

Growth through acquisition of innovations[¶]

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June 3, 2019

Abstract

The paper develops a model of growth driven by the acquisition of domestic firms by their peers, treating innovations endogenously. The model builds on microeconomic evidence concerning acquisitions in a technology economy, where the acquirers are innovative firms, which regard acquisitions as a complementary strategy to their R&D investments. The targets are small firms with leading positions on markets for their products. The acquirers are capable of further improving the products of their targets. The model includes the government, which collects corporate profit tax and redistributes it to provide subsidies for innovation and for acquisitions.

We quantify the model using 1999–2013 financial data for Japanese firms, matched with their patents. The estimates bear out the predictions of positive effect of acquisitions on economic growth. The impact of acquisitions on R&D intensity is negative under the substitutability between innovation and acquisition strategies. The effect of government subsidies to encourage acquisitions is linked to the parameters of the cost function and reflects the association between the cost of acquisitions and of R&D.

JEL Classification Codes: O11, O38, O40, O53

Keywords: innovation, endogenous growth, acquisition, social planner, patents

[¶]We are indebted to Kyoji Fukao, Takeo Hoshi, Petros Sofronis, Sjak Smulders; participants of the EEA-ESEM (2017–2018), the Japan program seminars at Stanford University Freeman Spogli Institute for International Affairs (2016–2017) and Kyushu University I2CNER lectures (2017) for helpful comments.

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1 Introduction

Dynamic industrial organization is characteristic of an economy undergoing technological change. Acquisitions are an important part of firm dynamics and there is abundant microeconomic evidence of links between innovative behavior and growth of the acquiring and acquired firms.¹ However, there is no general macroeconomic framework that adequately evaluates the impact of acquisitions on the innovation strategy of companies and on growth in an economy undergoing technological change. The existing models that incorporate innovation and acquisitions describe partial equilibrium (Phillips and Zhdanov (2013), Rhodes-Kropf and Robinson (2008)), while a few attempts to consider acquisitions using a general equilibrium approach abstracted from innovation (Xu (2017), David (2017)). Also, the findings of microeconomic research on acquisitions and innovations are sometimes hard to interpret. In particular, there is ambiguity as to the estimated effect of acquisitions on the ratio between a firm's R&D expenditure and its sales (Ahuja and Novelli (2013), Gantumur and Stephan (2012), Zhao (2009), Hall (1999), Hitt et al. (1991), Hall et al. (1990)). The lack of general agreement in the literature suggests that analysis is required in order to explain or reconcile the conflicting results.

In this paper we propose a growth model with acquisitions for endogenous innovations. The novel feature of our theoretical approach is that it models acquisitions by taking account of the underlying microeconomic evidence on acquiring and acquired firms. Specifically, we investigate how companies add new product lines through strategic acquisitions, which we define as an increase in the acquired company's stock or capital, varying from zero or a minority stake to a majority share.² We regard acquirers as innovative firms, which use acquisitions to complement the effect of internal development through innovation. The targets are small firms, which have the best technology on the market for their goods. The acquirers raise target's productivity by improving the quality of target's products after the acquisition.

The acquirers incur acquisition costs, spending on market research and evaluation in their search for potential targets.³ The government may use various policy instruments to subsidize the acquisition costs. Firstly, companies may be allowed to capitalize certain types of expenses as transaction costs or to deduct them from taxable corporate income. So extension of the list of activities, which fall under transaction costs of acquisition, may be regarded as an increase in government subsidies. Secondly, government may support joint market research by several companies and hence enable a cheaper search for targets.

Our model allows decomposition of growth and analysis of the impact of industrial policy, designed to incentivize acquisitions and innovations. The first prediction states that acquisitions have a positive effect on growth. The second prediction relates the impact of acquisitions on R&D intensity to the type of complementarity between innovation and acquisition strategies.

¹See review in Andrade et al. (2001) and recent case studies in Bena and Li (2014), Sheen (2014), Gantumur and Stephan (2012), Zhao (2009), Kapoor and Lim (2007), Higgins and Rodriguez (2006), Cassiman et al. (2005).

²Buyouts of stressed companies carried out due to pressure from the regulator and rare events of one-to-one mergers are not considered.

³It should be noted that a company's decision to bear the acquisition cost may not necessarily lead to a successful acquisition. So the value of a completed deal is considered as a separate type of cost in our model.

Finally, we predict that the effect of subsidies for acquisitions is associated with the parameters of the cost function and reflects the relationship between the costs of acquisitions and R&D.

We use data on the Japanese economy, which offers several advantages for quantification of the model. Firstly, Japan has experienced an unprecedented boom in acquisitions since the end of 1990s, which has dramatically altered the industrial organization of the national economy. These acquisitions have contributed to productivity in various industries and often compensated for lack of internal innovation by the acquirers (Watanabe et al. (2009), Miyajima (2007), Fukao et al. (2005)). Secondly, standard approach, which approximated innovation using patent statistics in endogenous growth models is particularly applicable to Japan, where patenting is a more prevalent business strategy than in the U.S. (Cohen et al. (2002)).

We take our sample from innovative Japanese firms, their deals and their patents in 1999–2013. Our main sample consists of firms from the *Nikkei Economic Electronic Databank System*. The auxiliary sample is composed of innovative firms as defined by the *Japan National Innovation Survey* of the National Institute of Science and Technology Policy. Additionally, we employ the data on the universe of Japanese firms from the *Orbis* database. The unique empirical part of our work with data is the establishment of a link between the firms in our samples and their patent applications. For this purpose, we use bulk data from the Japan Patent Office and available at the National Center for Industrial Property Information and Training (the *J-PLAT-PAT* platform) and the Institute of Intellectual Property (*IIP Patent Database*). The data on acquisitions come from the *Zephyr* database.

We equate the data-generated moments to the predicted moments to quantify the model, and discover a good fit between the model and the data. Our estimates yield a positive relation between costs of R&D and costs of acquisitions for Japanese firms. The results of our counterfactual policy analysis show that government subsidies for acquisitions increase both innovation and acquisition rates, and lead to growth of the economy. The finding is in line with the predictions of positive impact of acquisitions on growth contained in the theoretical model. However, we discover that limiting of government policy to the subsidizing of acquisitions is relatively unproductive per unit of government spending. When the cost of government subsidies is evaluated against percentage-point increase in GDP, we find that acquisition subsidies are slightly cheaper than R&D subsidies to incumbents, but more expensive than R&D subsidies to entrants or uniform R&D subsidies to all firms.

The present paper is related to several literatures. The microeconomic foundations of our model are the Wernerfelt (1984) resource-based theory of the firm and the Rubin (1973) framework, where expansion is associated with acquiring necessary resources from another firm. In the context of a technology economy, Aghion and Tirole (1994) similarly interpret acquisition as a substitute for a firm’s own innovation. Our model can also be linked to the game-theoretic approach by Phillips and Zhdanov (2013), who suggest that large firms use acquisitions as a substitute strategy to win the innovation race.

As regards the macroeconomic literature, our approach builds on the endogenous growth models of Klette and Kortum (2004) and Lentz and Mortensen (2008), where the quality of

product lines in the economy is affected by innovation processes. The firm, which achieves best quality, has monopoly power over the market for its product. Accordingly, firms innovate to maximize profits through adding new products or improving the quality of existing products. This approach originates from the Schumpeterian concept of monopoly competition through quality increase, for instance, as described in [Schumpeter \(1942\)](#), [Romer \(1990\)](#), [Grossman and Helpman \(1991\)](#), [Aghion and Howitt \(1992\)](#). The second current in earlier macroeconomic research is that of endogenous growth models with multiple types of innovation and different sizes of quality improvement ([Akcigit and Kerr \(2018\)](#), [Acemoglu et al. \(2016\)](#), [Acemoglu and Cao \(2015\)](#)). Finally, our model is linked to analyses of industrial policy, imposed by social planner in a macroeconomic framework with endogenous innovation ([Acemoglu et al. \(2018\)](#), [Akcigit et al. \(2017\)](#), [Akcigit et al. \(2016\)](#), [Lentz and Mortensen \(2016\)](#)).

It should be noted that endogenous growth models with technological change usually account for firm dynamics only in terms of market entry and exit. The models make a good fit with empirical evidence on firm size, R&D activity and growth in such countries as the U.S., France, Denmark, Japan, Chili and Indonesia ([Acemoglu et al. \(2018\)](#), [Akcigit and Kerr \(2018\)](#), [Acemoglu et al. \(2016\)](#), [Lentz and Mortensen \(2016\)](#), [Ates and Saffie \(2016\)](#), [Peters \(2013\)](#), [Lentz and Mortensen \(2008\)](#), [Grossman \(1990\)](#)), but acquisitions are not captured in this framework. To the best of our knowledge, the only exception is the [Smulders and Klundert \(1995\)](#) approach, which considers the [Romer \(1990\)](#) endogenous growth model with firm-specific R&D and regards the multiproduct firms as the firms, appearing after the horizontal merger.

The empirical part of our paper follows research, based on the [Lentz and Mortensen \(2008\)](#) methodology for quantifying general equilibrium models with endogenous growth along a balanced growth path ([Acemoglu et al. \(2018\)](#), [Akcigit and Kerr \(2018\)](#), [Akcigit et al. \(2017\)](#), [Lentz and Mortensen \(2016\)](#), [Acemoglu and Cao \(2015\)](#)).

As regards literature on the general equilibrium analysis of acquisitions, increase of a target's productivity up to the level of the acquirer in our model is similar to the approach used by [Xu \(2017\)](#).

The remainder of the paper is structured as follows. Section 2 presents stylized microeconomic facts about the association between acquisitions, innovation and growth. Section 3 builds a growth model with endogenous innovations, acquisitions and heterogeneous firms. A version of the model with multiple types of innovations is given in section 4. Section 5 deals with quantification of the model and testing of predictions. The data on Japanese firms and their patents are given in section 5.1. Section 5.2 presents the empirical specifications for firm-level estimates, while the calibration of our model and policy experiments within the general equilibrium framework are conducted in section 5.3. The model proofs, details on data work and matching algorithms are given in the appendices.

2 Stylized facts on acquisitions and innovations

This section outlines microeconomic evidence on the association between acquisitions, innovations and firm growth. We focus on established facts about the causes and consequences of acquisitions in the economy with technological change. See reviews in [Ahuja and Novelli \(2013\)](#) and [Andrade et al. \(2001\)](#).

2.1 Drivers of acquisitions

1. Acquisitions are driven by uniqueness/high value of technological knowledge ([Carayannopoulos and Auster \(2010\)](#), [Villalonga and McGahan \(2005\)](#), [Schilling and Steensma \(2002\)](#)), technological/managerial economies of scale and overall synergies in technology ([Betton et al. \(2008\)](#), [Andrade et al. \(2001\)](#), [Teece \(1982\)](#), [Mueller \(1969\)](#)).
2. Acquisitions are an alternative strategy to firm's internal development through innovation or diversification ([Teece \(1982\)](#), [Aghion and Tirole \(1994\)](#), [Miller \(2004\)](#), [Rhodes-Kropf and Robinson \(2008\)](#), [Phillips and Zhdanov \(2013\)](#)).

2.2 The acquiring and the acquired firms

1. Acquisitions mostly happen across innovative firms ([Bena and Li \(2014\)](#), [Phillips and Zhdanov \(2013\)](#), [Gantumur and Stephan \(2012\)](#), [Cassiman and Veugelers \(2006\)](#)).
2. Acquirers use acquisition of knowledge as a substitution ([Phillips and Zhdanov \(2013\)](#), [Watanabe et al. \(2009\)](#), [Rhodes-Kropf and Robinson \(2008\)](#), [Arikawa and Miyajima \(2007\)](#), [Higgins and Rodriguez \(2006\)](#), [Aghion and Tirole \(1994\)](#)) or a complementary ([Kaul \(2012\)](#), [Cassiman and Veugelers \(2006\)](#), [Rubin \(1973\)](#)) strategy to in-house R&D.
3. Acquisitions have a positive impact on the quality of products ([Sheen \(2014\)](#), [Clodt et al. \(2006\)](#), [Ahuja and Katila \(2001\)](#)).
4. The acquired firms are small ([Arikawa and Miyajima \(2007\)](#)). They have high quality products, which may be proxied in the empirical literature by valuable knowledge ([Carayannopoulos and Auster \(2010\)](#), [Villalonga and McGahan \(2005\)](#), [Schilling and Steensma \(2002\)](#)), superior technologies ([Villalonga and McGahan \(2005\)](#), [Hitt et al. \(1996\)](#)), and recent/highly-cited patents ([Ransbotham and Mitra \(2010\)](#)).

2.3 Effect on firm growth and innovation intensity

1. Acquisitions lead to growth of the acquiring firm, for instance in terms of sales ([Watanabe et al. \(2009\)](#), [Higgins and Rodriguez \(2006\)](#)) or assets/market value ([Gantumur and Stephan \(2012\)](#)).

2. The effect of acquisitions on the R&D intensity (i.e. R&D expenses per firm size) may be insignificant (Hall et al. (1990), Watanabe et al. (2009)), negative (Hitt et al. (1991), Hitt et al. (1996)) or positive (Gantumur and Stephan (2012)).

3 The model with heterogeneous firms

The main version of our model introduces acquisitions within the classic innovation strategy of Klette and Kortum (2004) and Lentz and Mortensen (2008), where heterogeneous firms perform innovations not focused on a particular product. Another variant of the model follows the approaches of Akcigit and Kerr (2018), Acemoglu et al. (2016) and Acemoglu and Cao (2015) to distinguish internal and external innovations. Both versions of our model are built on the premise that the R&D costs scale down with the firm size (Klette and Kortum (2004)). In other words, the R&D is less expensive for large firms.

The major novelty of our model is an explicit introduction of acquisitions as a mean for technological development of a firm. We assume that in addition to an innovation, a firm can expand its product lines through acquiring another firm and getting ownership over the products of the target.

Our model builds on the microeconomic evidence concerning the behavior of the acquiring and the acquired firms. Firstly, we account for the complementarity between the costs of R&D and acquisition. Namely, both depend on the innovation intensity and acquisition intensity, and the complementarity parameter may be zero, negative or positive. Secondly, the acquisition costs decline with firm size, so acquisitions are less expensive for large firms. Thirdly, the acquired firms are small, so they have only one product line in the model. Finally, we let acquisitions have a positive impact on the quality of product originally produced by the target. The step size of innovation in our model is higher for the acquirer than for its target. Therefore, the acquirer is capable to raise the productivity of the target by improving target's technology after the acquisition. Accordingly, an acquisition results in a quality improvement of the acquired products.

We regard acquisition expenditure as a part of firm's operational costs: market research, membership dues in business associations, and the burden of other activities, focused on organizing and implementing deals. Note that our inclusion of the costs for searching a potential target resembles the analysis in other studies of acquisition activity (David (2017), Rhodes-Kropf and Robinson (2008)). The price of deal is not viewed as the cost of acquisition in our model. Instead, we take the deal value as the firm value of the target, which is paid to the previous owner by the acquirer.

The model includes the government which can subsidize the R&D and provide various means to incentivize acquisitions by lowering their costs. Examples of the latter policies relate to relaxing the legal framework on implementation of acquisitions, easing the regulations on accounting of acquisition expenses or facilitating search of potential targets through government support of collaborative projects. Although these examples of acquisition subsidy may not

directly come in the monetary terms, in the model we consider their monetary equivalent for a firm. The government revenues come from the corporate profit tax.

Below we outline the optimization problems for all agents and define a balanced growth equilibrium path within the main model. The full solution of the model can be found in Appendix A.

3.1 Consumer

The representative consumer maximizes the intertemporal utility function:

$$\begin{aligned} \max \int_0^{+\infty} e^{-\rho t} U(C_t) dt, \\ \text{s.t.: } \dot{S}_t + nS_t = r_t S_t + w_t - C_t + \pi_t + T_t, \\ S_0 \text{ is given, no-Ponzi-scheme condition is applied.} \end{aligned} \quad (1)$$

All consumers own an equal share of all firms in the economy. The representative consumer gets the lump-sum profit π_t of the producer of the final good, holds the amount S_t of the intermediate goods producers stock and receives an interest income $r_t S_t$. C_t is consumption, T_t is the lump-sum transfer by the government, and n is the rate of growth of population.

