:

$$D_i^H(p_1, p_2) \equiv \frac{1 - p_i + p_j}{2}.$$

This case arises as long as $|p_1 - p_2| < 1$ (to ensure that the market is shared) and $v + \sigma \geq (1 + p_1 + p_2)/2$ (to ensure that the market is covered). The two goods are then substitutes: $\partial_2 D_i = 1/2 > 0$.

By contrast, for high enough prices, firms are local monopolies and each firm i faces a demand satisfying:

$$q_i = v + \sigma Q - p_i,$$

where now $Q = q_1 + q_2 < 1$. As long as both firms remain active, their demands are then given by:

$$D_i^m(p_1, p_2) \equiv \frac{v - (1 - \sigma) p_i - \sigma p_j}{1 - 2\sigma}.$$

This case arises as long as $Q < 1$ and $q_i \geq 0$, which amounts to $p_1 + p_2 > 2v - (1 - 2\sigma)$ and $(1 - \sigma) p_i + \sigma p_j \leq v$ for $i \neq j \in \{1, 2\}$. The goods are then complements: $\partial_j D_i = -\sigma/(1 - 2\sigma) < 0$.

B.2 Best-responses

We now study firm i's best-response to the price p charged by firm j. For the sake of exposition, we will focus on the range $p \in [0, v]$.[1]

Consider first the case where firm j charges a price $p_j \in [0, v - (1 - \sigma)]$, so that it would serve the entire market if firm i were to charge a prohibitive price.[2] In this case, the market remains fully covered whatever price firm i chooses to charge, and it is optimal for firm i to obtain a positive share of that market. Firm i will thus seek to maximize $p_i D_i^H(p_1, p_2)$ and choose to charge:

$$p_i = R^H(p_j) \equiv \arg\max_{p_i} \left\{ p_i D_i^H(p_i, p_j) \right\} = \frac{1 + p_j}{2}.$$

Consider now the case where firm j charges a price $p_j \in (v - (1 - \sigma), v]$; the market is then covered only if firm i charges a sufficiently low price, namely, if

$$p_i \leq \tilde{p}(p_j) \equiv 2v - (1 - 2\sigma) - p_j.$$

In this range, firm i seeks to maximize $p_i D_i^H(p_1, p_2)$, which is maximal for $p_i = R^H(p_j)$. If instead firm i chooses to charge a higher price, the firms are local monopolies; furthermore,

[1]It can be checked that the Nash prices and the monopoly prices lie indeed in this range.

[2]To see this, note that under full participation (i.e., $Q = 1$), the consumer who the farthest away from firm j is willing to pay $v + \sigma - 1 \geq p_j$ for firm j's product.

as $p_j \leq v$, firm j's market share remains positive, whatever price firm i chooses to charge.[3] As it is optimal for firm i to maintain a positive market share as well, the demand will be given by $D_i^m(p_i, p_j)$. In this range, firm i will thus seek to maximize $p_i D_i^m(p_i, p_j)$, which is maximal for:

$$p_i = R^m(p_j) \equiv \arg\max_{p_i} \{p_i D_i^m(p_i, p_j)\} = \frac{v - \sigma p_j}{2(1-\sigma)}.$$

Note that:

- The profit functions $p_i D_i^H(p_1, p_2)$ and $p_i D_i^m(p_1, p_2)$ are concave with respect to p_i in their respective relevant ranges.
- $\tilde{p}(v - (1-\sigma)) = v + \sigma > R^m(v - (1-\sigma)) = R^H(v - (1-\sigma)) = (v+\sigma)/2$.
- $\tilde{p}' = -1 < (R^m)' = -\sigma/2(1-\sigma) < 0 < \left(R^H\right)' = 1/2$.

It follows that $R^H(p_j) > R^m(p_j)$ in the range $p_j \in (v - (1-\sigma), v]$, and $\tilde{p}(p_j) > R^H(p_j)$ in the beginning of that range. This, in turn, implies that firm i's best response is given by:

$$p_i = R(p_j) \equiv \min\left\{R^H(p_j), \max\{\tilde{p}(p_j), R^m(p_j)\}\right\}.$$

More precisely:

- If $\tilde{p}(v) \geq R^H(v)$, which amounts to $v \geq 3 - 4\sigma$, then $(R^m(p_j) <) R^H(p_j) < \tilde{p}(p_j)$, and thus $R(p_j) = R^H(p_j)$;
- If $R^H(v) > \tilde{p}(v) \geq R^m(v)$, which amounts to $3 - 4\sigma > v \geq 2 - 4\sigma$, then there exists $\check{p} = 4(v+\sigma)/3 - 1$ such that $R^H(\check{p}) = \tilde{p}(\check{p})$, and thus:
  - For $p_j \leq \check{p}$, we have again $(R^m(p_j) <) R^H(p_j) \leq \tilde{p}(p_j)$; hence, $R(p_j) = R^H(p_j)$;
  - For $p_j > \check{p}$, $R^m(p_j) < \tilde{p}(p_j) < R^H(p_j)$; hence, $R(p_j) = \tilde{p}(p_j)$;
- Finally, if $R^m(v) > \tilde{p}(v)$, which amounts to $2 - 4\sigma > v$, then there also exists $\mathring{p} = [(3-4\sigma)v - 2(1-\sigma)(1-2\sigma)]/(2-3\sigma)$ such that $R^m(\mathring{p}) = \tilde{p}(\mathring{p})$, and thus:
  - For $p_j \leq \check{p}$, we still have $(R^m(p_j) <) R^H(p_j) \leq \tilde{p}(p_j)$; hence, $R(p_j) = R^H(p_j)$;
  - For $\check{p} < p_j < \mathring{p}$, we still have $R^m(p_j) < \tilde{p}(p_j) < R^H(p_j)$; hence, $R(p_j) = \tilde{p}(p_j)$;
  - For $p_j \geq \mathring{p}$, we now have $\tilde{p}(p_j) \leq R^m(p_j) < R^H(p_j)$; hence, $R(p_j) = R^m(p_j)$.

[3] To see this, note that even if $q_i = 0$, the consumer who's the nearest to firm j is willing to pay at least $v \geq p$ for firm j's product.

It follows that for low prices (namely, for $p_j < \min\{\check{p}, v\}$), goods are substitutes and prices are strategic complements: $\partial_j D_i = \partial_j D_i^H > 0$ and $R' = \left(R^H\right)' > 0$. By contrast, whenever $v < 2 - 4\sigma$ (implying $\mathring{p} < v$), then for high enough prices (namely, for $p_j > \mathring{p}$), goods are complements and prices are strategic substitutes: $\partial_j D_i = \partial_j D_i^m < 0$ and $R' = (R^m)' < 0$.

B.3 Monopoly prices

Conditional on covering the entire market, it is optimal to raise prices until the marginal consumer is indifferent between buying or not: indeed, starting from a situation where $q_1 + q_2 = 1$, and the marginal consumer strictly prefers buying, increasing both prices by the same amount does not affect the firms quantities, q_1 and q_2, but increase both of their margins. Hence, without loss of generality, we can focus on situations such that, for some $q_1 \in [0,1]$: $q_2 = 1 - q_1$ and $p_i = v + \sigma - q_i$ for $i = 1, 2$. Total profit, as a function of q_1, is then given by:

$$\Pi^H(q_1) = (v + \sigma - q_1) q_1 + [v + \sigma - (1 - q_1)](1 - q_1) = v + \sigma - q_1^2 - (1 - q_1)^2.$$

This profit is concave in q_1 and reaches its maximum for $q_1 = 1/2$, where it is equal to

$$\Pi_H^M \equiv v + \sigma - \frac{1}{2}.$$

Alternatively, the firms may choose to cover only part of the market, in which case they are both local monopolies. For any $Q = q_1 + q_2 \in [0, 1)$ and any $q_1 \in [0, Q]$, the associated quantity for firm 2 is then $q_2 = Q - q_1$ and the associated prices are $p_i = v + \sigma Q - q_i$; the resulting industry profit is therefore given by:

$$\Pi = (v + \sigma Q - q_1) q_1 + [v + \sigma Q - (Q - q_1)](Q - q_1) = (v + \sigma Q) Q - q_1^2 - (Q - q_1)^2.$$

For any $Q \in [0, 1)$, it is thus optimal to choose $q_1 = q_2 = Q/2$, which yields an industry profit equal to:

$$\Pi^m(Q) = (v + \sigma Q) Q - \frac{Q^2}{2}.$$

This profit is concave in Q and coincides with Π_H^M for $Q = 1$; furthermore, ignoring the constraint $Q \leq 1$, it is maximal for $Q = v/(1 - 2\sigma)$. Therefore, the monopoly outcome is $q_i^M = q^M = Q^M/2$, where:

$$Q^M = \min\left\{\frac{v}{1 - 2\sigma}, 1\right\},$$

and the associated prices are $p_1^M = p_2^M = p^M$, where:

$$\begin{aligned} p^M &= v + \sigma Q^M - q^M = v - (1 - 2\sigma)\, q^M \\ &= \begin{cases} \frac{v}{2} & \text{if} \quad v < 1 - 2\sigma \\ v - \left(\frac{1}{2} - \sigma\right) & \text{if} \quad v \geq 1 - 2\sigma \end{cases} = \max\left\{\frac{v}{2}, v - \left(\frac{1}{2} - \sigma\right)\right\}. \end{aligned}$$

B.4 Nash equilibrium

We first note that, in equilibrium, both firms must obtain a positive market share. Starting from a situation where all consumers are inactive, each firm could profitably attract some consumers by charging a price slightly below v. Furthermore, in a candidate equilibrium in which only one firm attracts consumers, this firm must charge a non-negative price, otherwise it would have an incentive to raise its price in order to avoid making a loss; but then, the other firm could profitably deviate, as charging a price only slightly higher would attract some consumers.

It can also be checked that, when one firm charges $p \leq v$, then the profit of the other firm is globally quasi-concave in the relevant price range $[0, v + \sigma]$. To see this, note that the profit functions $p_i D_i^H (p_i, p_j)$ and $p_i D_i^m (p_i, p_j)$ are both strictly concave in their relevant ranges; the conclusion then follows from the fact that, at the boundary between these two ranges, $D_i^H (p_1, p_2) = D_i^m (p_1, p_2)$ and $\partial_i D_i^H (p_1, p_2) > \partial_i D_i^m (p_1, p_2)$.

Suppose first that, at the Nash equilibrium prices, the market is fully covered *and* the marginal consumer strictly prefers buying (from either firm) to not buying. As both firms must be active, their demands are given by $D_i^H (\cdot)$, and remain so around the Nash prices. Therefore, their best-responses are given by $R^H (\cdot)$. It follows that the Nash equilibrium price is then symmetric, with both firms charging the standard Hotelling price

$$p^H = 1.$$

Conversely, both firms charging p^H is indeed an equilibrium if and only if the consumer that is at equal distance from the two firms (thus facing a transportation cost equal to 1/2) then strictly prefers to be active,[4] that is, if and only if $p^H + 1/2 < v + \sigma Q|_{Q=1}$, which amounts to:

$$v > \frac{3}{2} - \sigma \left(> 1 = p^H\right).$$

When instead the market is not fully covered at the Nash equilibrium prices, firms' best-responses are given by $R^m (\cdot)$. It follows that the Nash equilibrium price is again

[4] The profit functions $p_i D_i^H (p_1, p_2)$ and $p_i D_i^m (p_1, p_2)$ are both strictly concave with respect to p_i in their relevant ranges; in addition, at the boundary between these two ranges, $D_i^H (p_1, p_2) = p_i D_i^m (p_1, p_2)$ and $\partial_i D_i^H (p_1, p_2) > \partial_i D_i^m (p_1, p_2)$; the conclusion follows.

symmetric, with both firms charging

$$p^m = \frac{v}{2-\sigma}\,(< v)\,.$$

Conversely, both firms charging p^m is indeed an equilibrium if and only if the market is not fully covered at these prices, that is, if and only if $p^m + 1/2 > v + \sigma Q|_{Q=1}$, which amounts to:

$$v < \left(\frac{1}{2} - \sigma\right)\frac{2-\sigma}{1-\sigma}.$$

Finally, the Nash equilibrium can also be such that the entire market is "barely" covered, in that the marginal consumer is just indifferent between buying or not. The prices are then such that $p_i^N = v + \sigma - q_i^N$ and satisfy (as $q_1^N + q_2^N = 1$):

$$p_1^N + p_2^N = 2\,(v+\sigma) - 1. \tag{7}$$

Furthermore, no firm $i = 1, 2$ should benefit from a small deviation; as the market would remain covered if firm i lowers its price, but not so if it increases its price, we must have:

$$\begin{aligned}
\partial_i \pi_i\,(p_1,p_2)|_{p_j=p_j^N, p_i=p_i^{N-}} &= \frac{\partial}{\partial p_i}\left\{p_i D_i^H\,(p_1,p_2)\right\}\Big|_{p_j=p_j^N, p_i=p_i^N} = q_i^N - \frac{p_i^N}{2} \geq 0,\\
\partial_i \pi_i\,(p_1,p_2)|_{p_j=p_j^N, p_i=p_i^{N+}} &= \frac{\partial}{\partial p_i}\left\{p_i D_i^m\,(p_1,p_2)\right\}\Big|_{p_j=p_j^N, p_i=p_i^N} = q_i^N - \frac{1-\sigma}{1-2\sigma}p_i^N \leq 0.
\end{aligned}$$

Summing-up these conditions for $i = 1, 2$, and using $q_1^N + q_2^N = 1$ and (7), yields:

$$\left(\frac{1}{2} - \sigma\right)\frac{2-\sigma}{1-\sigma} \leq v \leq \frac{3}{2} - \sigma.$$

Figure 1 from Section 2.2 illustrates three possible configurations.

