Valuing patents using renewal data :
An inquiry into the feasability of an automated patent scoring method

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Abstract--In this paper, we address the problem of patent valuation. With this aim in view, we focus on the feasibility and reliability of a patent rating system. This leads us to develop a structural model of patent renewal decisions based on real options that links patent renewals and patent value and to estimate it on micro level data. Results for a sample of European patents show that unobserved heterogeneity is too high to efficiently discriminate among patents and cast some doubt on the possibility to develop a reliable rating system based only on patent metrics.

Key words : patent valuation, real options, duration model
JEL classification : O34, C41

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Intellectual property rights (IPRs) have today reached a level of financial and strategic visibility that offers the possibility of an emerging class of IP-based transactions. Managed for a long time like a simple tool for protection, they are now used more and more like a primary source of value creation. In a context of open innovation, companies are indeed encouraged to manage their IPRs, not only in a defensive way but also as a tool of financial valorization. Whereas, traditionally, companies saw in IP laws a budget item and a simple legal mean to protect their investments from competitors, much of them now consider IPRs, and in particular patents, as a source of competitive advantage and as an important constituent of their capacity to create value and attract external financing[3]**.** It is therefore important for policy-makers to think about new ways to increase the flows of, and therefore the benefits from, transfers of intellectual property rights[4]. In particular, this implies that the markets for trading IPRs become less opaque and fragmented so that IPRs buyers and sellers can find each other efficiently and transactions take place on fair terms.

Among the set of initiatives that are currently being explored on both sides of the Atlantic, the proposal of the European Commission to create a patent rating agency gives rise to debates on the proposed methods and the desirability of deploying U.S. practices in Europe that are sometimes considered as controversial.

[3] By acting as "quality signals", patents can be used as collaterals for technology-based firms to attract external financing. Even if they are still in an early stage of development, patent backed financial instruments like patent loans, patent sales and lease-back or patent securitizations can be used by companies to leverage their most valuable assets to finance their development.

[4] Few empirical papers deal with patent transfers. A noticeable exception is Serrano (2010) but no data on patent value is provided.

The valuation of assets concerned with IPRs corresponds without any doubt to a true need, for companies as for the financial sphere. Indeed, from an investor's point of view, a correct understanding of the drivers of patent valuation may favor a better allocation of capital and a reduction of investment risks. However, the process by which the management of IPRs becomes a major source of value for companies is still in its infancy and the evaluation of IPRs continues to butt against serious difficulties linked to the lack of generally accepted methodologies of valuation and the fact that it is difficult to allot a value to a patent at the time of its deposit or shortly after[5]. The problem is emphasized by the fact that the algorithms at the basis of existing commercial methods[6] for rating patents are proprietary and therefore cannot be disclosed, which disregards the critical requirement of transparency.

Generally speaking, three basic requirements are needed for a trustworthy patent evaluation process, namely transparency, repeatability and objectivity. In this respect and although it was initially aimed at developing reliable measures of the pace of innovation beyond simple patent counts, the econometric literature on patent value offers interesting avenues for research on patent rating. At least three streams of research may be identified.

The first stream of patent valuation analyses relies on surveys (Harhoff, Scherer and Vopel, 2003; Gambardela, Harhoff and Verspagen, 2008). These studies collect information on the reservation price of patent holders. Ordered discrete choice econometric models are then estimated to link the stated value or interval of value with patent metrics. Broadly speaking, these studies show that patent metrics have significant individual or group impacts

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[5] For a discussion on what it is meant by the term « value », see Hall (2009). Broadly speaking, the value of a patent is the future commercial utility of the patented innovation as perceived by the person/co' interested in the valuation.

[6] Such as the Intellectual Property Quotient (IPQ®) built upon patented methodologies by *Ocean Tomo Patent Ratings*® or the Pantros IP patent Factor Index analytics (PFI)™ developed by *PatentCafe*®.

on patent value but that they only explain a small fraction of the total variance of patent value[7]. An interesting characteristic of these survey methods is that the information asymmetry problem between patent holders and potential buyers is avoided, or at least reduced to its minimum. However these methods are probably the most costly of all assessment methods and are time-consuming. These two features are obviously prejudicial to the choice of such a method of evaluation given that the underlying econometric model has to be re-estimated frequently if we consider the case of patents whose value constantly needs assessing during their lifetime.

The second stream of patent valuation analyses is often referred to as Tobin's q methods (Hall, Jaffe and Trajtenberg, 2005; Bessen, 2009). The keystone of these valuation methods is that they rely on the information efficiency hypothesis of stock markets. This assumption is all the more objectionable in a context of patent valuation where the role of information asymmetries between patent holders and other economic agents is generally stressed[8]. Moreover, there would be a paradox in implementing patent scorings in direction of external investors considering the fact that the method assumes they already have efficiently dealt with all the information available to them. Most empirical studies in that field are limited to single patent counts and thus only give an average patent value, which does not correspond to the needs of patent scoring. To our knowledge, Bloom and Van Reenen (2002), Lanjouw and Schankerman (2004) and Hall and alii (2005) are the only ones using patent metrics. This allows these authors to capture heterogeneity in patent values. However, the studies by Bloom and Van Reenen (2002) and Hall and alii (2005) are limited to the sole forward citations. If this metric is fundamental, survey methods show that it does not suffice to explain the variance of the value. For their part, Lanjouw and Schankerman (2004) capture patents

[7] As stressed by Gambardela, Harhoff and Verspagen (2008) "*the measure of our ignorance is still sizable*".

[8] Event studies methods (Austin, 1993) rely on the same hypothesis and encompass the same drawbacks.

heterogeneity with an index of patent quality estimated as the latent variable of a factor model. Their results show that such an index improves the quality of regression for several technological fields but their factor model is restrained to a limited number of metrics (claims, backward and forward citations). More generally, existing methods evaluate patent portfolios but the hypothesis which is made is then based on a pure additivity of the patents, i.e. these studies apprehend patent portfolios from metrics calculated as weighted sums of the patent metrics within the portfolio. For example, they describe a patent portfolio by the sum of the constituent patent citations. A major drawback of existing methods is thus that they do not allow to capture any over- or under additivity of the portfolio value with respect to the value of its constituent patents[9].

The third stream of econometric studies looks at patent value from the patentees' point of view using patent renewal data. Originally developed by Schankerman and Pakes (1986), models of patent renewals encompass many advantages. Firstly, significant insights can be gained from analyzing past renewal decisions of patent owners[10]. Indeed, it is fair to assume that patent owners are uniquely knowledgeable and well-qualified to make internal patent value and risk assessments since the patentee has better information than the stock market does. They are also economically motivated to make timely and relevant assessments and to take sound decisions based thereon. This means that they will choose to pay renewal fees only when the perceived value of the expected remaining economic benefit secured by the patent exceeds the amount of the maintenance fee. Thus renewal decisions are a way of measuring the patentees' assessment of a patent worth. Secondly, if one wants to fulfill the requirement of objectivity, methods based on revealed value have to be preferred to those based on stated

[9] Our paper does not address this sensitive issue which remains open for further research.

[10] For a patent to be kept in force, renewal fees have to be paid on an annual basis in Europe versus every 4 year in the U.S. by all patent owners whatever their status.

value like survey methods. Lastly, as determinants of patent renewals are restricted to observed patent metrics and variables related to the macro-economic context, the third requirement of repeatability is also fulfilled.

With these requirements in view, our paper tries to implement an original approach for comparatively rating and benchmarking patents. Indeed, firms cannot effectively manage that which they cannot measure. To do so, we develop an econometric modeling based on an analysis of observed renewal decisions conditional on a number of identified predictor variables including patent metrics.

The originality of our paper is to propose a structural model of patent renewals decisions that links patent renewals and patent value and that can be estimated on micro-level data with observed and unobserved heterogeneity affecting both the initial rent and its dynamics[11]. Existing models of patent renewals are not as such implementable for individual patent scoring as they rely on aggregated data for particular types or cohorts of patents and, for most of them, on the hypothesis that the rent is purely determinist (Schankerman, 1998; Deng, 2007; Bessen, 2008; Grönqvist, 2009). Option models proposed by Pakes (1986) and further developed by Lanjouw (1998) and Baudry & Dumont (2006) introduce a stochastic dynamics of the rent but are still estimated on aggregated data. Moreover, such models have the disadvantage to be complex to estimate and micro-level explicative variables are difficult to introduce.

As stressed by Schankerman and Pakes (1986, p. 1075) "*The next step [in the estimation of patent value] is to examine the empirical characteristics and the theoretical determinants of variation in the quality dimension at a more disaggregated level, among different industries,*

[11] The rationale for evaluating individual patents is that the empirical literature underlines that the distribution of patent value is highly skewed.

and between different types of patents". To our knowledge, the only articles that try to extend the renewal method to micro-level renewal data are Barney (2002) and Bessen (2008) but again their approach encompass two drawbacks: firstly, the depreciation rate of the rent is invariant and as such is not suitable to capture in a consistent way the effect of variables that change with patent age, meaning that it is impossible to revise the estimation of patent value according to new information affecting the dynamics of the rent during the patent life. Secondly, the possibility of random chocks affecting the dynamics of the rent and generating an option value is not taken into consideration. Yet, as outlined by Oriani and Sereno (2011), there are multiple sources of uncertainty that generate a patent option value. It is thus a key issue to correctly address their role in patent valuation. Our paper attempts to remedy these problems.