The representative consumer inelastically supplies her labor at any wage rate w_t . We use the consumption good as a numeraire, so its price equals to one. The utility function is constant relative risk aversion:

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0. \quad (2)$$

The solution to the consumer problem yields the following Euler equation:

Proposition 3.1. *The growth rate for the per capita consumption along the equilibrium path is:*

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - n - \rho}{\sigma}. \quad (3)$$

3.2 Producer of the final good

Intermediate goods are used as inputs for the production of the consumption good, which is denoted below as the final good:

$$Y_t = \exp \int_0^1 \ln X_t(j) dj. \quad (4)$$

Here Y_t is the amount of the final good, a continuum of intermediate goods is normalized to $[0, 1]$, so $X_t(j)$ is the quantity of intermediate good j , $j \in [0, 1]$.

The profit maximization problem is:

$$\max \Pi_t = \max \left[\exp \int_0^1 \ln X_t(j) dj - \int_0^1 P_t(j) X_t(j) dj \right], \quad (5)$$

where maximization is over $\{X_t(j): j \in [0, 1]\}$. The producer of the final good takes the prices of intermediate goods $P_t(j)$ as given. Note that the producer of the final good is a competitive firm and the production function exhibits constant returns to scale. Therefore, the profit is zero in the equilibrium.

Solving (5), we can prove the following propositions:

Proposition 3.2. *The equilibrium prices of intermediate goods satisfy*

$$\exp \int_0^1 \ln P_t(j) dj = 1.$$

Proposition 3.3. *The demand for intermediate good j equals*

$$X_t(j) = \frac{Y_t}{P_t(j)}.$$

3.3 Producers of intermediate goods

A producer of intermediate goods is a firm which at any time period t has the leading technology for producing goods $j_1, \dots, j_m \in [0, 1]$. Each producer acts as a monopolist on the market for these goods and collects non-zero profits, which are spent on R&D or acquisition activity, or are distributed among consumers. Hence the problem can be separated into two parts: choosing the optimum quantity of a good to maximize instantaneous profits, and choosing the strategy for R&D and acquisitions to maximize the intertemporal value of the firm. There are incumbent firms and potential entrants. The latter conduct R&D to enter the market through a successful innovation.

Government co-finances the R&D and acquisition expenditure by firms. The government revenue comes from a corporate profit tax. Accordingly, subsidizing of the R&D may be interpreted as a tax credit mechanism. The government subsidizes the share $\phi \in (0, 1)$ of R&D expenditure by incumbents, the share $\chi \in (0, 1)$ of R&D expenditure by entrants, and the share $\psi \in (0, 1)$ of expenditure on acquisition activity by incumbents. The government balances its budget in every period. The government revenue, which is non-distributed across firms, goes to consumer via the lump-sum transfer T_t . The first order condition of consumer's maximization program is not affected, so there is no change in the relationship between the growth rate of consumption \dot{C}_t/C_t and the interest rate r_t . The R&D and acquisition subsidies by the government change incentives for the producers of intermediate goods.

3.3.1 Incumbents

Following [Klette and Kortum \(2004\)](#), we denote the default quality of good j $z(j, 0) = 1$, so if there were $J_t(j)$ innovations before time t then the whole set of quality levels available is: $z(j, 0) < z(j, 1) < z(j, 2) < \dots < z(j, J_t(j))$ where the second argument represents the number of successive innovations for the good. Each innovation improves the quality. The quality enhancement, i.e. the innovative step for the k -th innovation is $q(j, k) = z(j, k)/z(j, k - 1)$.

The producers of intermediate good j engage in Bertrand competition with the following tie breaker rule: when several firms set the same price, only the good with highest quality is consumed.

Using the approach of [Lentz and Mortensen \(2008\)](#), we consider the quality level $z(j, k)$ as productivity. This enables interpreting the growth of quality as economic growth.

The production function is linear in labor:

$$X(j) = z(j, k)l(j).$$

Accordingly, the cost function is

$$\frac{w_t X(j)}{z(j, k)},$$

where w_t is the wage rate, and $X(j)$ is the amount produced.

Given the demand structure from [Proposition 3.3](#), we get

Proposition 3.4. *Only the firm with the highest quality $z_t(j) = z(j, J_t(j))$ produces good j , and the price it sets equals $P_t(j) = \frac{w_t}{z_t(j, J_t(j) - 1)} = \frac{w_t q_t(j)}{z_t(j)}$, where $q_t(j)$ is the quality improvement (i.e. innovation step).*

In conjunction with [proposition 3.2](#), this yields

Proposition 3.5. *The equilibrium wage rate is:*

$$w_t = \exp\left(\int_0^1 \ln z_t(j) dj - \int_0^1 \ln q_t(j) dj\right) = \frac{\exp(\overline{\ln z_t})}{\exp(\overline{\ln q_t})}.$$

The instantaneous profit is:

$$\left(P_t(j) - \frac{w_t}{z_t(j)}\right) X_t(j) = \left(1 - \frac{1}{q_t(j)}\right) P_t(j) X_t(j) = \left(1 - \frac{1}{q_t(j)}\right) Y_t. \quad (6)$$

Define the firm profitability at time t as

$$\pi_t(j) = 1 - \frac{1}{q_t(j)}. \quad (7)$$

Assume that profitability is firm-specific and is the same for each good, produced by the firm.

The firms are heterogeneous in their profitability. We consider the firm profitability π as a random variable, distributed over $(0, 1)$ with some CDF $F(\pi)$. Denote $\bar{\pi} = \mathbf{E}(\pi)$.

A firm participates in two types of activities. Firstly, it invests in R&D. A successful innovation makes a firm the leader in the production of an extra good. Secondly, a firm can acquire another small firm (with only one good).

The hazard rate of innovations is denoted by I and the full cost of innovation is

$$R \frac{\pi}{\bar{\pi}} m^{-1} (aI^2 + 2bIJ) Y_t,$$

where m is the number of goods, produced by the firm, and J is the hazard rate of acquisitions. Similarly, to maintain the hazard rate of J of acquisitions the firm spends

$$\frac{\pi}{\bar{\pi}}m^{-1}(2cIJ + dJ^2)Y_t.$$

An acquisition implies that the owner of the target gets the full firm value, and the acquirer starts to make full profits π from the acquired firm. Accordingly, it is unreasonable to acquire a firm which has the profitability higher than that of the potential acquirer. Hence a firm acquires only firms with profitability $\pi' \leq \pi$.

Consider the firm with profitability π , which currently produces m goods. Since its instantaneous profits do not depend on the goods quality as shown in (6), the value of the firm $V_\pi(m)$ does not depend on the goods quality either. So the Bellman equation for the intertemporal profit maximization is:

$$\begin{aligned} r_t V_\pi(m) - \dot{V}_\pi(m) = \max_{I \geq 0, J \geq 0} \left\{ \right. \\ (1 - \tau) \left(\pi Y_t m - \frac{\pi}{\bar{\pi}} m^{-1} (aI^2 + 2bIJ) Y_t - \frac{\pi}{\bar{\pi}} m^{-1} (2cIJ + dJ^2) Y_t \right) \\ + \phi \frac{\pi}{\bar{\pi}} m^{-1} (aI^2 + 2bIJ) Y_t + \psi \frac{\pi}{\bar{\pi}} m^{-1} (2cIJ + dJ^2) Y_t \\ + I [V_\pi(m+1) - V_\pi(m)] \\ + J [V_\pi(m+1) - \mathbf{E}(V_{\pi'}(1) \mid \pi' \leq \pi) - V_\pi(m)] \\ \left. + \mu_t \sum_{i=1}^m [V_\pi(m-1) - V_\pi(m)] \right\}. \end{aligned} \quad (8)$$

Here μ_t is the rate of creative destruction or the rate of appropriation of a good, due to innovations by incumbents and new entrants. Note that the acquirer pays the full value to the owner of the target, so the Bellman equation does not contain any terms to account for the acquisition.

The first line represents the firm's profits less the corporate profit tax, where the tax rate is denoted by τ . The second line is the government subsidy to compensate the share ϕ of the R&D expenses and the share ψ of the M&A expenses.

3.3.2 Entrants

Entrants form a pool which can achieve a Poisson process of innovations over any good $j \in [0, 1]$. Similarly to incumbents, the entrants receive the R&D subsidy. So the hazard rate η of innovations requires the flow of costs $\frac{\pi}{\bar{\pi}} \nu \eta Y_t$ which share χ is compensated by the government. The Bellman equation for the profit maximization problem is:

$$r_t V_\pi(0) - \dot{V}_\pi(0) = \max_{\eta \geq 0} \left\{ \eta [V_\pi(1) - V_\pi(0)] - (1 - \chi) \frac{\pi}{\bar{\pi}} \nu \eta Y_t \right\}. \quad (9)$$

Here, $V_\pi(0)$ is the value of newly formed firm, for which the free entry condition holds, so $V_\pi(0) = 0$ in an equilibrium.

The profitability π of start-ups have distribution $F_E(\pi)$ which in general differs from the distribution of incumbents $F(\pi)$.

3.4 Balanced growth equilibrium

Definition 3.1. An *allocation* in the economy consists of time paths of interest rates and wage rates $\{r_t, w_t\}$, per capita consumption, saving and transfer levels $\{C_t, S_t, T_t\}$, production of the final good $\{Y_t\}$, qualities, prices and production of intermediate goods $\{z_t(j), P_t(j), X_t(j), j \in [0, 1]\}$, hazard rates for R&D and M&A events by incumbents $\{I_t(m, \pi), J_t(m, \pi), \pi \in [0, 1], m \in \{1, 2, \dots\}\}$, hazard rate for R&D events by entrants $\{\eta_t\}$, profitability distributions among incumbents and entrants $\{F_t(\pi), F_{Et}(\pi)\}$, where $t \in [0, +\infty)$

Definition 3.2. An *equilibrium* is an allocation where

- (i) a representative consumer maximizes her utility;
- (ii) producers of the final good maximize profits;
- (iii) incumbents maximize their net present discounted value with respect to hazard rates of R&D and M&A, prices and output;
- (iv) entrants maximize their net present discounted value with respect to hazard rate of R&D;
- (v) markets for labor, savings, and the final good are cleared;
- (vi) government balances its budget.

Now, we focus on a balanced growth path.

Definition 3.3. A *balanced growth path* is an equilibrium where the hazard rates for the innovations by incumbents λ , by entrants η , and acquisitions κ are constant. So output Y_t , per capita consumption C_t , and the geometric mean of quality $\exp(\overline{\ln z_t})$ grow at constant rates.

The full list of market clearing conditions on a balanced growth path can be found in Appendix A.

3.5 Profitability distribution

Note that the profitability distributions for incumbents and entrants are stationary along a balanced growth path and the rate of creative destruction μ is constant. Additionally, we use the following assumption to ensure the existence of a balanced growth path:

Assumption 3.1. Let the profitability of firms producing intermediate goods be distributed according to the following CDF:

$$F(\pi) = \begin{cases} 0, & \pi < 0, \\ \pi^{\alpha/(1-\alpha)}, & 0 \leq \pi \leq 1, \\ 1, & \pi > 1, \end{cases}$$

for $0 < \alpha < 1$.

This allows computing the expected profitability for firms with $\pi' \leq \pi$.

Proposition 3.6.

$$\mathbf{E}(\pi' \mid \pi' \leq \pi) = \alpha\pi. \quad (10)$$

Accordingly, $\bar{\pi} = \mathbf{E}(\pi) = \alpha$.

Next, we focus on the profitability distribution of the acquired firms. Recall that only firms with one product can become targets in our model. An acquirer only buys firms of lower profitability, so very profitable firms with $m = 1$ are rarely acquired. Denote $F_T(x)$ the distribution of profitability for targets. Then using the law of full probability we can compute this distribution.

$$\begin{aligned} F_T(x) = \mathbf{P}\{\pi' \leq x\} &= \int_0^1 \mathbf{P}\{\pi \leq x \mid \pi\} dF(\pi) = \int_0^x dF(\pi) + \int_x^1 \frac{F(x)}{F(\pi)} dF(\pi) \\ &= F(x) + F(x) \int_x^1 \frac{dF(\pi)}{F(\pi)} = F(x)[1 - \ln F(x)]. \end{aligned} \quad (11)$$

To ensure that the profitability distribution is invariant, we impose the following requirement:

Assumption 3.2. *The profitability of the new entrants is distributed as*

$$F_E(\pi) = \frac{\eta - \kappa}{\eta} F(\pi) + \frac{\kappa}{\eta} F_T(\pi),$$

where η is the entry rate (or the rate of innovations by entrants) and κ is the acquisition rate.

3.6 Innovation and acquisition rates

Proposition 3.6 about the expected profitability implies the following value function on a balanced growth path:

Proposition 3.7. *Under the Assumptions 3.1 and 3.2, the per capita value function of a firm on a balanced growth path is*

$$V_\pi(m) = \frac{\pi}{\pi} mvY_t,$$

where v is a constant.

The value of v together with λ , κ , and μ can be derived from the following system of equations

under the nonzero entry rate η :

$$v = (1 - \chi)\nu, \quad (12)$$

$$\lambda = \frac{(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)(1 - \alpha)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}, \quad (13)$$

$$\kappa = \frac{(1 - \tau - \phi)a(1 - \alpha) - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}, \quad (14)$$

$$\begin{aligned} & (r - g - n + \mu - (\lambda + \kappa(1 - \alpha)))v \\ & = (1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2). \end{aligned} \quad (15)$$

In short, the equation (12) follows from the free entry condition and the first order condition in (9), equations (13) and (14) represent the first order conditions in (8), and equation (15) holds under the firm value function in Proposition 3.7. See full proof in Appendix A.

Note that the optimal values of λ , κ , and v are firm-independent, but implicitly depend on the interest rate r , the growth rate of economy $g + n$ and on the rate of creative destruction μ . The effect of subsidy rates ϕ and ψ on λ and κ depend on the signs of b and c . So government policy, promoting R&D or company mergers, may have a negative or a positive impact on the innovation and acquisition rates.

3.7 Distribution of the firm size

Following Klette and Kortum (2004), we consider the balanced growth path with a time invariant distribution of the mass of firms with m goods, denoted M_m , $m \geq 1$.

Consider the entry rate $\eta = \mu - \lambda$. We assume the strict inequality $\eta > \kappa$, otherwise the steady distribution of the firm sizes does not exist.

The following proposition holds:

Proposition 3.8. *The time invariant distribution of the firm sizes on a balanced growth path is*

$$M_m = B \left(\frac{\lambda + \kappa}{\mu} \right)^{m-1} \frac{1}{m}, \quad m = 1, 2, \dots, \quad (16)$$

where B equals to

$$B = \frac{\mu - \lambda}{\mu} \cdot \frac{\mu - \lambda - \kappa}{\mu - \lambda} = \frac{\mu - \lambda - \kappa}{\mu}, \quad (17)$$

and the total mass of the firms is

$$M = \frac{\mu - \lambda - \kappa}{\lambda + \kappa} \ln \left(\frac{\mu}{\mu - \lambda - \kappa} \right).$$

See proof in Appendix A.

3.8 Aggregate growth

The model attributes the growth of the economy to innovations and acquisitions. In particular, acquisitions enable the acquirers to produce higher quality product after buying a low profit firm with the highest quality on the corresponding market. Denote g the rate of growth of C_t and n the rate of population growth. Using Assumption 3.2, we can express the growth rate of the economy (i.e. of Y_t) as $g + n$, where

$$g = \lambda \overline{\ln q} + ((\eta - \kappa) \overline{\ln q} + \kappa \overline{\ln q'}) + \kappa (\overline{\ln q} - \overline{\ln q'}) = (\lambda + \kappa) \overline{\ln q} = \mu \overline{\ln q}, \quad (18)$$

where

$$\overline{\ln q} = \int_0^1 \ln \frac{1}{1 - \pi} dF(\pi)$$

is the mean of log innovation steps for all firms in the economy, and

$$\overline{\ln q'} = \int_0^1 \ln \frac{1}{1 - \pi} dF_T(\pi) = \int_0^1 \ln \frac{1}{1 - \pi} d(F(\pi)[1 - \ln F(\pi)]) = - \int_0^1 \ln F(\pi) \ln \frac{1}{1 - \pi} dF(\pi)$$

is the mean of log innovation steps for all firms which are being acquired by other firms.

