Cooperation vs. Collusion: How Essentiality Shapes Co-opetition*

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Abstract

The assessment of public policies regarding oligopolies requires forming an opinion on whether such policies are likely to hinder or facilitate tacit collusion. Yet, products rarely satisfy the axiom of perfect substitutability that underlies our rich body of knowledge on the topic. We study tacit coordination for a class of demand functions allowing for the full range between perfect substitutes and perfect complements. In our nested demand model, the individual users must select a) which products to purchase within the technological class and b) whether they adopt the technology at all.

We first derive general results about the sustainability of tacit coordination under independent marketing. We then study the desirability of joint marketing alliances, such as patent pools. We show that a combination of two information-free regulatory requirements, mandated unbundling by the joint marketing entity and unfettered independent marketing by the firms, makes joint-marketing alliances always socially desirable, whether tacit coordination is feasible or not. We provide the analysis both for fixed offerings and for an endogenous product set.

Keywords: tacit collusion, cooperation, substitutes and complements, essentiality, joint marketing agreements, patent pools, independent licensing, unbundling, co-opetition.

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1 Introduction

1.1 Paper's contribution

The assessment of public policies regarding oligopolies (structural remedies and merger analysis, regulation of transparency and other facilitating practices, treatment of joint marketing alliances such as patent pools...) requires forming an opinion on whether such policies are likely to hinder or facilitate tacit collusion. Yet, products rarely satisfy the axiom of perfect substitutability that underlies our rich body of knowledge on the topic. Competitors in a technological class exhibit various forms of differentiation; furthermore they often also are complementors: network externalities facilitate the adoption of their technology and deter the emergence of rivals using alternative approaches. This paper's first contribution is to provide a study of tacit collusion for a class of demand functions allowing for the full range between perfect substitutes and perfect complements.

To achieve this while preserving tractability, we adopt a nested demand model in which the individual users must select a) which products to purchase in the technological class and b) whether they adopt the technology at all. The first choice depends on the extent of product substitutability within the class, while the second captures the complementarity dimension. We capture the "essentiality" of offerings through an essentiality parameter; with two firms, say, the essentiality parameter is the reduction in the user's value of the technology when he foregoes an offering. Users differ along one dimension: the cost of adopting the technology, or equivalently their opportunity cost of not adopting another technology. Within this class, we derive general results about the sustainability of "tacit collusion" (coordinated increase in price) or "tacit cooperation" (coordinated decrease in price), that is, about bad and good collusion.

When essentiality is low, firms are rivals and would like to raise price; yet, and unlike in the perfect-substitutes case, such tacit collusion leads users to forego part of the technology, as the price of the component does not vindicate acquiring all. This inefficiency both acts as a partial deterrent to collusion and makes the latter, if it happens, socially even more costly. Yet collusion is feasible when firms are patient enough and essentiality is limited.

Beyond some essentiality threshold, firms become complementors and would like to lower price toward the joint-profit-maximizing price. Such tacit cooperation is feasible provided that the firms are patient enough; it is also easier to enforce, the higher the essentiality parameter.

It is often pointed out that when products exhibit complementarities, joint marketing alliances ("JMAs," hereafter) have the potential of preventing multiple marginalization. Yet authorities are never quite sure whether products are complements or substitutes; such knowledge requires knowing the demand function and the field of use; the pattern of complementarity/substitutability may also vary over time. Suggestions therefore qualify the recommendation of leniency toward JMAs with the caveat that firms keep ownership of their products and be able to freely market them outside the common marketing scheme.

The paper's second contribution is the analysis of tacit collusion under joint and independent marketing. In particular we would like to know whether the "perfect screen" result obtained in Lerner-Tirole (2004) extends to the possibility of tacit coordination. Lerner and Tirole showed that in the absence of tacit coordination, joint marketing is always socially desirable if firms keep ownership of, and thereby are able to independently market their offering; and thus authorities need no information about essentiality when considering pools.

We derive the optimal tacit coordination when firms are allowed to form a pool with the independent licensing provision. The pool enables the firms to lower price when firms are complementors. It prevents the collusion inefficiency stemming from selling an incomplete technology at a high quality-adjusted price when firms are strong substitutes. However, the pool may also facilitate collusion. By eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive. Thus, unless the authorities are reasonably convinced that firms are complementors, they run the risk of approving a JMA when firms are weak substitutes, generating some welfare loss along the way.

Tacit coordination thus poses a new challenge: Independent licensing no longer is a perfect screen. We show that another information-free instrument, the "unbundling requirement" that the JMA markets individual pieces at a total price not exceeding the bundle price, can be appended so as to re-create a perfect screen, and that both instruments are needed to achieve this.

The paper is organized as follows. We first provide further motivation through the case in which "products" are licenses to existing patents held by different companies; and we relate our contribution to the existing literature. Section 2 develops the nested-demand framework in the absence of joint marketing and derives the uncoordinated equilibrium. Section 3 studies tacit coordination in this framework as essentiality increases, making firms rivals, then weak complementors and finally strong complementors. Section 4 introduces joint marketing subject to the firms keeping ownership of their product; it analyses whether this institution has the potential to raise or lower price. Section 5 derives the information-free regulatory requirement. Section 6 adds an ex-ante investment. Section 7 extends the model to asymmetric essentiality and to an arbitrary number of products. Section 8 concludes.

1.2 Illustration: the market for intellectual property

In industries such as software and biotech, the recent inflation in the number of patents has led to a serious concern about the ability of users to build on the technology without infringing on intellectual property. The patent thicket substantially increases the transaction costs of assembling licenses and raises the possibility of numerous marginalizations or unwanted litigation. To address this problem, academics, antitrust practitioners and policy-makers have proposed that IP owners be able to bundle and market their patents within patent pools. And indeed, since the first review letters of the US Department of Justice in the 90s and similar policies in Europe and Asia, patent pools are enjoying a revival (before WWII, most of the high-tech industries of the time were organized around patent pools; patent pools almost disappeared in the aftermath of adverse decisions by the US Supreme Court).

Patent pools however are under sharp antitrust scrutiny as they have the potential to enable the analogue of "mergers for monopoly" in the IP domain. Focusing on the two polar cases, patent pools are socially detrimental in the case of perfectly substitutable patents (they eliminate Bertrand competition) and beneficial for perfectly complementary patents (they prevent Cournot n^{th} marginalization). More generally, they are more likely to raise welfare, the more complementary the patents involved in the technology. But in this grey zone, antitrust authorities have little information as to the degree of complementarity, which furthermore changes over time. Demand data are rarely available and to make matters worse patents can be substitute at some prices and complements at others. Thus patent pool regulation occurs under highly incomplete information. Yet a covenant requiring no information- specifying that patent owners keep property of their patent, so that the pool only performs common marketing- can perfectly screen in welfare-enhancing pools and out welfare-reducing pools; this result, due to Lerner and Tirole (2004), holds even if patents have asymmetric importance. Interestingly, this "independent licensing" covenant has been required lately by antitrust authorities in the US, Europe, and Japan for instance.

Because of the simplicity of this screening device and the importance of patent pools for the future of innovation and its diffusion, its efficacy should be explored further. Indeed, nothing is known about its properties in a repeated-interaction context (the literature so far has focused on static competition). Independent licensing enables deviations from a collusive pool price when patents are sufficiently substitutable as to make the pool welfare-reducing; but it also facilitates the punishment of deviators.

1.3 Relation to the literature

There is no point reviewing here the rich literature on repeated interactions with and without observability of actions. By contrast, applications to non-homogeneous oligopolies are scarcer, despite the fact that antitrust authorities routinely consider the possibility of tacit collusion in their merger or commercial alliances decisions. Exceptions to this overall neglect include Deneckere (1983), Wernerfelt (1989) and Ross (1992). For instance, the latter paper studies tacit collusion with Nash reversal in two models (Hotelling, quadratic payoffs with substitute products).

The conventional view is that, in a context of horizontal differentiation, homogeneous cartels are more stable than non-homogeneous ones (what Jéhiel (1992) calls the principle of minimum differentiation). Stability however does not monotonically grow as substitutability decreases. As stressed by Ross (1992), product differentiation lowers the payoff from deviation, but also reduces the severity of punishments (if one restricts attention to Nash reversals; Häckner (1996) shows that Abreu's penal codes can be used to provide more discipline than Nash reversals).[1] Building on these insights, Lambertini *et al.* (2002) argue that, by reducing product variety, joint ventures can actually destabilize collusion.

In a context of vertical differentiation, where increased product diversity also implies greater asymmetry among firms, Häckner (1994) finds that collusion is instead easier to sustain when goods are more similar (and thus firms are more symmetric). Building on this insight, Ecchia and Lambertini (1997) note that introducing or raising a quality standard can make collusion less sustainable.

This paper departs from the existing literature in several ways. First, it studies collusion with (varying degrees of) complementarity and not only substitutability. It characterizes optimal tacit coordination when products range from perfect substitutes to perfect complements. Second, it allows for JMAs and for tacit collusion not to undermine these alliances. Finally, it derives regulatory implications.

[1] Raith (1996) emphasizes another feature of product differentiation, which is to reduce market transparency; this, in turn, tends to hinder collusion.

2 The model

2.1 Framework

For expositional purposes and because we will later want to extend the model to JMAs (patent pools), it is natural to develop the model using the language of intellectual assets and licensing instead of goods and sales; but the model applies more broadly to general repeated interactions within industries. We assume that the technology is covered by patents owned by separate firms (two in the version below). To allow for the full range between perfect substitutes and perfect complements while preserving tractability, we adopt a nested demand model in which the individual users must select a) which patents to acquire access to if they adopt the technology and b) whether they adopt the technology at all.

Users differ in one dimension: the cost of adopting the technology or, equivalently, their opportunity cost of adopting another technology. There are thus two elasticities in this model: the intra-technology elasticity which reflects the ability/inability of users to opt for an incomplete set of licenses; and the inter-technology elasticity. The simplification afforded by this nested model is that, conditionally on adopting the technology, users have identical preferences over license bundles. This implies that under separate marketing all adopting users select the same set of licenses; furthermore, a JMA need not bother with menus of offers (second-degree price discrimination).

There are two firms, $i = 1, 2$, and a mass 1 of users. Each firm owns a patent pertaining to the technology. While users can implement the technology by building on a single patent, it is more effective to combine both: users obtain a gross benefit V from the two patents, and only $V - e$ with either patent alone. The parameter $e \in [0, V]$ measures the essentiality of individual patents: these are clearly not essential when e is low (in the limit case $e = 0$, the two patents are perfect substitutes), and become increasingly essential as e increases (in the limit case $e = V$, the patents are perfect complements, as each one is needed in order to develop the technology). The extent of essentiality is assumed to be known by IP owners and users; for policy purposes, it is advisable to assume that policymakers have little knowledge of the degree of essentiality.

Adopting the technology involves an opportunity cost, θ, which varies across users and has full support $[0, V]$ and c.d.f $F(\theta)$. A user with cost θ adopts the technology if and only if $V \geq \theta + P$, where P is the total licensing price. The demand for the bundle of the two patents licensed at price P is thus

$$D(P) \equiv F(V - P).$$

Similarly, the demand for a single license priced p is

$$D(p + e) = F(V - e - p).$$

That is, an incomplete technology sold at price p generates the same demand as the complete technology sold at price $p + e$; thus $p + e$ will be labelled the "quality-adjusted price:"[2]

Users obtain a net surplus $S(P)$ when they buy the complete technology at total price P, where $S(P) \equiv \int_0^{V-P} (V - P - \theta)\, dF(\theta) = \int_P^V D(\tilde{P}) d\tilde{P}$, and a net surplus $S(p + e)$ from buying an incomplete technology at price p.

To ensure the concavity of the relevant profit functions, we will assume that the demand function is well-behaved:

Assumption A: $D(.)$ is twice continuously differentiable and, for any $P \geq 0$, $D'(P) < 0$ and $D'(P) + PD''(P) < 0$.

If users buy the two licenses at unit price p, each firm obtains

$$\pi(p) \equiv pD(2p),$$

which is strictly concave under Assumption A;[3] let $p^m \in [0, V]$ denote the

[2] A slightly more general version of this model was introduced by Lerner-Tirole (2004), in which the users' gross surplus, $V(\Sigma_{i=1}^{n} w_i x_i) + \theta$, is separable between a user idiosyncratic characteristic θ, and a benefit that depends on each patent's weight or relative importance w_i and on which licenses are acquired ($x_i = 1$ if the user has a license to patent i, and $x_i = 0$ otherwise).

[3] We have:

$$\pi''(p) = 4[D'(2p) + pD''(2p)],$$

per-patent monopoly price:

$$p^m \equiv \arg\max_p \{\pi(p)\}.$$

If instead a single firm licenses its patent at price p, then the resulting profit is

$$\tilde{\pi}(p) \equiv pD(p+e),$$

which is also strictly concave under Assumption A; let $\tilde{p}^m(e)$ denote the monopoly price for an incomplete technology:

$$\tilde{p}^m(e) \equiv \arg\max_p \{\tilde{\pi}(p) = pD(p+e)\}.$$

Finally, let

$$\pi^m \equiv \pi(p^m) = p^m D(2p^m)$$

and

$$\tilde{\pi}^m(e) \equiv \tilde{\pi}(\tilde{p}^m(e)) = \tilde{p}^m(e) D(\tilde{p}^m(e) + e)$$

denote the highest possible profit per *licensing* firm when two or one patents are licensed.

2.2 Static non-cooperative pricing

Consider the static game in which the two IP firms simultaneously set their prices. Without loss of generality we require prices to belong to the interval $[0, V]$. When a firm raises its price, either of two things can happen: First, the technology adopters may stop including the license in their basket; second, they may keep including the license in their basket, but because the technology has become more expensive, fewer users adopt it.

Let us start with the latter case. In reaction to price p_j set by firm j, firm i sets price $r(p_j)$ given by:

$$r(p_j) \equiv \arg\max_{p_i} \{p_i D(p_i + p_j)\}.$$

which is clearly negative if $D'' \leq 0$; if $D'' > 0$, then $\pi'' < 4[D'(2p) + 2pD''(2p)]$, which is negative under Assumption A. A similar reasoning applies to $\tilde{\pi}(p)$ (defined shortly).