Our main contribution compared to the existing literature is that our model is flexible enough to capture the influence on patent value of both the characteristics of the patent known at the application date and those that change with patent age. This means that the depreciation rate of the rent varies with patent metrics or macro-level variables. The dynamics of the depreciation rate is also allowed to be partly stochastic. Taking into account the fluctuations of the rent over time allows us to re-assess the patent value year after year. This point is important as it constitutes an essential criterion to update patent scoring on a regular basis and for tapping the economic potential of patent information to support decision-making. By contrast with previous attempts to assess patent value at the micro level, our model also incorporates an option value of patents. It is finally stressed that the link between patents duration and patents value is not as simple as one may intuitively think. *Ad hoc* specifications of patents duration model are thus not able to provide reliable estimates of patents value. Our structural model is introduced as an explicit option problem. Conditions under which the option problem can be simplified are discussed. The specification used to derive the econometric duration model is more general than the very peculiar specification corresponding to the model initially proposed by Schankerman and Pakes (1986).

The paper is organized as follows: the next section presents a model of patent renewals and discusses theoretical issues that lead us to develop a structural model based on patent renewal data to predict patent value. Section 3 describes data collection and variables. Section 4 presents estimation results for a panel of European patents designating France. Our results corroborate those obtained with survey methods. Patent metrics have significant impacts on the decision to renew or withdraw patents, but a too high fraction of patent duration and thus of patent value is still unexplained. Thus, our results cast some doubts on the possibility to derive reliable patents scorings from patent metrics. Section 5 concludes.

1. A micro-economic model of patent renewals and patent value

This section first offers a general model to analyze patent renewal decisions. We then derive a structural model of patent renewal decisions that links patent renewals and patent value and that can be estimated on micro-level data with observed and unobserved heterogeneity affecting both the initial rent and its dynamics.

1.1. Modelling patent renewal decisions

1.1.1. A general approach to patent renewal decisions

A basic but key assumption to estimate patent value on the basis of patent renewal decisions is that patent owners act rationally to maximise the value of their patent conditional on information available. More precisely, at each renewal date, a patent owner has to decide whether to discontinue or not the payment if the required renewal fees exceed the value of the patent. This option to renew exists because the payment of renewal fees is discretionary.

Costs of renewing a patent are in fact multi-facets. They encompass internal costs to assess the usefulness of the patent, enforcement costs and those corresponding to the payment of renewal fees to the patent office. In practice, renewal fees depend on the age a of the patent and are revised in the course of time. Thereafter, $f_t^a \geq 0$ denotes the fee charged at age a for a patent with application date t to be renewed up to age $a+1$. Patent offices in Europe charge increasing fees each year (i.e. $f_t^{a+1} > f_t^a > 0 \; \forall a \in \{0, \cdots, A-1\}$ where A is the statutory life limit of patents). Most renewal costs like legal expenses are not directly observed nor easily measured, except renewal fees that are published by patent offices. As a result, a common practice consists in subtracting the unobserved renewal costs from the gross rent associated to the exclusivity right conferred by a patent on all industrial and commercial applications of the patented invention. The resulting net rent for a patent of age a applied for at time t is denoted by R_t^a. Gains that accrue from renewing a patent are obtained by adding the current flow of net benefits given by $R_t^a - f_t^a$ and the expected and discounted value $E_{t+a}\left[V_t^{a+1}\right]/1+r$ of the patent at age $a+1$ where r stands for the discount rate and E_{t+a} for the mathematical expectation conditional on all information available to the patent owner at date $t+a$. Renewing a patent is optimal if and only if the associated gains are positive. Thus, at any renewal date before the statutory life limit A, the value of a patent is recursively defined by the following expression:

$$V_t^a = Max \begin{cases} R_t^a - f_t^a + \dfrac{E_{t+a}\left[V_t^{a+1}\right]}{1+r} & \text{if the patent is renewed} \\ \\ 0 & \text{if the patent is withdrawn} \end{cases} \qquad \forall a < A \qquad (1.a)$$

At the statutory life limit A, the expected future value of the patent falls to zero and the value of the patent is given by

$$V_t^A = Max\begin{cases} R_t^A - f_t^A & \text{if the patent is renewed} \\ \\ 0 & \text{if the patent is withdrawn} \end{cases} \qquad (1.b)$$

The optimal age of withdrawal for a patent is the first age such that, conditional on information available at the current time, renewing the patent generates a net loss. Formally, it is the optimal stopping time associated with the dynamic programming problem (1):

$$a^* = Inf\left\{a \in \{0,\cdots,A\};\ R_t^a - f_t^a + \frac{E_{t+a}\left[R_t^{a+1}\right]}{1+r} < 0\right\} \qquad (2)$$

Whether a^* is deterministic or random depends on assumptions about the dynamics of the rent and renewal fees. For the optimal age of withdrawal to be random, either the rent or renewal fees must be affected by unexpected shocks; the observation of which constitutes new information. In that case, some authors call V_t^a the option value of patents in reference to the real option theory that analyses irreversible decisions when facing risk or uncertainty. Pakes (1986), Lanjouw (1998) or Baudry and Dumont (2006) for instance use this terminology. However, the stochastic nature of the dynamics of the rent in these approaches complicates the determination of the optimal withdrawal date. As a result, the impact of observed patent characteristics on the decision rule is not accounted for and the analysis is confined to the determination of the value distribution of patent rights within a cohort. Our paper tries to solve this problem by extending the method to micro-level renewal data.

1.1.2. A simplified renewal decision rule

A reason for the complexity of decisions rules in real option models of patent renewals is that the rent is allowed to vary upwards and downwards. As a result, even if the rent falls far below the renewal fee, this situation may be reversed in the short or medium term due to future positive shocks. Hence, comparing only the current value of the rent and renewal fees is not a relevant decision-rule. For such a decision-rule to be efficient, the additional Assumption 1 as regards the dynamics of the rent and renewal fees is required.

> <u>Assumption 1</u>*: The gap* $R_t^a - f_t^a$ *between the rent and the renewal fee decreases monotonically from an initial positive value to a possibly negative value with patent age.*

Assumption 1 neither precludes that the dynamics of the rent is stochastic nor that the rate of decrease is identical across patents. By contrast, a more restrictive form is generally used in most articles that attempt to assess patent value on the basis of renewal decisions. It states that the rent itself monotonically and deterministically decreases at a similar rate for patents belonging to a same cohort or a same technological class whereas, as observed for most patent offices, renewal fees are assumed to monotonically increase. Pakes and Schankerman (1986) were the first to introduce this strong version of Assumption 1 followed by Schankerman (1998) and more recently Deng (2007), Bessen (2008) or Grönqvist (2009). Assumption 1 is supported by empirical evidence. Indeed, the few models that do not rely on Assumption 1 and let the rent evolves stochastically (Pakes, 1986; Lanjouw, 1998) find that most leaning is over by the fifth year of protection and few patents yield higher returns after that point. A more restrictive case, with a constant rent is considered by Barney (2002). In this very specific case, the interval of value of the rent is directly deduced from patent duration. What is effectively captured by the depreciation of the rent depends on the fact that the patented technology is embodied or disembodied[12]. For embodied patents, the rent is generated by sales of the patented product or by a drop of production costs resulting from the new production process. Thus, the decrease of the rent directly reflects the intuitive idea that as the patented product or process ages, competitors with newer inventions progressively erode the sources of benefits. The rent that accrues from a disembodied patent is for its part linked to indirect pecuniary advantages. It may result, for instance, from a signaling effect that makes the external financiers of a patenting firm more confident about its ability to innovate and generate cash flow in the future. Maintaining a patent in its disembodied form for a long term alters the quality of the signal and thus the subsequent pecuniary advantages, causing the decrease of the associated rent. Note that the switch from the disembodied form to the embodied one is based on the relative values of the rent in the two respective forms and is thus not inconsistent with the fact that the rent decreases with patent age whatever the form considered[13]. Whatever the exact form considered, Assumption 1 implies Proposition 1 and Corollary 1:

> <u>Proposition 1</u>*: Under assumption 1, the rent will never exceed the renewal fee once it falls below it. As a result, maintaining the patent alive is optimal if and only if the rent exceeds the renewal fee.*

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[12] Following the terminology used by Bloom and van Reenen (2002), embodied patents are patents that protect those products and process innovations which a firm has invested in. Conversely, disembodied patents are patents for which a firm has not yet committed into actual production.

[13] Bloom and Van Reenen (2002) simplify the analysis and consider that a positive rent exists only in the embodied form. As a result, it is required in their model that the potential rent in the embodied state increases for the switch to occur.

<u>Corollary 1</u>: Under assumption 1, if a patent applied for at date t is withdrawn at age a, then the rent has always exceeded the renewal fee from age 0 to age $a-1$.

Figure 1 illustrates these key results in the case of renewal fees that start at $10€$ and increase at a constant rate of 10% whereas the rent for patents A, B and C respectively starts at $90€$, $100€$ and $40€$ and decreases at a constant rate that respectively amounts to 15%, 10% and 5%. The associated optimal withdrawal age is 9 years for patent A, 12 years for patent B and 10 years for patent C. If monetary flows are discounted at a 3% discount rate, patents A, B and C are respectively worth $304{,}81€$, $458{,}61€$ and $148{,}72€$.