• In the first situation, v is sufficiently high (namely, $v > 3 - 4\sigma\,(> 3/2 - \sigma)$) that firms always compete for consumers in the relevant price range $[0, v]$. The goods are thus substitutes $\left(\partial_j D_i = \partial_j D_i^H = 1/2 > 0\right)$, and their prices are strategic complements $(R_i' = \left(R^H\right)' = 1/2 > 0)$. Furthermore, the monopoly prices lie above the Nash level.

• In the second, intermediate situation, firms compete again for consumers when prices are low, as in the previous situation. However, for higher price levels, firms best-respond to each other so as to maintain full participation; as a result the goods are at the boundary between substitutes and complements[5] and their prices become strategic substitutes ($R_i' = \tilde{p}' = -1 < 0$). While there are multiple Nash equilibria, they all involve the same total price, and the symmetric Nash equilibrium coincides with the monopoly outcome.

[5]Namely:
$\partial_j D_i(p_i^N, p_j^{N-}) = \partial_j D_i^H\left(p_i^N, p_J^N\right) = 1/2 > 0 > \partial_j D_i(p_i^N, p_j^{N+}) = \partial_j D_i^m\left(p_i^N, p_J^N\right) = -\sigma/\left(1 - 2\sigma\right)$.

• Finally, in the last situation v is sufficiently low (namely, $v < 2 - 4\sigma$) that firms become local monopolies for high enough prices. The goods then become complements ($\partial_j D_i = \partial_j D_i^m = -\sigma / (1 - 2\sigma) < 0$) and their prices are again strategic substitutes ($R_i' = (R^m)' = -\sigma/2(1-\sigma) < 0$); the monopoly prices then lie below the Nash level.

C On Assumption A

We show here that Assumption A holds under the following condition:

Assumption A': For any $i \neq j \in \{1, 2\}$ and any prices $p_i \in [0, p_i^N)$ and $p_j > p_j^N$:

$$D_j(p_j, p_i)\, \partial_{11}^2 D_j(p_j, p_i) < 2\left(\partial_1 D_j(p_j, p_i)\right)^2 - C_j''\left(D_j(p_j, p_i)\right)\left(\partial_1 D_j(p_j, p_i)\right)^3,$$

and:

$$\begin{aligned} & D_j(p_j, p_i)\left[\partial_1 D_j(p_j, p_i)\, \partial_2 D_j(p_j, p_i) - D_j(p_j, p_i)\, \partial_{12}^2 D_j(p_j, p_i)\right] \\ < \; & D_i(p_i, p_j)\left[2\left(\partial_1 D_j(p_j, p_i)\right)^2 - D_j(p_j, p_i)\, \partial_{11}^2 D_j(p_j, p_i)\right] \\ & + C_j''(D_j)\left(\partial_1 D_j\right)^2\left[D_j(p_j, p_i)\, \partial_2 D_j(p_j, p_i) - D_i(p_i, p_j)\, \partial_1 D_j(p_j, p_i)\right]. \end{aligned}$$

The first part of this assumption amounts to say that, for any given price of the other firm, the profit of a given firm is concave with respect to the price of that firm. It is satisfied, for instance, when the cost function is weakly convex (i.e., $C_i''(\cdot) \geq 0$) and the elasticity of the inverse of the residual demand is lower than 2, as is the case for the demand functions usually considered in oligopoly theory – in particular, it holds whenever the residual demand is log-concave (or equivalently, that the elasticity of its inverse is lower than 1), or it is exponential (and thus log-convex) with an elasticity higher than 1.

The second part of the assumption holds, for instance, when the goods are close to being perfect complements.[6]

Firm j's best-response, $R_j(p_i)$, is characterized by the first-order condition:

$$\left[R_j(p_i) - C_j'\left(D_j\left(R_j(p_i), p_i\right)\right)\right] \partial_1 D_j\left(R_j(p_i), p_i\right) + D_j\left(R_j(p_i), p_i\right) = 0,$$

[6] Remember that Assumption A holds trivially when prices are strategic complements, as is usually the case for substitutable goods. Hence, considering Assumption A' is useful only when prices are strategic substitutes, which in turn is mostly relevant when the goods are complements. But for perfect complements, demands are of the form $D_i(p_i, p_j) = D(p_1 + p_2)$, and Assumption A' then boils down to

$$\begin{aligned} D(\cdot)\left[\left(D'(\cdot)\right)^2 - D(\cdot)\, D''(\cdot)\right] \; < \; & D(\cdot)\left[2\left(D'(\cdot)\right)^2 - D(\cdot)\, D''(\cdot)\right] \\ & + C_j''(\cdot)\left(D'(\cdot)\right)^2\left[D(\cdot)\, D'(\cdot) - D(\cdot)\, D'(\cdot)\right], \end{aligned}$$

or $D(\cdot)\left(D'(\cdot)\right)^2 > 0$, which is trivially satisfied. By continuity, this strict inequality still holds when the above demands are only slightly modified.

which yields (dropping the argument $(R_j(p_i), p_i)$):

$$R_j'(p_i) = -\frac{\partial_1 D_j \partial_2 D_j - D_j \partial_{12}^2 D_j - C_j''(D_j)(\partial_1 D_j)^2 \partial_2 D_j}{2(\partial_1 D_j)^2 - D_j \partial_{11}^2 D_j - C_j''(D_j)(\partial_1 D_j)^3},$$

where the denominator of the right-hand side is positive under Assumption A'.

Therefore, Assumption A amounts to (dropping the argument $(R_j(p_i), p_i)$):

$$\begin{aligned} & D_i \left[2(\partial_1 D_j)^2 - D_j \partial_{11}^2 D_j - C_j''(D_j)(\partial_1 D_j)^3\right] \\ & \quad > D_j \left[\partial_1 D_j \partial_2 D_j - D_j \partial_{12}^2 D_j - C_j''(D_j)(\partial_1 D_j)^2 \partial_2 D_j\right], \end{aligned}$$

which follows from Assumption A'.

D Platform & apps

There is a platform, a unit-mass continuum of independent applications running on that platform, indexed by $i \in [0,1]$, and a unit-mass continuum of consumers with heterogeneous values for the platform and for each application. More precisely, for each consumer: (i) the value V for the platform is drawn from a distribution with c.d.f. $F(V)$ over some interval $[\underline{V}, \bar{V}]$; and: (ii) the value v of each application is drawn from a distribution with c.d.f. $G(v)$ over some interval $[\underline{v}, \bar{v}]$. The bounds can be finite or infinite, and such that $\underline{V} < \bar{V}$, $\underline{v} < \bar{v}$, and both $\bar{v}$ and $\bar{V} + \bar{v}$ are positive; all values are drawn independently (across users, as well as across the platform and the applications), $F(V)$ and $G(v)$ are both C^1, and their hazard rates are increasing over their respective supports.[7] Finally, all costs are normalized to zero.

For the sake of exposition, we will refer to the platform as "good 1" and to the applications as (variants of) "good 2". The timing is as follows:

- Stage 1: The platform sets its price p_1 and, simultaneously, each application i sets its price p_{2i}.
- Stage 2: Each consumer decides whether to adopt the platform and, if so, which applications to buy.

D.1 Complementarity between the platform and the applications

Let

$$d(p) \equiv \Pr[v \geq p] = 1 - G(p) \tag{8}$$

[7] That is, the hazard rates $F'(V)/[1-F(V)]$ and $G'(v)/[1-G(v)]$ are increasing, respectively, over $[\underline{V}, \bar{V}]$ and $[\underline{v}, \bar{v}]$.

denote the expected demand that a given application faces from each platform user, as a function of its price p. In stage 1, each application i chooses p_{2i} so as to maximize its profit, $Np_{2i}d(p_{2i})$, where N denotes the anticipated number of platform users.[8] Hence, all applications face the same profit function, which is moreover strictly quasi-concave given the monotonicity of the hazard rate of the distribution $G(v)$; it follows that all applications will charge the same price p_2.

Let

$$s(p) \equiv \int_p^{\bar{v}} (v-p)\, dG(v)$$

denote the additional surplus that platform users derive from applications. The number of these users, as a function of the platform price p_1 and the application price p_2, is given by:

$$D_1(p_1, p_2) \equiv \Pr[V + s(p_2) \geq p_1] = D(p_1 - s(p_2)),$$

where $D(\tilde{p}) \equiv 1 - F(\tilde{p})$ denotes the demand for the platform, as a function of the "quality-adjusted" price $\tilde{p} = p_1 - s(p_2)$. This, in turn, implies that the demand for each application is given by:

$$D_2(p_2, p_1) \equiv D(p_1 - s(p_2))\, d(p_2),$$

which satisfies:

$$\partial_1 D_1(p_1, p_2) = D'(p_1 - s(p_2)) < 0.$$

It follows that *the platform and the applications are complements*: using $s'(p) = -d(p)$,[9] we have:

$$\partial_2 D_1(p_1, p_2) = \partial_1 D_2(p_2, p_1) = D'(p_1 - s(p_2))\, d(p_2) < 0.$$

D.2 Best-responses

Consider first the platform's pricing decision, in response to an application price p_2. The profit of the platform can be expressed as:

$$\pi_1(p_1, p_2) \equiv p_1 D_1(p_1, p_2).$$

The monotonicity of the hazard rate of the distribution $F(V)$ ensures that this profit is strictly quasi-concave in p_1; the platform will thus respond to the application price p_2 by

[8] As each application has a negligible impact on consumers' surplus, the number of platform users is not affected by the price of an individual application.

[9] From (8), we have:

$$s'(p) = \int_p^{\bar{v}} -dG(p) = -[1 - G(p)] = -d(p).$$

charging:

$$p_1 = R_1(p_2) \equiv \arg\max_p pD(p - s(p_2)).$$

We first note that, from the platform's standpoint, prices are strategic substitutes (i.e., $R_1'(\cdot) < 0$). To see this, consider two prices application prices p_2 and $p_2' > p_2$, and let denote by $s = s(p_2)$ and $s' = s(p_2') < s$ the corresponding levels of surplus generated by the applications, and by $p = R_1(p_2)$ and $p' = R_1(p_2')$ the platform's optimal price responses. Using a revealed preference argument, we have:

$$p'D(p' - s') \geq pD(p - s') \text{ and } pD(p - s) \geq p'D(p' - s).$$

Furthermore, as the profit functions are strictly concave and the demand $D(\cdot)$ is strictly decreasing, these inequalities must be strict.[10] Taking the logarithms of the above expressions, we thus have:

$$\log(p') + \log(D(p' - s')) > \log(p) + \log(D(p - s')),$$

and

$$\log(p) + \log(D(p - s)) > \log(p') + \log(D(p' - s)).$$

Let $L(\tilde{p}) \equiv \log(D(\tilde{p}))$; combining these inequalities yields:

$$\int_p^{p'} L'(x - s')\,dx = L(p' - s') - L(p - s') > L(p' - s) - L(p - s) = \int_p^{p'} L'(x - s)\,dx,$$

leading to:

$$\int_p^{p'} \int_s^{s'} -L''(x - y)\,dydx > 0.$$

As the monotonicity of the hazard rate of the distribution $F(\cdot)$ implies $L''(\cdot) < 0$, and $s' < s$ by assumption, the above inequality yields $p' < p$. Hence, $R_1'(p_2) < 0$.

Finally, we can check that Assumption (A) is satisfied. It is obvious for the applications, as $R_2'(p_1) = 0$. Turning to the platform's pricing policy, it will be useful to consider its quality-adjusted price response to a given application surplus s:

$$\tilde{R}_1(s) = \arg\max_{\tilde{p}} \{(\tilde{p} + s) D(\tilde{p})\}.$$

Using again a revealed preference argument, it can be checked that $\tilde{R}_1'(s) < 0$ (that is, in response to a decrease in the application price p_2, and thus to an increase in the surplus s, the platform increases its own price – as $R_1'(p_2) < 0$ – but not so as to offset

[10] If, for instance, $p'D(p' - s') = pD(p - s')$, then the strict quasi-concavity of the profit function implies $p' = p$. But as $D(\cdot)' < 0$, the first-order condition cannot be satisfied for both s and s', a contradiction.

entirely the consumers' benefit from the reduction in the price p_2). To see this, consider two prices application surpluses s and $s' > s$, and denote by $\tilde{p} = \tilde{R}_1(s)$ and $\tilde{p}' = \tilde{R}_1(s')$ the platform's optimal quality-adjusted price responses. We now have:

$$(\tilde{p}' + s') D(\tilde{p}') > (\tilde{p} + s') D(\tilde{p}) \text{ and } (\tilde{p} + s) D(\tilde{p}) > (\tilde{p}' + s) D(\tilde{p}'),$$

leading to

$$(s' - s)(D(\tilde{p}') - D(\tilde{p})) > 0,$$

which, using $s' > s$ and $D'(\cdot) < 0$, implies $\tilde{p}' < \tilde{p}$. Hence, $\tilde{R}_1'(s) < 0$.

We now show that, along the platform's best-response, consumer total surplus increases as p_2 decreases. Total consumer surplus can be expressed as

$$CS(p_1, p_2) = S(p_1 - s(p_2)),$$

where

$$S(\tilde{p}) \equiv \int_{\tilde{p}}^{\bar{V}} (V - \tilde{p}) \, dF(V),$$

and thus:

$$\frac{d}{dp_2}[CS(R_1(p_2), p_2)] = S'\left(\tilde{R}_1(s(p_2))\right) \tilde{R}_1'(s(p_2)) \, s'(p_2) < 0.$$

D.3 Monopoly outcome

We first note that, in order to maximize the industry profit, it is optimal to sell the applications at cost. To see this, let us index the applications by $i \in [0,1]$; for a given platform price p_1 and given application prices $(p_{2i})_{i \in [0,1]}$, the industry profit can then be expressed as:

$$\Pi(p_1, (p_{2i})_i) = \left[p_1 + \int_0^1 p_{2i} d(p_{2i}) \, di\right] D(p_1 - s),$$

where

$$s = \int_0^1 s(p_{2i}) \, di$$

denotes consumers' expected surplus from the applications. Replacing these prices with $\tilde{p}_2 = 0$ and $\tilde{p}_1 = p_1 + s(0) - s$ does not affect the number of platform users (the quality-adjusted price remains equal to $p_1 - s$), and thus the impact on industry profit is equal to:

$$\begin{aligned} \Delta\Pi &= \left\{s(0) - s - \int_0^1 p_{2i} d(p_{2i}) \, di\right\} D(p_1 - s) \\ &= \left\{\int_0^1 [s(0) - s(p_{2i}) - p_{2i} d(p_{2i})] \, di\right\} D(p_1 - s) \\ &> 0, \end{aligned}$$

where the inequality stems from the fact that the total surplus from the applications, $s(p) + pd(p)$, is maximal under marginal cost pricing, i.e., for $p = 0$.