3.9 Interest rate

The per capita growth rate of consumption C_t equals the average quality growth, which in turn equals g . Using proposition 3.1, we close the model by establishing a relationship between g and the interest rate r :

$$g = \frac{r - n - \rho}{\sigma}. \quad (19)$$

The following proposition summarizes the equations to determine the parameters for the balanced growth equilibrium path:

Proposition 3.9. *The values of λ , κ , μ , g , r , v on the balanced growth path can be determined by solving the system of equations (12), (13), (14), (15), (18), (19).*

See proof in Appendix A.

3.10 Comparative statics

In this section we compare the rates of innovations by entrants and incumbents and the rates of acquisitions under parameter changes. Namely, we consider changes in innovation costs of incumbents and entrants, and changes in acquisition costs of incumbents.

The first proposition describes the comparative statics with respect to innovation cost by entrants ν .

Proposition 3.10. *If we consider values of λ , κ and μ on the balanced growth path, then*

$$\frac{\partial \lambda}{\partial \nu} > 0, \quad \frac{\partial \kappa}{\partial \nu} > 0,$$

$$\frac{\partial \mu}{\partial \nu} = - \frac{(1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2)}{(\sigma - 1)\overline{\ln q} + 1}.$$

See proof in Appendix A.

Note that ν may be interpreted as the cost of entering the market. The entry cost equals the rent, which incumbents receive on corresponding markets. Lower rent point to a higher competitiveness in the economy, as rent is zero under perfect competition. Accordingly, we may interpret low values of ν as a reflection of high competition on the markets. As is shown in the proposition, innovations and acquisitions by incumbents are inhibited when competition (proxied by ν) increases. At the same time, the effect of competition on the rate of creative destruction depends on the values of α and σ . When agents are risk-averse ($\sigma \geq 1$) or when the average log of innovation step size is sufficiently small ($\alpha < 0.5$), then the sign of $\partial\mu/\partial\nu$ is equal to the sign of $(1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2)$. Recall that $\bar{\pi} = \alpha$. So if α is very small, μ goes down when competitiveness raises (i.e. ν decreases). The effect is opposite for large values of α .

The second proposition considers the change in incumbent R&D costs, i.e. growth in both a and b .

Proposition 3.11. *Consider value of μ on a balanced growth path. Then, similarly to Proposition 3.10,*

$$a \frac{\partial \mu}{\partial a} + b \frac{\partial \mu}{\partial b} < 0 \quad \text{iff } (\sigma - 1)\overline{\ln q} + 1 > 0,$$

so the total rate of creative destruction depends negatively on the incumbent innovation costs if the coefficient of risk aversion is large ($\sigma \geq 1$) or average firm profitability is sufficiently small ($\alpha < 0.5$).

See proof in Appendix A.

Note that we cannot predict the signs of

$$a \frac{\partial \lambda}{\partial a} + b \frac{\partial \lambda}{\partial b}, \quad \text{and} \quad a \frac{\partial \kappa}{\partial a} + b \frac{\partial \kappa}{\partial b}$$

as they depend on parameters b and c , which define the complementarity between the costs of innovation and acquisition.

Next, similarly to Proposition 3.11 we find the dependency on the acquisition costs (parameters c and d).

Proposition 3.12. *Consider value of μ on a balanced growth path. Then, similarly to Proposition 3.10,*

$$c \frac{\partial \mu}{\partial c} + d \frac{\partial \mu}{\partial d} < 0 \quad \text{iff } (\sigma - 1)\overline{\ln q} + 1 > 0,$$

so the rate of creative destruction depends negatively on the incumbent acquisition costs if the coefficient of risk aversion is large ($\sigma \geq 1$) or average firm profitability is sufficiently small ($\alpha < 0.5$).

See proof in Appendix A.

Again, the signs of

$$c \frac{\partial \lambda}{\partial c} + d \frac{\partial \lambda}{\partial d}, \quad \text{and} \quad c \frac{\partial \kappa}{\partial c} + d \frac{\partial \kappa}{\partial d}$$

depend on parameters b and c .

3.11 Firm growth

Define the size of a firm $Q_t = \sum_j P_t(j) X_t(j) = m_t Y_t$ and consider the dynamics of it. A number of events may happen before the period $t + \Delta t$. Firstly, every good of the firm can be taken away due to an innovation by another firm with probability $\mu \Delta t$. At the same time, the firm can acquire a new good with probability $m_t \lambda \Delta t$. The firm can buy another firm with probability $m_t \kappa \Delta t$. Finally, if the firm is small, i. e. $m = 1$, it can be acquired by another firm with probability

$$\kappa(M_1 + 2M_2 + 3M_3 + \dots)(1 - F(\pi))\Delta t = \kappa(1 - F(\pi))\Delta t,$$

where π is the firm profitability.

Hence the difference in the firm size may be approximated as

$$\mathbf{E}_t Q_{t+\Delta t} - Q_t \approx (n + g - \mu)Q_t \Delta t - \kappa(1 - F(\pi))Q_t \mathbf{1}\{m_t = 1\} \Delta t + \lambda Q_t \Delta t + \kappa Q_t \Delta t. \quad (20)$$

Dividing (20) it by Δt and letting Δt to zero, we get

$$\dot{Q}_t = (n + g - \mu)Q_t - \kappa(1 - F(\pi))Q_t \mathbf{1}\{m_t = 1\} + \lambda Q_t + \kappa Q_t.$$

So the expected rate of firm growth $\mathcal{G}(Q_t) = \dot{Q}_t/Q_t$ is:

$$\mathcal{G}(Q_t) = n + g - \mu - \kappa(1 - F(\pi)) \mathbf{1}\{m_t = 1\} + \lambda + \kappa. \quad (21)$$

3.12 R&D and acquisition intensity

Given the optimal values of λ , κ , and v from (13) (14) and (15), we derive the R&D and acquisition intensities:

$$\mathcal{R}(Q_t) = \frac{(\pi/\bar{\pi})m_t(a\lambda^2 + 2b\lambda\kappa)Y_t}{Q_t} = (\pi/\bar{\pi})(a\lambda^2 + 2b\lambda\kappa), \quad (22)$$

$$\mathcal{M}(Q_t) = \frac{(\pi/\bar{\pi})m_t(2c\lambda\kappa + d\kappa^2)Y_t}{Q_t} = (\pi/\bar{\pi})(2c\lambda\kappa + d\kappa^2). \quad (23)$$

3.13 Predictions

1. The impact of the innovation rate by entrants ($\eta = \mu - \lambda$) on growth of a firm is negative.
2. The impact of the innovation rate by incumbents (λ) on growth of a firm consists of two parts: a negative effect from losing a product, and a positive effect from expanding the

product line. The expanding effect is inversely related with the firm size and is proportional to the firm innovation step size (or profitability).

3. The impact of the acquisition rate on growth of a firm depends negatively on the firm size and positively on the firm profitability.

4 The model with homogeneous firms

4.1 Outline

In this section we model acquisitions along the lines of the [Akcigit and Kerr \(2018\)](#), [Acemoglu et al. \(2016\)](#) and [Acemoglu and Cao \(2015\)](#) to distinguish different types of innovation. Internal innovations are based primarily on firm's prior art and targeted at improving existing products, while external innovations are focused at products not currently produced by the firm ([Akcigit and Kerr \(2018\)](#)).

We formulate the optimization problems for all agents, define an equilibrium balanced growth path and give predictions. The full solution of the model can be found in Appendix B.

4.2 Consumer

The representative consumer maximizes her intertemporal utility function:

$$\begin{aligned} \max \int_0^{+\infty} e^{-\rho t} U(C_t) dt, \\ \text{s.t.: } \dot{S}_t + nS_t = r_t S_t + w_t - C_t + \pi_t + T_t, \\ S_0 \text{ is given, no-Ponzi-game condition is applied} \end{aligned} \quad (24)$$

All consumers own an equal share of all firms in the economy. The representative consumer gets the lump-sum profit π_t of the producer of the final good, holds the amount S_t of intermediate goods producers stock and gets an interest income $r_t S_t$. C_t is consumption, T_t is the lump-sum transfer by the government, and n is the rate of growth of population.

The representative consumer inelastically supplies her labor at any wage rate w_t . We use the consumption good as a numeraire, so its price equals to one.

The utility function is constant relative risk aversion

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0. \quad (25)$$

The representative consumer's problem implies

Proposition 4.1. *The following condition holds on a balanced growth path:*

$$r - n = \rho + \sigma g.$$

It immediately follows from (B.3) derived in Appendix B.

4.3 Producer of the final good

To formulate the problem of the producer of the final good, we start with specifying the production function:

$$Y_t = \frac{L_t^\beta}{1-\beta} \int_0^1 q_t^\beta(j) k_t^{1-\beta}(j) dj.$$

Here Y_t is the amount of the final good, $q_t(j)$ is the quality of intermediate good $j \in [0, 1]$, $k_t(j)$ is the amount of intermediate good j used in production, L_t is the amount of labor.

The profit maximization problem is:

$$\max \Pi_t = \max \frac{L_t^\beta}{1-\beta} \int_0^1 q_t^\beta(j) k_t^{1-\beta}(j) dj - \int_0^1 p_t(j) k_t(j) dj - w_t L_t, \quad (26)$$

where maximization is over $\{k_t(j) : j \in [0, 1]\}$, L_t . The producer of the final good takes the prices of intermediate goods $p_t(j)$ and the wage rate w_t as given. Note that the profit is zero in the equilibrium.

These conditions imply that the following proposition holds

Proposition 4.2. *The inverse demand for intermediate good j equals*

$$p_t(j, k_t(j)) = q_t^\beta(j) k_t^{-\beta}(j) L_t^\beta. \quad (27)$$

See proof in Appendix B.

4.4 Producer of the intermediate good

4.4.1 Instantaneous profit from good j

The producer of any intermediate good j has a linear technology $k_t(j) = \bar{q}_t l_t(j)$, where $\bar{q}_t = \int_0^1 q_t(j) dj$ is the mean quality for all intermediate goods on the market. Similarly to the producer of the final good, the producer of the intermediate good hires labor at a competitive rate w_t .

The instantaneous profit maximization problem is:

$$\pi_t(j) = \max_{k_t(j)} \left\{ \left(p_t(j, k_t(j)) - \frac{w_t}{\bar{q}_t} \right) k_t(j) \right\}, \quad (28)$$

where $p_t(j, k_t(j))$ is the inverse demand for intermediate good j , given by (27). There are many firms with technology for producing any particular good j , but we assume that only the firm with the highest quality is actually producing and selling the good.

Combining the solutions to the problems of the producer of the final good, the producers of intermediate goods and the labor market clearing condition, we can prove the following proposition:

Proposition 4.3. *In an equilibrium the instantaneous profit for the producer of a set of goods \mathbf{q} equals*

$$\sum_{j:q(j)\in\mathbf{q}} \pi_t(j) = \pi N_t \sum_{j:q(j)\in\mathbf{q}} q_t(j),$$

where $\pi = \frac{\beta^{1+\beta}(1-\beta)^{2(1-\beta)}}{(1-\beta)^2 + \beta}$.

See proof in Appendix B.

4.4.2 Intertemporal value of a firm

There are incumbent and entrant firms in the economy.

Innovation step size

We start with the evolution of quality for intermediate goods. The quality may change owing to internal innovation, external innovation by entrants and an acquisition. Following [Akcigit and Kerr \(2018\)](#) and [Acemoglu and Cao \(2015\)](#), we assume that internal and external innovations lead to a proportional increase the quality of good j by factors $1 + \lambda$ and $1 + \eta$, respectively. We do not distinguish the subtypes of external innovations as they do not affect the dynamics of the economy in our model.

Entrants

Entrants form a pool which can achieve a Poisson process of external innovations with hazard rate x_e if they keep the flow of costs $x_e \nu \bar{q}_t N_t$. An innovation applies to a random good $j \in [0, 1]$. The Bellman equation for the profit maximization problem is:

$$r_t V_0 - \dot{V}_0 = \max_{x_e \geq 0} \{x_e [\mathbf{E}_j V(\{q_t(j)(1 + \eta)\}) - V_0] - (1 - \phi)x_e \nu \bar{q}_t N_t\}. \quad (29)$$

Here V_0 is the value of a newly formed firm, $V(\{q\})$ is the value of the firm which produces a single good of quality q , and ϕ is the share of R&D costs compensated by the government.

The free entry condition in presence of innovations (i.e. under $x_e > 0$) is: $V_0 = 0$.

A firm exits the market when its last product is taken away due to an external innovation or when the whole firm is acquired by another firm.

Incumbents

Internal R&D. Internal R&D constitute a Poisson process. To maintain the rate $z_t(j)$ of internal R&D for good j a firm has to bear the flow of costs

$$R_z(z_t(j), q_t(j)) = \hat{\chi} z_t^{\hat{\psi}}(j) q_t(j) N_t.$$

External R&D. External innovations are a Poisson process with rate x_{mt} . The flow of costs is

$$R_x(x_{mt}, y_{mt}, \bar{q}_t, m_t) = (\tilde{\chi}x_{mt}^2 + \tilde{\delta}x_{mt}y_{mt})\bar{q}_tm_t^{-1}N_t.$$

The costs negatively depend on the number of goods m_t , produced by the firm. Note that costs also depend on the hazard rate of acquisitions y_{mt} .

Acquisition. If a firm maintains the flow of costs

$$R_y(x_{mt}, y_{mt}, \bar{q}_t, m_t) = (\check{\chi}y_{mt}^2 + \check{\delta}x_{mt}y_{mt})\bar{q}_tm_t^{-1}N_t,$$

then acquisitions are a Poisson process with rate y_{mt} . Acquisitions are cheaper for large firms. The quality of the product of the acquired firm goes up by $\kappa q_t(j)$, owing to an acquisition. The quality increase gives additional profits to the acquirer.

Loss of a product due to an external innovation. Denote μ_t the rate of transfer of intermediate goods across firms, owing to external innovations. μ_t is constant on a balanced growth path and is endogenously determined.

Loss of a firm due to an acquisition. The acquirer pays the full value of the target to its owner.

Bellman equation for an incumbent. Denote $\mathbf{q} = \{q(j_1), \dots, q(j_m)\}$ a tuple of intermediate goods the firm produces. The Bellman equation is⁴

$$\begin{aligned} rV(\mathbf{q}) - \dot{V}(\mathbf{q}) = & \max_{x_m \geq 0, y_m \geq 0, \{z(j) \geq 0\}} \left\{ \right. \\ & (1 - \tau) \left(\sum_{j: q(j) \in \mathbf{q}} [\pi(j)q(j) - \hat{\chi}z^{\hat{\psi}}(j)q(j)N] \right. \\ & \quad \left. - (\tilde{\chi}x_m^2 + \tilde{\delta}x_my_m)\bar{q}m^{-1}N - (\check{\chi}y_m^2 + \check{\delta}x_my_m)\bar{q}m^{-1}N \right) \\ & + \hat{\phi} \sum_{j: q(j) \in \mathbf{q}} \hat{\chi}z^{\hat{\psi}}(j)q(j)N + \tilde{\phi}(\tilde{\chi}x_m^2 + \tilde{\delta}x_my_m)\bar{q}m^{-1}N + \check{\phi}(\check{\chi}y_m^2 + \check{\delta}x_my_m)\bar{q}m^{-1}N \\ & + \sum_{j: q(j) \in \mathbf{q}} \left[z(j)[V(\mathbf{q} \setminus \{q(j)\} \cup \{q(j)(1 + \lambda)\}) - V(\mathbf{q})] + \mu[V(\mathbf{q} \setminus \{q(j)\}) - V(\mathbf{q})] \right] \\ & + x_m[\mathbf{E}_j V(\mathbf{q} \cup \{q(j)(1 + \eta)\}) - V(\mathbf{q})] \\ & + y_m[\mathbf{E}_j (V(\mathbf{q} \cup \{q(j)(1 + \kappa)\}) - V(q(j))) - V(\mathbf{q})] \\ & \left. - \Phi\bar{q}mN \right\}. \end{aligned} \tag{30}$$

The first two lines in (30) show the current profits owing to production less the corporate profit tax which is imposed at rate τ . The third line represents a R&D and M&A subsidy by the

⁴Index t is omitted for brevity.

government. The fourth line shows a payoff due to an increase in the firm value owing to an internal innovation and a decrease in the firm value due to a loss of a product. The fifth line is the gain from obtaining a new product as a result of the external innovation. Here η is the step size for an external innovation. The sixth line indicates the profit from acquiring another firm. The last term is a fixed cost of managing a firm, which ensures a linear form for the value function. The equation does not provide for any loss by the target firm, since the acquirer pays its full value.