<u>Insert Figure 1</u>

Figure 1 also highlights that patent value is tightly linked to patent duration but this link is not that simple. In Figure 1, the height separating the decreasing profiles of the rent and the increasing profile of renewal fees measures the net revenue that accrues from a patent at the corresponding age on the abscissa. The total value of a patent at the application date (age $a=1$) is obtained by discounting and summing up the heights between the two profiles for all ages on the left of the crossing point between the two profiles. The first date on the right of the crossing point corresponds to the patent withdrawal date. Due to discounting, the total value of two patents cannot systematically be compared graphically. Nevertheless, simple cases that can be easily compared are considered on Figure 1. For instance, patent B has a rent that exceeds the rent of patents A and C and this rent also lasts longer than for patents A and C. This leads us to conclude that patent B has the highest total value. Patent A is withdrawn slightly before patent C but its rent is far higher than that of patent C at almost all dates before the withdrawing dates. Therefore one can reasonably assert that the total value of patent A exceeds that of patent C. Thus, patent ranking from the earliest non-renewal date to the latest is A-C-B but the ranking from the lowest total value to the highest is C-A-B. Obviously, both rankings depart. Note that the different rankings obtained from patent duration in the one hand, and patent value on the other hand, follow on from heterogeneity in terms of depreciation rates. If depreciation rates were arbitrarily constrained to be identical and constant (Schankerman and Pakes, 1998; Bessen, 2008) or null (Barney, 2002), then a similar ranking for both duration and value would have been observed. Thus, if we allow for heterogeneity in depreciation rates, a structural model of patent duration is required to derive estimates of patent value from observed characteristics affecting the date of patent withdrawal and then to correctly rank patents in terms of economic value.

1.2. A structural model of patent duration

1.2.1. Basic specification with a constant depreciation rate

The traditional solution adopted in seminal papers on patents valuation to account for patent heterogeneity in terms of the observed date of withdrawals, consists in assuming that patents only differ in their initial rent at the filing date. The rent is assumed to decrease at a same decay rate for all patents so that the simplified decision rule described in Proposition 1 applies. As a result, given the initial value of the rent, the optimal withdrawal age defined in (2) is deterministic. The probability distribution of the withdrawal age is then derived from the probability distribution of the initial rent. The basic specification proposed here slightly departs from previous approaches. Indeed, the probability distribution of the initial rent is conditional on patent characteristics. This aims at introducing a source of observed initial heterogeneity between patents that explains differences in terms of expected withdrawal age at the application date. Patent characteristics influencing the probability distribution of the initial rent have to be time invariant to be consistent with the idea that they capture differences in the initial conditions influencing both the duration and value of a patent. The corresponding variables are thus referred to as static variables. Indicators of the technology field, number of claims, backward citations, patent family size are variables that typically fulfil this condition.

Formally, the value $R_{t\,i}^{a+1}$ of the net rent at age $a+1$ for a patent i applied for at date t is written as the value of the net rent $R_{t\,i}^{a}$ at the previous age, affected by a decay or depreciation factor δ_t^{a+1}. Note that the index i is introduced to capture the fact that the initial and following values of the rent may be patent specific. Conversely, the decay factor is independent of the patent characteristics but may be contingent to the application date t and the age a. Proceeding recursively, we have

$$R_{t\,i}^{a} = R_{t\,i}^{0} \prod_{s=1}^{a} \delta_t^{s} \tag{3}$$

Furthermore, heterogeneity of patents as regards the initial rent $R_{t\,i}^{0}$ follows on from observed and unobserved factors. Observed heterogeneity is captured by a vector $X_i = \{x_{1\,i}, \cdots, x_{k\,i}, \cdots, x_{K\,i}\}$ of K objectively measurable characteristics of the patent that are time-invariant. Unobserved heterogeneity, for its part, is taken into account by assuming that $R_{t\,i}^{0}$ is drawn independently for each patent from a same probability distribution, one or more parameters of which depend on X_i. Though not necessary, it is convenient to assume that observed and unobserved heterogeneity affecting the initial rent interact multiplicatively and that observed heterogeneity is correctly captured by a Cobb-Douglas functional form of the static characteristics of each patent. Accordingly, we have :

$$R_{t\,i}^{0} = \alpha_0 \left(\prod_{k=1}^{K} x_{k\,i}^{\;\alpha_k} \right) \varepsilon_i \tag{4}$$

where α_k (with $k \in \{0, \cdots, K\}$) are parameters and ε_i is a i.i.d random term. The probability distribution of the initial rent directly follows on from the probability distribution of ε_i that captures unobserved heterogeneity.

Imposing $\delta_t^a \in [0\,,1]\, \forall t\ \forall a$ in (3) guarantees that that the rent never increases. This ensures the validity of the simplified decision rule defined in Proposition 1. Then, given that the dynamics of the rent fulfils Assumption 1, Corollary 1 implies that any patent applied for at date t which is still alive at age a satisfies the following properties:

$$R_{t\,i}^{s} \geq f_{t\,i}^{\,s} \quad \forall s \in \{0, \cdots, a-1\} \tag{5}$$

Moreover, Assumption 1 implies that $R_{t\,i}^{a-1} \geq f_{t\,i}^{\,a-1}$ is a sufficient condition for all inequalities in (5) to be satisfied. Therefore, the information revealed by observing that a patent applied for at date t is still alive at age a may be synthesised by this last condition. Combining this result with (3) and (4), we finally obtain that a patent i applied for at date t is renewed up to (at least) age a if and only if

$$\varepsilon_i \geq Q_{t\,i}^{a-1} \quad \text{with} \qquad Q_{t\,i}^{a-1} = \frac{f_t^{\,a-1}}{\alpha_0 \left(\prod_{k=1}^{K} x_{k\,i}^{\;\alpha_k} \right) \prod_{s=1}^{a-1} \delta_t^s} \tag{6}$$

Once expressed in logarithms, (6) is similar to the key condition that Schankerman and Pakes (1986) or Schankerman (1998) use to obtain their econometric model. Nevertheless, for the estimation method proposed by these authors to work, the dataset of patents has to be partitioned in such a way that the threshold value $Q_{t\,i}^{a-1}$ is identical for all patents within a same subset. Moreover, the size of each subset of patents has to be sufficiently large to obtain reliable measures of the proportion of patents withdrawn at each age. This means that this econometric method works for patent cohorts (Schankerman and Pakes 1986) or patents belonging to large technological classes (Schankerman 1998) as long as none of the characteristics that distinguish patents within a subset is used as an explanatory variable. For these reasons, we suggest an alternative econometric approach that also relies on condition (6) but that is adapted to the use of micro-level patent characteristics. For this purpose, note that Assumption 1 implies that the value of the threshold $Q_{t\,i}^{a-1}$ decreases with age a. Furthermore, a patent i applied for at date t is optimally withdrawn at age a if and only if condition (6) prevails at age $a-1$ but not at age a. This yields the probability $\mathrm{Pr}_{t\,i}^{a}$ of an optimal withdrawal at age a conditionally on a renewal up to age a:

$$\mathrm{Pr}_{t\,i}^{a} = \frac{\Phi\left(Q_{t\,i}^{a}\right) - \Phi\left(Q_{t\,i}^{a-1}\right)}{1 - \Phi\left(Q_{t\,i}^{a-1}\right)} \tag{7}$$

where Φ denotes the cumulative density function of ε_i. In the terminology of duration models, $\mathrm{Pr}_{t\,i}^{a}$ is nothing else than the hazard rate characterising the econometric model of patent duration. The corresponding survival function is $1-\Phi\left(Q_{t\,i}^{a-1}\right)$. Let Ω_a denote the subset of patents renewed up to at least age a whatever their application date and let I_i^a be a variable that takes value 1 if patent $i \in \Omega_a$ is renewed at age a and value 0 otherwise. The log-likelihood of withdrawal *versus* renewal at age a for a patent i, conditional on the fact that we know that $i \in \Omega_a$, is given by:

$$L_i^a = \mathrm{I}_i^a \ln \mathrm{Pr}_{t\,i}^{a} + \left(1-\mathrm{I}_i^a\right) \ln\left(1-\mathrm{Pr}_{t\,i}^{a}\right) \tag{8}$$

Summing over all ages and all patents, we obtain the following log-likelihood

$$L_{tot} = \sum_{a=1}^{A} \sum_{i \in \Omega_a} L_i^a \tag{9}$$

Note that a same patent appears several times in (9) but at different ages. Estimates of parameters α_k ($k \in \{0, \cdots, K\}$) and of parameters of the probability distribution of ε are obtained by maximising (9) with respect to all these parameters. The advantage of estimating the discrete time duration model developed above rather than an *ad hoc* duration model of patents relies on its structural specification that directly provides estimates of all parameters required to assess patent value. Another advantage of our duration model compared to ordered discrete choice models such as those proposed by Barney (2002) and Bessen (2008) is that our basic specification can be extended in order to take into account heterogeneity in the depreciation rate.

1.2.2. Real option specification with heterogeneity in the depreciation rate

The basic specification of patent renewal decisions presented so far is featured by a deterministic and homogenous dynamics of the rent. In such a framework, a patent owner is indifferent in choosing between a mechanism based on renewal fees or a mechanism based on a menu of patent duration and upfront fees corresponding to the discounted sum of renewal fees. In other words, commitment to a predefined withdrawal date is costless because no additional information is supposed to be revealed as the patent ages.

This is no longer true if the time path of the rent can differ *ex post* from the time path expected *ex ante*. Practitioners stress the importance of this type of uncertainty. An extra value then accrues from the renewal mechanism compared to a commitment to a predefined withdrawal age because it makes it possible to adapt the optimal decision to unforeseen shocks that modify the time path of the rent. This extra value is the value of flexibility that is typically accounted for in real option models (Dixit and Pindyck, 1994; Trigeorgis, 1996).

Uncertainty affecting the dynamics of the rent makes the decay factor that is applied to obtain the rent at age a from its value at age $a-1$ in (3) specific to each patent. Both observed and unobserved heterogeneity in this decay factor are considered.