The monopoly prices are thus $p_2^M = 0$ and

$$p_1^M \equiv \arg\max_{p_1} \{\Pi(p_1, p_2 = 0)\} = \arg\max_{p_1} \{p_1 D(p_1 - s(0))\} = R_1(0).$$

The monopoly outcome therefore lies (weakly) below firms' best-responses (more precisely, $p_2^M < R_2(p_1^M) =$ and $p_1^M = R_1(p_2^M)$). However, as $p_2^M = 0 < p_2^N$, the application price is (strictly) lower than its Nash level $(p_2^M < p_2^N)$, but the opposite holds for the platform: as $R_1'(\cdot) < 0$, we have:

$$p_1^M = R(0) > R(p_2^N) = p_1^N.$$

E Nash equilibrium in the technology adoption model

Consider, in the technology adoption environment described in Section 4.2, the static game in which the two firms simultaneously set their prices. Without loss of generality we can require prices to belong to the interval $[0, V]$.

From the discussion presented in the text, firm i's best response to firm j setting price $p_j \leq e$ is to set

$$p_i = \min\{e, r(p_j)\},$$

where

$$r(p) = \arg\max_p \{p_i D(p_i + p)\}$$

satisfies

$$-1 < r'(p_j) < 0$$

and has a unique fixed point $\hat{p} > p^M$.

When instead $p_j > e$, then firm i faces no demand if $p_i > p_j$ (as users buy only the lower-priced license), and faces demand $D(p_i + e)$ if $p_i < p_j$. Competition then drives prices down to $p_1 = p_2 = e$. Hence, the Nash equilibrium is unique and such that both firms charge $p^N \equiv \min\{e, \hat{p}\}$.

Figure 6 summarizes this analysis.

F Proof of Lemma 5

To establish part (i), note that firm i can secure its presence in the users' basket by charging e, thus obtaining $eD(e + p_j)$ if $p_j \leq e$ and $eD(2e)$ if $p_j > e$. Either way it can secure at least $\pi(e) = eD(2e)$. Because for $e \leq \hat{p}$ this lower bound is equal to the static Nash profit, we have $\underline{\pi} = \pi^N = \pi(e)$.

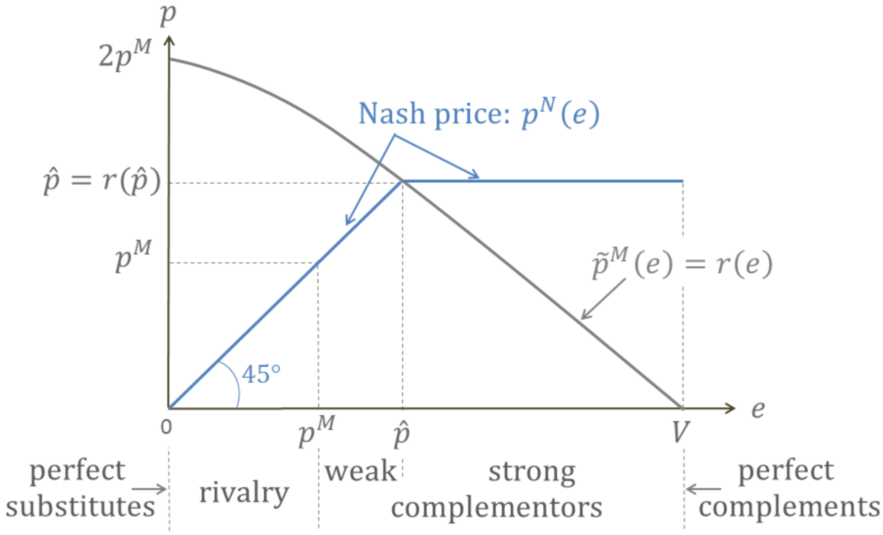

Figure 6: Nash equilibrium in the technology model

We now turn to part (*ii*). If firm j sets a price $p_j \geq e$, firm i can obtain at most $\max_{p \leq p_j} pD(e+p) = \tilde{\pi}^M(e)$ (as $\tilde{p}^M(e) = r(e) < \hat{p} < e \leq p_j$). Setting instead a price $p_j < e$ allows firm i to obtain at least $\max_{p \leq e} pD(p_j + p) > \max_{p \leq e} pD(e+p) = \tilde{\pi}^M(e)$. Therefore, setting any price above e minmaxes firm i, which then obtains $\tilde{\pi}^M(e)$.

G Proof of Proposition 8

Let $\pi_i(p_i, p_j)$ denote firm i's profit. Prices such that $\min\{p_1, p_2\} \leq e$ cannot yield greater profits than the static Nash:

- If $p_1, p_2 \leq e$, total price P is below $2e$; as the aggregate profit $PD(P)$ is concave in P and maximal for $P^M = 2p^M > 2e$, total profit is smaller than the Nash level.
- If instead $p_i \leq e < p_j$, then

$$\pi_1(p_1, p_2) + \pi_2(p_2, p_1) = p_i D(e + p_i) \leq eD(2e) \leq 2eD(2e) = 2\pi^N,$$

  where the first inequality stems from the fact that the profit $\tilde{\pi}(p) = pD(e+p)$ is concave in p and maximal for $\tilde{p}^M(e) = r(e)$, which exceeds e in the rivalry case (as then $e < p^M < \hat{p} = r(\hat{p})$).

Therefore, to generate more profits than the static Nash profit in a given period, both firms must charge more than e; this, in turn, implies that users buy at most one license, and thus aggregate profits cannot exceed $\tilde{\pi}^M(e)$. It follows that collusion cannot enhance profits if $\tilde{\pi}^M(e) \leq 2\pi^N = 2\pi(e)$. Keeping V and thus p^M constant, increasing e from 0 to p^M decreases $\tilde{\pi}^M(e) = \max_p pD(p+e)$ but increases $\pi(e)$; as $\tilde{\pi}^M(0) = 2\pi\left(p^M\right) = 2\pi^M$, there exists a unique $\underline{e} < p^M$ such that, in the range $e \in \left[0, p^M\right]$, $\tilde{\pi}^M(e) < 2\pi^N$ if and only if $e > \underline{e}$.

Thus, when $e > \underline{e}$, the static Nash payoff π^N constitutes an upper bound on average discounted equilibrium payoffs. But the static Nash equilibrium here yields minmax profits, and thus also constitutes a lower bound on equilibrium payoffs. Hence, π^N is the unique average discounted equilibrium payoff, which in turn implies that the static Nash outcome must be played along any equilibrium path.

Consider now the case $e < \underline{e}$, and suppose that collusion raises profits: $v^* > \pi^N$, where, recall, v^* is the maximal average discounted equilibrium payoff. As v^* is a weighted average of per-period profits, along the associated equilibrium path there must exist some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2v^*$. This, in turn, implies that users must buy an incomplete version of the technology; thus, there exists p^* such that:

$$\tilde{\pi}(p^*) = \pi_1^\tau + \pi_2^\tau \geq 2v^*.$$

By undercutting its rival, each firm i can obtain the whole profit $\tilde{\pi}(p^*)$ in that period; as this deviation could at most be punished by reverting forever to the static Nash behavior, a necessary equilibrium condition is, for $i = 1, 2$:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\tilde{\pi}(p^*) + \delta\underline{\pi},$$

where $v_i^{\tau+1}$ denotes firm i's continuation equilibrium payoff from period $\tau + 1$ onwards. Combining these conditions for the two firms yields:

$$(1-\delta)\tilde{\pi}(p^*) + \delta\underline{\pi} \leq (1-\delta)\frac{\pi_1^\tau + \pi_2^\tau}{2} + \delta\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq (1-\delta)\frac{\tilde{\pi}(p^*)}{2} + \delta\frac{\tilde{\pi}(p^*)}{2},$$

where the second inequality stems from $v_1^{\tau+1} + v_2^{\tau+1} \leq 2v^* \leq \pi_1^\tau + \pi_2^\tau = \tilde{\pi}(p^*)$. This condition amounts to

$$\left(\delta - \frac{1}{2}\right)\tilde{\pi}(p^*) \geq \delta\underline{\pi} = \delta\pi(e), \tag{9}$$

which requires $\delta \geq 1/2$ (with a strict inequality if $e > 0$). This, in turn, implies that (9) must hold for $\tilde{\pi}^M(e) = \max_p \tilde{\pi}(p)$:

$$\left(\delta - \frac{1}{2}\right)\tilde{\pi}^M(e) \geq \delta\pi(e). \tag{10}$$

Conversely, if (10) is satisfied, then the stationary path $\left(\tilde{p}^M(e), \tilde{p}^M(e)\right)$ (with equal market shares) is an equilibrium path, as the threat of reverting to the static Nash behavior ensures that no firm has an incentive to deviate:

$$\frac{\tilde{\pi}^M(e)}{2} \geq (1-\delta)\tilde{\pi}^M(e) + \delta\pi(e),$$

or

$$\delta \geq \delta^{R}(e) \equiv \frac{1}{2} \frac{1}{1-\frac{\pi(e)}{\tilde{\pi}^{M}(e)}}.$$

Finally, $\delta^{R}(e)$ increases with e, as $\pi(e)$ increases with e in that range, whereas $\tilde{\pi}^{M}(e) = \max_{p}\{pD(p+e)\}$ decreases as e increases.

H Proof of Proposition 9

To establish Proposition 9, we first study the effectiveness of Abreu's penal codes, before considering successively the case of weak and strong complementors.

The following lemma shows that Abreu's penal codes (even when restricting attention to symmetric on- and off-equilibrium paths) can sustain minmax profits when firms are sufficiently patient:

Lemma 7 (minmax with strong complementors) *The minmax payoff is sustainable whenever*

$$\delta \geq \underline{\delta}(e) \equiv \frac{\tilde{\pi}^{M}(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)},$$

where $\underline{\delta}(e) \in (0,1)$ *for* $e \in (\hat{p}, V)$*, and* $\underline{\delta}(V) = \lim_{e \longrightarrow \hat{p}} \underline{\delta}(e) = 0$.

Proof. In order to sustain the minmax profit $\underline{\pi} = \tilde{\pi}^{M}(e)$, consider the following two-phase, symmetric penal code. In the first phase (periods $t = 1, ..., T$ for some $T \geq 1$), both firms charge e, so that the profit is equal to $\pi(e)$. In the first period of the second phase (i.e., period $T+1$), with probability $1-x$ both firms charge e, and with probability x they switch to the best collusive price that can be sustained with minmax punishments, which is defined as:

$$p^{C}(e,\delta) \equiv \arg\max_{p} pD(2p),$$

subject to the constraint

$$(1-\delta) \max_{\tilde{p} \leq e} \tilde{p}D(p+\tilde{p}) + \delta \underline{\pi} \leq pD(2p). \tag{11}$$

Then, in all following periods, both firms charge p^{C}. Letting $\Delta = (1-\delta) x \delta^{T} + \delta^{T+1} \in (0, \delta)$ denote the fraction of (discounted) time in the second phase, the average discounted per-period punishment profit is equal to

$$\pi^{p} = (1-\Delta)\pi(e) + \Delta \pi\left(p^{C}\right),$$

which ranges from $\pi(e) < \underline{\pi} = \tilde{\pi}^{M}(e)$ (for $T = +\infty$) to $(1-\delta)\pi(e) + \delta\pi\left(p^{C}\right)$ (for $T = 1$ and $x = 1$). Thus, as long as this upper bound exceeds $\tilde{\pi}^{M}(e)$, there exists $T \geq 1$ and $x \in [0,1]$ such that the penal code yields the minmax: $\pi^{p} = \tilde{\pi}^{M}(e) = \underline{\pi}$.

As p^C satisfies (11), the final phase of this penal code (for $t > T+1$, and for $t = T+1$ with probability x) is sustainable. Furthermore, in the first $T+1$ periods the expected payoff increases over time (as the switch to p^C comes closer), whereas the maximal profit from a deviation remains constant and equal to $\max_{p \leq e} pD(e+p) = \tilde{\pi}^M(e)$ (as $\tilde{p}^M(e) = r(e) < e$ for $e > \hat{p}$). Hence, to show that the penal code is sustainable it suffices to check that firms have no incentive to deviate in the first period, which is indeed the case if deviations are punished with the penal code:

$$\tilde{\pi}^M(e) = (1-\Delta)\pi(e) + \Delta\pi\left(p^C\right) \geq (1-\delta)\tilde{\pi}^M(e) + \delta\tilde{\pi}^M(e) = \tilde{\pi}^M(e).$$

There thus exists a penal code sustaining the minmax whenever the upper bound $(1-\delta)\pi(e) + \delta\pi\left(p^C\right)$ exceeds $\tilde{\pi}^M(e)$; as by construction $\pi\left(p^C\right) \geq \pi^N = \pi(\hat{p})$, this is in particular the case whenever

$$(1-\delta)\pi(e) + \delta\pi(\hat{p}) \geq \tilde{\pi}^M(e),$$

which amounts to $\delta \geq \underline{\delta}(e)$. Finally:

- $\underline{\delta}(e) \in (0,1)$ for any $e \in (\hat{p}, V)$, as then:

$$\pi(\hat{p}) = \max_p pD(\hat{p}+p) > \tilde{\pi}^M(e) = \max_p pD(e+p) > \pi(e) = eD(2e);$$

- $\underline{\delta}(V) = 0$, as $\tilde{\pi}^M(V) = \pi(V) = 0$, and

$$\lim_{e \longrightarrow \hat{p}} \frac{\tilde{\pi}^M(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)} = \left. \frac{\frac{d\tilde{\pi}^M(e)}{de} - \frac{d\pi(e)}{de}}{-\frac{d\pi(e)}{de}} \right|_{e=\hat{p}} = \frac{D(2\hat{p}) + \hat{p}D'(2\hat{p})}{D(2\hat{p}) + 2\hat{p}D'(2\hat{p})} = 0,$$

  where the last equality stems from $\hat{p} = r(\hat{p}) = \arg\max_p pD(\hat{p}+p)$.