To simplify the solution, similarly to [Akcigit and Kerr \(2018\)](#) we use Φ from the following proposition:

Proposition 4.4. *There exists Φ such that the firm value function can be expressed as*

$$V(\mathbf{q}) = AN_t \sum_{j:q(j) \in \mathbf{q}} q(j).$$

See proof in Appendix B.

Propositions 4.5–4.6 describe the solution to the problem for the producer of intermediate goods.

Proposition 4.5. *The optimum values of $z(j)$, x_m , and y_m on a balanced growth path are*

$$\begin{aligned} z(j) = z &= \left(\frac{A\lambda}{(1-\tau-\hat{\phi})\hat{\chi}\hat{\psi}} \right)^{1/(\hat{\psi}-1)}, \\ x_m = mx &= m \frac{2(1-\tau-\check{\phi})\check{\chi}A(1+\eta) - ((1-\tau-\check{\phi})\check{\delta} + (1-\tau-\check{\phi})\check{\delta})A\kappa}{4((1-\tau-\check{\phi})\check{\chi} + (1-\tau-\check{\phi})\check{\chi}) - ((1-\tau-\check{\phi})\check{\delta} + (1-\tau-\check{\phi})\check{\delta})^2}, \\ y_m = my &= m \frac{-((1-\tau-\check{\phi})\check{\delta} + (1-\tau-\check{\phi})\check{\delta})A(1+\eta) + 2(1-\tau-\check{\phi})\check{\chi}A\kappa}{4((1-\tau-\check{\phi})\check{\chi} + (1-\tau-\check{\phi})\check{\chi}) - ((1-\tau-\check{\phi})\check{\delta} + (1-\tau-\check{\phi})\check{\delta})^2}. \end{aligned}$$

Proposition 4.6. *If the solution to the Bellman equation of entrant (29) is internal and there is a free entry condition, then along a balanced growth path $A = \frac{(1-\phi)\nu}{1+\eta}$.*

See proof in Appendix B.

4.5 Balanced growth equilibrium

Definition 4.1. An *allocation* in the economy consists of time paths of interest rates and wage rates $\{r_t, w_t\}$, per capita consumption, saving and transfer levels $\{C_t, S_t, T_t\}$, production of the final good $\{Y_t\}$, qualities, prices and production of intermediate goods $\{q_t(j), p_t(j), k_t(j), j \in [0, 1]\}$, hazard rates for R&D and M&A events by incumbents $\{z_t(j), x_{mt}, y_{mt}, j \in [0, 1], m \in \{1, 2, \dots\}\}$, hazard rate for R&D events by entrants $\{x_{et}, t \in [0, +\infty)\}$, where $t \in [0, +\infty)$.

Definition 4.2. An *equilibrium* is an allocation where

- (i) a representative consumer maximizes her utility;
- (ii) producers of the final good maximize profits;

- (iii) incumbents maximize their net present discounted value with respect to hazard rates of R&D and M&A, prices and output;
- (iv) entrants maximize their net present discounted value with respect to hazard rate of R&D decisions;
- (v) markets for labor, savings, and the final good are cleared;
- (vi) government balances its budget.

Now, we focus on a balanced growth path.

Definition 4.3. A *balanced growth path* is an equilibrium where the hazard rates of internal innovations z , external innovations by incumbents x and by entrants x_e , and acquisitions y are constant. So the per capita consumption C_t , output Y_t , and the mean quality \bar{q}_t grow at constant rates.

The list of market clearing conditions on a balanced growth path can be found in Appendix B.

4.6 Firm size

Following [Akcigit and Kerr \(2018\)](#), we consider the balanced growth path with a time invariant distribution of the number of products produced by a firm.

Proposition 4.7. *The share s_m of firms, which produce m goods on the balanced growth path, equals*

$$s_m = B \left(\frac{x+y}{\mu} \right)^{m-1} \frac{1}{m}, \quad m = 1, 2, \dots,$$

where $B = \left(\frac{\mu}{x+y} \right) / \ln\left(\frac{\mu}{\mu-x-y} \right)$, μ is the rate of creative destruction, x is the rate of external innovations by incumbents and y is the rate of acquisitions. The total number of firms is

$$F = \frac{\mu - x - y}{x + y} \ln \left(\frac{\mu}{\mu - x - y} \right).$$

The hazard rate for external innovations by entrants equals

$$x_e = \mu - x.$$

See proof in Appendix B.

4.7 Aggregate growth rate and rate of creative destruction

The average quality increase of the good of the acquired firm is $\kappa \bar{q}_t$. The rate of acquisitions equals y , so we derive the growth rate for the average quality in the economy.

Proposition 4.8. *On the balanced growth path, the rate of growth for the average quality \bar{q}_t equals the per capita growth rate for the whole economy:*

$$g = z\lambda + \mu\eta + y\kappa. \tag{31}$$

Using propositions 4.1, 4.5 and 4.6, we obtain the expression for the rate of creative destruction:

Proposition 4.9. *On a balanced growth path*

$$\mu = \frac{\pi(1+\eta)}{(1-\phi)\nu} + (\hat{\psi} - 1) \left(\frac{(1-\phi)\nu}{(1+\eta)\hat{\chi}} \right)^{1/(\hat{\psi}-1)} \left(\frac{\lambda}{\hat{\psi}} \right)^{\hat{\psi}/(\hat{\psi}-1)} - \rho - \sigma g. \quad (32)$$

Solving equations (31) and (32), we find the aggregate growth rate g and the rate of creative destruction μ .

4.8 Firm growth

Define the size of a firm $Q_t = \sum_{j:q_t(j) \in \mathbf{q}_t} q_t(j)N_t$ and consider the dynamics of it. A number of events may happen before the period $t + \Delta t$. Firstly, the firm grows due to an increase of labor supply with rate n . Regarding the number of product lines and innovation decisions, every good of the firm can be taken away owing to an external innovation by another firm with probability $\mu\Delta t$. At the same time, the firm can acquire a new good with probability $m_t x \Delta t$. The firm can implement an internal innovation with probability $z\Delta t$ per product. The firm can buy another firm with probability $m_t y \Delta t$. Finally, if the firm is small, i.e. $\{m = 1\}$, it can be acquired by another firm with probability $y\Delta t$.

Hence the difference in the firm size may be approximated as:

$$\begin{aligned} \mathbf{E}_t Q_{t+\Delta t} - Q_t \\ \approx (n - \mu)Q_t \Delta t - yQ_t \mathbf{1}\{m_t = 1\} \Delta t + m_t x (1 + \eta) \bar{q}_t N_t \Delta t + m_t y (1 + \kappa) \bar{q}_t N_t \Delta t + \lambda z Q_t \Delta t. \end{aligned}$$

Dividing by Δt and letting Δt to zero, we get

$$\dot{Q}_t = (n + \lambda z - \mu)Q_t - y \mathbf{1}\{m_t = 1\} Q_t + m_t [x(1 + \eta) + y(1 + \kappa)] \bar{q}_t N_t,$$

or for the expected firm growth rate $\mathcal{G}(Q_t) = \dot{Q}_t/Q_t$:

$$\mathcal{G}(Q_t) = n + \lambda z - \mu - y \mathbf{1}\{m_t = 1\} + \frac{m_t [x(1 + \eta) + y(1 + \kappa)] \bar{q}_t N_t}{Q_t}. \quad (33)$$

4.9 R&D intensity

The R&D expenditure of the firm is the sum of its expenditure on internal and external innovation: $\hat{\chi} z^{\hat{\psi}} Q_t + (\tilde{\chi} x^2 + \tilde{\delta} x y) \bar{q}_t m_t N_t$.

The sales of the firm are

$$\begin{aligned} \sum_{j:q_t(j) \in \mathbf{q}_t} p_t(j)k_t(j) &= \sum_{j:q_t(j) \in \mathbf{q}_t} \frac{w_t}{(1-\beta)\bar{q}_t} \left(\frac{(1-\beta)\bar{q}_t}{w_t} \right)^{1/\beta} L_t q_t(j) \\ &= \sum_{j:q_t(j) \in \mathbf{q}_t} [\beta^\beta (1-\beta)^{-2\beta}]^{1-1/\beta} L_t q_t(j) = \beta^{\beta-1} (1-\beta)^{2-2\beta} \frac{L_t}{N_t} Q_t = \frac{\pi}{\beta} \frac{L_t}{N_t} Q_t. \end{aligned}$$

The R&D intensity is the R&D expenditure divided by the sales:

$$\mathcal{R}(Q_t) = \frac{\beta}{\pi L_t/N_t} \left[\hat{\chi} z^{\hat{\psi}} + (\tilde{\chi} x^2 + \tilde{\delta} xy) \frac{\bar{q}_t m_t N_t}{Q_t} \right], \quad (34)$$

where the ratio L_t/N_t is constant because of (B.25)

4.10 Acquisition intensity

The acquisition intensity is the acquisition expenditure divided by the firm sales

$$\mathcal{M}(Q_t) = \frac{\beta}{\pi L_t/N_t} (\check{\chi} y^2 + \check{\delta} xy) \frac{\bar{q}_t m_t N_t}{Q_t}. \quad (35)$$

5 Quantitative analysis

5.1 Data work

Sources

The empirical analysis focuses around the Japanese innovative companies from *The Nikkei Economic Electronic Databank System (The Nikkei NEEDS)* – the database with financial and corporate information on large and medium-sized firms.⁵

We match *The Nikkei NEEDS* companies to the *The Orbis/Zephyr* databases by Bureau van Dijk. *The Orbis* provides exhaustive information on the company industrial classification (if compared to the data in *The Nikkei NEEDS*), while *The Zephyr* database enables a comprehensive definition of an acquisition.⁶

Concerning patent data, we use the data from The Japan Patent Office data on the *The J-PLAT-PAT* platform and *The IIPP Database (2017)*. Note that *The IIPP Database* is the NBER-type version of *The Japan Patent Office* data and covers all applications submitted since 1964.⁷ It contains information on patents, main technological field, inventors and citations.

Sample

The main sample are firms of *The Nikkei NEEDS* – the database with corporate information on large and medium-sized firms. The auxiliary sample are innovative firms according to *The Japan National Innovation Survey*.⁸

We start with 6,083 firms from *The Nikkei NEEDS*, which provide data in 1999 onwards. For each firm we take the consolidated financial statement for each corresponding fiscal year (if present) or use non-consolidated statements.

The Nikkei NEEDS firms account for most of Japan’s GDP and the R&D expenditure. Using the national data from *The Survey of Research and Development* by The Japan Statistical

⁵The creation of the database was launched by Nikkei Shimbun in March 1967, with the first publication of post 1963-year data in December 1968 and regular data collection since March 1969 (Sueyoshi (2008)). The data collection originally focused on large companies (i.e. largest 282 companies over 1971–1976, Tada (1977)), but companies of smaller size were added in later years. The database gives non-consolidated statements since March 1964 and both consolidated and non-consolidated statements since March 1978.

⁶We compared the data on acquisition deals of our sample across *The Zephyr* database, the acquisition events in *The Japan National Innovation Survey* and the M&A block of *The Nikkei NEEDS*. While *The Japan National Innovation Survey* and *The Nikkei NEEDS* only report the deals, where acquirers and targets are within the corresponding database, *The Zephyr* gives information on all the deals, in which a database company is a target or an acquirer. Additionally, *The Zephyr* explicitly states the acquired percentages of company or capital, which enables us to drop acquisitions with minority stakes.

⁷See details about the creation of *The IIPP Database* in Goto and Motohashi (2007).

⁸Overall, *The Japan National Innovation Survey* started in 2003 with 45 thousand firms. The second wave of 2009 focused only on 15 thousand firms: a random sample of all 330,000 companies in Japan with over 10 employees. The third wave of 2012 are 25 thousand firms: the 2009 sample plus all firms with 1000+ employees. The disclosed lists of firms are available for the 2014–2016 waves of the survey according to the below criteria. The 2014 wave considers traded firms with over 100 patents in 1970–2010, while the waves of 2015–2016 include traded firms with an increase in the (total) number of patents, relative to the previous 3-5-7 year bands. The 2016 data additionally includes/specifies the names of University-Public Foundations, which may be non-listed on a stock exchange and may have zero patents.

Agency,⁹ we derive that the Nikkei NEEDS compose 80% of the R&D in most industries in 1999–2013.

We establish a firm-to-patent crosswalk, using the matching algorithms across the name and location of firms in *The Nikkei NEEDS*, The Japan Patent Office data on the *The J-PLAT-PAT* platform and *The IIPP Database (2017)*. There are 2,294 *Nikkei NEEDS* firms with patents in 1999–2013. External patents and internal patents are present among 2,037 and 1,616 *Nikkei NEEDS* firms, respectively. The mean annual numbers of patents for patenting firms are: 49.6 (st.dev.228.4) for all patents, 18.3 (st.dev.66.7) for external patents and 5.9 (st.dev.24.5) for internal patents.

Next, we match *The Nikkei NEEDS* firms to *The Orbis* database. The overlap consists of 5,305 firms, which is 87% of the firms in Nikkei NEEDS. Finally, we search the deals of *The Nikkei NEEDS* firms in *The Zephyr* database. The database provides the deals starting 1997, and the data for 1997–1998 may not be well reported. Accordingly, we start our analysis with 1999. We focus exclusively on domestic acquisitions. There are 2,219 acquirers among *The Nikkei NEEDS* firms and the mean annual number of deals per acquirer is 1.2 (st.dev.0.64).

As robustness check of the information on company deals, we match *The Nikkei NEEDS* firms to the firms in *The Japan National Innovation Survey*. The majority of *The Nikkei NEEDS* firms conduct R&D. Accordingly, the overlap of *The Nikkei NEEDS* firms with the firms from *The Japan National Innovation Survey* is over 80% in any year.

Our analysis employs data for 1999–2013. The period may be regarded uniform in terms of institutional context of the Japanese patent system, as it accounts exclusively for multi-claim applications after the 1988 revision of the Patent Law.

The microeconomic estimates and quantification of the general equilibrium model are based on the sample of 5,763 *Nikkei NEEDS* firms with financial data, of which 4,882 appear in *Orbis*. This gives, respectively, 66,676 and 58,346 longitudinal observations in fiscal years 1999–2013.

Firm size

We proxy firm size by total annual revenues as reported in *The Nikkei NEEDS* financial statements. Note that firm value could not be exploited as a proxy for firm size, since a considerable share of companies is non-listed.

Acquisitions

The Zephyr database gives details on the types of deals over the time period of 1997–2017. It contains an explicit variable, which reports whether a deal is: 1) acquisition and the percent of the target acquired; 2) acquisition increase and the pre-/post-acquisition percentages of the target; 3) capital increase and the percent of capital increased; 4) demerger; 5) institutional buyout; 6) joint venture; 7) merger; 8) minority stake and its percentage; 9) minority stake

⁹The collection of *Historic data* for 1959–2006, 2007–2010 and annual data for 2011–2013 from *The Japan Statistical Yearbooks*.

increase and the pre-/post-deal percentages; 10) share buyback and its percentage. Additionally, the database allows recovering the full corporate financial information for the targets.