Observed heterogeneity is associated to the time path of observed variables that affect the depreciation of the rent. These variables may either be sector-based or macro-economic indicators of market conditions for patented inventions or patent specific characteristics like forward citations and patent litigation. The main point here is that their values change with the age of the patent. In reference to this property, such variables are referred to as dynamic variables. Let $Z_i^{t+a} = \left\{ z_{1i}^{t+a}, \cdots, z_{mi}^{t+a}, \cdots, z_{Mi}^{t+a} \right\}$ denote the vector of values taken by the M dynamic variables z_{mi}^{t+a} ($m \in \{1, \cdots, M\}$) affecting at age a the depreciation of the rent for patent i applied for at date t. This vector conditions the depreciation rate $g_{t\,i}^{a}$ associated to the observed component of the decay rate. In order to be consistent with the fact that this depreciation rate ranges between 0 and 1, a logistic specification is more specifically convenient:

$$g_{t\,i}^{a} = \frac{1}{1 + \exp\left(\beta_0 + \sum_{m=1}^{M} \beta_m \, z_{m\,i}^{t+a} \right)} \tag{10}$$

where β_m (with $m \in \{0, \cdots, M\}$) are parameters to be estimated. Unobserved heterogeneity is associated to factors affecting the depreciation of the rent and that are known to the patent owner but not to other economic agents. “Good” or “bad” information about technological opportunities offered by the patented invention typically belongs to this category. As a result, unobserved heterogeneity is captured by idiosyncrasic random terms. It is more precisely assumed that, between two renewal dates, there is a series of N random events that reduce the rent in a multiplicative form. Observed and unobserved heterogeneity are also assumed to interact multiplicatively for a technical reason that will be made explicit latter on. Accordingly, the decay factor of the rent between age $a-1$ and age a may be written as

$$\delta_{t\,i}^{a}=\left(1-g_{t\,i}^{a}\right)\prod_{n=1}^{N}\left(1-\tilde{\theta}_{n\,i}^{a}\right) \tag{11}$$

The depreciation rates $\tilde{\theta}_{n\,i}^{a}$ ($n\in\{1,\cdots,N\}$ and $a\in\{0,\cdots,A\}$) associated to the random events affecting the rent for each patent i are assumed to be identically and independently distributed in the range $[0\,,1]$ so that Assumption 1, Proposition 1 and Corollary 1 are satisfied. Uncertainty about the future time path of the rent arises from events underlying both unobserved and observed heterogeneity. Nevertheless, patent owners can make rational expectations about future values of $\delta_{t\,i}^{a}$ conditional on the information avalaible to them at the current time. “Good” news (respectively “bad” news) are then defined as realisations of the $\tilde{\theta}_{n\,i}^{a}$ and $z_{m\,i}^{t+a}$ that generate higher (respectively lower) than initially expected values of $\delta_{t\,i}^{a}$. Substituting (11) in (3) and taking the natural logarithm we obtain that

$$\ln R_{t\,i}^{a}=\ln R_{t\,i}^{0}+\sum_{s=1}^{a}\ln\left(1+g_{t\,i}^{a}\right)+\sum_{s=1}^{a}\sum_{n=1}^{N}\ln\left(1-\tilde{\theta}_{n\,i}^{s}\right) \tag{12}$$

Combining with (4) yields

$$\ln R_{t\,i}^{a}=\ln\alpha_{0}+\sum_{k=1}^{K}\alpha_{0}\ln x_{k\,i}+\sum_{s=1}^{a}\ln\left(1+g_{t\,i}^{a}\right)+\ln\varepsilon_{i}+\sum_{s=1}^{a}\sum_{n=1}^{N}\ln\left(1+\tilde{\theta}_{n\,i}^{s}\right) \tag{13}$$

Condition (6) for a patent i applied for at date t to be still alive at age a then becomes

$$\ln\varepsilon_{i}+\sum_{s=1}^{a}\sum_{n=1}^{N}\ln\left(1+\tilde{\theta}_{n\,i}^{s}\right)>\ln Q_{t\,i}^{a-1} \tag{14}$$

with $Q_{t\,i}^{a-1}$ already defined in (6) except that $1-g_{t\,i}^{a}$ has to be substituted to δ_t^a.

The next step in specifying the econometric duration model consists in obtaining the probability distribution of the left hand side of (14). At this stage, the assumption of a multiplicative interaction between random events affecting the unobserved component of depreciation is useful. Indeed, for a sufficiently high number N of i.i.d. random events between two successive renewal dates, the central limit theorem applies and (14) may be rewritten as

$$\ln \varepsilon_i + \tilde{\omega}_i^a > \ln Q_{t\,i}^{a-1} \tag{15}$$

Where $\tilde{\omega}_i^a$ is a Gaussian random term with a negative expected value $-\mu\, a$ and variance $\upsilon^2\, a$. Parameters $\mu > 0$ and $\upsilon > 0$ correspond respectively to the opposite of the expected value and to the standard deviation of the unobserved component $\sum_{n=1}^{N} \ln\left(1+\tilde{\theta}_{n\,i}^{s}\right)$ of the decay factor at age a. In order to proceed to the computation of the hazard rate of the discrete time model of patent duration as in (7), it is necessary to determine the cumulative probability distribution Φ of the sum of the two random variables that appear in the left hand side of (15). An immediate solution is to postulate a normal distribution for $\ln \varepsilon$ (with expected value 0 and variance σ^2) so that the probability distribution of the sum of the two terms in the left hand side of (15) is a normal probability distribution with expected value $-\mu\, a$ and variance $\sigma^2 + \upsilon^2\, a$.

2. Data collection and variables

The dataset used in this paper consists of all EPO patents designating France and applied for in 1989. The choice of this year was motivated by the fact that all patents in this cohort have reached the statutory life limit. Therefore, there is no censored data in the duration model. The choice to study French patents relies on the fact that they have been regularly studied in the economic literature (Schankerman and Pakes, 1986), thus allowing for comparisons. Another advantage is that contrary to some countries, renewal fees have been used for a long time and since the first age of the patent in France. Our comprehensive dataset of 26 904 patents was courteously provided by *Qwestel*®. Table 1 details the frequencies of withdrawals for the eight main technological fields in the International Patent Classification (IPC)[14]. The official code and description of each technological field are reported in Table 1. The profiles of withdrawal frequencies are similar across technological fields, except in the case of technological field E ("*fixed constructions*") where more patents were withdrawn earlier. Table 1 also displays the average renewal fees to be paid at the different ages of a patent.

Insert Table 1

Existing automated scorings that aim at gauging the overall quality of patents try to provide a yardstick for measuring and comparing patent value. The premise of these rating models is that combining a number of predictor variables revealed by the patent document itself helps identifying patents that are statistically either more likely or less likely to produce

[14] For a comprehensive description of IPC, consult the website of the World International Patent Office at http://www.wipo.int/classifications/en/.

economic returns. A brief description of the predictor factors used in our model is provided below. Two kinds of predictor variables have been distinguished and used to measure patent quality. On the one hand, static variables that have an impact on the initial rent but not on the dynamics of the rent. On the other hand, dynamic variables, i.e. variables that conversely have an impact on the dynamics of the rent and which, as such, play a crucial role when it comes to reassess the patent over time.

Static Variables (or time-invariant variables) :

- *Size* : number of patents from the same applicant recorded in the database. The underlying idea is to capture the size or the innovative intensity of applicants.

- *nbIPC:* number of IPC subclasses (at the 4-digit IPC level) as declared in the patent application. This variable measures whether the patent is broad or narrow. If numerous subclasses are applied for by the applicant, one may expect diffusion of the innovation in different domains.

- *speIPC:* percent of declared IPC subclasses that belong to the main IPC class of the patent. This variable measures whether what is declared as the main IPC class of the patent (the one that concentrates the highest number of subclasses declared by the patent) largely dominates other IPC classes or not. This variable can be interpreted as a measure of patent specialisation.

- *entropyIPC:* entropy of declared IPC classes (measured on the basis of the eight main IPC classes). This variable completes speIPC with the advantage that it takes into account the fact that declared IPC classes may concentrate on some classes but with the disadvantage that only the eight main IPC codes are used.

- *Claims*: number of independent and dependent claims in logarithm. The underlying idea is that the more claims a patent has, the broader the likely scope of protection and the better the likelihood of surviving a validity trial.
- *Keywords:* number of words in some key sentences in logarithm (information on this specific metric was provided by Qwestel®).
- *Ncte :* number of backwards citations. For EP publications, this field also contains opposition citations (reasons for opposition) and observer citations (i.e. examiner references). This variable describes the scope of prior art and signals valuable technological knowledge (De Carolis, 2003).
- *Ncta:* number of backwards citations by the applicant.
- *Npr:* number of recorded priority claims, in logarithm. Intuitively, more priority claims probably means a patent is entitled to an earlier filing date, which can be beneficial in fending off art-based validity attacks. It can also indicate a greater level of overall interest and investment by the patentee.
- *Npn:* family size, in logarithm. Lanjouw and Schankerman (2004) found that family size is highly correlated with other indicators of patent value.
- *Ncc:* number of countries in the patent family, in logarithm.
- *Pct:* dummy variable taking value one if and only if the patent is applied for via the PCT procedure.
- *Ndsep:* number of European countries in which the patent is taken out, in logarithm. The geographical scope of the patents reveals the expectations of the patent applicant concerning patent value (Reitzig, 2004).

- *Univ*: dummy variable related to patent ownership and taking value one if and only if the applicant is an academic institution. This variable is not available for technological fields D *("Textiles; Paper*"), E (*"Fixed constructions*") and F (*"Mechanical engineering; Lightening; Heating; Weapons; Blasting"*) because too few patents were coded one with the result that the dummy could capture idiosyncratic shocks rather than the ownership effect.