Under Assumption A:[4]

$$-1 < r'(p_j) < 0.$$

The two patents are then both complements (the demand for one decreases when the price of the other increases) and strategic substitutes: An increase in the price of the other patent induces the firm to lower its own price. Furthermore, $r'(.) > -1$ implies that $r(.)$ has a unique fixed point, which we denote by $\hat{p}$:

$$\hat{p} = r(\hat{p}).$$

Double marginalization implies[5] that $\hat{p} > p^m$.

Being in this regime, in which each firm can raise its price without being dropped from the users' basket, requires that, for all i, $p_i \leq e$. The best response of firm j setting price $p_j \leq e$ is to set $p_i = \min\{e, r(p_j)\}$. When instead $p_j > e$, then firm i faces no demand if $p_i > p_j$ (as users buy only the lower-priced license), and faces demand $D(p_i + e)$ if $p_i < p_j$. Competition then drives prices down to $p_1 = p_2 = e$.

It follows that the Nash equilibrium is unique and symmetric: Both patent holders charge price

$$p^N \equiv \min\{e, \hat{p}\},$$

and face positive demand. We will denote the resulting profit by

$$\pi^N \equiv \pi\left(p^N\right).$$

In what follows, we will vary e and keep V constant; keeping the technology's value V constant keeps invariant the reaction function $r(.)$ and its fixed point $\hat{p}$, as well as the optimal price and profit, p^m and π^m, which all depend only on V. By contrast, $\tilde{p}^m(e)$ and $\tilde{\pi}^m(e)$, and possibly the Nash

[4] See Appendix A.

[5] By revealed preference,

$$p^m D(2p^m) \geq \hat{p} D(2\hat{p}) \geq p^m D(\hat{p} + p^m)$$

and thus

$$D(2p^m) \geq D(\hat{p} + p^m) \text{ implying } \hat{p} \geq p^m.$$

Assumption A moreover implies that this inequality is strict.

price and profit, p^N and π^N, vary with e.

3 Tacit coordination

We now suppose that the two firms play the same game repeatedly, with discount factor $\delta \in (0,1)$, and we look for the best (firm optimal) tacit coordination equilibria. Let $v_i = (1-\delta)\,\Sigma_{t\geq 0}\delta^t \pi_i^t$ denote firm i's average discounted profit over the entire equilibrium path, and

$$\bar{v} \equiv \max_{(v_1,v_2)\in\mathcal{E}} \left\{ \frac{v_1+v_2}{2} \right\}$$

denote the maximal per firm equilibrium payoff in the set $\mathcal{E}$ of pure-strategy equilibrium payoffs.[6] Tacit coordination enhances profits only if $\bar{v} > \pi^N$.

The location of e with respect to p^m drastically affects the nature of this tacit coordination:

- If $e < p^m$, which implies $e < \hat{p}$ and thus $p^N = e$, through tacit coordination the firms will seek to *raise* the price above the static Nash level; we will refer to such tacit coordination as *collusion*, as it benefits the firms at the expense of users. But charging a price above $p^N = e$ induces users to buy at most one license. We will assume that firms can share the resulting profit $\tilde{\pi}(p)$ as they wish: in our setting, they can do so by charging the same price $p > e$ and allocating market shares among them; more generally, introducing a dose of heterogeneity among users' preferences would allow the firms to control market shares by differentiating their prices appropriately. In this incomplete-technology region, it is optimal for the firms to raise the price up to $\tilde{p}^m(e)$, if feasible, and share the resulting profit, $\tilde{\pi}^m(e)$.
- If $e > p^m$, through tacit coordination the firms will seek to *lower* the

[6] This maximum is well defined, as the set $\mathcal{E}$ of subgame perfect equilibrium payoffs is compact; see, e.g., Mailath and Samuelson (2006), chapter 2. Also, although we restrict attention to pure-strategy subgame perfect equilibria here, the analysis could be extended to public mixed strategies (where players condition their strategies on public signals) or, in the case of private mixed strategies, to perfect public equilibria (relying on strategies that do not condition future actions on private past history); see Mailath and Samuelson (2006), chapter 7.

total price $p_1 + p_2$ below the static Nash level; we will refer to such tacit coordination as *cooperation*, as it benefits users as well as the firms. Ideally, the firms would reduce the per patent price down to p^m, and share the profit π^m – and they can share any way they want by adjusting p_1 and p_2, keeping the average price equal to p^m.

Likewise, the location of e with respect to $\hat{p}$ affects the scope for punishments:

Lemma 1 (minmax) *Let $\underline{\pi}$ denote the minmax payoff.*

i) If $e \leq \hat{p}$, the static Nash equilibrium (e, e) gives each firm the minmax profit and thus constitutes the toughest punishment: $\underline{\pi} = \pi^N = \pi(e)$.

ii) If $e > \hat{p}$, each firm can only guarantee itself the incomplete-technology per-period monopoly profit: $\underline{\pi} = \widetilde{\pi}^m(e) < \pi^N = \pi(\hat{p})$.

Proof. To establish part *i)*, note that firm i can secure its presence in the users' basket by charging e, and obtain in this way $eD(e + p_j)$ if $p_j \leq e$ and $eD(2e)$ if $p_j > e$. Thus a firm can guarantee itself $\pi(e) = eD(2e)$. But this lower bound is equal to π^N for $e \leq \hat{p}$ and can thus be reached through the repeated occurrence of the static Nash outcome. Hence, $\underline{\pi} = \pi^N = \pi(e)$.

We now turn to part *ii)*. If firm j sets a price $p_j \geq e$, firm i can obtain at most $\max_{p \leq p_j} pD(e + p) = \tilde{\pi}^m(e)$ (as $\tilde{p}^m(e) = r(e) < \hat{p} < e \leq p_j$). Setting instead a price $p_j < e$ allows firm i to obtain at least $\max_{p \leq e} pD(p_j + p) > \max_{p \leq e} pD(e + p) = \tilde{\pi}^m(e)$. Therefore, setting any price above e minmaxes firm i, who then obtains $\tilde{\pi}^m(e)$. ■

Thus, the location of e with respect to $\hat{p}$ affects the scope for punishments. When $e \leq \hat{p}$, the static Nash equilibrium (e, e) yields the minmax profit and thus constitutes the toughest punishment for both firms. When instead $e > \hat{p}$, each firm can only guarantee itself the incomplete-technology monopoly profit $\widetilde{\pi}^m$, which is lower than the profit of the static Nash equilibrium $(\hat{p}, \hat{p})$; as shown by Lemma 3 below, Abreu's optimal penal codes may still sustain the minmax profit, in which case it constitutes again the toughest punishment.

It is therefore useful to distinguish three cases, depending on the location of e with respect to p^m and $\hat{p}$.

3.1 Rivalry: $e < p^m$

Ignoring sustainability, collusion can be profitable in the rivalry region only if competition is strong enough. To understand why, note that collusion requires raising the price above the Nash price, e, which is also the price above which the users opt for a partial basket. The implied loss in demand grows with essentiality. For e close to 0, the Nash profit is negligible and so collusion, if feasible, is attractive for the firms; conversely for e close to p^m, the Nash equilibrium yields approximately the highest possible profit π^m, while a higher price does not. The following Lemma confirms that collusion cannot enhance profits when the patents are weak substitutes:

Lemma 2 *Let* $\underline{e} < p^m$ *denote the unique solution to*

$$\tilde{\pi}^m(\underline{e}) = 2\pi(\underline{e}).$$

Then in the range $e \in [\underline{e}, p^m]$ *the unique equilibrium is the repetition of the static Nash one.*

Proof. Let $\pi_i(p_i, p_j)$ denote firm i's profit, for $i = 1, 2$, when the two firms charge prices p_1 and p_2. We first note that charging prices p_1, p_2 such that $\min\{p_1, p_2\} \leq e$ cannot yield greater profits than the static Nash:

- If $p_1, p_2 \leq e$, then:

$$\pi_1(p_1, p_2) + \pi_2(p_2, p_1) = (p_1 + p_2) D(p_1 + p_2) \leq 2eD(2e) = 2\pi^N,$$

where the inequality stems from the fact that the aggregate profit $PD(P)$ is concave in P and maximal for $P^m = 2p^m > 2e$.

- If instead $p_i \leq e < p_j$, for $i \neq j \in \{1, 2\}$, then:

$$\pi_1(p_1, p_2) + \pi_2(p_2, p_1) = p_i D(e + p_i) \leq eD(2e) \leq 2eD(2e) = 2\pi^N,$$

where the first inequality stems stems from the fact that the profit $\tilde{\pi}(p) = pD(e + p)$ is concave in p and maximal for $\tilde{p}^m(e) = r(e)$, which exceeds e in the rivalry case (as then $e < p^m < \hat{p} = r(\hat{p})$).

Therefore, to generate more profits than the static Nash profit in a given period, both firms must charge a price higher than e; this, in turn, implies that users buy at most one license, and thus aggregate profits cannot exceed $\tilde{\pi}^m(e)$. It follows that collusion cannot enhance profits if $\tilde{\pi}^m(e) \leq 2\pi^N = 2\pi(e)$. Keeping V and thus p^m constant, increasing e from 0 to p^m decreases $\tilde{\pi}^m(e) = \max_p pD(p+e)$ but increases $\pi(e)$; as $\tilde{\pi}^m(0) = 2\pi(p^m) = 2\pi^m$, there exists a unique $\underline{e} < p^m$ such that, in the range $e \in [0, p^m]$, $\tilde{\pi}^m(e) < 2\pi^N$ if and only if $e > \underline{e}$.

Thus, when $e > \underline{e}$, the static Nash payoff π^N constitutes an upper bound on average discounted equilibrium payoffs. To conclude the proof, it suffices to recall that the static Nash equilibrium yields here minmax profits $\left(\pi^N = \pi(e) = \underline{\pi}\right)$, and thus also constitutes a lower bound on equilibrium payoffs. Hence, π^N is the unique average discounted equilibrium payoff, which in turn implies that the static Nash outcome must be played along any equilibrium path. ■

Consider now the case $e < \underline{e}$, and suppose that collusion does enhance profits: $\bar{v} > \pi^N$, where, recall, $\bar{v}$ is the maximal average discounted equilibrium payoff. As $\bar{v}$ is a weighted average of per-period profits, along the associated equilibrium path there must exist some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2\bar{v}$. This, in turn, implies that users must buy an incomplete version of the technology; thus, there exists $\bar{p}$ such that:

$$\tilde{\pi}(\bar{p}) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v}.$$

By undercutting its rival, each firm i could obtain the whole profit $\tilde{\pi}(\bar{p})$ in that period; as this deviation could at most be punished by reverting forever to the static Nash behavior, a necessary equilibrium condition is, for $i = 1, 2$:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\tilde{\pi}(\bar{p}) + \delta\underline{\pi},$$

where $v_i^{\tau+1}$ denotes firm i's continuation equilibrium payoff from period $\tau+1$ onwards. Adding these conditions for the two firms yields:

$$(1-\delta)\,\tilde{\pi}(\bar{p})+\delta\underline{\pi}\leq(1-\delta)\,\frac{\pi_1^\tau+\pi_2^\tau}{2}+\delta\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2}\leq(1-\delta)\,\frac{\tilde{\pi}(\bar{p})}{2}+\delta\frac{\tilde{\pi}(\bar{p})}{2},$$

where the second inequality stems from $v_1^{\tau+1}+v_2^{\tau+1}\leq 2\bar{v}\leq\pi_1^\tau+\pi_2^\tau=\tilde{\pi}(\bar{p})$. This condition amounts to

$$\left(\delta-\frac{1}{2}\right)\tilde{\pi}(\bar{p})\geq\delta\underline{\pi}=\delta\pi(e)\,. \tag{1}$$

and thus implies $\delta\geq 1/2$ (with a strict inequality if $e>0$). This, in turn, implies that (1) must hold for $\tilde{\pi}^m(e)=\max_{\bar{p}}\tilde{\pi}(\bar{p})$:

$$\left(\delta-\frac{1}{2}\right)\tilde{\pi}^m(e)\geq\delta\pi(e)\,. \tag{2}$$

Conversely, if (2) is satisfied, then the stationary path $(\bar{p},\bar{p})$ (with equal market shares) is an equilibrium path, as the threat of reverting to the static Nash profit $\pi^N=\pi(e)$ ensures that no firm has an incentive to deviate:

$$\frac{\tilde{\pi}^m(e)}{2}\geq(1-\delta)\,\tilde{\pi}^m(e)+\delta\pi(e)\,.$$

As this collusion yields the maximal profits, we have:

Proposition 1 (rivalry) *If $e<p^m$, then tacit collusion is feasible if and only if*

$$\delta\geq\delta^N(e)\equiv\frac{1}{2}\frac{1}{1-\frac{\pi(e)}{\tilde{\pi}^m(e)}}. \tag{3}$$

The threshold $\delta^N(e)$ is increasing in e and exceeds 1 if $e\geq\underline{e}$, in which case collusion is therefore not sustainable. If $e<\underline{e}$, then the most profitable collusion occurs at price $\tilde{p}^m(e)$.

As depicted in Figure 1, the threshold $\delta^N(e)$ increases with e in the range $e\in[0,p^m]$ (as $\pi(e)$ increases with e in that range, whereas $\tilde{\pi}^m(e)=\max_p\{pD(p+e)\}$ decreases as e increases). Therefore, for any given $\delta\in$

$(1/2, 1)$, there exits a unique $\hat{e}(\delta) \in (0, \underline{e})$ such that collusion is feasible if and only if $e < \hat{e}(\delta)$.