The Zephyr reports 27,108 domestic deals for the overlap of *The Nikkei NEEDS* and *The Orbis* firms, where a firm can be an acquirer, a target or a vendor. We define a (new) acquisition as an increase in the acquired stakes or capital, so that the final stake is over 50% and the initial stake was below 50% (or initial stake is unknown). Such acquisitions account for 42.6% of all deals. Note that such deals as “Minority stake increased from 49.98% to 54.73%” also fall under our definition of an acquisition.

R&D expenditure

We take the R&D in the cash flow statement as a proxy for firm’s R&D expenditure. Note that this value coincides with the R&D expenditure of Japanese firms in the profit-loss statement according to the internationally comparable methodology, as reported in *The Orbis* database.

Overall, the R&D expenditure is reported in different sheets of the consolidated annual financial statements of Japanese firms *The Nikkei NEEDS*. R&D may be entered as: 1) a part of asset stock (code B106), 2) expenditure on R&D in the R&D statement (code H033), 3) expenditure on R&D in the cash flow statement (code K082). The presence of the R&D stock does not necessarily imply that the company had any type of R&D expenditure in the corresponding year. Indeed, the stock may represent an imputed financial value of the firm’s intangibles. The values of the R&D expenditure and cash flow R&D may be different. Arguably, cash-flow R&D may comprise intramural R&D expenditure which is defined in the national statistics as the sum of firm’s self-financed R&D and net of inflow and outflow of the financial sources on R&D. At the same time, R&D expenditure per se may be taken as self-financed R&D.

Patent statistics

We define the flow of patents as the number of patent applications submitted by each firm during the corresponding year. Joint patents with several owners are assigned to each owner. Following [Akcigit and Kerr \(2010\)](#) and [Galasso and Simcoe \(2011\)](#), internal patent is defined to have at least 50% of backwards citations to patents of the same firm, while external patent has no backwards citations to patents of the firm.

5.2 Firm-level analysis

In this section we estimate the firm growth equation, using the sample of *The Nikkei NEEDS* firms with at least one acquisition during 1999–2013.

Model with heterogeneous firms

The growth equation (21) from our model becomes

$$QG_{it} = \alpha_0 + (\alpha_1 x_{it} + \alpha_2 x_{it} \pi_{it} + \alpha_3 y_{it} + \alpha_4 y_{it} \pi_{it}) / Q_{it}, \quad (36)$$

where QG_{it} is the percentage growth of the firm size, x_{it} is a proxy for the rate of external innovations – number of external patents of firm i in year t , y_{it} is a proxy for the acquisition intensity – number of firms acquired in year t , and π_{it} is the firm profitability per one produced good.

Our theoretical model predicts that x_{it} and y_{it} should be positively significant for growth of the firm size in (36).

Model with homogeneous firms

The growth equation (33) from our model becomes

$$QG_{it} = \alpha_0 + \alpha_1 \frac{z_{it}}{Q_{it}} + \frac{\alpha_2 + \alpha_3 x_{it} + \alpha_4 y_{it}}{Q_{it}/Y_t}, \quad (37)$$

where z_{it}/Q_{it} is a proxy for the rate of internal innovations – number of internal patents in year t per firm size, the other variables have the same meaning as in (36), and Y_t is the country GDP which captures the overall quality $\bar{q}_t N_t$ from (33).

Following the predictions of our theoretical model, we expect the positive signs of the coefficients for z_{it}/Q_{it} , $x_{it}/(Q_{it}/Y_t)$ and $y_{it}/(Q_{it}/Y_t)$ in (37).

Industry-year fixed effects are added to the right-hand side in each of the two specifications.

Table 1 demonstrates the results of the estimates for firm size, proxied by firm sales. As expected, the rates of external and internal innovations, as well as the rate of acquisitions have a positive effect on growth. Yet, the coefficient for the rate of internal innovations is insignificant.

Table 1: **Estimating the firm growth equation**

Heterogeneous firms		Homogeneous firms	
x/Q	5.098*** (0.419)	z/Q	0.711 (1.263)
$x\pi/Q$	-0.0940*** (0.0269)	$1/(Q/Y)$	-0.0123*** (0.00334)
y/Q	9.879*** (0.614)	$x/(Q/Y)$	0.0109*** (0.000943)
$y\pi/Q$	-0.192 (1.583)	$y/(Q/Y)$	0.0283*** (0.00265)
Constant	10.17*** (1.146)	Constant	9.820*** (1.076)
Observations	7529	Observations	7561
Firms	985	Firms	990
R^2	0.279	R^2	0.282

Notes: The table reports the results of panel data regressions with firm-level and industry-year fixed effects. Standard errors are in parentheses. *, ** and *** show significance at 0.1, 0.05 and 0.01 level, respectively. Q indicates the firm size, which is proxied here by firm sales. All regressors with Q/Y in the denominator are scaled down by 1000, so firm sales are in million yen, while the GDP is in billion yen.

5.3 Calibrating the model with heterogeneous firms

5.3.1 Identification

The model has 10 parameters listed in Table 2. We apply external calibration to the social

Table 2: Parameters of the model

Parameter	Description	Identification
ρ	social discount rate	external calibration
n	growth rate of population	macro data
τ	nominal rate of the corporate profit tax	macro data
ϕ, χ	R&D subsidy rate to entrants and incumbents	macro data
ψ	acquisition subsidy rate	external calibration
σ	consumer risk aversion	indirect inference
α	shape of the profitability distribution	indirect inference
ν	cost of innovation for entrants	indirect inference
a, b	parameters of the R&D cost function	indirect inference
c, d	parameters of the acquisition cost function	indirect inference

discount rate ρ and to the rate of subsidizing acquisitions ψ . The growth rate of population n and the R&D subsidy ϕ are calibrated using the macro data on Japan’s economy. Following [Lentz and Mortensen \(2008\)](#), we use the indirect inference to calibrate the remaining parameters $\sigma, \alpha, \eta, a, b, c, d$. Namely, we choose a set of moment conditions and find the parameter values, which provide for the closest correspondence between the model and the data. Our estimates exploit the financial data on Japanese firms in 1999–2016 (*The Nikkei NEEDS*), the information about their deals and the number of products (*The Zephyr* and *The Orbis*), the patent statistics (*The IIP database*).

5.3.2 External calibration

Following [Akcigit and Kerr \(2018\)](#), we use the social discount rate $\rho = 0.02$. We set $\psi = 0$ as the status quo absence of any government support towards acquisitions.

5.3.3 Macro data

We assume that the rate of R&D subsidies to entrants and incumbents are equal. So $\phi = \chi$. The values are computed as the ratio of the R&D subsidies to the total R&D expenditure. The R&D expenditure come from the Statistics Bureau of the Ministry of Internal Affairs and Communications: an aggregate intramural expenditure on innovations by business enterprises, non-profit organizations, universities and colleges. We exploit the 2000–2004 historical data ([Statistics Bureau of Japan \(2018a\)](#)) and the 2005–2013 data from Japan Statistical Yearbooks for each corresponding year ([Statistics Bureau of Japan \(2018b\)](#)). We obtain the average value of $\phi = \chi = 0.0455$.

We use the population statistics from the [Statistics Bureau of Japan \(2018b\)](#) and measure n as the average growth rate of population in 1999–2013. The resulting value is close to zero: $n = 0.000356$.

We follow the data from the [Ministry of Finance \(2018\)](#) and measure the average nominal rate for profit tax over the period from 2000 to 2015. The resulting value is $\tau = 0.37$.

5.3.4 Indirect inference

We minimize:

$$\sum_{i=1}^9 \left(\frac{\text{model}(i) - \text{data}(i)}{(\text{model}(i) + \text{data}(i))/2} \right)^2, \quad (38)$$

where the moments are indexed with i . The nine moment conditions are as follows.

Real GDP growth rate

The growth rate equals $n + g$ in our model. We target the moment using the 2000–2013 annual data from the system of national accounts ([Economic and Social Research Council at the Cabinet Office \(2018\)](#)). Excluding a short-term economic recession due to the 2008/2009 global financial crisis, we compute annual growth rates of the GDP (in 2011 real terms) and take their mean value.

Real interest rate

We discipline the real interest rate r using the estimates by the World Bank together with the International Monetary Fund. Namely, we calculate the mean of the annual values, which are reported in the [World Bank \(2018\)](#) for years in 2000–2013.

R&D expenditure to GDP ratio

The model defines sales (as well as costs and profits) as multiples of the final good Y . Accordingly, the moment for calibrating the R&D intensity is the ratio of the R&D expenditure to GDP. We target the moment with an average value of the ratio in 2000–2013, with exclusion of 2008 and 2009.

The R&D expenditure come from the [Statistics Bureau of Japan \(2018a\)](#)) and the [Statistics Bureau of Japan \(2018b\)](#). The nominal GDP is taken from the [Economic and Social Research Council at the Cabinet Office \(2018\)](#).

Mean firm profitability

We calibrate the shape parameter α of the profitability distribution using the pooled data for the universe of Japanese firms from the *Orbis* database.¹⁰ Following the definition of profits in our model, we compute profits as the sum of the R&D expenditure and the accounting value of

¹⁰Data are available for 2010–2017.

profits before tax. Next, we calculate profitability as the ratio of profits to the firm turnover. The average annual value of the profitability, taken over the sample of firms with non-zero R&D expenditure, becomes the data-generated moment in our analysis. Note that we remove a few outliers with extreme values of profitability (i.e. falling out of the range of $(-10, 10)$).

Number of acquisitions per firm

We discipline the acquisition rate κ by equating κ/M to the annual values for the number of acquisitions per firm, as reported in *The Zephyr* database for the matched sample of *The Nikkei NEEDS* firms.

Entry rate

We equate the ratio of η/M to the annual number of new firms divided by the total number of firms in Japan. The data come from the Economic Census for Business Activity ([Ministry of Internal Affairs and Communications, Ministry of Internal Affairs \(2018\)](#)). This enables calibration of the per product entry rate η .

Shares of firms with one, two and three goods

The share of firms with m goods equals M_m/M , where the expressions for the numerator and denominator are given in Proposition 3.8. The data-generated moments come from the number of 4-digit codes from the US Standard Industrial Classification (US SIC), as reported in the *Orbis* database. We use the moment conditions for $m \leq 3$.

5.3.5 Calibration results

Table 3 reports the values of the parameters found through indirect inference. Note that the parameter b is positive, so acquisitions increase the costs of innovation. Similarly, the positive value of the parameter c implies that the R&D makes acquisition activity more expensive for firms.

Table 3: **Calibrated parameters**

σ	α	ν	a	b	c	d
0.6157	0.0787	0.6484	7.5632	1.2224	4.5203	5.9980

The empirical and targeted moments, given in Table 4, show that the model provides a close fit to the data.

Table 4: **Moments**

Moment	Data	Model
Real GDP growth rate	0.0152	0.0139
Real interest rate	0.0287	0.0287
R&D expenditures to GDP ratio	0.0330	0.0330
Mean firm profitability	0.0722	0.0787
Number of acquisitions per firm	0.0990	0.0866
Entry rate	0.0234	0.0292
Share of firms with only one good	0.5523	0.5353
Share of firms with two goods	0.3396	0.2026
Share of firms with three goods	0.0770	0.1022

Figure 1 demonstrates the distributions of the number of goods per firm according to the data on Japanese firms and on data, constructed within model calibration.

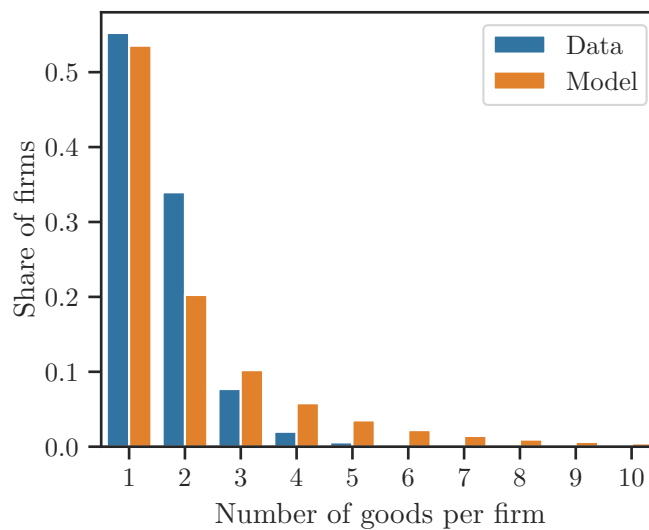


Figure 1: **Distribution of the number of goods produced by a firm**

5.3.6 Policy experiments

In this section we perform a counterfactual policy analysis, targeted at establishing a relationship between the government policy and growth through innovation and acquisition on the balanced growth path. The government may provide an innovation subsidy through co-financing the R&D expenditure by incumbents or entrants. The policy instruments are parameters ϕ and χ , respectively, and negative values of the parameters should be interpreted as innovation tax. The second policy instrument is parameter ψ , which is the share of acquisition costs born by the government. Finally, we consider lowering the nominal rate of the corporate profit tax τ as another type of policy, targeted at enhancing economic growth.

Our analysis examines the balanced growth equilibrium paths under different values of the policy instruments ϕ , χ , ψ and τ . We focus on the values of the growth rate g . As regards the consumption on the balanced growth paths, it depends both on g and the initial value C_0 . Recall that government subsidies towards innovation and acquisition are financed by taxes. So higher economic growth through higher subsidies may come at the cost of reduced production and consumption in the initial period (namely, C_0 may go down). Therefore, along with contrasting the growth rate under different policies, we conduct the welfare analysis and evaluate the policy effect on consumers.

Denote the policy vector $\Theta = (\phi, \chi, \psi, \tau)$.

Following [Acemoglu et al. \(2018\)](#), we define the welfare function as the utility function of a representative consumer on an equilibrium balanced growth path:

$$U_0(\Theta) = \int_0^{+\infty} e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} dt.$$

Let the mean value of quality equal unity at $t = 0$: $\mathbf{E} q_0 = 1$. We focus on balanced growth paths, where $C_t = C_0 e^{gt}$ and

$$U_0(\Theta) = \int_0^{+\infty} e^{-\rho t} \frac{C_0^{1-\sigma} e^{(1-\sigma)gt} - 1}{1-\sigma} dt = \frac{1}{1-\sigma} \left(\frac{C_0^{1-\sigma}}{\rho - (1-\sigma)g} - \frac{1}{\rho} \right) = U_0(C_0(\Theta), g(\Theta)).$$

Similarly to [Acemoglu et al. \(2018\)](#), we compare two policies Θ_1 and Θ_2 through introducing a multiplier ξ to C_0 . The approach equates the utility of ξC_0 under the first policy to the utility of C_0 under the second policy:

$$U_0(\xi C_0(\Theta_1), g(\Theta_1)) = U_0(C_0(\Theta_2), g(\Theta_2)).$$

So policy Θ_1 is less efficient than Θ_2 when $\xi > 1$, and we interpret $\xi - 1$ as the welfare gain due to the changeover from the first to the second policy.

In the empirical analysis in this section we focus on the comparative statics of innovation and acquisition rates, growth rate and welfare gain under the five combinations of policy instruments:

1. Uniform R&D subsidy, $\phi = \chi \in [0, 0.1]$,
2. R&D subsidy to incumbents, $\phi \in [0, 0.1]$,

3. R&D subsidy to entrants, $\chi \in [0, 0.1]$,
4. Acquisition subsidy, $\psi \in [0, 0.1]$,
5. Corporate profit tax, $\tau \in [0.25, 0.4]$.

Finally, we compare economic efficiency of the four policies, related to co-financing innovations and acquisitions. For this purpose we take a given increase in the amount of government support to a firm (as % of GDP), and compute a required rise in the subsidy rate for innovation or acquisition. Next, we calculate the values for growth rate and consumer welfare under the required rate for each policy instrument (See Figure 3). Note that lowering the corporate profit tax induces various changes in the economy, which may lead to moves across different balanced growth paths within one policy experiment. Accordingly, the economic efficiency of the decrease of the corporate profit tax would be upwards biased. So the policy becomes incomparable to the government subsidies for innovations and acquisitions.