- *Firm&Univ*: dummy variable taking value one if and only if the applicant is a firm associated to an academic institution. Again, this variable is not available for technological fields D, E and F.

Insert Table 2

Some descriptive statistics on static variables are displayed in Table 2. No major differences appear between technological fields as regards these variables. However, similarities as regards the mean of static variables may hide different distributions of these variables. As it is not expected that parameter estimates for renewal models are systematically similar across technological fields, the model described in the previous section is estimated separately for each technological field.

Dynamic Variables (or time-dependent variables) :

- *Age:* age of the patent. This variable captures the fact that *ceteris paribus* the depreciation rate of the rent may be greater in the early ages.

- *Gdp:* GDP growth rate of the current year. This variable captures the business cycle effect.

- *Fcit:* forward citations rate, i.e. number of new citations received at the different ages of the patent. Some studies[15] have suggested that the number of citations or references made to an issued patent by other subsequently issued patents (forward citations) may have a positive correlation with economic value. Intuitively, a high forward citation rate could indicate a high level of commercial interest or activity in the patented technology.

Insert Figure 2

Figure 2 shows the average profile of forward citations received at each age as a function of patent age for the different technological fields. Forward citations are mainly received between age four and fifteen in all technological fields. The maximum average number of forward citations is received early, at around age four or five and generally, it slightly decreases up to age fifteen. A more drastic drop is then observed. Technological fields A (*"Human necessities*"), C ("*Chemistry; Metallurgy"*) and D (*"Textiles; Paper"*) on the one hand, and B ("*performing operations; transporting*"), F ("*Mechanical engineering; Lightening; Heating; Weapons; Blasting*") and E ("*Fixed constructions*") on the other hand respectively exhibit the highest and the lowest average number of citations. The average number of new citations for technological field A is about twice that of field E at most ages.

3. Results

3.1. Estimation results

Maximisation of the likelihood function (9) with the stochastic specification (15) of the probability of an optimal withdrawal at a given age conditional on renewal up to that age has

[15] Harhoff, D. & al. (1999) and Trajtenberg (1990).

been implemented to the dataset described in the previous section. The model has been estimated separately for each IPC technological field.

Insert Figure 3

Figure 3 provides some general insights into how the model fits the data. For this purpose, it follows a common practice that consists in comparing observed and simulated cumulated frequencies of withdrawals for a same cohort of patents at different ages. Three different methods are used to generate simulated frequencies of withdrawals.

The first method consists in generating one thousand random draws per patent of the initial value of the rent and its time path. Simulations for the time path of the rent are based on additional assumptions as regards dynamic variables which exact values are treated as unknown at the beginning of the patent life. The annual growth rate of the economy is assumed to follow a Gaussian probability distribution and it is supposed that the stochastic process of forward citations obeys a Poisson process[16]. The withdrawal age is computed for each random draw and the optimal withdrawal age finally forecasted for a patent represents the average of these different ages. The use of numerous random draws per patent minimises the role played by rare events in differentiating the dynamics of the rents. As a result, the first method may be thought of as a method that tends to neutralise unexplained important factors affecting the decision to renew or not a patent. On the one hand, this method makes sense to forecast patent duration at the beginning of their life. On the other hand, it induces very little

[16] More precisely, forward citations received at age a by a patent are modelled as the outcome of a Poisson count data model which parameter is a function of the cumulated forward citations received by the patent up to age $a-1$. Such a Poisson model has been estimated age by age. The fact of conditioning the model on the cumulative citations received turns out to have a statistically highly significant impact. Detailed estimation results are available from the authors upon request.

variations in the forecasted optimal age of withdrawal so that, as shown by Figure 3, the frequencies of withdrawal for the cohort are much more concentrated than observed frequencies, whatever the technological field considered.

The second and third methods attempt to provide further evidence of this result. The second method is similar to the first one with the noticeable exception that only one initial value and one time path of the rent are generated for each patent. Therefore, the role of rare events is emphasised, some patents benefiting from important unexplained positive shocks while others suffer from important unexplained negative shocks. The resulting cumulated frequency of withdrawal for the cohort moves closer to the one that is observed compared to the first method. This result argues in favour of the role of these unexplained shocks. Nevertheless, simulated frequencies of withdrawals remain much more concentrated than the observed ones.

The third method goes one step further by replacing the series of simulated forward citations and simulated economic growth rates by the observed ones. As a consequence, the simulated cumulated frequency of withdrawals for a cohort overlaps almost perfectly with that which is observed whatever the technological field considered. This third method thus outlines the importance of correctly forecasting the time path of dynamic variables which requires a more complex modelling than what was postulated in the first two methods.

<u>Insert Figure 4</u>

In order to give a general idea of the estimated initial value and dynamics of the rent, Figure 4 shows the mean and median time paths of the rent that are obtained with the third method already used to generate Figure 3[17]. For each technological field, these time paths are compared to the average profile of renewal fees on the period covered by the dataset. In order

[17] Broadly speaking, results obtained with the first and second methods do not drastically differ.

to facilitate the comparison between initially high rents and low renewal fees, both are expressed in natural logarithm. A first common feature of all technological fields is that the mean time path always exceeds the median time path, thus suggesting an asymmetry in the distribution of the rents with some very high values whatever the age considered[18]. The absolute gap between the natural logarithm of the mean and median time paths is approximately constant over all ages. Consequently, the ratio of the mean to the median time path is also constant, indicating that the degree of asymmetry does not change drastically as patents age. A second common feature is that the mean time path of the rent never goes below the annual fees, thus justifying maintaining the patent in force up to the statutory life limit. By contrast, the median time path falls below the renewal fee at around age thirteen for all technological fields.

Figure 4 also highlights some differences between technological fields. Technological field D ("*Textiles and Paper*") appears to be the most atypical one with a high initial mean but a substantially lower median value of the rent. A higher depreciation rate of the rents largely counterbalances the higher initial values of the rents so that, for instance, the median time path does not pass below annual fees at a later age compared to what is observed for other technological fields. The high gap between mean and median time paths indicates that the asymmetry between patents with the highest values and those (the majority) with middle or low values is important. To a lesser extent, technological field H ("*Electricity*") exhibits a similar profile in terms of the dynamics of the rents. This atypical profile suggests that patents protect strategically important and independent inventions but that the pace of innovation is faster in these two technological fields compared to other technological fields. The profile of the dynamics of the rents for technological fields E ("*Fixed constructions*"), F ("*Mechanical engineering; Lighting; Heating; Weapons; Blasting*") and B ("*Performing operations;*

[18] This result corroborates the high skewed distribution of rents observed by previous empirical studies.

Transporting") is just opposite to the profile observed for technological fields D and F. Initial values of the rents are low and not highly different (as indicated by the low gap between the mean and median) but are counterbalanced by a slower depreciation rate. This is typical of technological fields where incremental and highly complementary innovations dominate. The other technological fields have intermediate profiles in terms of the dynamics of rents.

<u>Insert Table 3</u>

Table 3 reports indicators related to the global quality of estimates. The percentage of total variance of the natural logarithm of the initial rent explained by the model is used to assess whether patent heterogeneity in terms of the initial rent is correctly captured by the static variables of the model. The log linearity of the expression of the initial rent makes its computation easy. Similarly, the log linearity of expression (13) facilitates the computation of the percentage of total variance of the natural logarithm of depreciation up to a given age that is explained by the dynamic variables of the model. Though relatively standard when dealing with micro level data, these two percentages are clearly too low to enable a reliable forecasting of patent duration. This is confirmed by the comparison between the average probability of withdrawal at age ten for patents effectively withdrawn at this age and for patents renewed at this age. Patents still alive at age ten only have been used to compute the two average probabilities. The focus on the midterm of the maximum patent duration offers an interesting compromise between the necessity to take into account the realisations of the dynamic variables of the model on a sufficiently long period and the requirement of a sufficiently large subset of patents still alive for results to be statistically reliable. The difference between the two average probabilities has the expected sign but is very low. More importantly, the probability of withdrawal for patents effectively withdrawn never exceeds 0.5 so that a prediction of withdrawals on the basis of this probability can hardly be performed.

This is not in contradiction with the almost perfect overlap of the observed cumulated frequencies of withdrawals and the simulated cumulated frequencies obtained with the third simulation method in Figure 3. It just reveals that, due to the role of unobserved heterogeneity, the profile of withdrawals at the level of the cohort is correctly predicted but withdrawals at the patent level are not.

This result is important because it casts some doubts about the possibility to implement a reliable patent scoring with some mitigation depending on the technological field considered. Technological fields A ("*Human necessities*") and H ("*Electricity*") for instance are the two technological fields where the initial rent is the less correctly explained by the model but, by contrast, the dynamics of the rent is better explained than for all the other technological fields except technological field D ("*Textiles; Paper*"). A patent scoring at a too early age would thus probably be affected by important errors but results could be improved as time passes and that sufficient information is revealed about the realisation of dynamic variables that could then allow discriminating among patents. At the opposite, the best explanation for the initial rent is obtained for technological field E ("*Fixed constructions*") but at the expense of the dynamics of the rent which is then the worst correctly explained. This leads us to conclude that there is little to be expected from improving patent scoring by using patent age and new information. The technological fields exhibiting the best compromise between a correct explanation of the initial rent, on the one hand and a correct explanation of its dynamics, on the other hand are technological fields B ("*Performing operations; Transporting*") and G ("*Physics*"). More details about the impact of static variables and dynamic variables on respectively the initial rent and its dynamics are provided in the two following tables.