Proposition 1 echoes the literature on product differentiation, in that greater differentiation (here, greater essentiality) tends to impede collusion. Like in this literature, greater essentiality reduces the scope for punishment; even the toughest punishment (i.e., minmax profits), which here coincides with the profit from the static Nash equilibrium, becomes less effective as e increases. Although the gains from deviation (equal to $\widetilde{\pi}^m(e)/2$) also decrease, which tends to facilitate collusion, this effect is always dominated by the impact on punishments. This comes from the fact that the gain from a deviation is here proportional to the collusive profit, as in the standard case with perfect substitutes.[7]

3.2 Weak complementors: $p^m < e \leq \hat{p}$

In the case of weak complementors, the static Nash equilibrium (e, e) still yields minmax profits and thus constitutes again the toughest punishment in case of deviation. Furthermore, selling the incomplete technology cannot be more profitable than the static Nash outcome, as

$$\tilde{\pi}^m(e) = \tilde{p}^m(e) D(e + \tilde{p}^m(e)) < (e + \tilde{p}^m(e)) D(e + \tilde{p}^m(e)) \leq 2eD(2e),$$

where the first inequality stems from $e > 0$ and the second one from the fact that the aggregate profit $PD(P)$ is concave in P and maximal for $P^m = 2p^m < 2e \leq e + \tilde{p}^m(e)$ (as $\tilde{p}^m(e) = r(e) \geq r(\hat{p}) = p \geq e$). Therefore, to generate more profits than in the static Nash equilibrium, both firms must charge a price not exceeding e, so as to ensure that users buy both licenses; aggregate profits are thus equal to $PD(P)$, where $P = p_1 + p_2$ denotes the

[7] More precisely, the profit from a deviation, π^D, is twice as large as the collusive profit, $\pi^C - \pi^D = \widetilde{\pi}^m = 2\pi^C$. Therefore, the sustainability condition, which can be expressed as

$$\frac{\delta}{1-\delta} \geq \frac{\pi^D - \pi^C}{\pi^C - \underline{\pi}} = \frac{\pi^C}{\pi^C - \underline{\pi}} = \frac{1}{1 - \frac{\underline{\pi}}{\pi^C}},$$

becomes more stringent as $\underline{\pi} = \pi(e)$ increases and/or $\pi^C = \widetilde{\pi}^m(e)$ decreases.

total price charged by the two firms.

Suppose now that collusion enhances profits: $\bar{v} > \pi^N = \pi(e)$. In the most profitable collusive equilibrium, there exists again some period τ in which the average profit is at least $\bar{v}$. And as $\bar{v} > \pi(e) > \tilde{\pi}^m(e)$, users must buy the complete technology in that period; thus, each firm i must charge a price p_i^τ not exceeding e, and the average price $\bar{p} = \frac{p_1^\tau + p_2^\tau}{2}$ must moreover satisfy

$$\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq \bar{v}.$$

As $p_j \leq e < \hat{p} = r(\hat{p}) < r(p_j)$, firm i's best deviation, for $i \neq j \in \{1, 2\}$, consists in charging e. Hence, to ensure that firm i has an incentive not to deviate, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta) eD\left(p_j^\tau + e\right) + \delta \underline{\pi}.$$

Combining these conditions for the two firms yields, using $\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2}$ and $\underline{\pi} = \pi(e)$:

$$(1-\delta) e\frac{D(p_1^\tau + e) + D(p_2^\tau + e)}{2} + \delta\pi(e) \leq (1-\delta)\pi(\bar{p}) + \delta\frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(\bar{p}),$$

where the inequality stems from $\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \leq \bar{v} \leq \pi(\bar{p})$. If the demand function is (weakly) convex (i.e., $D'' \geq 0$ whenever $D > 0$), then this condition implies:

$$\pi(\bar{p}) \geq (1-\delta) eD(\bar{p} + e) + \delta\pi(e). \tag{4}$$

Conversely, if (4) is satisfied, then the stationary path $(\bar{p}, \bar{p})$ is an equilibrium path. Building on this, we have:

Proposition 2 (weak complementors) *If $p^m < e \leq \hat{p}$, then:*

i) When $p^m < e \leq \hat{p}$, perfect cooperation on price p^m is feasible if and only if

$$\delta \geq \overline{\delta}^N(e) \equiv \frac{eD(p^m + e) - \pi^m}{eD(p^m + e) - \pi(e)},$$

where $\overline{\delta}^N(e)$ lies strictly below 1 for $e > p^m$, and is decreasing for e

close to p^m.

ii) *Furthermore, if* $D'' \geq 0$*, then profitable cooperation is sustainable if and only if*

$$\delta \geq \underline{\delta}^N(e),$$

where $\underline{\delta}^N(e)$ *lies below* $\overline{\delta}^N(e)$*, strictly decreases in* e*, and is equal to* 0 *for* $e = \hat{p}$*. The set of sustainable Nash-dominating per-firm payoffs is then* $[\pi(e), \overline{\pi}(e, \delta)]$*, where* $\overline{\pi}(e, \delta) \in (\pi(e), \pi^m]$ *is (weakly) increasing in* δ*.*

Proof. We first establish part *i)*. From the above analysis, perfect cooperation (i.e., $p_1^t = p_2^t = p^m$ for $t = 0, 1, ...$) is sustainable when $H(p^m; e, \delta) \geq 0$, where

$$H(p; e, \delta) \equiv \pi(p) - (1 - \delta) eD(p + e) - \delta\pi(e), \tag{5}$$

where, for any $p < e$:

$$\frac{\partial H}{\partial \delta}(p; e, \delta) = e[D(p + e) - D(2e)] > 0.$$

In addition:

$$H(p^m; e, 0) = p^m D(2p^m) - eD(p^m + e) < 0,$$

as $p^m < e \leq \hat{p} < r(p^m)$, and

$$H(p^m; e, 1) = \pi^m - \pi(e) > 0.$$

Therefore, perfect cooperation is feasible if δ is large enough, namely, if:

$$\delta \geq \overline{\delta}^N(e) = \frac{eD(p^m + e) - \pi^m}{eD(p^m + e) - \pi(e)} = \frac{1}{1 + \frac{\pi^m - \pi(e)}{eD(p^m + e) - \pi^m}}.$$

For $e \in (p^m, \hat{p}]$, $\pi^m > \pi(e)$ and $eD(p^m + e) > \pi^m$ (as $r(p^m) > r(e) \geq \hat{p} \geq e$);

therefore, $\overline{\delta}^N(e) < 1$. Also, for ε positive but small, we have:

$$\overline{\delta}^N(p^m + \varepsilon) \simeq \frac{1}{1 - \frac{\pi''(p^m)}{D(2p^m)+p^m D'(2p^m)} \frac{\varepsilon}{2}},$$

which decreases with ε, as $\pi''(p^m) < 0$ and $D(2p^m)+p^m D'(2p^m) = -p^m D'(2p^m) > 0$.

Turning to part *ii)*, the above analysis shows that, when $D'' \geq 0$, some profitable cooperation is feasible (i.e., $\bar{v} > \pi^N$) if there exists $\bar{p} < e$ satisfying $\pi(\bar{p}) > 2p^N$ and $H(\bar{p}; e, \delta) \geq 0$. By construction, $H(e; e, \delta) = 0$. In addition,

$$\frac{\partial H}{\partial p}(p; e, \delta) = D(2p) + 2pD'(2p) - (1-\delta)\, eD'(p+e).$$

Hence, $D'' \geq 0$ and Assumption A (which implies that $PD'(P)$ decreases with P) ensure that

$$\frac{\partial^2 H}{\partial p^2}(p; e, \delta) < 0.$$

Therefore, if $J(e, \delta) \geq 0$, where:

$$J(e, \delta) \equiv \frac{\partial H}{\partial p}(e; e, \delta) = D(2e) + (1+\delta)\, eD'(2e),$$

then no cooperation is feasible, as then $H(p; e, \delta) < 0$ for $p < e$. Conversely, if $J(e, \delta) < 0$, then tacit cooperation on $\bar{p}$ is feasible for $\bar{p} \in \left[\underline{p}(e, \delta), e\right]$, where $p = \underline{p}(e, \delta)$ is the unique solution (other than $p = e$) to $H(p; e, \delta) = 0$. Note that

$$\frac{\partial J}{\partial \delta}(e, \delta) = eD'(2e) < 0,$$

and

$$J(e, 0) = D(2e) + eD'(2e) \geq 0,$$

as $e \leq \hat{p} \leq r(e)$, whereas

$$J(e, 1) = D(2e) + 2eD'(2e) < 0,$$

as $e > p^m$. Therefore, there exists a unique $\underline{\delta}^N(e)$ such that tacit cooperation can be profitable for $\delta > \underline{\delta}^N(e)$. Furthermore, Assumption A implies that $eD'(2e)$ is decreasing and so

$$\frac{\partial J}{\partial e}(e, \delta) = 2D'(2e) + (1+\delta)\frac{d}{de}(eD'(2e)) < 0.$$

Hence the threshold $\underline{\delta}^N(e)$ decreases with e; furthermore, $\underline{\delta}^N(\hat{p}) = 0$, as $J(\hat{p}, 0) = D(2\hat{p}) + \hat{p}D'(2\hat{p}) = 0$ (as $\hat{p} = r(\hat{p})$).

Finally, when $\delta > \underline{\delta}^N(e)$, the set of sustainable Nash-dominating per-firm payoffs is $[\pi(e), \overline{\pi}(e, \delta)]$, where $\overline{\pi}(e, \delta) \equiv \pi\left(\max\left\{p^m, \underline{p}(e, \delta)\right\}\right)$, and $\underline{p}(e, \delta)$ is the lower solution to $H(p; e, \delta) = 0$; as H increases in δ, $\underline{p}(e, \delta)$ decreases with δ and thus $\overline{\pi}(e, \delta)$ weakly increases with δ. ∎

3.3 Strong complementors: $e > \hat{p}$

With strong complementors, the static Nash equilibrium $(\hat{p}, \hat{p})$ no longer yields the minmax payoff, which is equal to the incomplete-technology monopoly profit: $\underline{\pi} = \tilde{\pi}^m(e)$. However, Abreu (1988)'s penal codes can provide more severe punishments than the static Nash outcome. Abreu showed that optimal penal codes have moreover a particularly simple structure in the case of symmetric behaviors on- and off-the equilibrium path, as punishment paths then have two phases: a finite phase with a low payoff and then a return to the equilibrium cooperation phase. These penal codes can indeed be used to sustain more severe punishments here, and may even yield minmax profits:

Lemma 3 (minmax with strong complementors) *The minmax payoff is sustainable when the discount factor is not too small; this is in particular the case when*

$$\delta \geq \underline{\delta}(e) \equiv \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)},$$

where $\underline{\delta}(e) \in (0, 1)$ *for* $e \in (\hat{p}, V)$, *and* $\underline{\delta}(V) = \lim_{e \longrightarrow \hat{p}} \underline{\delta}(e) = 0$.

Proof. In order to sustain the punishment profit $\underline{\pi} = \tilde{\pi}^m(e)$, consider the following two-phase, symmetric penal code. In the first phase (periods $t = 1, ..., T$ for some $T \geq 1$), both firms charge e, so that the profit is equal to $\pi(e)$. In the first period of the second phase (i.e., period $T+1$), with probability $1 - x$ both firms charge e, and with probability x they switch to the best collusive price that can be sustained with minmax punishments, which is defined as:

$$p^C(e, \delta) \equiv \arg\max_p pD(2p),$$

subject to the constraint

$$(1 - \delta) \max_{\tilde{p} \leq e} \tilde{p}D(p + \tilde{p}) + \delta \underline{\pi} \leq pD(2p). \tag{6}$$

Then, in all following periods, both firms charge p^C. Letting $\Delta = (1 - \delta) x\delta^T + \delta^{T+1} \in (0, \delta)$ denote the fraction of (discounted) time in the second phase, the average discounted per-period punishment profit is equal to

$$\pi^p = (1 - \Delta) \pi(e) + \Delta\pi\left(p^C\right),$$

which ranges from $\pi(e) < \underline{\pi} = \tilde{\pi}^m(e)$ (for $T = +\infty$) to $(1 - \delta) \pi(e) + \delta\pi\left(p^C\right)$ (for $T = 1$ and $x = 1$). Thus, as long as this upper bound exceeds $\tilde{\pi}^m(e)$, there exists $T \geq 1$ and $x \in [0, 1]$ such that the penal code yields the minmax: $\pi^p = \tilde{\pi}^m(e) = \underline{\pi}$.

As p^C satisfies (6), the final phase of this penal code (for $t > T + 1$, and for $t = T + 1$ with probability x) is sustainable. Furthermore, in the first $T + 1$ periods the expected payoff increases over time (as the switch to p^C comes closer), whereas the maximal profit from a deviation remains constant and equal to $\max_{p \leq e} pD(e + p) = \tilde{\pi}^m(e)$ (as $\tilde{p}^m(e) = r(e) < e$ for $e > \hat{p}$). Hence, to show that the penal code is sustainable it suffices to check that firms have no incentive to deviate in the first period, which is indeed the case if deviations are punished with the penal code:

$$\tilde{\pi}^m(e) = (1 - \Delta) \pi(e) + \Delta\pi\left(p^C\right) \geq (1 - \delta) \tilde{\pi}^m(e) + \delta\tilde{\pi}^m(e) = \tilde{\pi}^m(e).$$

There thus exists a penal code sustaining the minmax whenever the upper bound $(1 - \delta) \pi(e) + \delta\pi\left(p^C\right)$ exceeds $\tilde{\pi}^m(e)$; as by construction $\pi\left(p^C\right) \geq$

$\pi^N = \pi(\hat{p})$, this is in particular the case whenever

$$(1-\delta)\,\pi(e) + \delta\pi(\hat{p}) \geq \tilde{\pi}^m(e),$$

which amounts to $\delta \geq \underline{\delta}(e)$. Finally:

- $\underline{\delta}(e) \in (0,1)$ for any $e \in (\hat{p}, V)$, as then:

$$\pi(\hat{p}) = \max_p pD(\hat{p}+p) > \tilde{\pi}^m(e) = \max_p pD(e+p) > \pi(e) = eD(2e);$$

- $\underline{\delta}(V) = 0$, as $\tilde{\pi}^m(V) = \pi(V) = 0$, and

$$\lim_{e \longrightarrow \hat{p}} \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi(\hat{p}) - \pi(e)} = \left.\frac{\frac{d\tilde{\pi}^m(e)}{de} - \frac{d\pi(e)}{de}}{-\frac{d\pi(e)}{de}}\right|_{e=\hat{p}} = \frac{D(2\hat{p}) + \hat{p}D'(2\hat{p})}{D(2\hat{p}) + 2\hat{p}D'(2\hat{p})} = 0,$$

  where the last equality stems from $\hat{p} = r(\hat{p}) = \arg\max_p pD(\hat{p}+p)$.