■

The following proposition characterizes the scope for tacit coordination in case of weak complementors:

Proposition 16 (weak complementors) *When $p^M < e \leq \hat{p}$:*

(i) Perfect cooperation on price p^M is feasible (i.e., $v^* = \pi^M$) if and only if

$$\delta \geq \overline{\delta}^C(e) \equiv \frac{eD\left(p^M+e\right) - \pi^M}{eD\left(p^M+e\right) - \pi(e)},$$

where $\overline{\delta}^C(e)$ lies strictly below 1 for $e > p^M$, and is decreasing for e close to p^M.

(ii) Furthermore, if $D'' \geq 0$, then profitable cooperation is sustainable (i.e., $v^* > \pi^N$) if and only if

$$\delta \geq \underline{\delta}^C(e),$$

where $\underline{\delta}^C(e)$ lies below $\overline{\delta}^C(e)$, is decreasing in e, and is equal to 0 for $e = \hat{p}$. The set of sustainable Nash-dominating per-firm payoffs is then $\mathcal{V}^+ = [\pi(e), v^*(e,\delta)]$, where $v^*(e,\delta) \in (\pi(e), \pi^M]$ is (weakly) increasing in δ.

Proof. (i) That perfect cooperation (on $p_i^t = p^M$ for $i = 1, 2$ and $t = 0, 1, ...$) is sustainable if and only if

$$\delta \geq \overline{\delta}^C(e) = \frac{eD\left(p^M + e\right) - \pi^M}{eD\left(p^M + e\right) - \pi(e)} = \frac{1}{1 + \frac{\pi^M - \pi(e)}{eD(p^M+e) - \pi^M}}$$

derives directly from (4).

For $e \in (p^M, \hat{p}]$, $\pi^M > \pi(e)$ and $eD\left(p^M + e\right) > \pi^M$ (as $r\left(p^M\right) > r(e) \geq \hat{p} \geq e$); therefore, $\overline{\delta}^C(e) < 1$. Also, for ε positive but small, we have:

$$\overline{\delta}^C\left(p^M + \varepsilon\right) \simeq \frac{1}{1 - \frac{\pi''(p^M)}{D(2p^M) + p^M D'(2p^M)} \frac{\varepsilon}{2}},$$

which decreases with ε, as $\pi''\left(p^M\right) < 0$ and

$$D\left(2p^M\right) + p^M D'\left(2p^M\right) = -p^M D'\left(2p^M\right) > 0.$$

(ii) Suppose that collusion enhances profits: $v^* > \pi^N = \pi(e)$. In the most profitable collusive equilibrium, there exists again some period τ in which the average profit is at least v^*. And as $v^* > \pi(e) > \tilde{\pi}^M(e)/2$,[11] users must buy the complete technology in that period; thus, each firm i must charge a price p_i^τ not exceeding e, and the average price $p^\tau = \frac{p_1^\tau + p_2^\tau}{2}$ must moreover satisfy

$$\pi(p^\tau) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq v^*.$$

As $p_j^\tau \leq e \leq \hat{p} = r(\hat{p}) \leq r(p_j^\tau)$, firm i's best deviation consists in charging e. Hence, to ensure that firm i has no incentive to deviate, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta) eD\left(p_j^\tau + e\right) + \delta \underline{\pi}.$$

Combining these conditions for the two firms yields, using $\pi(p^\tau) = \frac{\pi_1^\tau + \pi_2^\tau}{2}$ and $\underline{\pi} = \pi(e)$:

$$(1-\delta) e \frac{D(p_1^\tau + e) + D(p_2^\tau + e)}{2} + \delta\pi(e) \leq (1-\delta)\pi(p^\tau) + \delta \frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(p^\tau),$$

where the second inequality stems from $\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq v^* \leq \pi(p^\tau)$. If the demand function is (weakly) convex (i.e., $D'' \geq 0$ whenever $D > 0$), then this condition implies $H(p^\tau; e, \delta) \geq$

[11] See footnote 40.

0, where

$$H\left(p;e,\delta\right)\equiv\pi(p)-\left(1-\delta\right)eD\left(p+e\right)-\delta\pi\left(e\right). \tag{12}$$

Conversely, if $H\left(p^{*};e,\delta\right)\geq 0$, then the stationary path $\left(p^{*},p^{*}\right)$ is an equilibrium path.

Summing-up, when $D''\geq 0$, $v^{*}>\pi^{N}$ if and only if there exists $p^{*}<e$ satisfying $\pi\left(p^{*}\right)>\pi^{N}$ and $H\left(p^{*};e,\delta\right)\geq 0$. By construction, $H\left(e;e,\delta\right)=0$. In addition,

$$\frac{\partial H}{\partial p}(p;e,\delta)=D(2p)+2pD'(2p)-\left(1-\delta\right)eD'\left(p+e\right).$$

Hence, $D''\geq 0$ and Assumption A (which implies that $PD'\left(P\right)$ decreases with P) ensure that

$$\frac{\partial^{2}H}{\partial p^{2}}(p;e,\delta)<0.$$

Therefore, if $J\left(e,\delta\right)\geq 0$, where:

$$J(e,\delta)\equiv\frac{\partial H}{\partial p}(e;e,\delta)=D(2e)+\left(1+\delta\right)eD'\left(2e\right),$$

then no cooperation is feasible, as then $H\left(p;e,\delta\right)<0$ for $p<e$. Conversely, if $J\left(e,\delta\right)<0$, then tacit cooperation on p^{*} is feasible for $p^{*}\in\left[\underline{p}\left(e,\delta\right),e\right]$, where $p=\underline{p}\left(e,\delta\right)$ is the unique solution (other than $p=e$) to $H\left(p;e,\delta\right)=0$. Note that

$$\frac{\partial J}{\partial\delta}(e,\delta)=eD'\left(2e\right)<0,$$

and

$$J\left(e,0\right)=D(2e)+eD'\left(2e\right)\geq 0,$$

as $e\leq\hat{p}\leq r\left(e\right)$, whereas

$$J\left(e,1\right)=D(2e)+2eD'\left(2e\right)<0,$$

as $e>p^{M}$. Therefore, there exists a unique $\underline{\delta}^{C}\left(e\right)$ such that tacit cooperation can be profitable for $\delta>\underline{\delta}^{C}\left(e\right)$. Furthermore, Assumption A implies that $eD'\left(2e\right)$ is decreasing and so

$$\frac{\partial J}{\partial e}\left(e,\delta\right)=2D'(2e)+\left(1+\delta\right)\frac{d}{de}\left(eD'\left(2e\right)\right)<0.$$

Hence the threshold $\underline{\delta}^{C}\left(e\right)$ decreases with e; furthermore, $\underline{\delta}^{C}\left(\hat{p}\right)=0$, as $J\left(\hat{p},0\right)=D(2\hat{p})+\hat{p}D'\left(2\hat{p}\right)=0$ (as $\hat{p}=r\left(\hat{p}\right)$).

Finally, when $\delta>\underline{\delta}^{C}\left(e\right)$, the set of sustainable Nash-dominating per-firm payoffs is $\left[\pi\left(e\right),v^{*}\left(e,\delta\right)\right]$, where $v^{*}\left(e,\delta\right)\equiv\pi\left(\max\left\{p^{M},\underline{p}\left(e,\delta\right)\right\}\right)$, and $\underline{p}\left(e,\delta\right)$ is the lower solution to $H\left(p;e,\delta\right)=0$; as H increases in δ,[12] $\underline{p}\left(e,\delta\right)$ decreases with δ and thus $v^{*}\left(e,\delta\right)$ weakly

[12] For any $p<e$:

$$\frac{\partial H}{\partial\delta}\left(p;e,\delta\right)=e\left[D\left(p+e\right)-D\left(2e\right)\right]>0.$$

increases with δ. ■

Finally, the following proposition characterizes the scope for tacit coordination in case of strong complementors:

Proposition 17 (strong complementors) *When $e > \hat{p}$:*

(*i*) *$v^* > \pi^N$: some profitable cooperation is always sustainable. Perfect cooperation on price p^M is feasible (i.e., $v^* = \pi^M$) if $\delta \geq \overline{\delta}^C(e)$, where $\overline{\delta}^C(e)$ continuously prolongs the function defined in Proposition 16, lies strictly below 1, and is decreasing for e close to $\hat{p}$ and for e close to V.*

(*ii*) *Furthermore, if $D'' \geq 0$, then there exists $v^*(e,\delta) \in (\pi^N, \pi^M]$, which continuously prolongs the function defined in Proposition 16 and is (weakly) increasing in δ, such that the set of Nash-dominating sustainable payoffs is $\mathcal{V}^+ = [\pi(\hat{p}), v^*(e,\delta)]$.*

Proof. (*i*) We first show that, using reversal to Nash as punishment, firms can always sustain a stationary, symmetric equilibrium path in which they both charge constant price $p < \hat{p}$, for p close enough to $\hat{p}$. This amounts to $\hat{K}(p; e, \delta) \geq 0$, where

$$\hat{K}(p; e, \delta) \equiv \pi(p) - (1-\delta)\,\pi^D(p; e) - \delta\pi(\hat{p}),$$

where

$$\pi^D(p; e) \equiv \max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) = \begin{cases} r(p)\,D(p + r(p)) & \text{if } \; r(p) \leq e, \\ eD(p+e) & \text{if } \; r(p) > e. \end{cases}$$

Because $\pi^D(\hat{p}; e) = \pi(\hat{p})$, $\hat{K}(\hat{p}; e, \delta) = 0$ for any e, δ. Furthermore:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \pi'(\hat{p}) - (1-\delta)\,\hat{p}D'(2\hat{p}),$$

which using $\pi'(\hat{p}) = \hat{p}D'(2\hat{p})$, reduces to:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \delta\hat{p}D'(2\hat{p}) < 0.$$

Hence, for p close to $\hat{p}$, $\hat{K}(p; e, \delta) > 0$ for any $\delta \in (0, 1]$. If follows that cooperation on such price p is always sustainable, and thus $v^* > \pi^N$.

We now turn to perfect cooperation. Note first that it can be sustained by the minmax punishment $\underline{\pi} = \tilde{\pi}^M(e)$ whenever

$$\pi^M \geq (1-\delta)\,\pi^D\left(p^M; e\right) + \delta\tilde{\pi}^M(e),$$

or:

$$\delta \geq \overline{\delta}_1^C(e) \equiv \frac{\pi^D\left(p^M; e\right) - \pi^M}{\pi^D\left(p^M; e\right) - \tilde{\pi}^M(e)}.$$

Conversely, minmax punishments can be sustained using Abreu's optimal symmetric penal code whenever

$$(1-\delta)\,\pi(e) + \delta\pi^M \geq \tilde{\pi}^M(e), \tag{13}$$

or:

$$\delta \geq \overline{\delta}_2^C(e) \equiv \frac{\tilde{\pi}^M(e) - \pi(e)}{\pi^M - \pi(e)}.$$

Therefore, we can take $\overline{\delta}^C(e) \equiv \max\left\{\overline{\delta}_1^C(e), \overline{\delta}_2^C(e)\right\}$.

As $\overline{\delta}_1^C(\hat{p}) > \overline{\delta}_2^C(\hat{p}) = 0$ and $\overline{\delta}_1^C(V) > \underline{\delta}(V) = 0$, $\overline{\delta}^C(e) = \overline{\delta}_1^C(e) \geq \overline{\delta}_2^C(e)$ for e close to $\hat{p}$ and for e close to V. Furthermore, as $\tilde{\pi}^M(e)$ is continuous and coincides with $\pi(e)$ for $e = \hat{p}$, and $\pi^D\left(p^M; e\right) = eD\left(p^M + e\right)$ as long as $e < r\left(p^M\right)$ (where $r\left(p^M\right) > \hat{p}$), $\overline{\delta}_1^C(e)$ continuously prolongs the function $\overline{\delta}^C(e)$ defined in Proposition 16). Finally, both $\overline{\delta}_1^C(e)$ and $\overline{\delta}_2^C(e)$ lie below 1 (as $\tilde{\pi}^M(e) \leq \tilde{\pi}^M(\hat{p}) = \pi(\hat{p}) < \pi^M = \pi\left(p^M\right)$).

Next we show that

$$\overline{\delta}_1^C(e) = \frac{1}{1 + \dfrac{\pi^M - \tilde{\pi}^M(e)}{\pi^D\left(p^M; e\right) - \pi^M}}$$

decreases with e:

- For $e \geq r\left(p^M\right)$, $\pi^D\left(p^M; e\right) = r\left(p^M\right) D\left(p^M + r\left(p^M\right)\right)$ does not vary with e whereas $\tilde{\pi}^M(e) = \max_p pD(e+p)$ decreases with e; and so $\overline{\delta}_1^C(e)$ decreases with e.