Insert Table 4

Table 4 reports coefficients of static variables influencing the expected natural logarithm of the initial rent. When focusing on coefficients statistically significant at a 10% confidence level, it appears that each static variable generally impacts the initial rent in the same direction, whatever the technological field. The two exceptions are variables *Size* and *speIPC*. Among the four technological fields for which *Size* has a significant impact on the initial rent, two exhibit a positive impact (B ("*Performing operations; Transporting*") and G ("*Physics*")) while the two others (C ("*Chemistry; Metallurgy*") and D ("*Textile; Paper*")) are characterised by a negative impact. Thus, no general conclusions can be drawn as regards the impact of the innovative capacity of applicants on patent value. This impact varies from one technological field to the other. The variable *speIPC* has a positive impact for technological fields A ("*Human necessities*"), G ("*Physics*") and H ("*Electricity*") and a negative impact for technological field D ("*Textiles; Paper*"). The degree of specificity versus generality (as measured by the percentage of declared IPC subclasses that belong to the class of interest) is neutral for all other technological fields. The static variable that influences initial rents in the highest number of technological fields is the number of words in key sentences (*keywords*). Its impact is concordant and positive for all technological fields. It is followed by the number of claims (*Claims*). Other variables having a concordant and significant positive impact, although for a lower number of technological fields, are the number of IPC classes targeted by the patent (*nbIPC*), the number of backward citations made by the examiner (*Ncte*), the family size (*Npn*), the fact that the patent is applied for via the PCT procedure (*Pct*) and the number of European countries targeted by the patent (*Ndsep*). The other static variables have no significant influence on the initial rent.

Another way to read estimation results reported in Table 4 consists in looking at those technological fields for which the number of significant metrics is the highest. Technological field C ("*Chemistry; Metallurgy*") clearly emerges as the technological field with the highest number of significant coefficients followed by technological fields A ("*Human necessities*"), B ("*Performing operations; Transporting*"), G ("*Physics*") and H ("*Electricity*"). Note that these technological fields are not necessarily those with the highest percentage of total variance in the initial rent explained by the model.

<u>Insert Table 5</u>

Estimated values of coefficients related to observed variables that affect the dynamics of the rent are displayed in the upper part of Table 5. A striking result is that most of these coefficients are highly significant and modify the depreciation rates of the rent in the same way whatever the technological field considered. A positive sign of the coefficient associated to a dynamic variable means that the depreciation rate decreases with the variable (see expression (10)). Accordingly, the depreciation rate is lower *ceteris paribus* in the initial period of a patent life compared to the situation where the patent is close to its statutory life limit (variable *Age*) for technological field B ("*Performing operations; Transporting*") and F ("*Mechanical engineering; Lighting; Heating; Weapons; blasting*") .

Similarly, new forward citations (*Fcit*) received by a patent decrease its depreciation rate at the date these citations are received and thus yield higher values of the rent at all subsequent dates for all technological fields except technological field G ("*Physics*"). Finally, the depreciation rates of patents are systematically and negatively correlated to the GDP growth rate (variable *Gdp*). Indeed, the GDP growth rate of the current year always has a positive coefficient in Table 5.

The lower part of Table 5 helps understanding the net impact of dynamic variables on the depreciation rate of the rent. Average values of the estimated deterministic component of annual depreciation rates at age one and ten are first displayed. The fact that most patents are still alive at age ten and that, at the same time, a sufficient number of forward citations have been received at this age to generate heterogeneity in the evolution of the rents justifies the focus on this age. The comparison between age one and age ten clearly shows that, in absolute terms, the deterministic component of annual depreciation rates increases. Thus, the accumulation of forward citations (variable *Fcit*) and economic growth (variable *Gdp*) do not counterbalance a natural tendency for depreciation rates to accelerate (variable *Age*). The average sensitivity to changes in dynamic variables at age ten is quite similar from one technological field to another and for an additional citation on the one hand or a one percent more economic growth on the other hand. The estimated average drop for an additional citation ranges between -0.08 for technological field D ("*Textiles; Paper*") and -0.14 for technological field H ("*Electricity*"). These figures mean that if the depreciation rate was for instance 40% before the new citation is received, then it falls respectively at 32% or 26% after the new citation is received. As a result, patents receiving many forward citations may have a much lower depreciation rate than other patents. Technological fields that are the most sensitive to a one percent GDP increase are not systematically the same than those that are the most sensitive to the receipt of a new forward citation. Indeed, the estimated average drop of the depreciation rate associated to an additional point of economic growth ranges from -0.08 for technological field F ("*Mechanical engineering; Lighting; Heating; Weapons; Blasting*") up to -0.17 for technological field H ("*Electricity*").

Whether unobserved heterogeneity has a significant impact on the dynamics of the rents cannot be evaluated on the basis of t-statistics for two reasons. The first reason is that the significance of the two associated coefficients cannot be tested separately. The second reason is that coefficients reported in Table 5 are not directly the expected value and standard deviation of random shocks but their natural logarithm[19]. As a result, the role of unobserved heterogeneity is assessed on the basis of a log-likelihood ratio. This is why Table 3 reports the log-likelihood of the model with its dynamics restricted to observed heterogeneity and the chi square statistic to implement the test[20]. Unobserved heterogeneity plays a significant role in all technological fields except technological field D ("*Textiles; Paper*") and eventually technological field E ("*Fixed constructions*") if a low risk of error is imposed.

Note that even if the unobserved component of the dynamic is not statistically significant, the existence of a significant impact for one or several dynamic variables is sufficient to generate a patent option value. In this respect, the numerous and highly significant effects identified in Table 5, both for the observed and unobserved components of the dynamics, strongly advocate in favour of the existence of a patent option value whatever the technological field considered. Option value follows on from the important role played by the revealing of additional information as patents age. It thus justify the use of a renewal mechanism that provides greater flexibility in the withdrawal decision compared to a mechanism based on a upfront fee modulated in accordance to a predetermined withdrawal date. Meanwhile, the existence of an option value undermines the accuracy of any scoring system, at least in the early ages of a patent, due to the uncertainty that intrinsically affects future realisations of the rents.

[19] Instead of being specified as $\mu > 0$ and $\upsilon > 0$ the expected value and standard deviation are specified as $\exp(\mu')$ and $\exp(\upsilon')$ to make sure that they will have positive values.

[20] More precisely coefficients μ' and υ' have been restricted to -100 so that $\exp(\mu')$ and $\exp(\upsilon')$ are almost equal to zero.

3.2. Scoring results

As stressed by Barney (2002), although patent metrics with a significant coefficient are interesting and informative, individually they provide only limited guidance in determining overall patent quality. What we need to obtain is a single rating to be used to directly forecast or estimate the value of each patent. In the methodology used by Barney (2002), patents are positioned relative to each other. Raw scores are first produced on the basis of the estimated probability that each patent is maintained for the full statutory term. For convenience, raw scores are adjusted to provide a normalised mean or median score of 100. Thus, a score of 100 on the IPQ scale generally corresponds to an expected normal quality (average expected duration) while an IPQ score higher or lower than 100 indicates an expected above-average or below-average patent quality respectively. Of course, as with IQ, the IPQ score provides only part of the equation for determining patent quality/value. Thus, a high IPQ does not guarantee high quality/value and vice versa. It only establishes a statistical correlation based on the body of available data.

In our paper, we follow the same line of reasoning than Barney (2002) by positioning patents against each other. For this purpose, we compute the ratio of the estimated value of each patent to the median value times 100. However our methodology departs from that proposed by Barney (2002) as regards the basis for computing IPQs. Indeed, in accordance with the theoretical discussion on the link between patent renewals and patent value presented in section II, we compute our IPQ on the basis of an estimated monetary value for each patent rather than on the basis of maintenance rates. Monetary values are estimated by using one thousand random draws, conditional on observed static variables, of the initial rent and one thousand random draws of successive annual depreciation rates of the rent for each patent. The method used to generate these random draws is the same than the first method used to generate the cumulated frequencies of withdrawal in Figure 3. In order to stress the role of additional information acquired as patents age, IPQs have been computed at age one and ten. When computed at age one, IPQs are thus based on random simulations of cumulated forward citations and random simulations of GDP growth. Conversely, when computed at age ten, IPQs account for past realizations of dynamic variables that affect the probability distribution of the rent conditional on renewal up to age ten. Whatever the age considered, the use of numerous simulations for each patent enables us to generate an empirical distribution of IPQs for each patent and thus to determine the probability that a specific draw of the IPQ sharply departs from the expected IPQ. For each random draw, the optimal withdrawal age defined in Proposition 1 is determined, and then, the discounted sum of the rent net of renewal fees from the date of application to the optimal withdrawal date is computed[21]. The value affected to each patent is the average value obtained over the one thousand random draws.

Insert Figure 5

Figure 5 shows the empirical distribution of IPQs over all patents in a same technological field at age one and ten. Note that for graphical convenience, the distributions have been truncated at 800. Thus, the high frequencies of IPQs observed on the extreme right of each distribution only reflect the fact that there is still an important number of patents on the right tail of the distribution. The general shape of distribution does not sharply differ from one technological field to another and from year one to year ten. The distributions are systematically highly asymmetric with a mode associated to low values of the IPQ and a tail that spreads far on the right, up to more than 1000. This is in line with a well established

[21] The discount rate is set at 3%. Clearly, this choice is arbitrary and justifies working with a normalised score like the IPQ rather than on estimated monetary values.

result in the empirical literature. More interestingly, for most technological fields, the asymmetric distribution of IPQs is more pronounced at age ten than at age one. Indeed, the proportion of IPQs with low values and the proportion of IPQs above 800 increase whereas the proportion of patents with intermediate IPQs slightly decreases. As a result, patents are more discriminated at age ten than at age one, a result that follows on from the additional information available to compute IPQs. Technological field G ("*Physics*") and to a lesser extent technological field D ("*Textiles; Paper*") are two exceptions. Technological field G ("*Physics"*) is more specifically characterized by a decrease of the number of patents with an IPQ higher than 800 at age ten compared to age one; a characteristic which may be related to the sharp increase of the deterministic component of the depreciation rate reported in Table 5.