■

The above Lemma shows in particular that, for any $\delta > 0$, it is possible to sustain minmax punishments not only when e is close to $\hat{p}$ (as the static Nash outcome is then close to the minmax) but also when e is close to V, that is, when the patents are almost perfect complements – in which case $\pi^N = \pi(\hat{p}) > \underline{\pi} = 0$.[8] Indeed, when the patents are perfect complements ($e = V$), an optimal penal code consists in charging $p = e = V$ for ever (even after a deviation).

Following similar steps as for weak complementors, we can then establish:

Proposition 3 (strong complementors) *If $e > \hat{p}\,(> p^m)$, then:*

i) Some profitable cooperation is always sustainable. Perfect cooperation on price p^m is moreover feasible if $\delta \geq \overline{\delta}^N(e)$, where $\overline{\delta}^N(e)$ continuously

[8]Conversely, when $e \in (0,V)$ minmax punishments can only be sustained for large enough values of the discount factor. Although this is not formally established by the previous Lemma, it suffices to note that (i) as δ goes to 0, the best collusive price p^C tends to the static Nash price $p^N = \hat{p}$, and (ii) in the first phase of the penal code, the price cannot be lower than e, as the deviation profit would otherwise exceed $\underline{\pi} = \tilde{\pi}^m(e)$.

prolongs the function defined in Proposition 2, lies strictly below 1*, and is decreasing for* e *close to* $\hat{p}$ *and for* e *close to* V*.*

ii) Furthermore, if $D'' \geq 0$*, then there exists* $\overline{\pi}(e,\delta) \in (\pi(\hat{p}), \pi^m]$*, which continuously prolongs the function defined in Proposition 2 and is (weakly) increasing in* δ*, such that the set of Nash-dominating sustainable payoffs is* $[\pi(\hat{p}), \overline{\pi}(e,\delta)]$*.*

Proof. To demonstrate part *i)*, we first show that, using reversal to Nash as punishment, firms can always sustain a stationary, symmetric equilibrium path in which they both charge the price $p < \hat{p}$ over time, for p close enough to $\hat{p}$. This amounts to $\hat{K}(p; e, \delta) \geq 0$, where

$$\hat{K}(p; e, \delta) \equiv \pi(p) - (1-\delta)\,\pi^D(p; e) - \delta\pi(\hat{p}),$$

where

$$\pi^D(p; e) \equiv \max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) = \begin{cases} r(p)\, D(p + r(p)) & \text{if} \quad r(p) \leq e, \\ eD(p+e) & \text{if} \quad r(p) > e. \end{cases}$$

As $\pi^D(\hat{p}; e) = \pi(\hat{p})$, $\hat{K}(\hat{p}; e, \delta) = 0$ for any e, δ. Furthermore:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \pi'(\hat{p}) - (1-\delta)\,\hat{p}D'(2\hat{p}),$$

which using $\pi'(\hat{p}) = \hat{p}D'(2\hat{p})$, reduces to:

$$\frac{\partial \hat{K}}{\partial p}(\hat{p}; e, \delta) = \delta\hat{p}D'(2\hat{p}) < 0.$$

Hence, for p close to $\hat{p}$, $\hat{K}(p; e, \delta) > 0$ for any $\delta \in [0,1]$. If follows that cooperation on such price p is always sustainable.

We now turn to perfect cooperation. Note first that it can be sustained by the minmax punishment $\underline{\pi} = \tilde{\pi}^m(e)$ whenever

$$\pi^m \geq (1-\delta)\,\pi^D(p^m; e) + \delta\tilde{\pi}^m(e),$$

or:

$$\delta \geq \overline{\delta}_1^N(e) \equiv \frac{\pi^D(p^m; e) - \pi^m}{\pi^D(p^m; e) - \tilde{\pi}^m(e)}.$$

Conversely, adapting the proof of Lemma 3, minmax punishments can be sustained using Abreu's optimal symmetric penal code whenever

$$(1-\delta)\,\pi(e) + \delta\pi^m \geq \tilde{\pi}^m(e), \tag{7}$$

or:

$$\delta \geq \overline{\delta}_2^N(e) \equiv \frac{\tilde{\pi}^m(e) - \pi(e)}{\pi^m - \pi(e)}.$$

Therefore, we can take $\overline{\delta}^N(e) \equiv \max\left\{\overline{\delta}_1^N(e), \overline{\delta}_2^N(e)\right\}$. As $\overline{\delta}_1^N(\hat{p}) > \overline{\delta}_2^N(\hat{p}) = 0$ and $\overline{\delta}_1^N(V) > \underline{\delta}(V) = 0$, $\overline{\delta}^N(e) = \overline{\delta}_1^N(e) \geq \overline{\delta}_2^N(e)$ for e close to $\hat{p}$ and for e close to V. Furthermore, as $\tilde{\pi}^m(e)$ is continuous and coincides with $\pi(e)$ for $e = \hat{p}$, and $\pi^D(p^m; e) = eD(p^m + e)$ as long as $e < r(p^m)$ (where $r(p^m) > \hat{p}$), $\overline{\delta}_1^N(e)$ continuously prolongs the function $\overline{\delta}^N(e)$ defined in Proposition 2). Finally, both $\overline{\delta}_1^N(e)$ and $\overline{\delta}_2^N(e)$ lie below 1 (as $\tilde{\pi}^m(e) \leq \tilde{\pi}^m(\hat{p}) = \pi(\hat{p}) < \pi^m = \pi(p^m)$) and $\overline{\delta}_1^N(e)$ moreover decreases with e as, using

$$\overline{\delta}_1^N(e) = \frac{1}{1 + \dfrac{\pi^m - \tilde{\pi}^m(e)}{\pi^D(p^m; e) - \pi^m}},$$

we have:

- For $e \geq r(p^m)$, $\overline{\delta}_1^N(e)$ obviously decreases with e, as $\pi^D(p^m; e) = r(p^m)\,D(p^m + r(p^m))$ does not vary with e whereas $\tilde{\pi}^m(e) = \max_p pD(e+p)$ decreases as e increases.

- In the range $e \in [\hat{p}, r(p^m)]$, $\pi^D(p^m; e) = eD(p^m + e)$, and:

$$\begin{aligned} \frac{d}{de}\left(\frac{\pi^m - \tilde{\pi}^m(e)}{eD(p^m+e) - \pi^m}\right) &= \frac{\begin{array}{c}[eD(p^m+e) - \pi^m][-r(e)D'(e+r(e))] \\ -[\pi^m - \tilde{\pi}^m(e)][D(p^m+e) + eD'(p^m+e)]\end{array}}{(eD(p^m+e) - \pi^m)^2} \\ &= \frac{\begin{array}{c}[eD(p^m+e) - \pi^m]D(e+r(e)) \\ +[\pi^m - \tilde{\pi}^m(e)][-(D(p^m+e) + eD'(p^m+e))]\end{array}}{(eD(p^m+e) - \pi^m)^2} \\ &> 0, \end{aligned}$$

  where the second equality uses the first-order condition characterizing $r(e)$, and the inequality stems from all terms in the numerator being positive.

We now turn to part *ii)*. As in the case of weak complementors, selling the incomplete technology cannot be more profitable than the static Nash, as

$$\tilde{\pi}^m(e) = \max_p pD(e+p) < 2\pi^N = 2\pi(\hat{p}) = 2\max_p pD(\hat{p}+p).$$

Therefore, if collusion enhances profits ($\bar{v} > \pi^N$), there exists some period $\tau \geq 0$ in which each firm i charges a price p_i^τ not exceeding e, and the average price $\bar{p} = \frac{p_1^\tau + p_2^\tau}{2}$ moreover satisfies

$$\pi(\bar{p}) = \frac{\pi_1^\tau + \pi_2^\tau}{2} \geq \bar{v}.$$

To ensure that firm i has no incentive to deviate, and for a given punishment payoff $\underline{v}$, we must have:

$$(1-\delta)\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\pi^D(p_j^\tau; e) + \delta\underline{v}.$$

Combining these conditions for the two firms yields:

$$(1-\delta)\frac{\pi^D(p_i^\tau; e) + \pi^D(p_j^\tau; e)}{2} + \delta\underline{v} \leq (1-\delta)\pi(\bar{p}) + \delta\frac{v_2^{\tau+1} + v_2^{\tau+1}}{2} \leq \pi(\bar{p}), \quad (8)$$

where the inequality stems from $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq \bar{v} \leq \pi(\bar{p})$. But the deviation profit $\pi^D(p;e)$ is convex in p when $D'' \geq 0$,[9] and thus condition (8) implies $\underline{K}(\bar{p};e,\delta,\underline{v}) \geq 0$, where

$$\underline{K}(p;e,\delta,\underline{v}) \equiv \pi(p) - (1-\delta)\max_{\tilde{p}\leq e} \tilde{p}D(p+\tilde{p}) - \delta\tilde{\pi}^m(e). \tag{9}$$

Conversely, if $\underline{K}(\bar{p};e,\delta,\underline{v}) \geq 0$, then the stationary path $(\bar{p},\bar{p})$ is an equilibrium path.

For any δ, from Lemma 3 the minmax $\tilde{\pi}^m(e)$ can be used as punishment payoff for e close to $\hat{p}$; the sustainability condition then amounts to $K(p;e,\delta) \geq 0$, where

$$K(p;e,\delta) \equiv \pi(p) - (1-\delta)\max_{\tilde{p}\leq e} \tilde{p}D(p+\tilde{p}) - \delta\tilde{\pi}^m(e).$$

Using $\tilde{\pi}^m(e) = \max_p pD(e+p)$ and noting that $\hat{p} = r(\hat{p}) < e$ implies $\pi(\hat{p}) = \max_p pD(\hat{p}+p) = \max_{p\leq e} pD(\hat{p}+p)$, for $\delta > 0$ we have:

$$K(\hat{p};e,\delta) = \delta\left[\max_p pD(\hat{p}+p) - \max_p pD(e+p)\right] > 0.$$

Furthermore, K is concave in p if $\pi^D(p;e)$ is convex in p, which is the case when $D'' \geq 0$. Thus, there exists $\underline{p}(e,\delta) \in [p^m,\hat{p})$ such that cooperation at price p is feasible if and only if $\underline{p}(e,\delta) \leq p < \hat{p}$, and the set of sustainable Nash-dominating per-firm payoffs is then $[\pi(e), \overline{\pi}_1(e,\delta)]$, where $\overline{\pi}_1(e,\delta) \equiv \pi\left(\max\left\{p^m, \underline{p}(e,\delta)\right\}\right)$. Furthermore, using $\tilde{p}^m(e) = r(e) < \hat{p} < e$; we have,

[9] In the range where $r(p) < e$, $\frac{\partial \pi^D}{\partial p}(p;e) = r(p)D'(p+r(p))$ and thus

$$\frac{\partial^2 \pi^D}{\partial p^2}(p;e) = r'D' + rD''(1+r') = -\frac{(D')^2}{2D'+rD''} > 0.$$

In the range where $r(p) > e$, $\frac{\partial \pi^D}{\partial p}(p;e) = eD'(p+e)$ and thus π^D is convex if $D'' \geq 0$. Furthermore, the derivative of π^D is continuous at $p = p_e \equiv r^{-1}(e)$:

$$\lim_{\substack{p\to p_e \\ p<p_e}} \frac{\partial \pi^D}{\partial p}(p;e) = \lim_{p\to p_e} eD'(p+e) = eD'(p_e+e) = \lim_{p\to p_e} r(p)D'(p+r(p)) = \lim_{\substack{p\to p_e \\ p>p_e}} \frac{\partial \pi^D}{\partial p}(p;e).$$

for $p < \hat{p} < e$:

$$\begin{aligned}\frac{\partial K}{\partial \delta}(p;e,\delta) &= \delta\left[\pi^D(p;e)-\tilde{\pi}^m(e)\right]\\ &= \delta\left[\max_{\tilde{p}\leq e}\tilde{p}D(p+\tilde{p})-\max_{\tilde{p}}\tilde{p}D(e+\tilde{p})\right]>0.\end{aligned}$$

Therefore, $\underline{p}(e,\delta)$ decreases with δ, and thus $\overline{\pi}_1(e,\delta)$ weakly increases with δ. Finally, note that $K(p;\hat{p},\delta) = H(p;\hat{p},\delta)$, defined by (5); hence the function $\overline{\pi}_1(e,\delta)$ defined here prolongs that of Proposition 2.

The function $\overline{\pi}_1(e,\delta)$ remains relevant as long as the minmax $\tilde{\pi}^m(e)$ is sustainable. When this is not the case, then $\underline{v}$ can be replaced with the lowest symmetric equilibrium payoff, which, using Abreu's optimal symmetric penal code, is of the form $(1-\delta)\pi(p^p)+\delta\pi(p^*)$, where p^p is the highest price in $[\hat{p},e]$ satisfying $\pi^D(p^p;e)-\pi(p^p) \leq \delta[\pi(p^*)-\pi(p^p)]$, and p^* is the lowest price in $[p^m,\hat{p}]$ satisfying $\pi^D(p^*;e)-\pi(p^*) \leq \delta[\pi(p^*)-\pi(p^p)]$; we then have $\overline{\pi}_1(e,\delta) = \pi(p^*)$ and the monotonicity stems from p^* and p^p being respectively (weakly) decreasing and increasing with δ. ■

3.4 Welfare analysis

Figure 1 summarizes the analysis so far. In particular, it shows that tacit coordination is facilitated when the patents are close to being either perfect substitutes or perfect complements. In particular, tacit coordination is impossible when patents are weak substitutes; for, tacit coordination to raise price then leads users to adopt an incomplete version of the technology, and is thus quite inefficient when the essentiality parameter is not small. Collusion by contrast is feasible when patents are strong substitutes, and all the more so as they become better substitutes. Likewise, cooperation is not always feasible when patents are weak complementors, but the scope for cooperation increases as patents become more essential, and some cooperation is always possible when patents are strong complementors. In the same vein, the potential gain in profit from coordination (that is, the one that can be achieved by very patient firms) is also maximal when products are close to being either perfect substitutes (where profits would be zero absent coordination) or perfect complements (where per-firm profit would be $\pi(\hat{p})$ otherwise).