- When $e \in \left[\hat{p}, r\left(p^M\right)\right]$, $\pi^D\left(p^M; e\right) = eD\left(p^M + e\right)$, and:

$$\begin{aligned}
\frac{d}{de}\left(\frac{\pi^M - \tilde{\pi}^M(e)}{eD\left(p^M + e\right) - \pi^M}\right) &= \frac{\begin{array}{c}\left[eD\left(p^M + e\right) - \pi^M\right]\left[-r(e)\,D'(e + r(e))\right] \\ -\left[\pi^M - \tilde{\pi}^M(e)\right]\left[D\left(p^M + e\right) + eD'\left(p^M + e\right)\right]\end{array}}{\left(eD\left(p^M + e\right) - \pi^M\right)^2} \\
&= \frac{\begin{array}{c}\left[eD\left(p^M + e\right) - \pi^M\right] D(e + r(e)) \\ +\left[\pi^M - \tilde{\pi}^M(e)\right]\left[-\left(D\left(p^M + e\right) + eD'\left(p^M + e\right)\right)\right]\end{array}}{\left(eD\left(p^M + e\right) - \pi^M\right)^2} \\
&> 0,
\end{aligned}$$

  where the second equality uses the first-order condition characterizing $r(e)$, and the inequality stems from all terms in the numerator being positive.

(ii) As in the case of weak complementors, selling the incomplete technology cannot be more profitable than the static Nash:

$$\tilde{\pi}^M(e) = \max_p pD(e+p) < 2\pi^N = 2\pi(\hat{p}) = 2\max_p pD(\hat{p}+p).$$

Therefore, if collusion enhances profits ($v^* > \pi^N$), there must exist some period $\tau \geq 0$ in which each firm i charges a price p_i^* not exceeding e, and the average price $p^* = \frac{p_1^*+p_2^*}{2}$ moreover satisfies

$$\pi(p^*) = \frac{\pi_1^* + \pi_2^*}{2} \geq v^*.$$

To ensure that firm i has no incentive to deviate, and for a given punishment payoff $\underline{v}$, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\pi^D\left(p_j^*; e\right) + \delta \underline{v}.$$

Combining these conditions for the two firms yields:

$$(1-\delta)\frac{\pi^D(p_i^*; e) + \pi^D\left(p_j^*; e\right)}{2} + \delta\underline{v} \leq (1-\delta)\pi(p^*) + \delta\frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(p^*), \qquad (14)$$

where the last inequality stems from $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq v^* \leq \pi(p^*)$. But the deviation profit $\pi^D(p;e)$ is convex in p when $D'' \geq 0$,[13] and thus condition (14) implies $\underline{K}(p^*; e, \delta, \underline{v}) \geq 0$, where

$$\underline{K}(p; e, \delta, \underline{v}) \equiv \pi(p) - (1-\delta)\max_{\tilde{p}\leq e} \tilde{p}D(p+\tilde{p}) - \delta\underline{v}. \qquad (15)$$

Conversely, if $\underline{K}(p^*; e, \delta, \underline{v}) \geq 0$, then the stationary path (p^*, p^*) is an equilibrium path.

For any δ, from Lemma 7 the minmax $\tilde{\pi}^M(e)$ can be used as punishment payoff for e close to $\hat{p}$; the sustainability condition then amounts to $K(p; e, \delta) \geq 0$, where

$$K(p; e, \delta) \equiv \pi(p) - (1-\delta)\max_{\tilde{p}\leq e} \tilde{p}D(p+\tilde{p}) - \delta\tilde{\pi}^M(e).$$

Using $\tilde{\pi}^M(e) = \max_p pD(e+p)$ and noting that $\hat{p} = r(\hat{p}) < e$ implies $\pi(\hat{p}) = \max_p pD(\hat{p}+p) = \max_{p\leq e} pD(\hat{p}+p)$ for $\delta > 0$, we have:

$$K(\hat{p}; e, \delta) = \delta\left[\max_p pD(\hat{p}+p) - \max_p pD(e+p)\right] > 0.$$

Furthermore, K is concave in p if $\pi^D(p;e)$ is convex in p, which is the case when $D'' \geq 0$. Thus, there exists $\underline{p}(e,\delta) \in [p^M, \hat{p})$ such that cooperation at price p is feasible if and only if $\underline{p}(e,\delta) \leq p < \hat{p}$, and the set of sustainable Nash-dominating per-firm payoffs is then $[\pi(e), v^*(e,\delta)]$, where $v^*(e,\delta) \equiv \pi\left(\max\left\{p^M, \underline{p}(e,\delta)\right\}\right)$. Furthermore, using $\tilde{p}^M(e) =$

[13] In the range where $r(p) < e$, $\frac{\partial \pi^D}{\partial p}(p;e) = r(p)D'(p + r(p))$ and thus (using $-1 < r' < 0$):

$$\frac{\partial^2 \pi^D}{\partial p^2}(p;e) = r'D' + rD''(1+r') > 0.$$

In the range where $r(p) > e$, $\frac{\partial \pi^D}{\partial p}(p;e) = eD'(p+e)$ and thus π^D is convex if $D'' \geq 0$. Furthermore, the derivative of π^D is continuous at $p = p_e \equiv r^{-1}(e)$:

$$\lim_{\substack{p\to p_e\\ p<p_e}} \frac{\partial \pi^D}{\partial p}(p;e) = \lim_{p\to p_e} eD'(p+e) = eD'(p_e+e) = \lim_{p\to p_e} r(p)D'(p+r(p)) = \lim_{\substack{p\to p_e\\ p>p_e}} \frac{\partial \pi^D}{\partial p}(p;e).$$

$r(e) < \hat{p} < e$; we have, for $p < \hat{p} < e$:

$$\frac{\partial K}{\partial \delta}(p; e, \delta) = \pi^D(p; e) - \tilde{\pi}^M(e) = \max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) - \max_{\tilde{p}} \tilde{p} D(e + \tilde{p}) > 0.$$

Therefore, $\underline{p}(e, \delta)$ decreases with δ, and thus $v^*(e, \delta)$ weakly increases with δ. Finally, note that $K(p; \hat{p}, \delta) = H(p; \hat{p}, \delta)$, where H is defined by (12); hence the function $v^*(e, \delta)$ defined here prolongs that of Proposition 16.

The function $v^*(e, \delta) = \pi\left(\max\left\{p^M, \underline{p}(e, \delta)\right\}\right)$ remains relevant as long as the minmax $\tilde{\pi}^M(e)$ is sustainable. When this is not the case, then $\underline{v}$ can be replaced with the lowest symmetric equilibrium payoff, which, using Abreu's optimal symmetric penal code, is of the form $(1 - \delta)\pi(p^p) + \delta\pi(p^*)$, where p^p is the highest price in $[\hat{p}, e]$ satisfying $\pi^D(p^p; e) - \pi(p^p) \leq \delta[\pi(p^*) - \pi(p^p)]$, and p^* is the lowest price in $\left[p^M, \hat{p}\right]$ satisfying $\pi^D(p^*; e) - \pi(p^*) \leq \delta[\pi(p^*) - \pi(p^p)]$; we then have $v^*(e, \delta) = \pi(p^*)$ and the monotonicity stems from p^* and p^p being respectively (weakly) decreasing and increasing with δ. ■

Together, Propositions 16 and 17 establish the results stated in Proposition 9.

I Proof of Proposition 10

Proof. When users acquire both licenses at total price P, welfare has the familiar expression:

$$W(P) = S(P) + PD(P),$$

where $S(P) \equiv \int_P^V D(\tilde{P}) d\tilde{P}$. When instead users acquire a single license at price p, welfare is

$$\tilde{W}(p) = S(p + e) + pD(p + e).$$

Thus under rivalry ($e < p^M$), welfare is $W(2e)$ in the absence of collusion and $\tilde{W}(p)$ in the collusive outcome, for some $p > e$. Note that

$$\tilde{W}(p) = W(p + e) - eD(p + e).$$

This expression identifies the two facets of the collusive cost. First, the total price, $p + e$, exceeds the competitive price $2e$ as $p > e$. Second, there is a foregone surplus e on actual consumption $D(p + e)$ due to incomplete consumption. Collusion harms consumers and reduces total welfare under rivalry.

In the case of complementors, tacit coordination is profitable when firms cooperate in offering the complete technology at a price lower than the static Nash price; it then benefits users and increases total welfare. ■

J Proof of Propositions 11 and 13

We prove Proposition 13 in the extended setting described in Section 4.2.5, in which firms may have asymmetric offerings; this, in turn, establishes Proposition 11 for the case of symmetric offerings.

The case of complementors (part (ii), where $e_1 + e_2 \geq P^M = 2p^M$) is straightforward, as any vector of price caps $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_1 + \bar{p}_2 = P^M$ and $\bar{p}_i \leq e_i$ induces $\mathbf{p} = \bar{\mathbf{p}}$ as unique continuation equilibrium: starting from any price vector $\mathbf{p} \leq \bar{\mathbf{p}}$, any firm offering $p_i < \bar{p}_i$ would have an incentive to increase its price towards $\bar{p}_i$, as (using $-1 < r'(p_j) < 0$) $\bar{p}_i + \bar{p}_j = P^M = 0 + r(0) < \bar{p}_j + r(\bar{p}_j)$ implies $\bar{p}_i < r(\bar{p}_j) \leq r(p_j)$, for $i \neq j \in \{1, 2\}$.

We now turn to the case of rivalry (part (i), where $e_1 + e_2 < P^M$). We first show that, as noted in the text, this implies that both firms are constrained in the static Nash equilibrium. Indeed, if both firms were unconstrained, then we would have $p_1^N = p_2^N = \hat{p} = r(\hat{p}) \leq e_2 \leq e_1$ and thus $e_1 + e_2 \geq 2\hat{p} > P^M$, a contradiction. If instead firm i is unconstrained whereas firm j is constrained, for some $i \neq j \in \{1, 2\}$, then $p_j^N = e_j$ and $p_i^N = r(e_j) \leq e_i$; hence, $e_i + e_j \geq r(e_j) + e_j > 0 + r(0) = P^M$, again a contradiction. Therefore, it must be the case that both firms are constrained: $p_i^N = e_i \leq r(e_j)$ for $i \neq j \in \{1, 2\}$.

This, in turn, implies that reducing prices below their Nash levels would reduce both firms' profits: for any $\mathbf{p} \leq \mathbf{p}^N = \mathbf{e} = (e_1, e_2)$, we have $p_i \leq e_i \leq r(e_j)$ for $i \neq j \in \{1, 2\}$, and thus: $\pi_i(\mathbf{p}) \leq \pi_i(p_i, e_j) \leq \pi_i(e_i, e_j)$, where the first inequality stems from $p_j \leq e_j$, and the second one from $p_i \leq e_i \leq r(e_j)$ and quasi-concavity. Furthermore, offering a price $\bar{p}_i > V$ would be irrelevant. Thus, without loss of generality, suppose now that a price cap $\bar{p}_i \in [e_i, V]$ is introduced for each patent $i = 1, 2$.

Next, we show that the minmax profits: (a) are the same as without price caps, and (b) can be sustained by the repetition of the (unconstrained) static Nash outcome, $\mathbf{p}^N = \mathbf{e}$. To establish (a), it suffices to note that the minmaxing strategy $p_j = e_j \,(\leq \bar{p}_j)$ remains available to firm i's rival, and firm i's best response, $p_i = e_i \,(\leq \bar{p}_i)$, also remains available. To establish (b), it suffices to note that the static Nash outcome $\mathbf{p}^N = \mathbf{e}$ remains feasible, and that deviations are only more limited than in the absence of price caps.

We now show that any profitable collusion that is sustainable with price caps is also sustainable without them. Recall that the set of pure-strategy equilibrium payoffs can be characterized as the largest self-generating set of payoffs, where, as minmax profits are sustainable, a self-generating set of payoffs $\tilde{W}$ (where $\tilde{W} = W$ in the absence of price caps, and $\tilde{W} = W^c$ with price caps) is such that, for any payoff (π_1, π_2) in $\tilde{W}$, there exists a continuation payoff (π_1^*, π_2^*) in $\tilde{W}$ and a price profile $(p_1^*, p_2^*) \in \tilde{\mathcal{R}}_1 \times \tilde{\mathcal{R}}_2$, where $\tilde{\mathcal{R}}_i$ is the set of *relevant* prices for firm i (more on this below), that satisfy, for $i \neq j \in \{1,2\}$:

$$\pi_i = (1-\delta)\,\pi_i\left(p_i^*, p_j^*\right) + \delta\pi_i^* \geq \max_{p_i \in \tilde{\mathcal{R}}_i} \pi_i\left(p_i, p_j^*\right) + \delta\underline{\pi}_i. \tag{16}$$

To establish that the equilibrium payoffs that are weakly more profitable than Nash under price caps are also equilibrium payoffs without price caps, it suffices to show that any self-generating set with price caps $(\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_i \in [e_i, V]$ for $i = 1, 2$, is also a self-generating set in the absence of price caps.