Insert Figure 6

Figure 6 is intended to assess whether IPQs provide a reliable scoring or not. For this purpose, empirical distributions of the monetary values associated to the one thousand random draws have been generated for two patents with respectively the lowest and the highest IPQ in each technological field. Monetary values are normalized by the median value of the patents in the technological field, times one hundred, to be consistent with the computation of an IPQ. Comparisons between the two distributions have been made for age one and age ten. Nevertheless, because the results obtained for age ten systematically correspond to 100% of IPQs at the extreme left for the patent with the lowest average IPQ and 100% at the extreme right for the patent with the highest average IPQ, they are not displayed in Figure 6. This polarization of IPQs reveals an unambiguous ranking of these two patents and constitutes a positive signal as regards the reliability of an IPQ scoring when applied to patents at mid-term. Though less pronounced, this is also the case at age one for technological fields B ("*Performing operations; Transporting*"), E ("*Fixed constructions*"), F ("*Mechanical* *engineering; Lighting; Heating; Weapons; Blasting*") and G ("*Physics*"). Conversely, for technological fields A ("*Human necessities*"), C ("*Chemistry; Metallurgy*"), D ("*Textiles; Paper*") and H ("*Electricity*"), the patent with the highest average IPQ dominates the patent with the lowest IPQ only because of some abnormal IPQ draws that pull the average IPQ upwards. An IPQ scoring is thus not reliable at early ages for these technological fields.

4. Conclusion

A hallmark of a properly functioning marketplace for IPR is that there is a clear way to determine the price of the assets being bought and sold. It goes without saying that the creation of a rating scheme and of a robust and efficient financial market for intangibles should add considerable value to the on-going IP market. But this is challenging in itself and the task has been made even more difficult by the recent turmoil in the financial market and accusations that credit rating agencies (CRAs) are plagued by conflicts of interest that might inhibit them from providing accurate and honest ratings. Patent rating agencies are not exempt from such criticisms. This explains in part why policy-makers are considering transparency standards for patent rating agencies and the obligation for them to publish their valuation methods. It is indeed important for market operators and investors to understand how a specific rating was determined and to have assessments of the uncertainty surrounding scoring results.

Our paper was aimed at studying the feasibility and reliability of such a rating scheme. Results for a sample of EPO patents designating France show that some key observable patent characteristics significantly affect patent value. Nevertheless, unobserved heterogeneity is too high to efficiently discriminate among patents. Our results show that the most significant effects are those affecting the dynamics of the rent, not the initial rent itself. This is of course due to the fact that patent maintenance/abandonment decisions can only be observed *ex-post* and that uncertainty at the time of issue is very large meaning that patent value becomes visible only over time as uncertainty vanishes.

More precisely, we show that statistically derived patent performance benchmarks can provide objective measures of comparative patent quality and/or value but at the cost of potentially important uncertainties. Indeed, it is likely that some low or high scores could result from unexplained and random factors beyond the control of any rating agency. Moreover, it is worth keeping in mind that the existence of an option value undermines the accuracy of any scoring system, at least at the early ages of a patent, due to the uncertainty that intrinsically affects future realisations of the rents. Therefore, although an automatic scoring is intellectually stimulating in itself, our results cast some doubt on the possibility to develop a reliable rating system based only on patent metrics. Indeed, these scoring methods provide only a rough estimate of patent values, not a true economic evaluation. In other words, these methods do not give a reliable prediction of the monetary patent value but a measurement of patent quality ("exchange rate") for companies.

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TABLE I.
STATISTICS ON PATENT RENEWALS

Age	A Human necessities	B Performing operations; Transporting	C Chemistry; Metallurgy	D Textiles; Paper	E Fixed constructions	F Mechanical engineering; Lighting; Heating; Weapons; Blasting	G Physics	H Electricity	Average renewal fee on the period 1989 to 2009 In Euros
	Sample size								
	3641	5374	6235	680	638	2204	4595	3537	
	Observed frequencies of withdrawal								
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	49.55
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	26.81
3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	29.47
4	0.27%	0.19%	0.08%	0.29%	0.31%	0.23%	0.09%	0.08%	32.48
5	1.43%	1.30%	0.79%	2.21%	2.19%	2.00%	0.37%	0.59%	41.09
6	3.63%	5.08%	3.50%	3.24%	8.15%	6.03%	2.72%	1.41%	82.46
7	6.73%	7.15%	5.92%	4.12%	10.34%	7.67%	4.20%	4.44%	98.73
8	8.71%	8.49%	7.89%	8.09%	10.97%	7.17%	8.55%	7.72%	117.84
9	7.33%	7.13%	7.41%	8.24%	8.46%	6.94%	8.23%	8.37%	140.24
10	6.84%	6.74%	7.25%	7.50%	7.05%	6.58%	5.68%	6.64%	161.91
11	5.66%	6.59%	6.56%	5.29%	6.43%	6.90%	6.99%	6.87%	220.61
12	4.70%	5.38%	5.98%	6.91%	5.02%	5.76%	5.18%	5.80%	245.72
13	4.39%	5.10%	4.67%	4.85%	5.33%	6.08%	4.44%	4.89%	274.07
14	5.16%	5.95%	6.53%	5.74%	6.11%	5.85%	6.25%	6.93%	300.76
15	5.22%	5.32%	6.85%	8.09%	4.86%	5.76%	6.12%	7.35%	329.11
16	4.92%	4.69%	4.78%	4.26%	3.92%	5.13%	5.20%	5.63%	434.68
17	4.28%	3.67%	3.80%	3.82%	2.82%	4.17%	5.16%	4.66%	464.70
18	4.15%	4.24%	3.85%	5.00%	2.51%	4.17%	4.48%	4.07%	495.57
19	3.10%	4.06%	2.87%	4.85%	2.82%	2.99%	5.05%	4.18%	527.05
20	6.65%	6.29%	5.42%	6.62%	4.08%	4.76%	7.33%	6.93%	563.78
21	16.84%	12.65%	15.86%	10.88%	8.62%	11.80%	13.97%	13.43%	

TABLE II.
DESCRIPTIVE STATISTICS FOR STATIC VARIABLES

	A Human necessities	B Performing operations; Transporting	C Chemistry; Metallurgy	D Textiles; Paper	E Fixed constructions	F Mechanical engineering; Lighting; Heating; Weapons; Blasting	G Physics	H Electricity
Size	31.25 (68.11)	46.78 (86.56)	78.39 (101.98)	48.95 (80.56)	11.47 (37.20)	34.39 (67.27)	105.96 (127.47)	118.07 (133.48)
nbIPC	2,81 (1,50)	2,83 (1,71)	3,67 (1,91)	2,86 (1,71)	2,54 (1,48)	2,70 (1,40)	2,54 (1,51)	2,49 (1,40)
speIPC	59,55 (32,42)	52,39 (33,04)	58,12 (25,32)	42,40 (34,36)	60,44 (35,14)	50,33 (32,40)	48,31 (26,46)	48,17 (26,60)
entropyIPC	0,46 (0,38)	0,47 (0,41)	0,51 (0,37)	0,56 (0,39)	0,48 (0,43)	0,46 (0,42)	0,43 (0,40)	0,39 (0,41)
Claims	16,51 (17,59)	12,70 (9,40)	17,03 (17,39)	12,61 (9,12)	11,87 (8,05)	11,54 (8,24)	14,37 (12,44)	13,20 (11,51)
Keywords	463,72 (307,70)	512,47 (260,48)	440,05 (264,19)	462,42 (225,04)	494,82 (260,08)	519,71 (266,99)	533,84 (268,35)	527,07 (267,01)
Ncte	3,46 (2,54)	3,80 (2,47)	3,19 (2,29)	3,49 (2,14)	3,94 (2,85)	3,97 (2,66)	3,41 (2,74)	3,12 (2,21)
Ncta	0,03 (0,45)	0,02 (0,36)	0,05 (1,31)	0,01 (0,34)	0,02 (0,36)	0,00 (0,12)	0,05 (0,77)	0,01 (0,15)
Npr	2,27 (1,23)	2,11 (1,23)	2,18 (1,27)	1,94 (1,10)	2,25 (1,18)	2,04 (1,15)	1,94 (1,36)	1,81 (1,13)
Npn	9,31 (5,26)	7,27 (4,06)	8,43 (4,79)	7,73 (3,80)	7,57 (3,97)	6,86 (3,34)	6,29 (3,15)	6,20 (2,82)
Ncc	8,61 (4,70)	6,59 (3,36)	7,73 (4,26)	7,15 (3,37)	7,00 (3,74)	6,23 (2,88)	5,71 (2,59)	5,72 (2,52)
Pct	0,22 (0,41)	0,18 (0,38)	0,16 (0,37)	0,13 (0,34)	0,22 (0,42)	0,21 (0,41)	0,18 (0,38)	0,16 (0,36)
Ndsep	9,69 (3,38)	7,42 (3,51)	8,50 (3,50)	7,79 (3,20)	8,67 (3,40)	6,73 (3,34)	6,29 (3,39)	5,99 (3,23)
Univ	0,02 (0,14)	0,001 (0,05)	0,02 (0,13)	-	-	-	0,01 (0,08)	0,001 (0,06)
Firm&Univ	0,01 (0,08)	0,001 (0,03)	0,01 (0,08)	-	-	-	0,001 (0,04)	0,001 (0,04)