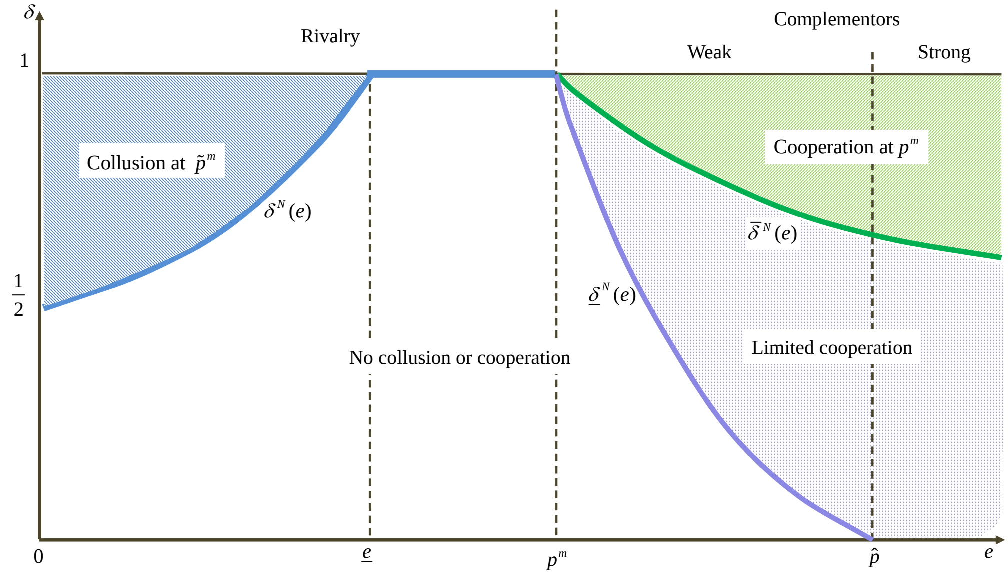

Figure 1: Tacit collusion and cooperation

We now consider the impact of tacit coordination on users and society. For the sake of exposition, we will assume that firms coordinate on the most profitable equilibrium.[10]

• Under rivalry ($e < p^m$), tacit coordination harms users and reduces total welfare: to increase their profits, firms must raise their prices, thereby inducing users to adopt an incomplete version of the technology. This adverse collusive effect is particularly potent in case of strong rivalry: Firms would then offer the complete technology at a low price in the absence of tacit coordination, and instead offer the incomplete technology at monopoly price whenever some coordination is sustainable. As firms' offerings become weaker substitutes, however, the impact of collusion is reduced, and this for two

[10]There always exists a symmetric equilibrium among those that maximize industry profit. It is therefore natural to focus on the symmetric equilibrium in which each firm obtains half of the maximal industry profit.

reasons. First, firms' prices and profits would be high even in the absence of collusion: the static Nash prices and profits increases with e (and coincide with the monopoly outcome in the borderline case where $e = p^m$). Second, the scope for collusion is also reduced ($\overline{\delta}^N(e)$ increases with e) and, when products are sufficiently weak substitutes (namely, when $e > \underline{e}$), collusion becomes so inefficient that it is no longer feasible.

• By contrast, tacit coordination is always desirable in case of complementors ($e > p^m$): to increase their profits, firms then aim at offering the complete technology at a price lower than what would prevail without cooperation. Furthermore, the scope for such desirable cooperation increases as products become more essential, and this again for two reasons. First, absent cooperation, in the case of weak complementors, double marginalization becomes more and more problematic as patents become more essential: the static Nash price increases with e in the range $e \in [p^m, \hat{p}]$. Second, more cooperation becomes feasible: $\underline{\delta}^N(e)$ decreases with e in the range $e \in [p^m, \hat{p}]$, and *some* cooperation is always feasible when $e \geq \hat{p}$.

Building on this yields:

Proposition 4 (welfare) *Suppose that firms coordinate on the most profitable equilibrium; then, compared the static Nash benchmark, tacit coordination:*

i) harms users and reduces total welfare under rivalry (i.e., when $e < p^m$).

ii) benefits users and increases total welfare in case of complementors (i.e., when $e > p^m$).

Proof. In the case of rivalry ($e < p^m$), whenever *some* collusion is sustainable, the most profitable collusive equilibrium consists in charging $\tilde{p}^m(e)$ and sharing the incomplete-technology monopoly profit; users then face an effective price $e + \tilde{p}^m(e) = e + r(e)$. Absent collusion, users face instead a total price of $2e$. Hence, collusion increases users' effective price (as $r(e) > \hat{p} > e$); it follows that tacit coordination harms users and reduces total welfare (by creating a deadweight loss, as more users are excluded, but also because only one patent generates profit).

In the case of complementors ($e > p^m$), some tacit coordination is always feasible, and the most profitable one consists in offering the complete technology at a price lower than the static Nash price. Hence, tacit coordination benefits users and increases total welfare (by reducing the deadweight loss stemming from the exclusion of users with high adoption costs). ■

4 Joint marketing

We now assume that the firms are allowed to set up a pool, providing access to the whole technology at some price $P = 2p$. If the pool can forbid independent licensing, then the pool can charge $P = P^m = 2p^m$ and each firm obtains π^m. From now on, we assume that, as is the case under current antitrust guidelines (in the US, Europe and Japan for example), independent licensing must be permitted by the pool. The pool can also offer individual licenses, and dividends can be shared according to any arbitrary rule (which can be contingent on the history of the sales made through the pool, either on a stand-alone basis or as part of the bundle). The game thus operates as follows:

1. At date 0, the firms form a pool and fix three pool prices, p_1^P, and p_2^P for the individual offerings, and $P^P \geq p_1^P + p_2^P$ for the bundle, as well as the sharing rule.

2. Then at dates $t = 1, 2, ...$, the firms non-cooperatively set prices p_i^t for their individual licenses; the profits of the pool are then shared according to the agreed rule.

It is here useful to distinguish two cases, depending on the location of e with respect to p^m.

4.1 Rivalry: $e < p^m$

The firms can of course collude as before, by not forming a pool or, equivalently, by setting pool prices at prohibitive levels ($P, p_i^P \geq V$, say); firms then collude on selling the incomplete technology if $\delta \geq \delta^N(e)$. Alternatively, they can use the pool to sell the bundle at a more profitable price.

Lemma 4 *In order to raise firms' profits, the pool must charge prices above the Nash price:* $\min\left\{\frac{P^P}{2}, p_1^P, p_2^P\right\} > e$.

Proof. See Appendix B ■

Thus, to be profitable, the pool must adopt a price $P^P > 2e$ for the complete technology, and a price $p_i^P > e$ for each patent $i = 1, 2$. This, in turn, implies that the repetition of static Nash outcome remains an equilibrium: if the other firm offers $p_j^{t+\tau} = e$ for all $\tau \geq 0$, buying an individual license from firm j (corresponding to quality-adjusted total price $2e$) strictly dominates buying from the pool, and so the pool is irrelevant (firm i will never receive any dividend from the pool); it is thus optimal for firm i to set $p_i^{t+\tau} = e$ for all $\tau \geq 0$. Furthermore, this individual licensing equilibrium, which yields $\pi(e)$, still minmaxes all firms, as in every period each firm can secure $eD\left(e + \min\left\{e, p_j^t\right\}\right) \geq \pi(e)$ by undercutting the pool and offering an individual license at a price $p_i^t = e$.

Suppose now that tacit coordination enhances profits: $\bar{v} > \pi^N = \pi(e)$, where as before $\bar{v}$ denotes the maximal average discounted equilibrium payoff. In the associated equilibrium, there exists some period $\tau \geq 0$ in which the aggregate profit, $\pi_1^\tau + \pi_2^\tau$, is at least equal to $2\bar{v}$. If users buy an incomplete version of the technology in that period, then there exists a price $\bar{p}$ such that:

$$\tilde{\pi}(\bar{p}) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v}.$$

The same reasoning as before then implies that (1) must hold, which in turn implies that collusion on $p_i^t = \tilde{p}^m$ is sustainable, and requires $\delta \geq \delta^N(e)$.

If instead users buy the complete technology in period τ, then they must buy it from the pool,[11] and the per-patent price $p^P = P^P/2$ must satisfy:

$$2\pi\left(p^P\right) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v} > 2\pi(e),$$

[11] Users would combine individual licenses only if they were offered at effective prices not exceeding e; hence, the total price P would not exceed $2e$. But $PD(P) = \pi_1^\tau + \pi_2^\tau \geq 2\bar{v} > 2\pi(e)$ implies $P > 2e$.

implying $p^P > e$. The best deviation then consists in offering an individual license at a price p^D such that users are indifferent between buying the individual license and buying from the pool:

$$(V - e) - p^D = V - 2p^P,$$

that is, $p^D = 2p^P - e\,(> e)$; the highest deviation profit is therefore:

$$\pi^D = \left(2p^P - e\right) D\left(2p^P\right) = \pi\left(p\right) + \left(p^P - e\right) D\left(2p^P\right) > \pi\left(p^P\right).$$

Thus, the price p^P is sustainable if, for $i = 1, 2$:

$$(1-\delta)\,\pi_i^\tau + \delta v_i^{\tau+1} \geq (1-\delta)\left[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)\right] + \delta\pi\left(e\right).$$

Adding these two conditions yields:

$$\begin{aligned}
(1-\delta)[\pi\left(p^P\right) + \left(p^P - e\right) D\left(2p^P\right)] + \delta\pi\left(e\right) &\leq (1-\delta)\frac{\pi_1^\tau + \pi_2^\tau}{2} + \delta\frac{v_1^{\tau+1} + v_2^{\tau+1}}{2} \\
&\leq (1-\delta)\,\pi\left(p^P\right) + \delta\pi\left(p^P\right) \\
&= \pi\left(p^P\right),
\end{aligned}$$

where the second inequality stems from using $\frac{v_1^{\tau+1}+v_2^{\tau+1}}{2} \leq \bar{v} \leq \frac{\pi_1^\tau+\pi_2^\tau}{2} = \pi\left(p^P\right)$. Conversely, if $L\left(p^P; e, \delta\right) \geq 0$, where

$$\begin{aligned}
L\left(p; e, \delta\right) &\equiv \pi(p) - (1-\delta)\left[\pi\left(p\right) + \left(p - e\right) D\left(2p\right)\right] - \delta\pi\left(e\right) \\
&= \delta p D\left(2p\right) - (1-\delta)\left(p - e\right) D\left(2p\right) - \delta e D\left(2e\right), \qquad (10)
\end{aligned}$$

then the pool price p^P is *stable*: setting the pool price $P^P = 2p^P$, together with high enough prices ($p_i^P \geq p^P$, say) and an equal profit-sharing rule, ensures that no firm has an incentive to undercut the pool; each pool member thus obtains $\pi\left(p^P\right)$.

Note that the ability to sustain a price $p^P \in (e, p^m]$ requires $\delta > 1/2$:

$$L\left(p; e, \delta\right) = (2\delta - 1)\left[pD\left(2p\right) - eD\left(2e\right)\right] + (1-\delta)\,e\left[D\left(2p\right) - D\left(2e\right)\right] < 0$$

if $\delta \leq 1/2$ and $p > e$.

Note also that $L(e; e, \delta) = 0$ for all e, and that L is concave in p if $D'' \leq 0$:

$$\frac{\partial^2 L}{\partial p^2} = (2\delta - 1)(pD(2p))'' + 4(1-\delta)eD''(2p),$$

as the first term is negative from Assumption A and $\delta > 1/2$. The sustainability of collusion then hinges on $I(e)$ being positive, where

$$I(e, \delta) \equiv \frac{\partial L}{\partial p}(e; e, \delta) = (2\delta - 1)D(2e) + 2\delta e D'(2e).$$

This leads to:

Proposition 5 (pool in the rivalry region) *Suppose $e \leq p^m$. As before, if $\delta \geq \delta^N(e)$ then the firms can sell the incomplete technology at the monopoly price $\tilde{p}^m$ and share the associated profit, $\tilde{\pi}^m$. In addition, a pool price p^P is stable if $L\left(p^P; e, \delta\right) \geq 0$, where $L(.)$ is defined by (10), in which case each pool member can obtain $\pi\left(p^P\right)$. In particular, if $D'' \leq 0$, then:*

i) Some collusion (i.e., on a stable pool price $p^P \in (e, p^m]$) is feasible if and only if

$$\delta \geq \underline{\delta}^P(e) \equiv \frac{1}{2} \frac{1}{1 + \frac{eD'(2e)}{D(2e)}}, \tag{11}$$

where the threshold $\underline{\delta}^P(e)$ is increasing in e.

ii) Perfect collusion (i.e., on a pool price $p^P = p^m$) is feasible if

$$\delta \geq \bar{\delta}^P(e) \equiv \frac{1}{2 - \frac{e}{p^m - e} \frac{D(2e) - D(2p^m)}{D(2p^m)}},$$

where the threshold $\bar{\delta}^P(e)$ is also increasing in e.