In the absence of price caps, without loss of generality the set of relevant prices for firm i is $\mathcal{R}_i \equiv [0, V]$; when a price cap $\bar{p}_i$ is introduced, then the set of relevant prices becomes $\mathcal{R}_i^c \equiv [0, \bar{p}_i]$. Consider now a self-generating set W^c for given price caps $(\bar{p}_1, \bar{p}_2)$ satisfying $\bar{p}_i \in [e_i, V]$ for $i = 1, 2$, and given payoffs $(\pi_1, \pi_2) \in W^c$, with associated payoffs $(\pi_1^*, \pi_2^*) \in W^c$ and prices $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{R}_1^c \times \mathcal{R}_2^c$ satisfying, for $i \neq j \in \{1,2\}$, $\underline{p}_i^* \leq \bar{p}_i$ and

$$\pi_i = (1-\delta)\,\pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right) + \delta\pi_i^* \geq \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) + \delta\underline{\pi}_i. \tag{17}$$

By construction, the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ also belongs to $\mathcal{R}_1 \times \mathcal{R}_2$. However, the gain from a deviation may be lower than in the absence of price caps, as the set of relevant deviating prices is smaller. To conclude the proof, we now show that, for any $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{R}_1^c \times \mathcal{R}_2^c$ satisfying (17), there exists $(p_1^*, p_2^*) \in \mathcal{R}_1 \times \mathcal{R}_2$ satisfying

$$\pi_i = (1-\delta)\,\pi_i\left(p_i^*, p_j^*\right) + \delta\pi_i^* \geq \max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, p_j^*\right) + \delta\underline{\pi}_i. \tag{18}$$

For this, it suffices to exhibit a profile $(p_1^*, p_2^*) \in \mathcal{R}_1 \times \mathcal{R}_2$ yielding the same profits (i.e., $\pi_i\left(p_i^*, p_j^*\right) = \pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right)$ for $i = 1, 2$) without increasing the scope for deviations (i.e., $\max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, p_j^*\right) \leq \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right)$ for $i = 1, 2$). We can distinguish four cases for the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$:

Case a: $\underline{p}_1^* \leq e_1, \underline{p}_2^* \leq e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$; as firm i's profit from deviating to p_i is then given by

$$\pi_i\left(p_i, \underline{p}_j^*\right) = \begin{cases} p_i D\left(\underline{p}_j^* + p_i\right) & \text{if } p_i \leq e_i \\ 0 \text{ otherwise} & \end{cases},$$

the best deviation is

$$\arg\max_{p_i \leq e_i} p_i D\left(\underline{p}_j^* + p_i\right) = e_i,$$

which belongs to both $\mathcal{R}_i$ and $\mathcal{R}_i^c$. Hence, $\max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, \underline{p}_j^*\right)$.

Case b: $\underline{p}_i^* - e_i \leq 0 < \underline{p}_j^* - e_j$, for $i \neq j \in \{1,2\}$. In that case, the profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ yields profits $\pi_j\left(\underline{p}_j^*, \underline{p}_i^*\right) = 0$ and $\pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right) = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, and best deviations are respectively given by:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, \underline{p}_i^*\right) &= \arg\max_{p_j \le e_j} p_j D\left(\underline{p}_i^* + p_j\right) = e_j, \\
\arg\max_{p_i} \pi_i \left(p_i, \underline{p}_j^*\right) &= \arg\max_{p_i \le \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\}.
\end{aligned}$$

As $e_j \in \mathcal{R}_j \cap \mathcal{R}_j^c$, $\max_{\underline{p}_j \in \mathcal{R}_j^c} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \in \mathcal{R}_j} \pi_j\left(p_j, \underline{p}_i^*\right)$. Therefore, if $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} \le \bar{p}_i$ (and thus $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c$), we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as then we also have $\max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{R}_i} \pi_i\left(p_i, \underline{p}_j^*\right)$. If instead $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^M\right\} > \bar{p}_i$, then we can pick $p_i^* = \underline{p}_i^*$ and $p_j^* \in (e_j, e_j + \bar{p}_i - e_i)$:[14] the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, and, as the best deviations are the same, with or without price caps:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, p_i^*\right) &= \arg\max_{p_j} \pi_j\left(p_j, \underline{p}_i^*\right) = e_j \in \mathcal{R}_j \cap \mathcal{R}_j^c, \\
\arg\max_{p_i} \pi_i \left(p_i, p_j^*\right) &= \arg\max_{p_i \le p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_j^* + e_i - e_j, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c,
\end{aligned}$$

as $\min\left\{p_j^* + e_i - e_j, p_i^M\right\} \le p_j^* + e_i - e_j < \bar{p}_i$.

Case c: $0 < \underline{p}_i^* - e_i = \underline{p}_j^* - e_j$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as best deviations consist in undercutting the other firm, and this is feasible with or without price caps.

Case d: $0 < \underline{p}_i^* - e_i < \underline{p}_j^* - e_j$, for $i \ne j \in \{1, 2\}$. In that case, the same payoff could be sustained with $p_i^* = \underline{p}_i^*$ and $p_j^* = \underline{p}_i^* + e_j - e_i \left(< \underline{p}_j^*\right)$, with the convention that technology adopters, being indifferent between buying a single license from i or from j, all favor i: the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, $\pi_j = 0$ and $\pi_i = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, but reduces the scope for deviations, which now boil down to undercutting the rival:

$$\begin{aligned}
\max_{p_j \in \mathcal{R}_j} \pi_j \left(p_j, p_i^*\right) &= \max_{\underline{p}_j \in \mathcal{R}_j^c} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \le \underline{p}_i^* + e_j - e_i} p_j D\left(e_i + p_j\right), \\
\max_{p_i \in \mathcal{R}_i} \pi_i \left(p_i, p_j^*\right) &= \max_{p_i \le p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) \le \max_{\underline{p}_i \in \mathcal{R}_i^c} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \le \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right).
\end{aligned}$$

This moreover implies that, as in case c above, these best deviations were already feasible with price caps. Indeed, as $p_k^* = p_h^* + e_k - e_h$, for $h \ne k \in \{1, 2\}$, we have:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, p_i^*\right) &= \arg\max_{\underline{p}_j} \pi_j\left(\underline{p}_j, \underline{p}_i^*\right) = \arg\max_{p_j \le \underline{p}_i^* + e_j - e_i} p_j D\left(e_i + p_j\right) = \min\left\{p_j^*, p_j^M\right\}, \\
\arg\max_{p_i} \pi_i \left(p_i, p_j^*\right) &= \arg\max_{p_i \le p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_i^*, p_i^M\right\},
\end{aligned}$$

where $\min\left\{p_j^*, p_j^M\right\} \in \mathcal{R}_j \cap \mathcal{R}_j^c$, as $\min\left\{p_j^*, p_j^M\right\} \le p_j^* < \underline{p}_j^* \in \mathcal{R}_j^c\,(\subset \mathcal{R}_j)$, and likewise

[14] This interval is not empty, as $\bar{p}_i \ge e_i$ by assumption.

$\min\left\{p_i^*, p_i^M\right\} \in \mathcal{R}_i \cap \mathcal{R}_i^c$, as $\min\left\{p_i^*, p_i^M\right\} \leq p_i^* = \underline{p}_i^* \in \mathcal{R}_i^c \, (\subset \mathcal{R}_i)$.

K Proof of Proposition 12

We first characterize the scope for tacit coordination for rival and complementary patents, before drawing the implications for the impact of a pool subject to independent licensing.

K.1 Rivalry: $e < p^M$

The firms can of course collude as before, by not forming a pool or, equivalently, by setting the pool price P at a prohibitive level ($P \geq V$, say); firms can then collude on selling the incomplete technology if $\delta \geq \delta^R(e)$. Alternatively, they can use the pool to sell the bundle at a higher price:

Lemma 8 *In order to raise firms' profits, the pool must charge a price* $P^P > 2p^N = 2e$.

Proof. Suppose that the pool charges a price $P^P \leq 2e$, and consider a period t, with individual licenses offered at prices p_1^t and p_2^t. Let $p^t = \min\{p_1^t, p_2^t\}$ denote the lower one.

• Users buy the complete technology from the pool only if $P^P \leq p^t + e$; the industry profit is then $P^P D\left(P^P\right) \leq 2\pi^N = 2\pi(e)$, as the aggregate profit function $PD(P)$ is concave and maximal for $2p^M > 2e \geq P^P$.

• Users buy the complete technology by combining individual licenses only if $p_i \leq e$ for $i = 1, 2$, in which case $p_1 + p_2 \leq 2e$ and the industry profit is $(p_1 + p_2)\, D(p_1 + p_2) \leq 2\pi^N$.

• Finally, users buy an incomplete version of the technology only if $p^t + e \leq P^P$, which in turn implies $p^t \leq e$ (as then $p^t \leq P^P - e$, and by assumption $P^P \leq 2e$); the industry profit is then $p^t D(p^t + e) \leq (p^t + e)\, D(p^t + e) \leq 2\pi^N$, as $p^t + e \leq 2e$.

Therefore, the industry profit can never exceed the static Nash level. ■

Thus, to be profitable, the pool must adopt a price $P^P > 2e$. This, in turn, implies that the repetition of static Nash outcome through independent licensing remains an equilibrium: If the other firm offers $p_j^t = e$ for all $t \geq 0$, buying an individual license from firm j (corresponding to quality-adjusted total price $2e$) strictly dominates buying from the pool, and so the pool is irrelevant (firm i will never receive any dividend from the pool); it is thus optimal for firm i to set $p_i^t = e$ for all $t \geq 0$. Furthermore, this individual licensing equilibrium, which yields $\pi(e)$, still minmaxes all firms, as in every period each firm can secure $eD\left(e + \min\left\{e, p_j^t\right\}\right) \geq \pi(e)$ by undercutting the pool and offering an individual license at price $p_i^t = e$.

Suppose that tacit coordination enhances profits: $v^* > \pi^N = \pi(e)$, where v^* denotes the maximal average discounted equilibrium per firm payoff. In the associated equilibrium, there exists some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2v^*$. If users buy an incomplete version of the technology in that period, then each firm can attract all users by undercutting the equilibrium price; the same reasoning as before then implies that collusion on $p_i^t = \tilde{p}^M(e)$ is sustainable, and requires $\delta \geq \delta^R(e)$.

If instead users buy the complete technology in period τ, then they must buy it from the pool,[15] and the per-patent price $p^P \equiv P^P/2$ must satisfy:

$$2\pi\left(p^P\right) = \pi_1^\tau + \pi_2^\tau \geq 2v^* > 2\pi(e),$$

implying $p^P > e$. In order to undercut the pool, a deviating firm cannot charge more for its individual license than p^D, the price that leaves users indifferent between buying the incomplete technology from the firm and buying the complete technology from the pool; that is, the price p^D is such that:

$$(V - e) - p^D = V - 2p^P,$$

or $p^D = 2p^P - e\,(> e)$; by offering its individual license at this price, the deviating firm obtains a profit equal to:

$$\pi^D = \left(2p^P - e\right) D\left(2p^P\right) = \pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right) > \pi\left(p^P\right). \tag{19}$$

Thus, for the price p^P to be sustainable, there must exist continuation payoffs $\left(v_1^{\tau+1}, v_2^{\tau+1}\right)$ such that, for $i = 1, 2$:

$$(1-\delta)\,\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\left[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)\right] + \delta\pi(e).$$

Combining these two conditions and using $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq v^* \leq \frac{\pi_1^\tau+\pi_2^\tau}{2} = \pi\left(p^P\right)$ yields:

$$\pi\left(p^P\right) \geq (1-\delta)[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)] + \delta\pi(e). \tag{20}$$

Conversely, a pool price $p^P \in (e, p^M]$ satisfying this condition is *stable*: a bundle price $P^P = 2p^P$, together with an equal profit-sharing rule and firms charging high enough individual prices (e.g., $p_i^t \geq V$ for all $t \geq 0$), ensures that no firm has an incentive to undercut the pool, and each firm obtains $\pi\left(p^P\right)$. To see this, it suffices to note that the expression of π^D given by (19) represents the highest deviation profit when $p^P \leq p^M$, as the deviating profit $pD(p+e)$ is concave and maximal for $\tilde{p}^M(e) = r(e)$, and $e + \tilde{p}^M(e) = e + r(e) \geq 0 + r(0) = 2p^M$ implies $\tilde{p}^M(e) > 2p^M - e \geq 2p^P - e$. Building on this insight yields:

Proposition 18 (pool in the rivalry region) *Suppose $e \leq p^M$. As before, if $\delta \geq \delta^R(e)$ the firms can sell the incomplete technology at the monopoly price $\tilde{p}^M$ and share*

[15] Users would combine individual licenses only if the latter were offered at prices not exceeding e; hence, the total price P would not exceed $2e$. But $PD(P) = \pi_1^\tau + \pi_2^\tau \geq 2v^* > 2\pi(e)$ implies $P > 2e$.

the associated profit, $\tilde{\pi}^M$. *In addition, a per-license pool price* p^P, *yielding profit* $\pi\left(p^P\right)$, *is stable if* (20) *holds.*

(*i*) *Perfect collusion (i.e., on a pool price* $p^P = p^M$*) is feasible if*

$$\delta \geq \bar{\delta}^P(e) \equiv \frac{1}{2 - \frac{e}{p^M - e}\frac{D(2e) - D(2p^M)}{D(2p^M)}},$$

where the threshold $\bar{\delta}^P(e)$ *is increasing in* e.

(*ii*) *If the firms can already collude without a pool (i.e., if* $\delta \geq \delta^R(e)$*), then the pool enables them to sustain a more profitable collusion, which benefits consumers as well.*

(*iii*) *There exists* $\underline{\delta}^R(e)$, *which coincides with* $\delta^R(e)$ *for* $e = 0$, *and lies strictly below* $\delta^R(e)$ *for* $e > 0$, *such that some collusion (i.e., on a stable pool price* $p^P \in (e, p^M]$*) is feasible when* $\delta \geq \underline{\delta}^R(e)$.

Proof. (*i*) We have established that a pool price p^P is stable if and only if $L\left(p^P; e, \delta\right) \geq 0$, where

$$\begin{aligned} L(p; e, \delta) &\equiv \pi(p) - (1-\delta)\left[\pi(p) + (p-e)D(2p)\right] - \delta\pi(e) \\ &= \delta p D(2p) - (1-\delta)(p-e)D(2p) - \delta e D(2e). \end{aligned}$$

In the particular case of perfect substitutes (i.e., $e = 0$), this expression reduces to $(2\delta - 1)\pi(p) \geq 0$. Therefore, any pool price $p^P \geq 0$ is stable – including the monopoly price p^M – if and only if $\delta \geq 1/2$. For $e > 0$, sustaining a price $p^P \in (e, p^M]$ requires $\delta > 1/2$:

$$L(p; e, \delta) = (2\delta - 1)\left[pD(2p) - eD(2e)\right] + (1-\delta)e\left[D(2p) - D(2e)\right],$$

where the second term is negative and, in the first term, $\pi(p) > \pi(e)$.