R&D subsidy to entrants

The effect of an increase in R&D subsidy to entrants is reported on Figure 2, left panel. The policy stimulates innovation and the effect is associated exclusively with the activity of entrants. The incumbents decrease both their innovation rate and acquisition rate. The overall effect on economic growth and consumer welfare is positive (Figure 2, right panel). R&D subsidies to entrants are the most efficient across the innovation and acquisition policies for increasing the economic growth and consumer welfare, as is shown on Figure 3.

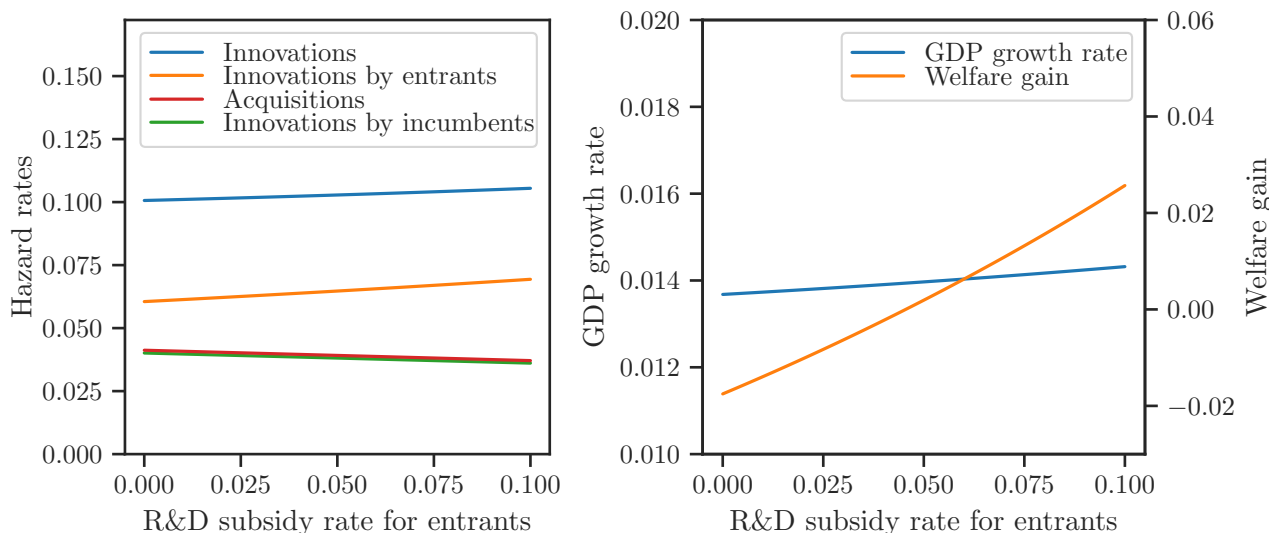


Figure 2: **Effect of the R&D subsidies to entrants**

Uniform R&D subsidy

A uniform value of R&D subsidy to all firms leads to a boost in the overall innovation rate (Figure 4, left panel). The effect may be attributed solely to incumbents, since the innovation

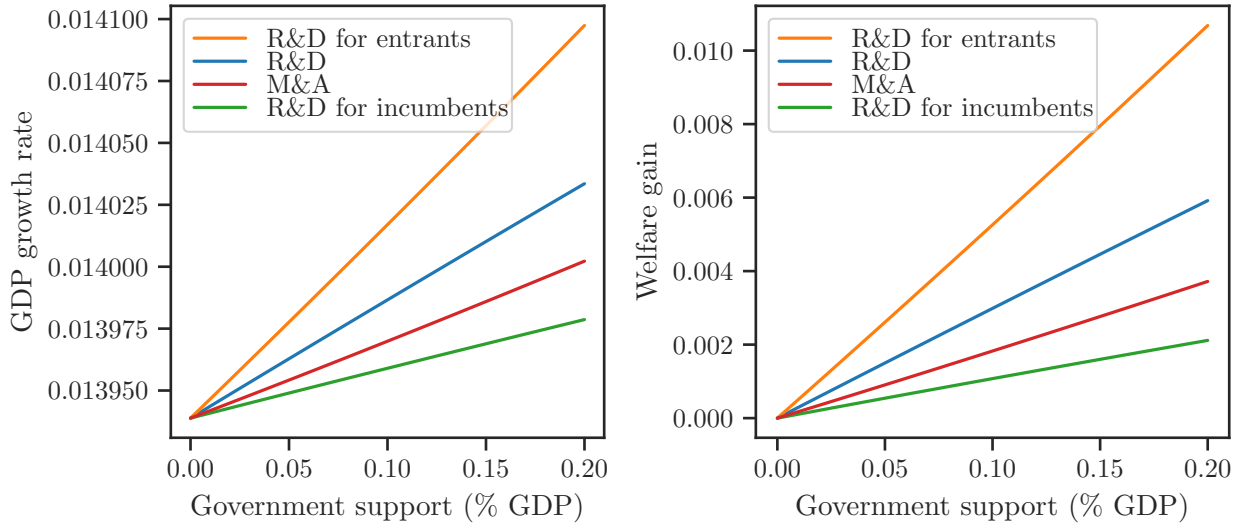


Figure 3: **Policy comparison**

rate by entrants drops appreciably. At the same time, higher R&D subsidies decrease the rate of acquisitions. Nonetheless, the overall effect on economic growth and consumer welfare is positive (Figure 4, right panel). As regards economic efficiency, the uniform R&D subsidies are more costly than the R&D subsidies for entrants. Yet, the policy is cheaper than the subsidy towards the acquisition costs or towards the R&D by incumbents (Figure 3).

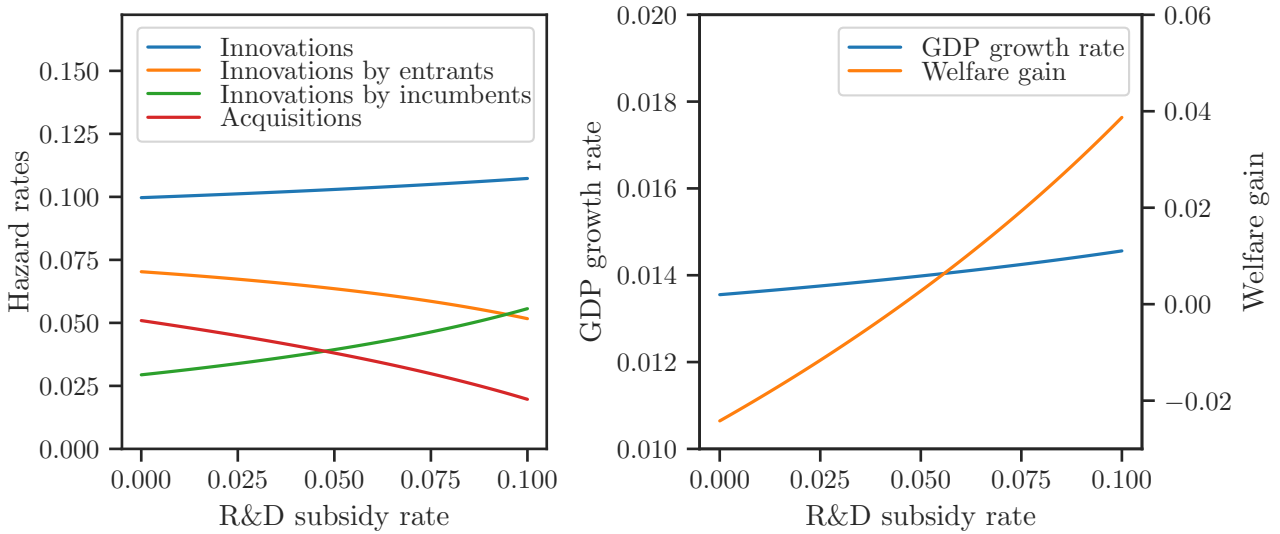


Figure 4: **Effect of the R&D subsidies both to incumbents and entrants**

R&D subsidy to incumbents

The innovation rate in the economy increases owing to a rise in R&D subsidy to incumbents (Figure 5, left panel). The effect may be attributed solely to incumbents, as the innovation rate by entrants declines. At the same time, higher R&D subsidies negatively affect the rate of acquisitions. The overall effect on economic growth and consumer welfare is positive (Figure 5, right panel). To sum up, providing the R&D subsidies to incumbents is effective for increasing the growth rate and raising consumer welfare. Yet, it is the worst policy in terms of economic

efficiency, as may be revealed from Figure 3.

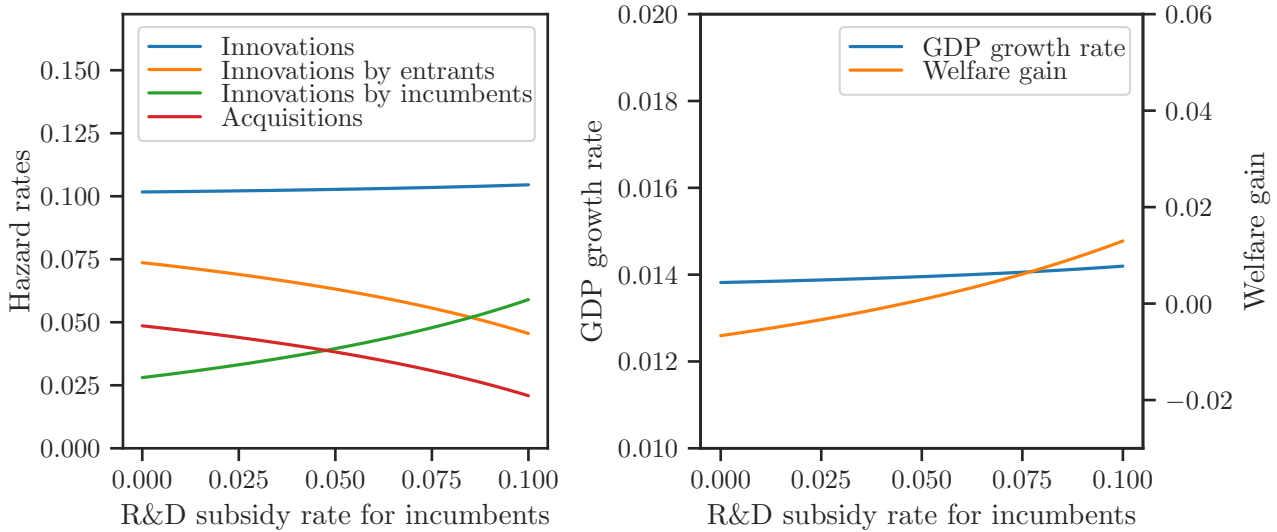


Figure 5: **Effect of the R&D subsidies to incumbents**

Subsidy towards acquisition costs

An increase in acquisition subsidy to incumbent firms raises both the innovation and acquisition rates (Figure 6, left panel). As a result, there is an acceleration of growth and a rise in consumer welfare (Figure 6, right panel). Subsidizing acquisitions provides a low value of economic efficiency. The policy is only slightly cheaper than the R&D subsidies to incumbents and requires higher government expenses than the R&D subsidies to entrants or the uniform R&D subsidies to all firms (Figure 3).

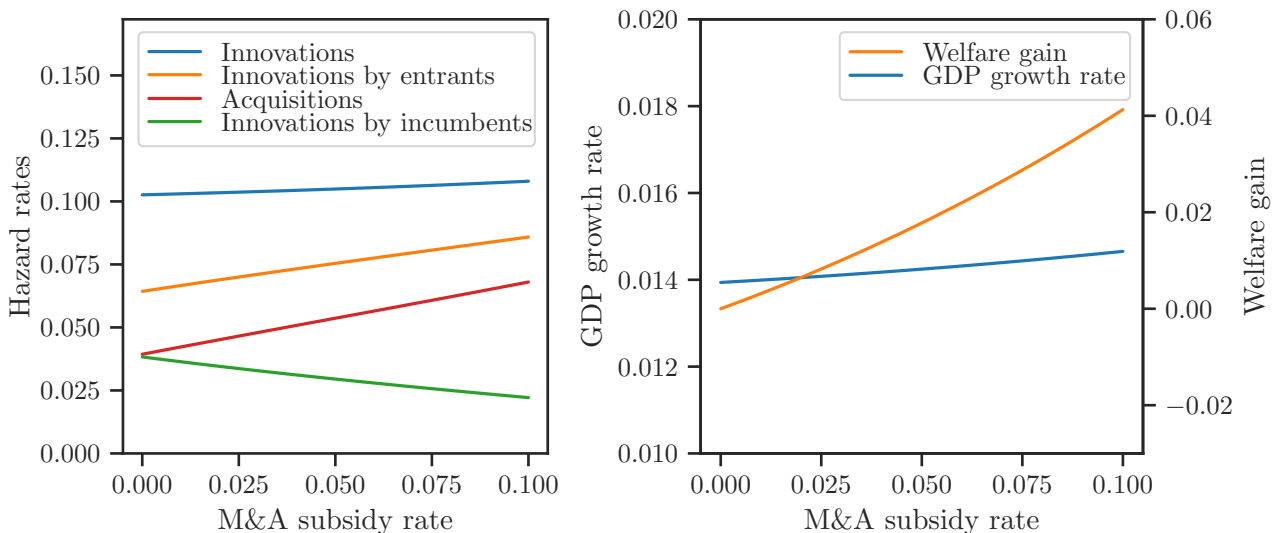


Figure 6: **Effect of the acquisition subsidies to incumbents**

Decreasing the rate of the corporate profit tax

Decreasing the rate of the corporate profit tax lowers the acquisition rate and the innovation rate by incumbents but raises the innovation rate by entrants (Figure 7). The overall innovation

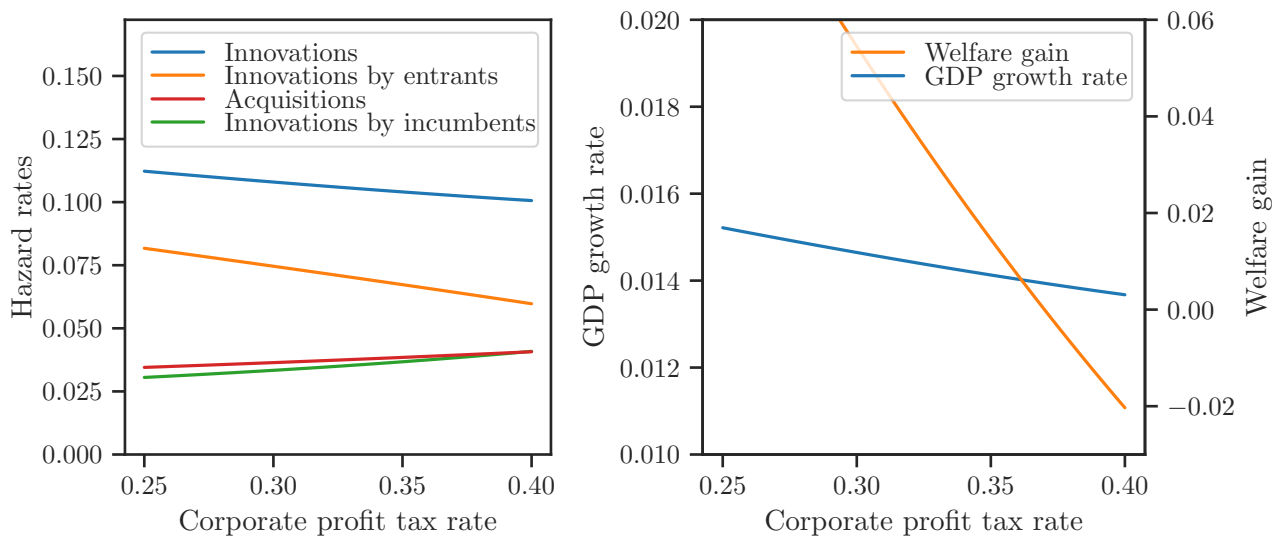


Figure 7: **Effect of decreasing the rate of the corporate profit tax**

rate by all firms in the economy goes up, leading to an increase of the growth rate and a rise of the consumer welfare.