Proof. We have:

$$\frac{\partial I}{\partial \delta}(e, \delta) = 2\left[D(2e) + eD'(2e)\right] > 0,$$

where the inequality follows from $e < r(e)$ (as here $e < p^m$ $(< \hat{p})$); as

$$I(e, 1/2) = eD'(2e) < 0 < I(e, 1) = D(2e) + 2eD'(2e),$$

where the last inequality stems from $e < p^m$, then some collusion is feasible if δ is large enough, namely, $\delta \geq \underline{\delta}^P(e)$. Furthermore:

$$\frac{\partial I}{\partial e}(e, \delta) = 2(3\delta - 1)\left[D'(2e) + \frac{\delta}{3\delta - 1} 2eD''(2e)\right].$$

But $D'(2e) + 2eD''(2e) < 0$ from Assumption A and $\delta/(3\delta - 1) < 1$ from $\delta > 1/2$; and so

$$\frac{\partial I}{\partial e}(e, \delta) < 0,$$

implying that the threshold $\underline{\delta}^P(e)$ increases with e. Finally, collusion on p^m is feasible if $L(p^m; e, \delta) \geq 0$, or:

$$\delta \geq \bar{\delta}^P(e) = \frac{(p^m - e)\, D(2p^m)}{(p^m - e)\, D(2p^m) + \pi^m - \pi(e)} = \frac{1}{2 - \frac{e}{p^m - e} \frac{D(2e) - D(2p^m)}{D(2p^m)}},$$

One has

$$\frac{d\overline{\delta}^P}{de}\left(e, \overline{\delta}^P(e)\right) = -\frac{\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right)}{\frac{\partial L}{\partial \delta}\left(p^m; e, \overline{\delta}^P(e)\right)}.$$

Clearly $\partial L/\partial \delta > 0$. Furthermore

$$\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right) = [1 - \overline{\delta}^P(e)]D(2p^m) - \overline{\delta}^P(e)\pi'(e).$$

Using the fact that $L\left(p^m; e, \overline{\delta}^P(e)\right) = 0$,

$$\frac{\partial L}{\partial e}\left(p^m; e, \overline{\delta}^P(e)\right) \propto [\pi^m - \pi(e) - (p^m - e)\pi'(e)] < 0,$$

from the concavity of π. And so

$$\frac{d\overline{\delta}^P}{de} > 0.$$

■

4.2 Weak or strong complementors: $p^m \leq e$

In that case, forming a pool enables the patent holders to collude perfectly, by charging $P^m = 2p^m$ for the whole technology, *not* offering (along the collusive path) independent licenses, and sharing the profit equally. No deviation is then profitable: the best price for an individual license is $\tilde{p} = 2p^m - e$ (that is, the pool price minus a discount reflecting the essentiality of the foregone license), which is here lower than p^m (since $p^m \leq e$) and thus yields:

$$(2p^m - e) D(2p^m) < p^m D(2p^m) = \pi^m.$$

Proposition 6 (pool with complements) *With weak or strong complementors, a pool allows for perfect cooperation (even if independent licensing remains allowed) and gives each firm a profit equal to π^m.*

4.3 Do independent licenses screen in good pools and out bad ones?

As mentioned in the introduction, a striking conclusion of the Lerner-Tirole (2004) analysis is that under static competition, independent licensing is a perfect screen: It restores competition in the rivalry region (making the pool neutral, or eliminating the pool if there is an arbitrarily small fixed cost of setting up a pool) and is irrelevant in the case of (weak or strong) complementors. Thus, even in the absence of information about the essentiality parameter, allowing the pool can do no harm and is conducive to lower prices in the complementors' case. This observation is important because authorities generally lack information about the nature of interactions among patents. First, licenses may be substitutes at some prices and complements at (lower) prices. Second, historical data often are not available. Third, the nature of the interaction may change over time: patents may be complements today and become substitutes tomorrow as the environment changes. Indeed in our model an increase in V has the potential to transform complementors into rivals; and technological progress or a different field of use may reduce e, with a similar effect. Clearly, regulatory rules that require little information (or none at all as in Lerner-Tirole) have practical appeal.

The desirability of pools with independent licensing carries over under dynamic competition, with a caveat in the case of (weak) rivalry. In that case:

• Whenever collusion would be sustained in the absence of a pool, i.e. whenever $\delta \geq \delta^N(e)$, then allowing for a pool can only benefit users. First, allowing for a pool has no effect if does not allow for a more profitable collusion. Second, allowing for a pool benefits users if it allows for a more profitable collusion, since users can now buy a license for the complete technology at a price $P \leq P^m = 2p^m$, which is preferable to buying a license for the incomplete technology at a price $\tilde{p}^m(e)$.[12]

• However, when collusion could not be sustained in the absence of a pool, i.e. if $\delta < \delta^N(e)$, then allowing for a pool can harm users if it allows for some collusion, i.e., if $\delta > \underline{\delta}^P(e)$, as they then face an increase in the price from e to some price $p > e$. Note that $\underline{\delta}^P(0) = 1/2 = \delta^N(0)$ and that $\overline{\delta}^P(p^m) = 1$. A sufficient condition for the collusion region to be larger under a pool than without a pool (i.e., $\underline{\delta}^P(e) < \delta^N(e)$) is that

$$\frac{\pi(e)}{\tilde{\pi}^m(e)} > -\frac{eD'(2e)}{D(2e)}.$$

Straightforward computations show that this is indeed the case when demand is linear, i.e., when the opportunity cost distribution is uniform.

By contrast, a pool is always desirable in the case of weak or strong complementors, as it then allows for perfect cooperation, which benefits users as well as the firms: in the absence of the pool, the firms would either not cooperate and thus set $p = p^N = \min\{\hat{p}, e\}$ (which here exceeds p^m, as $e > p^m$), or cooperate, in which case they license both technologies at some price $p \in [p^m, p^N)$; thanks to the pool, the firms can instead sustain

[12]Because $r' > -1$, $e + \tilde{p}^m = e + r(e) > 0 + r(0) = P^m = 2p^m$; hence users are better off when they buy both technologies at price p^m than when being offered a single technology at price $\tilde{p}^m$.

a (weakly) lower price, p^m. This leads to the following proposition (where "price" stands for "quality-adjusted price"):

Proposition 7 (screening through independent licensing) *Independent licensing provides a useful but imperfect screen:*

i) A pool with independent licensing has no impact if $\delta < \min\left\{\delta^N(e), \underline{\delta}^P(e)\right\}$ in case or rivalry ($e < p^m$), or if $\delta \geq \bar{\delta}^N(e)$ in case of complementors ($e > p^m$). In all other cases, a pool with independent licensing increases profit and:

- *in case of complementors, it always lowers price;*
- *in case of rivalry, it lowers price if $\delta \geq \delta^N(e)$, but raises price otherwise.*

ii) Appending independent licensing to a pool is always welfare-enhancing; unlike the case of low discount factor, however, it is not a perfect screen, as it allows rival firms to raise prices when (and only when) $\underline{\delta}^P(e) < \delta < \delta^N(e)$.

The analysis of this section is summarized in Figure 2, in the case of linear demand.

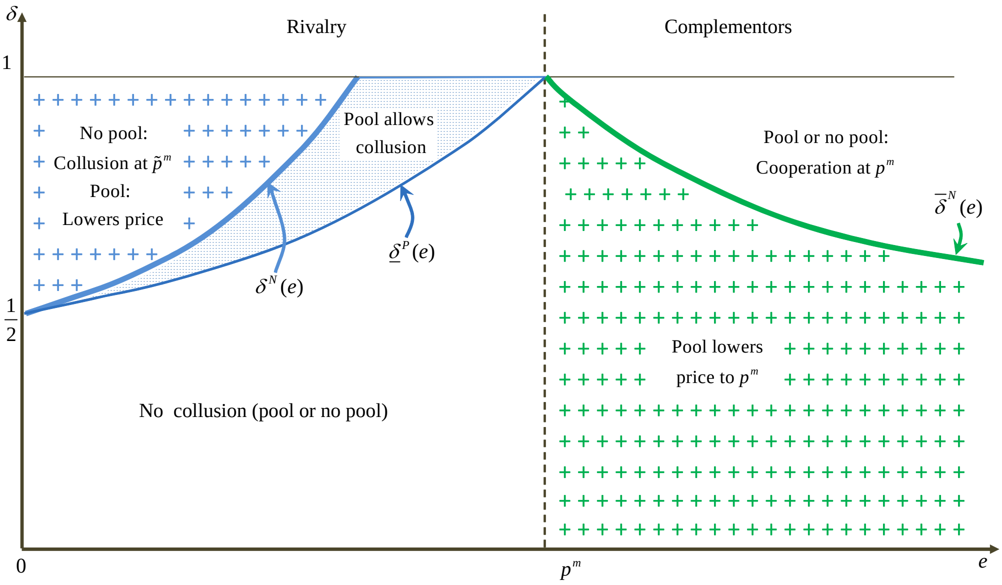

Figure 2: Impact of a pool
(+: beneficial; -: welfare reducing; blank: neutral)

5 A perfect screen for the validation of joint marketing alliances

As we earlier discussed, the independent licensing requirement screens in good pools and out bad ones when a low enough discount factor prevents tacit coordination. The requirement is a perfect screen in that it enables an unconcerned approval of pools even in the absence of perfect estimates of essentiality.

With patient, coordinating firms, the independent licensing provision still does a reasonable job: it preserves the pool's ability to lower price under weak or strong complements; and with substitutes, it prevents the collusion inefficiency stemming from selling an incomplete technology at a high quality-adjusted price. However, as depicted in Figure 2, in the case of weak substitutes there exists a region in which the pool can facilitate collusion. This is because, by eliminating the inefficiency from selling an incomplete technology (the corollary of an attempt to raise price in the absence of a pool), the pool makes high prices more attractive. Thus, unless the authorities are reasonably convinced that firms are complementors, they run the risk of approving a JMA when firms are weak substitutes, generating some welfare loss along the way.

Tacit coordination thus poses a new challenge: Independent licensing no longer is a perfect screen. This section shows that another information-free instrument, namely, requiring unbundling and pass-through, can be appended so as to re-establish the perfect screen property, and that both instruments are needed to achieve this.

Under the unbundling and pass-through requirement:

1. the pool prices, P^P for the bundle and p_1^P, p_2^P for the individual offerings, respectively, must satisfy[13]

$$P^P \geq p_1^P + p_2^P;$$

2. the dividend of an IP holder is equal to the revenue generated by its technology; that is, firm i's dividend is $p_i^P q_i^P$, where q_i^P denotes the number of users buying patent i from the pool (as part of the bundle, or on a stand-alone basis).

In essence, and because we ignore transaction costs associated with multiple licenses, appending the unbundling & pass-through requirement to independent licensing makes the pool act merely as a price-cap setter. Each firm remains free to undercut the pool, but cannot sell its technology at a price exceeding the pool's price. The situation is thus formally the same as under independent licensing, but for the fact that each firm i cannot charge

[13] While we model unbundling as imposing super-additivity on the price structure, in effect it amounts to strict additivity, as users would otherwise combine the individual offerings and pay $p_1^P + p_2^P$ rather than P^P for the bundle.

more than p_i^P for its technology.[14]

In the case of rivalry, setting a pool price below the static Nash level again cannot allow for any profitable collusion. Indeed, if $p_i^P \leq p^N = e$ for some patent $i \in \{1,2\}$, then in every period $t = 1, 2, ...$, the effective price $\underline{p}_i^t = \min\left\{p_i^P, p_i^t\right\}$, where p_i^t denotes the price charged by firm i in period t, will necessarily lie below e_i, which in turn implies that aggregate profits do not exceed that of the static Nash:

- If the effective price for patent j, $\underline{p}_j^t = \min\left\{p_j^P, p_j^t\right\}$, satisfies $\underline{p}_j^t \leq e$, then the industry profit is $\left(\underline{p}_i^t + \underline{p}_j^t\right) D\left(\underline{p}_i^t + \underline{p}_j^t\right) \leq 2\pi^N = 2eD(2e)$, as $PD(P)$ is concave and maximal for $P = 2p^m > 2e \geq \underline{p}_i^t + \underline{p}_j^t$.

- If instead $\underline{p}_j^t > e$, then the industry profit is $\underline{p}_i^t D\left(\underline{p}_i^t + e\right) \leq \left(\underline{p}_i^t + e\right) D\left(\underline{p}_i^t + e\right) \leq 2\pi^N$, as $\underline{p}_i^t + e \leq 2e$.

Therefore, to be profitable, the pool must charge individual prices above the static Nash level ($p_1^P, p_2^P > e$, and thus $P^P > P^N = 2e$), in order to sustain effective prices that are themselves above the static Nash level ($\underline{p}_1^t, \underline{p}_2^t > e$). But then, intuitively, a deviating member will aim at lowering its price rather than raising it, which in turn implies that profitable deviations are the same as in the absence of a pool. As the pool cannot make punishments more severe either (these are already maximal without a pool, as $\pi^N = \underline{\pi} = eD(2e)$), it follows that the pool cannot increase the scope for collusion.

By contrast, in the case of complementors:

- It is profitable to set the pool's prices below the static Nash level, so as to maintain effective prices closer to the cooperative level.

- Such prices also rule out any profitable deviation, as the deviator would need to raise the price above the pool's level.

Building on these insights yields the following Proposition.

[14] In practice, however, allowing the pool to offer the bundle at price $P\left(= p_1^P + p_2^P\right)$ can help save on transaction costs.

Proposition 8 (perfect screen) *Appending the independent licensing and the unbundling & pass-through requirements to the pool*

(i) has no impact on the pool's strategy and operations, and therefore raises welfare relative to the absence of pool, if firms are complementors;

(ii) restores the no-pool outcome, thus making the pool welfare-neutral, under rivalry.

Proof. See Appendix C. ■

Note that the unbundling & pass-through requirement alone is an imperfect screen. For, consider the case of perfect substitutes; then in the absence of independent licenses, the firms can obtain the monopoly profit through a pool with unbundling (by setting $p_1^P = p_2^P = \tilde{p}^m(0) = 2p^m$) even when the discount factor is low, whereas independent licensing guarantees perfectly competitive pricing.[15]

6 Investment incentives

Allowing pools can also foster investment incentives. We now show that subjecting pools to independent licensing and unbundling ensures that pools benefit users, both through greater investment and lower prices, and increase total welfare – even in the presence of business stealing. To see this, suppose that initially only one piece of technology is available, and thus offered at a price $\tilde{p}^m(e)$, and consider an innovator's incentive to invest $I/(1-\delta)$ in order to develop the other piece of technology and thereby enter the IP market with a second patent.

From the above analysis, allowing for a pool (subject to independent licensing and unbundling) has no effect on the profits that can be achieved in case of entry, and thus has no effect on investment, in case of rivalry ($e \leq p^m$), as the pool then does not affect the scope for collusion. The "perfect screen"

[15] A stand-alone unbundling requirement also prevents the pool from lowering the collusive price (by allowing more efficient collusion), when $\delta \geq \delta^N(e)$ in case of rivalry.

introduced in the previous section thus continues to make the pool welfare-neutral.