In particular, collusion on p^M is feasible if $L\left(p^M; e, \delta\right) \geq 0$, or:

$$\delta \geq \bar{\delta}^P(e) = \frac{\left(p^M - e\right)D\left(2p^M\right)}{\left(p^M - e\right)D\left(2p^M\right) + \pi^M - \pi(e)} = \frac{1}{2 - \frac{e}{p^M - e}\frac{D(2e) - D(2p^M)}{D(2p^M)}},$$

where

$$\frac{d\bar{\delta}^P}{de}\left(e, \bar{\delta}^P(e)\right) = -\frac{\frac{\partial L}{\partial e}\left(p^M; e, \bar{\delta}^P(e)\right)}{\frac{\partial L}{\partial \delta}\left(p^M; e, \bar{\delta}^P(e)\right)}.$$

Clearly $\partial L/\partial\delta > 0$. Furthermore

$$\frac{\partial L}{\partial e}\left(p^M; e, \bar{\delta}^P(e)\right) = [1 - \bar{\delta}^P(e)]D(2p^M) - \bar{\delta}^P(e)\pi'(e).$$

Using the fact that $L\left(p^M; e, \bar{\delta}^P(e)\right) = 0$,

$$\frac{\partial L}{\partial e}\left(p^M; e, \bar{\delta}^P(e)\right) \propto [\pi^M - \pi(e) - (p^M - e)\pi'(e)] < 0,$$

from the concavity of π. And so

$$\frac{d\bar{\delta}^P}{de} > 0.$$

(ii) In the absence of a pool, collusion is inefficient (users buy only one license) and is therefore unprofitable (and thus unsustainable) when $\tilde{\pi}^M(e) \leq 2\pi^N = 2\pi(e)$ (i.e., $e < \underline{e}$). When instead

$$\tilde{\pi}^M(e) > 2\pi^N(e) = 2\pi(e), \tag{21}$$

then (i) inefficient collusion on $p \in (e, \tilde{p}^M(e)]$ is profitable for p close enough to $\tilde{p}^M(e)$; in this case, maximal collusion (on $\tilde{p}^M(e)$) is sustainable whenever some collusion is sustainable, and it is indeed sustainable if $\delta \geq \delta^R(e)$. We now show that the pool then enables the firms to sustain a more efficient and more profitable collusion, which benefits consumers as well as the firms. To be as profitable, the pool must charge a price P^P satisfying:

$$P^P D\left(P^P\right) \geq \tilde{\pi}^M(e).$$

Let $\tilde{P}(e)$ denote the lowest of these prices, which satisfies $\tilde{P}D\left(\tilde{P}\right) = \tilde{\pi}^M(e)$.[16] The pool price $\tilde{p}(e) = \tilde{P}(e)/2$ is stable if and only if $L(\tilde{p}(e), e, \delta) \geq 0$, which amounts to:

$$\begin{aligned} 0 \leq G(e,\delta) &\equiv \delta\tilde{p}(e) D\left(\tilde{P}(e)\right) - (1-\delta)(\tilde{p}(e) - e) D\left(\tilde{P}(e)\right) - \delta e D(2e) \\ &= (2\delta - 1)\left[\frac{\tilde{\pi}^M(e)}{2} - \pi(e)\right] + (1-\delta) e \left[D\left(\tilde{P}(e)\right) - D(2e)\right]. \end{aligned}$$

We have:

$$\begin{aligned} \frac{\partial G}{\partial \delta}(e,\delta) &= \tilde{p}(e) D\left(\tilde{P}(e)\right) + (\tilde{p}(e) - e) D\left(\tilde{P}(e)\right) - eD(2e) \\ &= \left[\frac{\tilde{\pi}^M(e)}{2} - \pi(e)\right] + \left(1 - \frac{e}{\tilde{p}(e)}\right)\frac{\tilde{\pi}^M(e)}{2} \\ &> 0, \end{aligned}$$

[16] In the rivaly region, we have that $e < p^M < \hat{p} < r(e) = \tilde{p}^M(e)$; hence, the left-hand side increases from

$$2\pi^N = 2eD(2e) < 2\tilde{\pi}^M(e) = 2r(e) D(e + r(e))$$

to $\Pi^M = 2\pi^M > \tilde{\pi}^M(e)$ as P increases from $2e$ to $2p^M$; there thus exists a unique $P \in \left(2e, 2p^M\right)$ satisfying $PD(P) = \tilde{p}^M(e) D\left(\tilde{p}^M(e) + e\right)$.

where the inequality follows from $\tilde{\pi}^M > 2\pi(e)$ (using (21)), which in turn implies $e < \tilde{p}(e)$ (as $2\tilde{p}D(2\tilde{p}) = \tilde{\pi}^M > 2\pi(e) = 2eD(2e)$, and the profit function $PD(P)$ is concave); as

$$G(e, 1/2) = \frac{e}{2}\left[D\left(\tilde{P}(e)\right) - D(2e)\right] < 0 < G(e,1) = \frac{\tilde{\pi}^M(e)}{2} - \pi(e),$$

where the inequalities follow again from $\tilde{\pi}^M(e) > 2\pi(e)$ and $e < \tilde{p}(e)$, then some collusion is feasible if δ is large enough, namely, if $\delta \geq \underline{\delta}_1^R(e)$, where:

$$\underline{\delta}_1^R(e) \equiv \frac{[\tilde{p}(e) - e]\, D\left(\tilde{P}(e)\right)}{\tilde{\pi}^M(e) - \pi(e) - eD\left(\tilde{P}(e)\right)}.$$

From the proof of Proposition 8, the inefficient collusion on $\tilde{p}^M$ is instead sustainable (i.e., $\delta \geq \delta^R(e)$) when:

$$0 \leq \tilde{G}(e,\delta) \equiv (2\delta - 1)\frac{\tilde{\pi}^M(e)}{2} - \delta\pi(e).$$

In the case of perfect substitutes, this condition boils down again to $\delta \geq 1/2$. Therefore, when collusion is sustainable without the pool, the pool enables the firms to sustain perfect efficient collusion. Furthermore, for $e > 0$, $G(e,\delta) - \tilde{G}(e,\delta) = (1-\delta)\, eD\left(\tilde{P}(e)\right) > 0$ and thus, if some collusion is sustainable without a pool, then the pool enables again the firms to sustain a more efficient and more profitable collusion: as $G(e,\delta) > 0$ in this case, it follows that a pool price p^P slightly higher (and thus more profitable) than $\tilde{p}$ is also stable. Finally, note that the (quality-adjusted) price is lower when collusion is efficient: the most profitable sustainable price lies below P^M,[17] and

$$P^M = 0 + r(0) < e + r(e) = \tilde{p}^M + e.$$

(iii) Note that $L(e; e, \delta) = 0$ for all e. Therefore, some collusion is sustainable (i.e., there exists a stable pool price $p^P \in (e, p^M)$) whenever $I(e) > 0$, where

$$I(e,\delta) \equiv \frac{\partial L}{\partial p}(e; e, \delta) = (2\delta - 1)D(2e) + 2\delta e D'(2e).$$

We have:

$$\frac{\partial I}{\partial \delta}(e,\delta) = 2\left[D(2e) + eD'(2e)\right] > 0,$$

[17] A price $P^P > P^M$ cannot be the most profitable stable price:

$$L\left(p^M; e, \delta\right) - L\left(p^P; e, \delta\right) = (2\delta - 1)\left[\pi^M - \pi\left(p^P\right)\right] + (1-\delta)\, e\left[D\left(P^M\right) - D\left(P^P\right)\right],$$

which is positive for $P^P > P^M$, as $\pi^M \geq \pi\left(P^P\right)$ and $D\left(P^M\right) > D\left(P^P\right)$. Hence, whenever a pool price $P^P > P^M$ is stable, then $P = P^M$ is also stable.

where the inequality follows from $e < r(e)$ (as here $e < p^M$ $(< \hat{p})$); as

$$I(e, 1/2) = eD'(2e) < 0 < I(e, 1) = D(2e) + 2eD'(2e),$$

where the last inequality stems from $e < p^M$, then some collusion is feasible if δ is large enough, namely, if $\delta \geq \underline{\delta}_2^R(e)$, where:

$$\underline{\delta}_2^R(e) \equiv \frac{1}{2}\frac{1}{1 + \frac{eD'(2e)}{D(2e)}}. \tag{22}$$

Furthermore:

$$\frac{\partial I}{\partial e}(e, \delta) = 2(3\delta - 1)\left[D'(2e) + \frac{\delta}{3\delta - 1} 2eD''(2e)\right].$$

But $D'(2e) + 2eD''(2e) < 0$ from Assumption B and $\delta/(3\delta - 1) < 1$ from $\delta > 1/2$; and so

$$\frac{\partial I}{\partial e}(e, \delta) < 0,$$

implying that the threshold $\underline{\delta}_2^R(e)$ increases with e; it moreover coincides with $\delta^R(0) = 1/2$ for $e = 0$, and is equal to 1 for $e = p^M$ (in which case $D(2e) + 2eD'(2e) = 0$, and thus $I(p^M, \delta) = -(1 - \delta) D(2p^M)$).

To conclude the argument, it suffices to note that the statement of part (*iii*) holds for $\underline{\delta}^R(e) = \min\left\{\underline{\delta}_1^R(e), \underline{\delta}_2^R(e)\right\}$:

- For $e = 0$, perfect collusion is sustainable for $\delta \geq 1/2$, which coincides with the range where inefficient collusion at $\tilde{p}^M$ would be sustainable without a pool.

- For $e \in (e, \underline{e})$ (in which case, without a pool, inefficient collusion at $\tilde{p}^M$ is sustainable if and only if $\delta \geq \delta^R(e)$), $\underline{\delta}^R(e) \leq \underline{\delta}_1^R(e) < \delta^R(e)$.

- Finally, for $e \in [\underline{e}, p^M]$, no collusion is sustainable in the absence of a pool, whereas a pool enables the firms to collude on some price $p^P \in (e, p^M]$ whenever $\delta \geq \underline{\delta}^R(e)$, where $\underline{\delta}^R(e) \leq \underline{\delta}_2^R(e) < 1$.

■

Remark: If $D'' \leq 0$, then L is concave in p.[18] Hence, in that case, some collusion is feasible *if and only if* $\delta \geq \underline{\delta}_2^R(e)$, where $\underline{\delta}_2^R(e)$ lies strictly below $\delta^R(e)$ for $e \in \left(0, p^M\right)$ and increases from $\delta^R(0) = 1/2$ to 1 as e increases from 0 to p^M.

[18] As $pD(2p)$ is concave from Assumption B and $\delta > 1/2$, we have:

$$\frac{\partial^2 L}{\partial p^2}(p, e, \delta) = (2\delta - 1)(pD(2p))'' + 4(1 - \delta)eD''(2p) < 0.$$

K.2 Weak or strong complementors: $p^M \leq e$

In case of complementary patents, a pool enables the firms to cooperate perfectly:

Proposition 19 (pool with complements) *With weak or strong complementors, a pool allows for perfect cooperation (even if independent licensing remains allowed) and gives each firm a profit equal to π^M.*

Proof. Suppose that the pool charges $P^M = 2p^M$ for the whole technology and shares the profit equally. No deviation is then profitable: as noted above, the best price for an individual license is then $\tilde{p} = 2p^M - e$ (that is, the pool price minus a discount reflecting the essentiality of the foregone license), which is here lower than p^M (since $p^M \leq e$) and thus yields a profit satisfying:

$$\left(2p^M - e\right) D\left(2p^M\right) < p^M D\left(2p^M\right) = \pi^M.$$

■

K.3 Impact of a pool subject to independent licensing

Comparing the most profitable equilibrium outcomes with and without a pool (subject to independent licensing) yields the following observations:

- In the rivalry region, a pool can only benefit users whenever some collusion would already be sustained in the absence of a pool (i.e., when $\delta \geq \delta^R(e)$). In this case, a pool enables the firms to sustain a more efficient collusion, which is more profitable but also benefits users: they can then buy a license for the complete technology at a price $P \leq P^M = 2p^M$, which is preferable to buying a license for the incomplete technology at price $\tilde{p}^M(e)$: as $r'(\cdot) > -1$,

$$e + \tilde{p}^M(e) = e + r(e) > 0 + r(0) = P^M = 2p^M.$$

- By contrast, when collusion could not be sustained in the absence of a pool (i.e., when $\delta < \delta^R(e)$), then a pool harms users whenever it enables the firms to sustain some collusion, as users then face an increase in the price from $p^N(e) = e$ to some $p > e$. This happens in particular when $\delta \in [\underline{\delta}^R(e), \delta^R(e))$ (if $D''(\cdot) \leq 0$, it happens only in this case), where $\underline{\delta}^R(e)$ increases from $\delta^R(0) = 1/2$ to 1 as e increases from 0 to p^M, and lies strictly below $\delta^R(e)$ for $e \in \left(0, p^M\right)$.
- With weak or strong complementors, a pool enables perfect cooperation and benefits users as well as the firms: in the absence of the pool, the firms would either not cooperate and thus set $p = p^N(e) = \min\{\hat{p}, e\} > p^M$, or cooperate and charge per-license price $p \in [p^M, p^N)$, as opposed to the (weakly) lower price, p^M, under a pool.

Finally, note that, in the absence of the independent licensing requirement, a pool would always enables the firms to achieve the monopoly outcome. Appending independent licensing is therefore always welfare-enhancing, as it can only lead to lower prices in the case of rivalry, and does not prevent the firms from achieving perfect cooperation in the case of complementors.