6 Conclusion

The lack of a robust pattern of acquisitions may impede ability of the industrial base to innovate. It is therefore highly important to investigate how acquisitions can contribute to healthy R&D strategies of firms and to overall economic growth. An adequate economic environment may be another prerequisite for fostering technological change through the acquisition of innovations.

In this paper we focused on acquisitions in the context of endogenous growth with technological change. We developed a model with acquisition of innovations, which builds on key microeconomic evidence regarding the behavior of acquired and acquiring firms. The empirical part of the paper evaluated the effect of acquisitions on growth rates in the contemporary Japanese economy.

The results of our estimates offer persuasive evidence in support of the novel hypotheses of our model concerning positive impact of acquisitions on firm growth and growth in the economy. Our findings suggest that the social planner should provide incentives for acquisitions, since such incentives increase rates of both innovation and acquisition.

We let the R&D cost function depend simultaneously on the intensity of acquisitions and of innovations. Our estimated complementary parameter between acquisition and innovation intensities is positive for the contemporary Japanese economy. So an increase in the intensity of acquisitions rises R&D costs, but lowering the costs of acquisitions leads to increase in both acquisitions and innovations rates.

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Appendix A Solution of the model with heterogeneous firms

A.1 Consumer

Proof of proposition 3.1. The Hamiltonian for the consumer utility maximization problem is

$$H = e^{-\rho t}U(C_t) + \lambda_t((r_t - n)S_t + w_t - C_t + \pi_t + T_t).$$

The first order conditions are

$$\frac{\partial H}{\partial C_t} = e^{-\rho t}U'(C_t) - \lambda_t = 0, \quad (\text{A.1})$$

$$-\frac{\partial H}{\partial S_t} = -(r_t - n)\lambda_t = \dot{\lambda}_t. \quad (\text{A.2})$$

Combined with (2), they imply

$$\frac{\dot{C}_t}{C_t} = -\frac{U'(C_t)}{U''(C_t)C_t}(r_t - n - \rho) = \frac{r_t - n - \rho}{\sigma}. \quad (\text{A.3})$$

which finishes the proof. \square

A.2 Producer of the final good

Proof of proposition 3.2. The first order conditions for the problem (5) of the producer of the final good are:

$$\frac{1}{X_t(j)} \exp \int_0^1 \ln X_t(j) dj = P_t(j), \quad j \in [0, 1]. \quad (\text{A.4})$$

Taking both sides in logs and integrating over all the goods yields

$$\int_0^1 \ln X_t(j) dj - \int_0^1 \ln X_t(j) dj = \int_0^1 \ln P_t(j) dj, \quad (\text{A.5})$$

which immediately implies the proposed statement.¹¹ \square

Proof of proposition 3.3. The statement follows directly from (A.4) and (4). \square

A.3 Profitability distribution

Proof of proposition 3.6. Since

$$\mathbf{E}(\pi' \mid \pi' \leq \pi) = \frac{1}{F(\pi)} \int_0^\pi \pi' f(\pi') d\pi',$$

we can substitute the $F(\pi')$ definition and get

$$\mathbf{E}(\pi' \mid \pi' \leq \pi) = \frac{\alpha/(1-\alpha)}{\pi^{\alpha/(1-\alpha)}} \int_0^\pi \pi' \cdot \pi'^{(2\alpha-1)/(1-\alpha)} d\pi' = \frac{\alpha/(1-\alpha)}{\pi^{\alpha/(1-\alpha)}} (1-\alpha)\pi^{1/(1-\alpha)} = \alpha\pi,$$

¹¹More precisely, if the statement of proposition 3.2 does not hold, then the finite non-zero solution for the producer optimization problem does not exist.

which gives the statement of the proposition. \square

A.4 Innovation and acquisition rates

Proof of proposition 3.7. Conjecture the per capita firm value function in the form on a balanced growth path $V_\pi(m) = \frac{\pi}{\pi}mvY_t$, where v is a constant. The optimal values for hazard rates of innovation and acquisitions are, respectively, $I = \lambda m$ and $J = \kappa m$ for yet unknown constants λ and κ . Hence the Bellman equation on a balanced growth path is

$$\begin{aligned} rV_\pi(m) - \dot{V}_\pi(m) = \max_{\lambda, \kappa} \left\{ \right. \\ (1 - \tau)\pi m Y_t - \frac{\pi}{\pi}m(1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa)Y_t - (1 - \tau - \psi)\frac{\pi}{\pi}m(2c\lambda\kappa + d\kappa^2)Y_t \\ + \lambda m [V_\pi(m + 1) - V_\pi(m)] \\ + \kappa m [V_\pi(m + 1) - \mathbf{E}(V_{\pi'}(1) \mid \pi' < \pi) - V_\pi(m)] \\ \left. + \mu \sum_{i=1}^m [V_\pi(m - 1) - V_\pi(m)] \right\}. \end{aligned} \quad (\text{A.6})$$

Here μ is the rate of appropriation of goods, which happens because of innovations by rivals and new entrants.

Substituting $\frac{\pi}{\pi}mvY_t$ for $V_\pi(m)$ and dropping $\frac{\pi}{\pi}mY_t$, which enters as a multiplier for all the terms, we get the following problem:

$$\begin{aligned} (r - g - n)v = \max_{\lambda, \kappa} \left\{ (1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) \right. \\ \left. - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2) + \lambda v + \kappa(1 - \alpha)v - \mu v \right\} \end{aligned} \quad (\text{A.7})$$

The first order conditions are:

$$\begin{aligned} 2(1 - \tau - \phi)a\lambda + 2((1 - \tau - \phi)b + (1 - \tau - \psi)c)\kappa &= v, \\ 2((1 - \tau - \phi)b + (1 - \tau - \psi)c)\lambda + 2(1 - \tau - \psi)d\kappa &= (1 - \alpha)v, \end{aligned}$$

which can be solved for λ and κ to get

$$\lambda = \frac{(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)(1 - \alpha)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}, \quad (\text{A.8})$$

$$\kappa = \frac{(1 - \tau - \phi)a(1 - \alpha) - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}. \quad (\text{A.9})$$

The equation on v comes from the solution to (A.7):

$$(r - g - n + \mu - (\lambda + \kappa(1 - \alpha)))v = (1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2). \quad (\text{A.10})$$

We derive another equation on v from the entrant optimization problem. Assume nonzero entry rate on the balanced growth path, so the first order condition for (9) and with the free entry condition imply:

$$v = (1 - \chi)v.$$

This finishes the proof. \square

A.5 Distribution of the firm size

Proof of proposition 3.8. The steady state equations for the mass of firms with m products are:

$$\mu - \lambda + 2\mu M_2 = (\lambda + \kappa + \mu)M_1 + \kappa(M_1 + 2M_2 + 3M_3 + \dots), \quad (\text{A.11})$$

$$(m-1)(\lambda + \kappa)M_{m-1} + (m+1)\mu M_{m+1} = m(\lambda + \kappa + \mu)M_m, \quad m \geq 2. \quad (\text{A.12})$$

The solution to this system takes the form

$$M_m = B \left(\frac{\lambda + \kappa}{\mu} \right)^{m-1} \frac{1}{m}, \quad m = 1, 2, \dots,$$

and B can be found using (A.11):

$$\mu - \lambda + \mu B \left(\frac{\lambda + \kappa}{\mu} \right) = (\lambda + \kappa + \mu)B + \kappa B \left(1 + \left(\frac{\lambda + \kappa}{\mu} \right) + \left(\frac{\lambda + \kappa}{\mu} \right)^2 + \dots \right).$$

Therefore,

$$B = \frac{\mu - \lambda}{\mu} \cdot \frac{\mu - \lambda - \kappa}{\mu - \lambda} = \frac{\mu - \lambda - \kappa}{\mu}, \quad (\text{A.13})$$

and the total mass of firms is:

$$\begin{aligned} M &= M_1 + M_2 + M_3 + \dots = B \left(1 + \frac{1}{2} \left(\frac{\lambda + \kappa}{\mu} \right) + \frac{1}{3} \left(\frac{\lambda + \kappa}{\mu} \right)^2 + \dots \right) \\ &= B \ln \left(\frac{\mu}{\mu - \lambda - \kappa} \right) / \left(\frac{\lambda + \kappa}{\mu} \right) = \frac{\mu - \lambda - \kappa}{\lambda + \kappa} \ln \left(\frac{\mu}{\mu - \lambda - \kappa} \right). \end{aligned}$$

□

A.6 Closing the model

Proof of proposition 3.9. Consider the system of equations (12), (13), (14), (15), (18), (19).

$$v = (1 - \chi)\nu, \quad (\text{A.14})$$

$$\lambda = \frac{(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)(1 - \alpha)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}, \quad (\text{A.15})$$

$$\kappa = \frac{(1 - \tau - \phi)a(1 - \alpha) - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)}{(1 - \tau - \phi)a(1 - \tau - \psi)d - ((1 - \tau - \phi)b + (1 - \tau - \psi)c)^2} \frac{v}{2}, \quad (\text{A.16})$$

$$\begin{aligned} (r - g - n + \mu - (\lambda + \kappa(1 - \alpha)))v \\ = (1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2), \end{aligned} \quad (\text{A.17})$$

$$g = \mu \overline{\ln q}, \quad (\text{A.18})$$

$$g = \frac{r - n - \rho}{\sigma}. \quad (\text{A.19})$$

Equations (A.14)–(A.16) determine the values v^* , λ^* and κ^* . Next, substituting (A.18) and (A.19) into (A.17), we get

$$(\mu[(\sigma-1)\overline{\ln q}+1]+\rho-\lambda^*-\kappa^*(1-\alpha))v^* = (1-\tau)\bar{\pi}-(1-\tau-\phi)(a\lambda^{*2}+2b\lambda^*\kappa^*)-(1-\tau-\psi)(2c\lambda^*\kappa^*+d\kappa^{*2}),$$

which gives μ^* . Plugging it into (A.18) we find g^* , and then r^* from (A.19). This finishes the proof. \square

A.7 Comparative statics

Proof of proposition 3.10. Using (12), we conclude that $\partial v / \partial \nu = (1 - \chi) = v / \nu$. Then (13) and (14) imply

$$\frac{\partial \lambda}{\partial \nu} = \frac{\lambda}{\nu} > 0, \quad \frac{\partial \kappa}{\partial \nu} = \frac{\kappa}{\nu} > 0.$$

To compute $\partial \mu / \partial \nu$, we substitute (18) and (19) in (15)

$$(\mu[(\sigma - 1)\overline{\ln q} + 1] + \rho - \lambda - \kappa(1 - \alpha))v = (1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2). \quad (\text{A.20})$$

So

$$\begin{aligned} \frac{\partial \mu}{\partial \nu} &= \frac{\partial}{\partial \nu} \left[\frac{-\rho + \lambda + \kappa(1 - \alpha)}{(\sigma - 1)\overline{\ln q} + 1} \right. \\ &\quad \left. + \frac{(1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2)}{v((\sigma - 1)\overline{\ln q} + 1)} \right] \\ &= - \frac{(1 - \tau)\bar{\pi} - (1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa) - (1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2)}{(\sigma - 1)\overline{\ln q} + 1} \end{aligned}$$

\square

Proof of proposition 3.11. Using (12), we conclude that v does not depend on a or b . So we differentiate (A.20)

$$\begin{aligned} \frac{\partial \mu}{\partial a}((\sigma - 1)\overline{\ln q} + 1)v &= - \frac{\partial((1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa))}{\partial a} - \frac{\partial((1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2))}{\partial a} \\ &= -(1 - \tau - \phi)\lambda^2, \\ \frac{\partial \mu}{\partial b}((\sigma - 1)\overline{\ln q} + 1)v &= - \frac{\partial((1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa))}{\partial b} - \frac{\partial((1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2))}{\partial b} \\ &= -(1 - \tau - \phi)2\lambda\kappa. \end{aligned}$$

Hence,

$$a \frac{\partial \mu}{\partial a} + b \frac{\partial \mu}{\partial b} = \frac{-(1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa)}{(\sigma - 1)\overline{\ln q} + 1},$$

where the sign of the numerator is negative. This completes the proof. \square

Proof of proposition 3.12. Using (12), we conclude v does not depend on c or d . So we differentiate (A.20)

$$\begin{aligned} \frac{\partial \mu}{\partial c}((\sigma - 1)\overline{\ln q} + 1)v &= - \frac{\partial((1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa))}{\partial c} - \frac{\partial((1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2))}{\partial c} \\ &= -(1 - \tau - \psi)2\lambda\kappa, \\ \frac{\partial \mu}{\partial d}((\sigma - 1)\overline{\ln q} + 1)v &= - \frac{\partial((1 - \tau - \phi)(a\lambda^2 + 2b\lambda\kappa))}{\partial d} - \frac{\partial((1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2))}{\partial d} \\ &= -(1 - \tau - \psi)\kappa^2. \end{aligned}$$

Hence,

$$c \frac{\partial \mu}{\partial c} + d \frac{\partial \mu}{\partial d} = \frac{-(1 - \tau - \psi)(2c\lambda\kappa + d\kappa^2)}{(\sigma - 1)\ln q + 1},$$

where the sign of the numerator is negative. This completes the proof. \square

A.8 Market clearing conditions on a balanced growth path

The labor market:

$$L_t = N_t, \tag{A.21}$$

where N_t is population (labor supply), $L_t = \exp\left(\int_0^1 \ln l_t(j) dj\right)$ is the demand for labor by the producers of intermediate goods.

The market for the final good:

$$C_t N_t + (a\lambda^2 + 2b\lambda\kappa)Y_t + (2c\lambda\kappa + d\kappa^2)Y_t + \nu\eta Y_t = Y_t, \tag{A.22}$$

where the left hand side part lists the expenditures on consumption, the R&D and M&A.

The savings/investment market: As is shown in Proposition 3.7, the value function of an incumbent on a balanced growth path is $\frac{\pi}{\bar{\pi}} m v Y_t$, so the clearing condition for the savings market can be expressed as

$$S_t N_t = v Y_t. \tag{A.23}$$

The profits and dividends: The profits of the producer of the final goods are dividends

$$\Pi_t = \pi_t N_t$$

which are equal to zero in an equilibrium.

The government budget:

$$\begin{aligned} \phi(a\lambda^2 + 2b\lambda\kappa)Y_t + \chi\nu\eta Y_t + \psi(2c\lambda\kappa + d\kappa^2)Y_t + T_t N_t \\ = \tau(\bar{\pi} - (a\lambda^2 + 2b\lambda\kappa) - (2c\lambda\kappa + d\kappa^2))Y_t. \end{aligned} \tag{A.24}$$

Appendix B Solution of the model with homogeneous firms

B.1 Consumer

The Hamiltonian for the consumer utility maximization problem is

$$H = e^{-\rho t} U(C_t) + \lambda_t((r_t - n)S_t + w_t - C_t + \pi_t - T_t).$$

The first order conditions are

$$\frac{\partial H}{\partial C_t} = e^{-\rho t} U'(C_t) - \lambda_t = 0, \tag{B.1}$$

$$-\frac{\partial H}{\partial S_t} = -(r_t - n)\lambda_t = \dot{\lambda}_t. \tag{B.2}$$

Combined with (25), they yield

$$\frac{\dot{C}_t}{C_t} = -\frac{U'(C_t)}{U''(C_t)C_t}(r_t - n - \rho) = \frac{r_t - n - \rho}{\sigma}. \quad (\text{B.3})$$

B.2 Firm sizes

Proof of proposition 4.7. Recall that any firm can buy a firm with one good. The stationary conditions yield

# of products	Inflow	=	Outflow
$m = 0$	$Fs_1\mu + Fy(s_1 + 2s_2 + \dots)$	=	$x_e,$
$m = 1$	$Fs_2 \cdot 2\mu + x_e$	=	$Fs_1(x + \mu + y) +$ $Fy(s_1 + 2s_2 + \dots),$
$m \geq 2$	$Fs_{m+1}(m+1)\mu +$ $Fs_{m-1}(m-1)(x+y)$	=	$Fs_m m(\mu + x + y).$

or equivalently

$$\begin{aligned} s_1\mu + y(s_1 + 2s_2 + \dots) &= \frac{x_e}{F}, \\ s_2 \cdot 2\mu + \frac{x_e}{F} &= s_1(\mu + x + y) + y(s_1 + 2s_2 + \dots), \\ s_{m+1}(m+1)\mu + s_{m-1}(m-1)(x+y) &= s_m m(\mu + x + y). \end{aligned}$$

The solution to third equation is

$$s_m = B \left(\frac{x+y}{\mu} \right)^{m-1} \frac{1}{m}, \quad m = 1, 2, \dots,$$

where B is a constant. The mean value of the firm size is

$$M = s_1 + 2s_2 + 3s_3 + \dots = B \left(1 + \left(\frac{x+y}{\mu} \right) + \left(\frac{x+y}{\mu} \right)^2 + \dots \right) = B \frac{\mu}{\mu - x - y}.$$

The sum of the shares of all firms equals one, so

$$s_1 + s_2 + \dots = B \left(1 + \left(\frac{x+y}{\mu} \right) \frac{1}{2} + \left(\frac{x+y}{\mu} \right)^2 \frac{1}{3} + \dots \right) = B \ln \left(\frac{\mu}{\mu - x - y} \right) \Big/ \left(\frac{x+y}{\mu} \right) = 1,$$

which leads to the expression for B .