In case of complementors, allowing for a pool in case of entry has no effect either when the firms can already perfectly collude even without it (that is, when $\delta \geq \bar{\delta}^N(e)$), but otherwise allows the firms to lower prices, from some $p \in (p^m, p^N]$ down to p^m, and thus to increase profit, from $\pi(p)$ to π^m. Therefore:

- If $I > \pi^m$, there is no investment anyway, and thus the pool has again no impact.
- If $I < \pi(p)$, investment occurs both with and without the pool; allowing the pool however benefits users, whose surplus increase from $S(2p)$ to $S^m > S(2p)$.
- Finally, if $\pi(p) < I < \pi^m$, then the pool triggers entry, which benefits users whose surplus increase from $\tilde{S}^m$ (in the absence of entry) to S^m. Furthermore, despite some business stealing, the pool also increases total welfare; letting

$$W(2p) = S(2p) + 2p\pi(p) = \int_0^{D(2p)} \left[V - F^{-1}(q)\right] dq$$

denote total welfare, the impact of the pool on this welfare is given by:

$$\begin{aligned}
\Delta W^P &\equiv [W^m - I] - \tilde{W}^m \\
&= [\pi^m - I] + [W^m - \pi^m] - \tilde{W}^m \\
&\geq [W^m - \pi^m] - \tilde{W}^m \\
&= \int_0^{D(2p^m)} \left[V - p^m - F^{-1}(q)\right] dq - \int_0^{D(e+\tilde{p}^m(e))} \left[V - e - F^{-1}(q)\right] dq \\
&= \int_0^{D(e+\tilde{p}^m(e))} (e - p^m)\, dq + \int_{D(e+\tilde{p}^m(e))}^{D(2p^m)} \left[V - p^m - F^{-1}(q)\right] dq \\
&> 0,
\end{aligned}$$

where the inequality follows from $e > p^m$ and $V - p^m > F^{-1}(q)$ for $q < D(2p^m)$.

This can be summarized as follows:

Proposition 9 (investment) *Mandating independent licensing and unbundling & pass-through continues to make the pool welfare-neutral in case of rivalry, while allowing the pool to benefit users and society, both through lower prices and greater investment, in the case of complementors.*

7 Extensions

7.1 Asymmetric offerings

Suppose now that essentiality differs across firms: The technology has value $V - e_i$ if the user buys only patent j (for $i \neq j \in \{1,2\}$); without loss of generality, suppose that $e_1 \geq e_2$.

In the absence of tacit coordination,[16] firm i solves (for $i \neq j \in \{1,2\}$)

$$\max_{p_i \leq e_i} p_i D\left(p_j + p_i\right),$$

and thus charges $p_i = \min\left\{e_i, r\left(p_j\right)\right\}$. Therefore, the static Nash equilibrium $\left(p_1^N, p_2^N\right)$ can be of three types:[17]

- $\left(p_1^N, p_2^N\right) = (e_1, e_2)$; this arises when both firms' best responses are constrained, i.e., $e_i < r\left(e_j\right)$ for $i \neq j \in \{1,2\}$;
- $\left(p_1^N, p_2^N\right) = \left(r\left(e_2\right), e_2\right)$; this arises when firm 1 is no longer constrained $\left(e_1 > r\left(e_2\right)\right)$ but the other one is still constrained $\left(e_2 < r\left(r\left(e_2\right)\right)\right)$, which amounts to $e_2 < \hat{p}$);
- $\left(p_1^N, p_2^N\right) = (\hat{p}, \hat{p})$; this arises when no firm is constrained ($e_1 > r\left(e_2\right)$ and $e_2 > \hat{p}$).

[16] See Lerner-Tirole (2004), to which we refer for more detail and uniqueness.

[17] Note that $e_1 \geq e_2$ and $r' < 0$ imply $r\left(e_2\right) \geq r\left(e_1\right)$. Hence, $e_1 > r\left(e_2\right)$ implies $e_1 > r\left(e_2\right)$; as $r' > -1$, this in turn implies

$$e_1 + r\left(e_1\right) > r\left(e_1\right) + r\left(r\left(e_1\right)\right),$$

or $r\left(r\left(e_1\right)\right) < e_1$. Hence, the configuration $\left(p_1, p_2\right) = \left(e_1, r\left(e_1\right)\right)$ cannot arise – that is, firm 1 cannot be constrained if firm 2 is not.

Interestingly, in this static Nash equilibrium the total price $P^N = p_1^N + p_2^N$ exceeds the monopoly price $P^m = 2p^m$ whenever firm 1's best response is unconstrained (that is, $e_1 > r(e_2)$) and $e_2 > 0$ (implying double marginalization).[18] Furthermore, even when firm 1 is constrained (i.e. when $e_1 < r(e_2)$), the total Nash price P^N $(= e_1 + e_2)$ still exceeds the monopoly price as long as $e_1 + e_2 > P^m$; therefore:

- *rivalry* $\left(P^N < P^m\right)$ prevails when $e_1 + e_2 < P^m$,[19] in which case tacit coordination will aim at raising prices;
- conversely, the two patents are *complementors* $\left(P^N > P^m\right)$ whenever $e_1 + e_2 > P^m$, in which case tacit coordination will aim at reducing prices.

The following Proposition shows that combining the independent licensing and the unbundling & pass-through requirements still provides a perfect screen:

Proposition 10 (perfect screen) *Appending the independent licensing and the unbundling & pass-through requirements to the pool still provides a perfect screen, even with asymmetric offerings: The pool*

(i) allows firms to cooperate perfectly when they are complementors;

(ii) but does not affect the scope for collusion under rivalry.

Proof. See Appendix C. ■

7.2 Multiple offerings

[To be completed.]

[18] As $r' > -1$, we have:

$$P^m = r(0) < \min\{2\hat{p} = \hat{p} + r(\hat{p}), e_2 + r(e_2)\}.$$

[19] This condition generalizes the one obtained under symmetry (namely, $e < p^m$, or $2e < P^m$) and implies $e_1 < r(e_2)$, $e_2 < r(e_1)$.

7.3 Strategic JMA

We have so far assumed that the JMA set prices once and for all. But consider the following possibility: Firms offer substitutes, and by simple majority (one vote if $n = 2$) can decide to set the bundle's price forever at $P = 0$; they do not exercise this give-away option as long as no-one deviates from the collusive price, but all vote for setting forever $P = 0$ after a deviation. Because each individual vote is irrelevant after a deviation (the technology will be given away regardless of one's vote), such behavior is individually optimal. And, crucially, payoffs below the no-JMA minmax can be enforced thanks to the pool. And so, more collusion can be sustained than in the absence of pool.

As implausible as this collective hara-kiri example sounds (if only because the firms employ a weakly dominated strategy when they vote in favor of giving away the technology), it makes the theoretical point that a JMA might be used to increase the discipline on members. One can react to this point in several (consistent) ways. First, one can take it as a warning that antitrust authorities should keep an eye on instances in which a pool drastically lowers the price in reaction to low prices on individual licenses. Second, one can take a more ex-ante view of using regulation to prevent strategic pools from jeopardizing the information-free screens that were unveiled in this paper:

• *Charter regulation.* To prevent punishments below the minmax, it suffices to require unanimity among JMA members to change the pricing of joint offerings. In that case, each firm is able to guarantee itself $\pi(e)$ by refusing to renegotiate the initial deal. So, harsher punishments than the no-JMA minmax are infeasible.

• *Constraints on price flexibility.* In the stationary framework of this paper, a straightforward rule prevents the JMA from implementing such threats: it suffices that the JMA be prohibited from adjusting its price. This requirement however may be unreasonable in many instances, to the extent that members still face uncertainty about the nature of demand. Suppose for example that the initial demand for the technology results from distribution $F(\theta)$; with Poisson arrival rate λ, the final demand, corresponding to distribution $F(\theta + \omega)$, will be realized. Taking, say, the case of complementors, assuming that the hazard rate f/F is decreasing, the JMA will want to raise (lower) its bundle price when ω is realized and positive (negative). Thus flexibility might be desirable. In this simple environment, one can imagine a rule allowing limited price flexibility through a ratchet: the price, once lowered, cannot be raised again. In this case, and ruling out weakly dominated strategies (or imposing a unanimity rule), no-one would vote for a pool price below the minmax outcome.

The broader lesson is of course that some antitrust attention must be paid to JMA enforcement of punishments. Such enforcements however seem a bit far-fetched and rather easy to prevent, so we do not view them as a significant concern.

8 Concluding remarks

Competition policy guidelines and enforcement require a good understanding of factors that facilitate tacit collusion and cooperation. We were able to fully characterize optimal coordination among firms with an arbitrary pattern of rivalry or complementarity, parameterized in the context of our nested demand specification by one (or several) essentiality parameters. Coordination is easiest to achieve when offerings are strong substitutes or complements and most difficult in the intermediate range of weak rivalry and weak complementors.

The second and key contribution of the paper was the study of the treatment of joint marketing alliances such as patent pools. Critically, antitrust authorities must be able to screen in good (price-reducing) JMAs and out bad (price-increasing) ones. Such screening is important both to reduce the deadweight loss of under-usage and to provide innovators and potential entrants with incentives to bring to market essential new products, which create user value rather than steal rivals' business.

Authorities usually lack knowledge about the pattern of demand and therefore do not know whether such alliances are likely to be welfare-enhancing. It is therefore useful to devise acceptance rules that rely little or not at all on authorities' knowledge about product essentiality. We considered two information-free requirements: independent licensing (maintained ownership and ability to market outside the JMA), and unbundling & pass-through (super-additivity of the JMA's price structure, combined with the absence of cross-subsidy within the JMA).

Focusing initially on the firms' pricing behavior, we established three results. While independent licensing was known to be a perfect screen in the absence of tacit coordination, we showed that it is no longer so when rivals with weak substitutes take advantage of the JMA to raise price. But appending the unbundling & pass-through requirement re-establishes the perfect screen property. Finally, we showed that the two instruments still provide a perfect screen even with asymmetric offerings.

Perfect screens not only ensure that JMAs correct the under-usage inefficiency in the right direction. They were shown to further affect investment incentives in the right way, in particular by boosting incentives to bring essential innovations to market.

While this analysis brings the treatment of JMAs into safer territory, more research is desirable. The first area concerns generalizations of our nested demand function. While this specification affords much convenience, it also involves some restrictions. In particular, in the absence of separability between user characteristics and extent of essentiality, partners in a JMA would want to engage in second-degree price discrimination so as to better extract user rents. The formulation and properties of the unbundling requirement would then be an interesting alley for research; as we know from Maskin-Riley (1984) and Mussa-Rosen (1978), non-linear pricing often involves price discounts and thus violates the unbundling requirement. And even in the absence of JMA, it would be worthwhile producing a theory of tacit collusion in which consumers compose their basket à la carte, with different users selecting different baskets.

Second, the study of the so-called facilitating practices is a standard theme in antitrust economics. These include practices that enhance transparency, such as information exchanges through industry associations, as well as marketing practices such as advanced price announcements, product categorization that reduces the number of relevant prices, resale price maintenance, and so forth. The impact of such practices has been derived entirely in a perfect-substitutes world. Like in this paper, the extension to arbitrary degrees of substitutability or complementarity would much enhance our knowledge of their likely impact.

Appendix

A Analysis of best response functions

The function $r(p)$ is characterized by the FOC

$$D(p + r(p)) + r(p)\, D'(p + r(p)) = 0,$$

and thus:

$$r'(p) = -\frac{D'(p + r(p)) + r(p)\, D''(p + r(p))}{2D'(p + r(p)) + r(p)\, D''(p + r(p))},$$

where the numerator and the denominator are both negative under Assumption A: this is obvious is $D''(p + r(p)) \leq 0$ and, if $D''(p + r(p)) > 0$, then

$$D'(p + r(p)) + r(p)\, D''(p + r(p)) < D'(p + r(p)) + (p + r(p))\, D''(p + r(p)),$$

where the right-hand side is negative under Assumption A; therefore, $r'(p) < 0$. Since the denominator is obviously more negative, we also have $r'(p) > -1$.

We now use this function to characterize $p^r(p)$, the actual best response of an firm to its rival's price p. If the rival charges a price $p \leq e$, then users (weakly) favor relying on both technologies whenever they buy the firm's own technology, and they are willing to do so as long only if the firm does not charge more than e; therefore, the firm's best response is given by:

$$p^r(p) = \arg\max_{\tilde{p} \leq e} \tilde{p} D(p + \tilde{p}) = \min\{r(p), e\}.$$

If instead the rival charges a price $p > e$, then users use at most one technology, and look for the lowest price; therefore, the firm's best response is given by:

$$p^r(p) = \arg\max_{\tilde{p} < p} \tilde{\pi}(\tilde{p}) = \arg\max_{\tilde{p} < p} \tilde{p} D(e + \tilde{p}) = \min\{r(e), p_-\},$$

where p_- stands for slightly undercutting the rival's price p. It follows that the Nash equilibrium is unique, symmetric, and consists for both firms in charging $p^N = \{e, \hat{p}\}$; indeed:

- if $e \geq \hat{p}$, then $\hat{p} = r(\hat{p}) \leq e$ implies $p^r(\hat{p}) = \hat{p}$;
- if instead $e < \hat{p}$, then $r(e) > \hat{p} > e$ implies $p^r(e) = e$.

Finally, define $P^m \equiv 2p^m$ and $\hat{P} \equiv 2\hat{p}$. By construction, we have

$$P^m = 2p^m = \arg\max_P PD(P),$$

whereas

$$\hat{p} = r(p) = \arg\max_p pD(\hat{p} + p),$$

implies

$$\hat{P} = \arg\max_P (P - \hat{p}) D(P).$$

A revealed preference argument then implies $\hat{P} > P^m$, and thus $\hat{p} > p^m$.

B Proof of Lemma 4

Suppose that $\min\left\{\frac{P^P}{2}, p_1^P, p_2^P\right\} \leq e$, and consider a period t, with individual licenses offered at prices p_1^t and p_2^t. Let $\underline{p}_i^t = \min\left\{p_i^P, p_i^t\right\}$ denote the effective price for patent i, and $\underline{p}^t = \min\left\{\underline{p}_1^t, \underline{p}_2^t\right\}$ denote the lower one.

• Users buy the complete technology from the pool only if $P^P \leq \underline{p}^t + e$, which in turn implies $P^P \leq 2e$ (as $\underline{p}^t \leq \min\left\{p_1^P, p_2^P\right\}$, and by assumption, either $\frac{P^P}{2} \leq e$, or $\min\left\{p_1^P, p_2^P\right\} \leq e$); the industry profit is then $P^P D\left(P^P\right) \leq 2\pi^N = 2\pi(e)$, as the aggregate profit function $PD(P)$ is concave and maximal for $2p^m > 2e \geq P^P$.