L Proof of Proposition 14

We start by noting that, if all patents are priced below $\tilde{p}$, then technology adopters acquire all licenses:

Lemma 9 *Offering each license i at a price $p_i \leq \tilde{p}$ induces users to acquire all of them.*

Proof. Without loss of generality, suppose that the patents are ranked in such a way that $p_1 \leq ... \leq p_n$. If users strictly prefer acquiring only $m < n$ licenses, we must have:

$$V(m) - \sum_{k=1}^{m} p_k > V(n) - \sum_{k=1}^{n} p_k \iff \sum_{k=m+1}^{n} p_k > V(n) - V(m).$$

From from the definition of $\tilde{p}$, we also have:

$$V(n) - n\tilde{p} \geq V(m) - m\tilde{p} \iff V(n) - V(m) \geq (n-m)\tilde{p}.$$

Combining these conditions yields:

$$\sum_{k=m+1}^{n} p_k > (n-m)\tilde{p},$$

implying that some licenses are priced strictly above p. Conversely, if all licenses are priced below $\tilde{p}$, users are willing to acquire all of them. ■

To establish part (i) of Proposition 14, suppose that $p^N < p^M$ (which implies $p^N = \tilde{p}$ and $\pi^N = \tilde{p}D(n\tilde{p})$), that each firm faces a given price cap $\bar{p}_i$, and consider a stationary symmetric path in which all firms repeatedly charge the same price p^* (which thus must satisfies $p^* \leq \bar{p}_i$ for $i \in \mathcal{N}$), and obtain the same profit $\pi^* > \pi^N = \tilde{p}D(n\tilde{p})$. We first note that this last condition requires selling an incomplete bundle:

Lemma 10 *When $p^N < p^M$, generating more profit than the static Nash level requires selling less than n licenses.*

Proof. Suppose that a price profile $(p_1, ..., p_n)$ induces users to acquire all n licenses. The aggregate profit is then $\Pi(P) = PD(P)$, where $P = \sum_{k=1}^{n} p_k$ denotes the total price. But this profit function is concave in P under Assumption B, and thus increases with P

in the range $P \leq P^M = np^M$. From Lemma 9, selling all n licenses require $P \leq n\tilde{p}$, where by assumption $n\tilde{p} < P^M$; therefore, the aggregate profit $PD(P)$ cannot exceed that of the (unconstrained) static Nash, $n\tilde{p}D(n\tilde{p})$. ■

From Lemma 10, $\pi^* > \pi^N$ implies that users must buy $m^* < n$ patents; Lemma 9 then implies $p^* > \tilde{p}$; the per-firm equilibrium profit is then:

$$\pi^* = \frac{m^*}{n} p^* D\left(m^* p^* + V(n) - V(m^*)\right).$$

Furthermore, as $\bar{p}_i \geq p^* > \tilde{p}$ for all $i \in \mathcal{N}$, the price caps do not affect the static Nash equilibrium, in which all firms still charge $p^N = \tilde{p}$. The price p^* can therefore be sustained by reversal to Nash if and only if:

$$\pi^* \geq (1 - \delta)\, \pi^D(p^*) + \delta \pi^N,$$

where $\pi^N = \tilde{p}D(n\tilde{p})$ and $\pi^D(p^*)$ denotes the most profitable deviation from p^*, subject to charging a price $p^D \leq \bar{p}_i$. But as the deviating price must lie below p^* (otherwise, the member's patent would be excluded from users' basket), it is not constrained by the price cap $\bar{p}_i \geq p^*$; therefore, the deviation cannot be less profitable than in an alternative candidate equilibrium in which, in the absence of price caps, all members would charge p^*. Hence, price caps cannot sustain higher symmetric prices than what the firms could already sustain in a symmetric equilibrium in the absence of price caps.

To establish part (ii) of the Proposition, suppose that all firms face the same price cap $\bar{p} = p^M < p^N = \min\{\tilde{p}, \hat{p}\}$. As no firm can charge more than $\bar{p} < \tilde{p}$, Lemma 9 implies that, by charging $p_i = p^M$, each firm i can ensure that technology adopters acquire its license, and thus secure a profit at least equal to:

$$\pi_i = p^M D(p^M + \sum_{j \in \mathcal{N} \setminus \{i\}} p_j) \geq p^M D(p^M + (n-1)\,\bar{p}) = \pi^M = p^M D\left(P^M\right).$$

As each firm can secure π^M, and the industry profit is maximal for P^M, it follows that the unique candidate equilibrium is such that each firm charges $\bar{p} = p^M$. Conversely, all firms charging p^M indeed constitutes an equilibrium: a deviating firm can only charge a price $p < \bar{p} = p^M$, and the deviating profit is thus given by:

$$pD\left(p + (n-1)\, p^M\right).$$

The conclusion then follows from the fact that this profit is concave in p, and maximal for (using $r'(\cdot) < 0$ and $p^M < \hat{p}$):

$$r\left((n-1)\, p^M\right) > r\left((n-1)\,\hat{p}\right) = \hat{p} > p^M = \bar{p}.$$

M On Assumption C

This section first shows that Assumption C holds, for instance, when costs and demands are linear; it then provides a couple of other illustrating examples.

M.1 Linear demand

Suppose that all firms face the same constant unit cost of production, c, and that consumers are identical, with a gross surplus from any n goods given by:[19]

$$U_n(q_1, ..., q_n) = \sum_{i \in \mathcal{N}} \left[q_i - \frac{q_i^2}{2} + \sigma q_i \sum_{j \in \mathcal{N} \setminus \{i\}} q_j \right]$$

where $\sigma \geq 0$ represents the degree of complementarity. The inverse demand for good i is thus given by:

$$P_i(q_1, ..., q_n) = 1 - q_i + \sigma \sum_{j \in \mathcal{N} \setminus \{i\}} q_j.$$

To avoid exploding demands, we will focus on mild degrees of complementarity, namely:

$$\sigma < \frac{1}{n-1}.$$

When n products are offered, the monopoly outcome solves:

$$\max_{(q_1, ..., q_n)} \sum_{i \in \mathcal{N}} \{[1 - q_i + \sigma \sum_{j \in \mathcal{N} \setminus \{i\}} q_j - c] q_i\},$$

leading (as the objective function is strictly concave, and using the "aggregative game" feature of the problem) to, for $i = 1, ..., n$:

$$2q_i = 1 - c + 2\sigma \sum_{j \in \mathcal{N} \setminus \{i\}} q_j.$$

Therefore, the monopoly outcome is symmetric:

$$q_i = q_n^M = \frac{1}{2} \frac{1-c}{1-(n-1)\sigma}.$$

It follows that the monopoly price does not depend on the number of products being offered:

$$p_i = p^M = \frac{1+c}{2}.$$

Hence, thanks to product variety, consumer surplus increases with n: $S_n^M > S_{n-1}^M$ for $n = 1, ..., N$.

[19] Increasing the number of goods does not affect consumers' underlying preferences: $U_n(q_1, ..., q_n) = U_{n+1}(q_1, ..., q_n, 0)$. The same remark applies to the indirect demand functions $P_n(\cdot)$.

M.2 Technology adoption

Suppose now that consumers have unit demands, and their utility increases with the number of products consumed; however, each consumer must incur an adoption cost θ in order to enjoy the products; that is, when buying n goods at total price P, a consumer obtains a net surplus

$$v(n) - P - \theta$$

where θ is independent and identically distributed with c.d.f. $F(\cdot)$ over $\mathbb{R}_+$, $v(0) = 0$ and $v(n)/n$ is increasing in n.

This last condition ensures that all products are purchased when they are offered at the same price; indeed, if n products are offered at the same price $p = P/n$, then consumer θ's choice is given by:

$$\max_{m=0,\ldots,n} \left\{ m \left[\frac{v(m)}{m} - p \right] - \theta \right\}.$$

The term within bracket increases with m; hence, if $p > v(n)/n$ (implying $p > v(m)/m$ for $m < n$), the consumer buys no product. If instead $p < v(n)/n$, then any active consumer will buy all n products, and a consumer buys if $\theta \leq \tilde{\theta}_n(P) \equiv v(n) - P$. Total consumer surplus is then equal to:

$$S_n(P) = \int_0^{\tilde{\theta}_n(P)} \left[\tilde{\theta}_n(P) - \theta \right] dF(\theta),$$

and thus increases with $\tilde{\theta}_n(P)$.

Assuming costless production for notational simplicity, a monopolist thus solves:

$$\max_P PF(v(n) - P) = \max_{\tilde{\theta}} \left[v(n) - \tilde{\theta} \right] F\left(\tilde{\theta}\right).$$

A simple revealed-preference argument shows that the resulting optimal monopoly threshold, $\tilde{\theta}_n^M$, weakly decreases with n. It follows that consumer surplus can only increase with product variety.

M.3 Idiosyncratic taste for variety

Consider the same setting as in the previous example, except that now a consumer of type θ obtains a net surplus

$$\theta v(n) - P.$$

The assumption that $v(n)/n$ is increasing in n ensures that all products or none are purchased when they are offered at the same price: if n products are offered at the same price $p = P/n$, then by buying m products a consumer θ obtains a net surplus equal to

$$m\left[\theta\frac{v\left(m\right)}{m}-p\right],$$

where the term in brackets increases with the number m of products purchased; therefore, the consumer buys all products if $\theta \geq P/v\left(n\right)$, and none otherwise.

Assuming costless production for notational simplicity, a monopolist thus solves:

$$\max_{P} P\left[1-F\left(\frac{P}{v\left(n\right)}\right)\right]=\max_{\tilde{\theta}}\tilde{\theta}v\left(n\right)\left[1-F\left(\tilde{\theta}\right)\right].$$

As this objective is multiplicatively separable in n and $\tilde{\theta}$, the optimal monopoly threshold, $\tilde{\theta}^{M}$, does not depend on n; therefore, the resulting total consumer surplus, which is given by:

$$S_{n}^{M}=\int_{\tilde{\theta}^{M}}^{+\infty}\left(\theta-\tilde{\theta}^{M}\right)v\left(n\right)dF\left(\theta\right),$$

increases with n thanks to enhanced product variety.

N Proof of Proposition 15

(i) From Propositions 3 and 6, under (**S**) and (**SC**), price caps have no effect on the continuation equilibrium in the case of non-repeated interaction, and do not affect the scope for tacit collusion in the case of repeated interaction. Hence, if firms coordinate on the most profitable continuation equilibrium in case of multiplicity, price caps do not affect investment incentives.

(ii) From Propositions 3 and 6, under (**C**), price caps enable the firms to cooperate perfectly. Hence, focussing on the most profitable equilibrium in case of multiplicity, a potential entrant joining n other firms obtains some profit $\pi_{n+1} \leq \pi_{n+1}^{M}$ in the absence of price caps, and π_{n+1}^{M} with price caps. Therefore:

- In the single entrant scenario, the investment threshold below which entry occurs weakly decreases.
- In the free-entry scenario, price caps weakly increase the number of firms entering the market.

To show that allowing for price caps also benefits incumbents in the first scenario, we first establish the following lemma:

Lemma 11 (monopoly profits with complements) *If $C''(\cdot) \leq 0$, then π_{n}^{M} strictly increases with n.*

Proof. For $n \geq 2$, let $\tilde{D}_n(\tilde{p}, p)$ denote the demand faced by each of the first $n-1$ firms of a n-firm oligopoly when their products are all offered at the same price $\tilde{p}$, and the last product is offered at price p. We have, for $n \geq 2$:

$$\begin{aligned}
\pi_{n-1}^M &= p_{n-1}^M \tilde{D}_n\left(p_{n-1}^M, +\infty\right) - C\left(\tilde{D}_n\left(p_{n-1}^M, +\infty\right)\right) \\
&< p_{n-1}^M \tilde{D}_n\left(p_{n-1}^M, p_n^M\right) - C\left(\tilde{D}_n\left(p_{n-1}^M, p_n^M\right)\right) \\
&\leq p_n^M \tilde{D}_n\left(p_n^M, p_n^M\right) - C\left(\tilde{D}_n\left(p_n^M, p_n^M\right)\right) \\
&= \pi_n^M,
\end{aligned}$$

where the second inequality stems from the definition of p_n^M, and the first inequality stems from $\tilde{D}_n\left(p_{n-1}^M, +\infty\right) < \tilde{D}_n\left(p_{n-1}^M, p_n^M\right)$ (from (**C**)), $p_{n-1}^M > C'\left(\tilde{D}_n\left(p_{n-1}^M, +\infty\right)\right)$ (otherwise, a small increase in prices above p_{n-1}^M would raise profits in the $(n-1)$-firm oligopoly), and $C''(\cdot) \leq 0$, which together yield:

$$\begin{aligned}
\left[p_{n-1}^M \tilde{D}_n\left(p_{n-1}^M, p_n^M\right) - C\left(\tilde{D}_n\left(p_{n-1}^M, p_n^M\right)\right)\right] &- \left[p_{n-1}^M \tilde{D}_n\left(p_{n-1}^M, +\infty\right) - C\left(\tilde{D}_n\left(p_{n-1}^M, +\infty\right)\right)\right] \\
&= \int_{\tilde{D}_n\left(p_{n-1}^M, +\infty\right)}^{\tilde{D}_n\left(p_{n-1}^M, p_n^M\right)} \left[p_{n-1}^M - C'(q)\right] dq.
\end{aligned}$$

■

In the absence of price caps, the incumbents each obtain a profit equal to some $\pi_n \leq \pi_n^M$ in the absence of entry, and to some $\pi_{n+1} \leq \pi_{n+1}^M$ in case of entry, which occurs when $I \leq \pi_{n+1}$; with price caps, they each obtain π_n^M in the absence of entry, and π_{n+1}^M in case of entry, which occurs when $I \leq \pi_{n+1}^M$. Therefore:

- If $I > \pi_{n+1}^M$ (resp., if $I < \pi_{n+1}$), then entry does not occur (resp., entry occurs), regardless of whether prices caps are allowed or not; the conclusion then follows from $\pi_n \leq \pi_n^M$ (resp., from $\pi_{n+1} \leq \pi_{n+1}^M$).
- If $\pi_{n+1}^M > I > \pi_{n+1}$, then entry occurs only with price caps; the conclusion then follows from $\pi_n \leq \pi_n^M \leq \pi_{n+1}^M$, where the last inequality stems from Lemma 11.

Finally, Assumption C ensures that price caps benefit consumers, by reducing prices (from Propositions 5 and 7) if they have no impact on entry, and from greater product variety if they foster entry.