TABLE III.
GLOBAL INDICATORS OF ESTIMATION RESULTS

A	B	C	D	E	F	G	H
Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Log likelihood of the complete model							
$-9.908*10^{3}$	$-1.484*10^{4}$	$-1.682*10^{4}$	$-1.889*10^{3}$	$-1.739*10^{3}$	$-6.109*10^{3}$	$-1.252*10^{4}$	$-9.586*10^{3}$
Log likelihood of the model with dynamics restricted to observed heterogeneity							
$-1.829*10^{4}$	$-3.802*10^{4}$	$-3.024*10^{4}$	$-2.434*10^{3}$	$-4.948*10^{3}$	$-1.790*10^{4}$	$-3.207*10^{4}$	$-1.787*10^{4}$
Chi square statistic for the log-likelihood ratio test of restriction to a non stochastic dynamics							
16.76	46.36	26.84	1.09	6.42	23.58	39.10	16.56
% of total variance of the natural logarithm of the initial rent explained by the model							
8.58	16.81	13.90	12.60	28.84	12.95	16.4579	9.9758
% of total variance of the natural logarithm of depreciation up to age 10 explained by the model							
28.11	14.76	30.59	19.01	10.26	11.79	20.36	29.43
Average probability of withdrawal at age 10 for patents effectively withdrawn at age 10							
0.0834	0.0929	0.0912	0.0821	0.1199	0.0978	0.0776	0.0821
Average probability of withdrawal at age 10 for patents renewed at age 10							
0.0812	0.0908	0.0871	0.0798	0.1177	0.0967	0.0757	0.0789

TABLE IV.
ESTIMATION RESULTS FOR THE INITIAL RENT

	A	B	C	D	E	F	G	H
	Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Estimated coefficients for static variables (t-stats are reported in brackets, grey cells correspond to coefficients statistically significant at a 10% confidence level)								
Intercept	21.2660 (13.7637)	14.2947 (6.7753)	22.5300 (30.9868)	30.7070 (5.6114)	8.1877 (1.9651)	14.0031 (3.8652)	17.7109 (8.5633)	23.1772 (3.3055)
Standard deviation	4.2664 (8.1151)	3.1006 (18.5660)	4.1235 (8.1571)	7.0814 (2.4529)	2.4735 (10.3058)	3.0686 (8.6849)	3.2839 (11.8823)	4.5986 (6.0801)
Size	0.2179 (1.4226)	0.4719 (4.2490)	-0.4197 (-6.9914)	-0.7686 (-1.7202)	-0.2164 (-0.5563)	-0.1139 (-0.5065)	0.1845 (1.7525)	-0.0660 (-0.5164)
nbIPC	0.2605 (0.1063)	0.3442 (0.9659)	0.5414 (1.8900)	3.1060 (2.3040)	-0.3009 (-0.3126)	0.5461 (0.8679)	0.3489 (0.8708)	0.5216 (0.8990)
speIPC	0.0239 (2.7853)	0.0103 (1.3752)	-0.0056 (-0.9513)	-0.0625 (-2.3218)	-0.0074 (-0.4109)	-0.0016 (-0.1302)	0.0185 (2.4096)	0.0250 (2.5956)
entropyIPC	-0.2679 (-0.3561)	0.0281 (0.0521)	-0.4085 (-0.9643)	-1.7845 (-0.9373)	0.4279 (0.2940)	0.1045 (0.1208)	-0.4487 (-0.7799)	-0.4349 (-0.5668)
Claims	0.3044 (1.1430)	0.5439 (2.3710)	0.5151 (2.8818)	-0.8908 (-1.1747)	0.3182 (0.5766)	0.2707 (0.6832)	1.0703 (4.6271)	0.7117 (2.5632)
Keywords	0.4911 (1.5938)	1.0035 (3.4193)	0.5497 (2.7247)	0.7870 (0.7786)	1.7319 (2.6471)	1.3143 (2.3976)	0.8952 (2.3709)	1.1054 (2.6823)
Ncte	0.0316 (0.3913)	0.2846 (4.2616)	0.1135 (1.8363)	0.0656 (0.2497)	0.0684 (0.4427)	0.0671 (0.5526)	0.2155 (2.8401)	0.1445 (1.4161)
Ncta	0.1214 (0.2554)	0.3864 (0.6535)	0.0511 (0.4731)	-1.1308 (-0.9272)	0.0928 (0.0727)	1.2073 (0.3228)	0.2384 (0.7369)	0.3364 (0.1573)
Npr	-0.4312 (-0.8684)	0.0787 (0.1890)	-0.1819 (-0.5684)	-0.9561 (-0.6792)	-0.1776 (-0.1837)	-0.1495 (-0.1748)	0.3853 (0.8092)	0.2807 (0.4678)
Npn	3.2593 (1.9364)	1.1399 (0.8675)	2.9632 (2.6307)	0.5839 (0.1373)	1.6112 (0.5259)	2.1280 (0.8283)	1.8175 (1.3544)	3.2886 (1.7522)
Ncc	-1.8027 (-1.0729)	0.1704 (0.1284)	-1.4140 (-1.2610)	1.9939 (0.4548)	-0.1067 (-0.0373)	-0.9210 (-0.3544)	-1.4599 (-1.0263)	-2.2548 (-1.0153)
Pct	1.4424 (2.5790)	0.4472 (0.8691)	1.4454 (3.3925)	2.3339 (1.2277)	0.9695 (0.8586)	0.5031 (0.5385)	0.2262 (0.4019)	0.0740 (0.0997)
Ndsep	-0.3966 (-0.7281)	-0.1786 (-0.4673)	0.0245 (0.0751)	0.3424 (0.2463)	1.3958 (1.4766)	0.5125 (0.7207)	-0.0493 (-0.1190)	0.0056 (0.0102)
Univ	0.3394 (0.6643)	-1.3382 (-0.4696)	0.4550 (0.4202)	-	-	-	-1.1262 (-0.5638)	-2.0030 (-0.2930)
Firm&Univ	1.4594 (0.9366)	0.6355 (0.1224)	1.0101 (0.5851)	-	-	-	-0.1322 (-0.0413)	-0.7455 (-0.1498)

TABLE V.
ESTIMATION RESULTS FOR THE DYNAMICS OF THE RENT

	A	B	C	D	E	F	G	H
	Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Estimated coefficients for the observed component of the depreciation rate (t-stats are reported in brackets, grey cells correspond to coefficients statistically significant at a 10% confidence level)								
Intercept	-2.1056 (-6.1505)	-1.2277 (-5.8046)	-2.2855 (-8.6828)	-2.7270 (-2.3040)	-1.7116 (-3.0174)	-0.7164 (1.6511)	-1.5316 (-1.3642)	-2.6499 (-4.176)
Age	-0.0215 (-0.5419)	-0.0978 (-3.3161)	0.0174 (0.8223)	-0.0185 (-0.1942)	-0.0495 (-0.5740)	-0.1633 (-2.9322)	-0.0912 (-1.3722)	-0.0243 (-0.5452)
Gdp	0.7236 (6.5277)	0.6016 (8.2406)	0.6136 (5.8644)	0.6556 (6.5017)	0.6191 (11.4521)	0.3951 (7.5249)	0.7563 (2.1886)	0.7262 (3.7744)
Fcit	0.4829 (3.4291)	0.5809 (4.6599)	0.4999 (8.5962)	0.3495 (2.5960)	0.5531 (7.8719)	0.6134 (7.3627)	0.6299 (1.4277)	0.6033 (2.9220)
Estimated coefficients for the unobserved component of the depreciation rate (t-stats are reported in brackets)								
Expected value	-0.2768 (-0.6849)	-0.3470 (-1.2870)	-0.3365 (-0.9429)	-0.2834 (-0.3469)	-0.3264 (-0.6579)	-0.7302 (-1.0071)	-0.3290 (-0.2501)	-0.2623 (-0.3806)
Standard deviation	-0.4233 (-2.3326)	-0.2388 (-2.2897)	-0.6279 (-4.8544)	-0.6763 (-0.8986)	-0.3512 (-0.8057)	-0.0286 (-0.1569)	-0.3398 (-1.6054)	-0.4309 (-2.060)
Average value of the deterministic component of depreciation rates at age 1 (standard deviations are reported in brackets)								
	0.2909 (0.0101)	0.2346 (0.0074)	0.4268 (0.0169)	0.5031 (0.0081)	0.3059 (0.0071)	0.3165 (0.0110)	0.1775 (0.0099)	0.4119 (0.0160)
Average value of the deterministic component of depreciation rates at age 10 (standard deviations are reported in brackets)								
	0.3960 (0.0996)	0.4825 (0.0928)	0.4448 (0.0953)	0.6182 (0.0714)	0.4849 (0.0738)	0.6889 (0.0938)	0.4017 (0.0950)	0.5335 (0.1094)
Average impact on the deterministic component of depreciation rates at age 10 (standard deviations are reported in brackets)								
One forward citation more	-0.1045 (0.0216)	-0.1358 (0.0186)	-0.1143 (0.0187)	-0.0833 (0.0030)	-0.1313 (0.0148)	-0.1380 (0.0096)	-0.1345 (0.0260)	-0.1427 (0.0192)
1% more of GDP growth rate	-0.1504 (0.0322)	-0.1404 (0.0193)	-0.1383 (0.0232)	-0.1583 (0.0069)	-0.1460 (0.0167)	-0.0866 (0.0063)	-0.1580 (0.0312)	-0.1706 (0.0239)