• Users buy the complete technology by combining individual licenses only if $\underline{p}_i \leq e$ for $i = 1, 2$, in which case $\underline{p}_1 + \underline{p}_2 \leq 2e$ and the industry profit is $\left(\underline{p}_1 + \underline{p}_2\right) D\left(\underline{p}_1 + \underline{p}_2\right) \leq 2\pi^N$.

• Finally, users buy an incomplete version of the technology only if $\underline{p}^t + e \leq P^P$, which in turn implies $\underline{p}^t \leq e$ (as then $\underline{p}^t \leq \min\left\{p_1^P, p_2^P\right\}, P^P - e$, and by assumption, either $\min\left\{p_1^P, p_2^P\right\} \leq e$, or $P^P \leq 2e$); the industry profit is then $\underline{p}^t D\left(\underline{p}^t + e\right) \leq \left(\underline{p}^t + e\right) D\left(\underline{p}^t + e\right) \leq 2\pi^N$, as $\underline{p}^t + e \leq 2e$.

Therefore, the industry profit can never exceed the static Nash level.

C Proof of Propositions 8 and 10

We provide here a proof for Propositions 8 and 10. That is, we show below that, with symmetric or asymmetric offerings, when subject to the independent licensing and the unbundling & pass-through requirements, the pool:

i) has no impact on the pool's strategy and operations, and therefore raises welfare relative to the absence of pool, if firms are complementors;

ii) restores the no-pool outcome, thus making the pool welfare-neutral, under rivalry.

The case of complementors (part i), where $e_1 + e_2 > P^m = 2p^m$) is straightforward: pool members can generate the cooperative profits, $2\pi^m$, by charging $P^P = P^m$ for the bundle. Under unbundling, adopting any prices $\left(p_1^P, p_2^P\right)$ satisfying $p_1^p + p_2^p = P^m$ and $p_i^p < e_i$ (e.g., $p_i = p^m + \frac{e_i - e_j}{2}$ – in the symmetric case, $p_i = p^m$ would do) ensures that no user is attracted by the unbundled options; unbundling is irrelevant. Finally, no member has an incentive to undercut the pool's prices, as this would require offering $p_i < p_i^P$ and thus would yield $p_i D\left(p_j^P + p_i\right) < p_i^P D\left(p_j^P + p_i^P\right)$, as $p_1^P + p_2^P = P^m = r(0) + 0 < p_j^P + r\left(p_j^P\right)$ implies $r\left(p_j^P\right) > p_i^P$, for $i \neq j \in \{1, 2\}$.[20]

We now turn to the case of rivalry (part ii), where $e_1 + e_2 < P^m$). We have already noted in the text that the pool must charge prices above the static Nash level to be profitable. Furthermore, offering a price $p_i^P > V$ would be irrelevant. Thus, without loss of generality, suppose now that the pool adopted a price $p_i^P \in (e_i, V]$ for each patent $i = 1, 2$ (together with a price $P^P \geq p_1^P + p_2^P$ for the bundle). We first show that the minmax profits: (a) are the same as without a pool, and (b) can be sustained by the repetition of the static Nash outcome, (e_1, e_2). To establish (a), it suffices to note that the minmaxing strategy $p_j = e_j$ remains available to firm i's rival, and firm i's best response, $p_i = e_i$, also remains available: by setting a price $p_k = e_k$, firm $k \in \{1, 2\}$ generates an effective price $\underline{p}_k \equiv \min\left\{p_k, p_k^P\right\} = e_k$. To establish

[20]Without the pass-through requirement, a firm might have an incentive to undercut the pool, so as to secure all the revenue from its technology, and also obtain a share in the pool's revenue from the other technology.

(b), it suffices to note that the static Nash outcome (e_1, e_2) remains feasible, and that deviations are only more limited than in the absence of a pool (as the firms can no longer induce an effective price higher than the pool's price for their patents).

We now show that any collusion sustainable with the pool is also sustainable without a pool. For the sake of exposition, we restrict attention to pure-strategy equilibria, but the reasoning extends to mixed strategies,[21] at the cost of significant notational complexity. Recall that the set of pure-strategy equilibrium payoffs can be characterized as the largest self-generating set of payoffs, where, as minmax profits are sustainable, a self-generating set of payoffs W is such that, for any payoff (π_1, π_2) in W, there exists a continuation payoff (π_1^*, π_2^*) in W and a price profile $(p_1^*, p_2^*) \in \mathcal{P}_1 \times \mathcal{P}_2$, where $\mathcal{P}_i$ is the set of admissible prices for firm i (more on this below), that satisfy, for $i \neq j \in \{1, 2\}$:

$$\pi_i = (1-\delta)\,\pi_i\left(p_i^*, p_j^*\right) + \delta\pi_i^* \geq \max_{p_i \in \mathcal{P}_i} \pi_i\left(p_i, p_j^*\right) + \delta\underline{\pi}_i, \tag{12}$$

where $\pi_i\,(p_i, p_j)$ denotes firm i's profit when the two patents are offered at prices p_1 and p_2. To establish that the equilibrium payoffs generated by a pool are also equilibrium payoffs without the pool, it suffices to show that any self-generating set in the former situation is also a self-generating set in the latter situation.

In the absence of a pool, without loss of generality the set of admissible prices for firm i is $\mathcal{P}_i^N \equiv [0, V]$; when instead a pool offers a price p_i^P for patent i, then the admissible set for the *effective price* $\underline{p}_i = \min\left\{p_i, p_i^P\right\}$ is $\mathcal{P}_i^P \equiv \left[0, p_i^P\right]$. Consider now a self-generating set W^P for given pool prices $\left(p_1^P, p_2^P\right)$ satisfying $p_i^P \in (e_i, V]$ for $i = 1, 2$, and a given payoff $(\pi_1, \pi_2) \in W^P$,

[21] It is straightforward to apply the reasoning to public mixed strategies (i.e., when players randomize on the basis on public signals). In case of private mixed strategies (where only the realization is observed by the other players, and there is thus imperfect monitoring), the reasoning applies to public perfect equilibria, where strategies are based on public history (i.e., players do not condition their future decisions on their private choice of a lottery, but only on its realization). See e.g., Mailath and Samuelson (2006), Chapter 2 for the case of pure strategies and Chapter 7 for the extension to imperfect monitoring.

with associated payoff $(\pi_1^*, \pi_2^*) \in W^P$ and effective price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{P}_1^P \times \mathcal{P}_2^P$ satisfying

$$\pi_i = (1-\delta)\,\pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right) + \delta\pi_i^* \geq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) + \delta\underline{\pi}_i. \tag{13}$$

By construction, the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ also belongs to $\mathcal{P}_1^N \times \mathcal{P}_2^N$. However, the gain from a deviation may be lower than in the absence of a pool, as the set of admissible deviating prices is smaller. To conclude the proof, we now show that, for any $\left(\underline{p}_1^*, \underline{p}_2^*\right) \in \mathcal{P}_1^P \times \mathcal{P}_2^P$ satisfying (12), there exists $(p_1^*, p_2^*) \in \mathcal{P}_1^N \times \mathcal{P}_2^N$ satisfying

$$\pi_i = (1-\delta)\,\pi_i\left(p_i^*, p_j^*\right) + \delta\pi_i^* \geq \max_{p_i \in \mathcal{P}_i^N} \pi_i\left(p_i, p_j^*\right) + \delta\underline{\pi}_i. \tag{14}$$

For this, it suffices to exhibit a profile $(p_1^*, p_2^*) \in \mathcal{P}_1^N \times \mathcal{P}_2^N$ yielding the same profits (i.e., $\pi_i\left(p_i^*, p_j^*\right) = \pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right)$ for $i = 1, 2$) without increasing the scope for deviations (i.e., $\max_{p_i \in \mathcal{P}_i^N} \pi_i\left(p_i, p_j^*\right) \leq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right)$ for $i = 1, 2$). We can distinguish four cases for the associated price profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$:

Case a: $\underline{p}_1^* \leq e_1, \underline{p}_2^* \leq e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$; as firm i's profit from deviating to p_i is then given by

$$\pi_i\left(p_i, \underline{p}_j^*\right) = \begin{cases} p_i D\left(\underline{p}_j^* + p_i\right) & \text{if } p_i \leq e_i \\ 0 \text{ otherwise} & \end{cases},$$

the best deviation is

$$\arg\max_{p_i \leq e_i} p_i D\left(\underline{p}_j^* + p_i\right) = e_i,$$

which belongs to both $\mathcal{P}_i^N$ and $\mathcal{P}_i^P$. Hence, $\max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i\left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{P}_i^N} \pi_i\left(p_i, \underline{p}_j^*\right)$.

Case b: $\underline{p}_i^* - e_i \leq 0 < \underline{p}_j^* - e_j$, for $i \neq j \in \{1, 2\}$. In that case, the profile $\left(\underline{p}_1^*, \underline{p}_2^*\right)$ yields profits $\pi_j\left(\underline{p}_j^*, \underline{p}_i^*\right) = 0$ and $\pi_i\left(\underline{p}_i^*, \underline{p}_j^*\right) = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, and best deviations are respectively given by:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, \underline{p}_i^*\right) &= \arg\max_{p_j \leq e_j} p_j D\left(\underline{p}_i^* + p_j\right) = e_j, \\
\arg\max_{p_i} \pi_i \left(p_i, \underline{p}_j^*\right) &= \arg\max_{p_i \leq \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{\underline{p}_j^* + e_i - e_j, p_i^m\right\}.
\end{aligned}$$

As $e_j \in \mathcal{P}_j^N \cap \mathcal{P}_j^P$, $\max_{\underline{p}_j \in \mathcal{P}_j^P} \pi_j \left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \in \mathcal{P}_j^N} \pi_j \left(p_j, \underline{p}_i^*\right)$. Therefore, if $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^m\right\} \leq p_i^P$ (and thus $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^m\right\} \in \mathcal{P}_i^N \cap \mathcal{P}_i^P$), we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as then we also have $\max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \in \mathcal{P}_i^N} \pi_i \left(p_i, \underline{p}_j^*\right)$. If instead $\min\left\{\underline{p}_j^* + e_i - e_j, p_i^m\right\} > p_i^P$, then we can pick $p_i^* = \underline{p}_i^*$ and $p_j^* \in \left(e_j, e_j + p_i^P - e_i\right)$:[22] the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, and, as the best deviations are the same, with or without the pool:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, p_i^*\right) &= \arg\max_{p_j} \pi_j \left(p_j, \underline{p}_i^*\right) = e_j \in \mathcal{P}_j^N \cap \mathcal{P}_j^P, \\
\arg\max_{p_i} \pi_i \left(p_i, p_j^*\right) &= \arg\max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_j^* + e_i - e_j, p_i^m\right\} \in \mathcal{P}_i^N \cap \mathcal{P}_i^P,
\end{aligned}$$

as $\min\left\{p_j^* + e_i - e_j, p_i^m\right\} \leq p_j^* + e_i - e_j < p_i^P$.

Case c: $0 < \underline{p}_i^* - e_i = \underline{p}_2^* - e_2$. In that case, we can pick $(p_1^*, p_2^*) = \left(\underline{p}_1^*, \underline{p}_2^*\right)$, as best deviations consist in undercutting the other firm, and this is feasible with or without the pool.

Case d: $0 < \underline{p}_1^* - e_1 < \underline{p}_j^* - e_j$, for $i \neq j \in \{1, 2\}$. In that case, the same payoff could be sustained with $p_i^* = \underline{p}_i^*$ and $p_j^* = \underline{p}_i^* + e_j - e_i \left(< \underline{p}_j^*\right)$, with the convention that technology adopters, being indifferent between buying a single license from i or from j, all favor i: the profile (p_1^*, p_2^*) yields the same profits as $\left(\underline{p}_1^*, \underline{p}_2^*\right)$, $\pi_j = 0$ and $\pi_i = \underline{p}_i^* D\left(e_j + \underline{p}_i^*\right)$, but reduces the scope for

[22] This interval is not empty, as $p_i^P > e_i$ by assumption.

deviations, which now boil down to undercutting the rival:

$$\begin{aligned}
\max_{p_j \in \mathcal{P}_j^N} \pi_j \left(p_j, p_i^*\right) &= \max_{\underline{p}_j \in \mathcal{P}_j^P} \pi_j \left(\underline{p}_j, \underline{p}_i^*\right) = \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_j + p_j\right), \\
\max_{p_i \in \mathcal{P}_i^N} \pi_i \left(p_i, p_j^*\right) &= \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) \leq \max_{\underline{p}_i \in \mathcal{P}_i^P} \pi_i \left(\underline{p}_i, \underline{p}_j^*\right) = \max_{p_i \leq \underline{p}_j^* + e_i - e_j} p_i D\left(e_j + p_i\right).
\end{aligned}$$

This moreover implies that, as in case c above, these best deviations were already feasible with the pool. Indeed, as $p_k^* = p_h^* + e_k - e_h$, for $h \neq k \in \{1, 2\}$, we have:

$$\begin{aligned}
\arg\max_{p_j} \pi_j \left(p_j, p_i^*\right) &= \arg\max_{\underline{p}_j} \pi_j \left(\underline{p}_j, \underline{p}_i^*\right) = \arg \max_{p_j \leq \underline{p}_i^* + e_j - e_i} p_j D\left(e_j + p_j\right) = \min\left\{p_j^*, p_j^m\right\}, \\
\arg\max_{p_i} \pi_i \left(p_i, p_j^*\right) &= \arg \max_{p_i \leq p_j^* + e_i - e_j} p_i D\left(e_j + p_i\right) = \min\left\{p_i^*, p_i^m\right\},
\end{aligned}$$

where $\min\left\{p_j^*, p_j^m\right\} \in \mathcal{P}_j^N \cap \mathcal{P}_j^P$, as $\min\left\{p_j^*, p_j^m\right\} \leq p_j^* < \underline{p}_j^* \in \mathcal{P}_j^P \left(\subset \mathcal{P}_j^N\right)$, and likewise $\min\left\{p_i^*, p_i^m\right\} \in \mathcal{P}_i^N \cap \mathcal{P}_i^P$, as $\min\left\{p_i^*, p_i^m\right\} \leq p_i^* = \underline{p}_i^* \in \mathcal{P}_i^P \left(\subset \mathcal{P}_i^N\right)$.

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