The number of goods is normalized to one, so

$$F = \frac{1}{M} = \frac{\mu - x - y}{x + y} \ln \left(\frac{\mu}{\mu - x - y} \right).$$

The equation may be interpreted as a definition of the rate for creative destruction: a sum of the rates for external innovations by entrants and incumbents. \square

B.3 Producer of the final good

Proof of Proposition 4.2. The first order conditions for the producer of the final good are

$$\frac{\partial \Pi_t}{\partial L_t} = \beta \frac{Y_t}{L_t} - w_t = 0 \quad (\text{zero-profit condition}), \quad (\text{B.4})$$

$$\frac{\partial \Pi_t}{\partial k_t(j)} = q_t^\beta(j) k_t^{-\beta}(j) L_t^\beta - p_t(j) = 0. \quad (\text{B.5})$$

Equation (B.5) implies the demand for good j :

$$k_t(j) = p_t^{-1/\beta}(j) q_t(j) L_t \quad (\text{B.6})$$

and the inverse demand is:

$$p_t(j, k_t(j)) = q_t^\beta(j) k_t^{-\beta}(j) L_t^\beta. \quad (\text{B.7})$$

□

B.4 Producers of intermediate goods

Proof of Proposition 4.3. Insert the reverse demand (27) into the instantaneous profit maximization problem. The first order conditions become

$$(1 - \beta) q_t^\beta(j) L_t^\beta k_t^{-\beta}(j) = \frac{w_t}{\bar{q}_t}.$$

Rearranging the terms, we obtain the equations for the price p and quantity k :

$$p_t(j) = q_t^\beta(j) L_t^\beta k_t^{-\beta} = \frac{w_t}{(1 - \beta) \bar{q}_t}, \quad (\text{B.8})$$

$$k_t(j) = \left(\frac{(1 - \beta) \bar{q}_t}{w_t} \right)^{1/\beta} L_t q_t(j). \quad (\text{B.9})$$

Plugging in (B.8) and (B.9) into the expression for profits yields:

$$\pi_t(j) = \left(\frac{w_t}{(1 - \beta) \bar{q}_t} - \frac{w_t}{\bar{q}_t} \right) \left(\frac{(1 - \beta) \bar{q}_t}{w_t} \right)^{1/\beta} L_t q_t(j) = \beta (1 - \beta)^{(1 - \beta)/\beta} \left(\frac{\bar{q}_t}{w_t} \right)^{(1 - \beta)/\beta} L_t q_t(j). \quad (\text{B.10})$$

□

Proof of propositions 4.4, 4.5, and 4.6. Using the fact that a good with zero quality has no value, conjecture the firm value function as $V(\mathbf{q}) = AN_t \sum_{j:q(j) \in \mathbf{q}} q(j)$. Insert it in the Bellman equation of the firm

$$\begin{aligned} (r - n)A \sum_{j:q(j) \in \mathbf{q}} q(j) = \max_{x_m, y_m, \{z(j)\}} \{ & \\ & \sum_{j:q(j) \in \mathbf{q}} [(1 - \tau)\pi q(j) - (1 - \tau - \hat{\phi})\hat{\chi} z^\psi(j)q(j) + z(j)Aq(j)\lambda - \mu Aq(j)] \\ & + x_m A \bar{q} (1 + \eta) - (1 - \tau - \tilde{\phi})(\check{\chi} x_m^2 + \tilde{\delta} x_m y_m) \bar{q} m^{-1} \\ & + y_m A \bar{q} \kappa - (1 - \tau - \check{\phi})(\check{\chi} y_m^2 + \tilde{\delta} x_m y_m) \bar{q} m^{-1} \\ & - \Phi m \bar{q} \}. \end{aligned} \quad (\text{B.11})$$

Maximization of the right-hand side in $\{z(j)\}$ and (x_m, y_m) yields:

$$(r - n)A = \max_z \{(1 - \tau)\pi - (1 - \tau - \hat{\phi})\hat{\chi}z^{\hat{\psi}} + zA\lambda - \mu A\}, \quad (\text{B.12})$$

$$0 = \max_{x_m, y_m} \{x_m A(1 + \eta) - (1 - \tau - \tilde{\phi})(\tilde{\chi}x_m^2 + \tilde{\delta}x_m y_m)m^{-1} + y_m A\kappa - (1 - \tau - \check{\phi})(\check{\chi}y_m^2 + \check{\delta}x_m y_m)m^{-1} - \Phi m\}. \quad (\text{B.13})$$

There exists Φ that ensures that the last equation holds (this proves Proposition 4.4), otherwise the solution does not exist. Combining equation (29) and the free entry condition $V_0 = 0$, we obtain:

$$0 = (r - n)V_0 = \max_{x_e} \{x_e A\bar{q}(1 + \eta) - x_e(1 - \phi)\nu\bar{q}\} = \max_{x_e} \{x_e[A(1 + \eta) - (1 - \phi)\nu]\bar{q}\}.$$

If the solution is internal (i.e. $x_e > 0$), then $A = \frac{(1 - \phi)\nu}{1 + \eta}$. This finishes the proof of Proposition 4.6.

Maximizing (B.12), we get the values for all $z(j)$:

$$z(j) = \left(\frac{A\lambda}{(1 - \tau - \hat{\phi})\hat{\chi}\hat{\psi}} \right)^{1/(\hat{\psi}-1)} = \left(\frac{(1 - \phi)\nu\lambda}{(1 + \eta)(1 - \tau - \hat{\psi})\hat{\chi}\hat{\psi}} \right)^{1/(\hat{\psi}-1)}. \quad (\text{B.14})$$

The first order conditions for (B.13) become

$$2(1 - \tau - \tilde{\phi})\tilde{\chi}(x_m/m) + ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})(y_m/m) = A(1 + \eta), \quad (\text{B.15})$$

$$((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})(x_m/m) + 2(1 - \tau - \check{\phi})\check{\chi}(y_m/m) = A\kappa. \quad (\text{B.16})$$

The solution to this system yields x and y :

$$x_m = m \frac{2(1 - \tau - \check{\phi})\check{\chi}A(1 + \eta) - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})A\kappa}{4(1 - \tau - \tilde{\phi})(1 - \tau - \check{\phi})\tilde{\chi}\check{\chi} - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})^2}, \quad (\text{B.17})$$

$$y_m = m \frac{-((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})A(1 + \eta) + 2(1 - \tau - \tilde{\phi})\tilde{\chi}A\kappa}{4(1 - \tau - \tilde{\phi})(1 - \tau - \check{\phi})\tilde{\chi}\check{\chi} - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})^2}. \quad (\text{B.18})$$

Substituting A with $(1 - \phi)\nu/(1 + \eta)$ leads to

$$x_m = m \frac{2(1 - \tau - \check{\phi})\check{\chi}(1 - \phi)\nu(1 + \eta) - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})(1 - \phi)\nu\kappa}{(1 + \eta)(4(1 - \tau - \tilde{\phi})(1 - \tau - \check{\phi})\tilde{\chi}\check{\chi} - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})^2)}, \quad (\text{B.19})$$

$$y_m = m \frac{-((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})(1 - \phi)\nu(1 + \eta) + 2(1 - \tau - \tilde{\phi})\tilde{\chi}(1 - \phi)\nu\kappa}{(1 + \eta)(4(1 - \tau - \tilde{\phi})(1 - \tau - \check{\phi})\tilde{\chi}\check{\chi} - ((1 - \tau - \tilde{\phi})\tilde{\delta} + (1 - \tau - \check{\phi})\check{\delta})^2)}. \quad (\text{B.20})$$

So the rate of external R&D per product and the hazard rate of acquisitions are constant in the optimum. Denote them $x = x_m/m$ and $y = y_m/m$. This finishes the proof of Proposition 4.5 \square

B.5 Market equilibrium

Insert the amount of each intermediate good (B.9) into the production function of the producer of the final good:

$$\begin{aligned} Y_t &= \frac{L_t^\beta}{1-\beta} \int_0^1 q_t^\beta(j) k_t^{1-\beta}(j) dj = \frac{L_t^\beta}{1-\beta} \int_0^1 q_t^\beta(j) \left(\frac{(1-\beta)\bar{q}_t}{w_t} \right)^{(1-\beta)/\beta} L_t^{1-\beta} q_t^{1-\beta}(j) dj \\ &= \frac{L_t}{1-\beta} \left(\frac{(1-\beta)\bar{q}_t}{w_t} \right)^{(1-\beta)/\beta} \int_0^1 q_t(j) dj = L_t (1-\beta)^{(1-2\beta)/\beta} w_t^{(\beta-1)/\beta} \bar{q}_t^{1/\beta}. \end{aligned} \quad (\text{B.21})$$

Using zero profit condition (B.4), we get $w_t = \beta Y_t / L_t = \beta (1-\beta)^{(1-2\beta)/\beta} w_t^{(\beta-1)/\beta} \bar{q}_t^{1/\beta}$ which implies $w_t^{1/\beta} = \beta (1-\beta)^{(1-2\beta)/\beta} \bar{q}_t^{1/\beta}$ and finally

$$w_t = \beta^\beta (1-\beta)^{1-2\beta} \bar{q}_t. \quad (\text{B.22})$$

Inserting (B.22) into (B.21) yields

$$Y_t = L_t (1-\beta)^{(1-2\beta)/\beta} \beta^{\beta-1} (1-\beta)^{(1-2\beta)(\beta-1)/\beta} \bar{q}_t^{(\beta-1)/\beta} \bar{q}_t^{1/\beta},$$

or

$$Y_t = \frac{(1-\beta)^{1-2\beta}}{\beta^{1-\beta}} \bar{q}_t L_t. \quad (\text{B.23})$$

Finally, plugging in (B.22) into (B.10), we get the profits of the producer of the intermediate good:

$$\pi_t(j) = \beta (1-\beta)^{(1-\beta)/\beta} (\beta^\beta (1-\beta)^{1-2\beta})^{-(1-\beta)/\beta} q_t(j) L_t,$$

It implies

$$\pi_t(j) = \beta^\beta (1-\beta)^{2(1-\beta)} L_t q_t(j). \quad (\text{B.24})$$

Production of $k_t(j)$ units of good requires $l_t(j) = k_t(j) / \bar{q}_t$ units of labor. Formally,

$$l_t(j) = (1-\beta)^{1/\beta} (\beta^{-\beta} (1-\beta)^{2\beta-1})^{1/\beta} L_t \frac{q_t(j)}{\bar{q}_t} = \frac{(1-\beta)^2}{\beta} L_t \frac{q_t(j)}{\bar{q}_t}.$$

Accordingly, all producers of intermediate goods hire

$$\tilde{L}_t = \int_0^1 l_t(j) dj = \frac{(1-\beta)^2}{\beta} L_t \frac{\bar{q}_t}{\bar{q}_t} = \frac{(1-\beta)^2}{\beta} L_t.$$

Denote N_t the total population. Our model equates the total population and the total labor force. The labor market clearing condition becomes

$$N_t = L_t + \tilde{L}_t = \left(1 + \frac{(1-\beta)^2}{\beta} \right) L_t,$$

which implies

$$L_t = \frac{\beta}{(1-\beta)^2 + \beta} N_t. \quad (\text{B.25})$$

Using this expression for the labor force employed in the production of the final good, we simplify the profits of the producer of the intermediate good:

$$\pi_t(j) = \frac{\beta^{1+\beta} (1-\beta)^{2(1-\beta)}}{(1-\beta)^2 + \beta} q_t(j) N_t \equiv \pi q_t(j) N_t, \quad (\text{B.26})$$

where $\pi = \frac{\beta^{1+\beta}(1-\beta)^{2(1-\beta)}}{(1-\beta)^2 + \beta}$.

B.6 Rate of creative destruction

Using propositions 4.1, 4.5 and 4.6 we get the following equation on a balanced growth path:

$$\begin{aligned} \mu &= \frac{\pi}{A} + z\lambda - \frac{\hat{\chi}z^{\hat{\psi}}}{A} - \rho - \sigma g \\ &= \frac{\pi}{A} + \left(\lambda \left(\frac{(1-\phi)\nu\lambda}{(1+\eta)\hat{\chi}\hat{\psi}} \right)^{1/(\hat{\psi}-1)} - \frac{\hat{\chi}(1+\eta)}{(1-\phi)\nu} \left(\frac{(1-\phi)\nu\lambda}{(1+\eta)\hat{\chi}\hat{\psi}} \right)^{\hat{\psi}/(\hat{\psi}-1)} \right) - \rho - \sigma g \\ &= \frac{\pi}{A} + (\hat{\psi} - 1) \left(\frac{(1-\phi)\nu}{(1+\eta)\hat{\chi}} \right)^{1/(\hat{\psi}-1)} \left(\frac{\lambda}{\hat{\psi}} \right)^{\hat{\psi}/(\hat{\psi}-1)} - \rho - \sigma g. \end{aligned} \quad (\text{B.27})$$

B.7 Market clearing conditions on a balanced growth path

The labor market:

$$L_t + \tilde{L}_t = N_t, \quad (\text{B.28})$$

where N_t is population (labor supply), L_t is the labor demand for the producer of the final good, $\tilde{L}_t = \int_0^1 l_t(j) dj$ is the demand for labor by the producers of intermediate goods, and $l_t(j)$ is the demand for labor by the producer of a good j .

The market for the final good:

$$C_t N_t + \hat{\chi} z^{\hat{\psi}} \bar{q}_t N_t + (\tilde{\chi} x^2 + \tilde{\delta} xy) \bar{q}_t N_t + (\check{\chi} y^2 + \check{\delta} xy) \bar{q}_t N_t + x_e \nu \bar{q}_t N_t = Y_t, \quad (\text{B.29})$$

where C_t is a per capita consumption of the final good by the representative consumer, Y_t is an amount of the final good, and the remaining terms represent the total costs of internal innovations, acquisitions, and external innovations by incumbents and entrants.

The savings/investment market: Consumers hold their savings in the stocks of the firms, producing intermediate goods. The value of a firm $V(\mathbf{q}) = AN_t \sum_{j:q(j) \in \mathbf{q}} q(j)$. This implies the following market clearing condition on a balanced growth path

$$S_t N_t = A \bar{q}_t N_t, \quad (\text{B.30})$$

where S_t is the amount of savings held by consumers.

The profits and dividends: The profits equal the dividends

$$\Pi_t = \pi_t N_t$$

which are zero in equilibrium.

The government budget:

$$\begin{aligned} \hat{\phi} \hat{\chi} z^{\hat{\psi}} \bar{q}_t N_t + \tilde{\phi} (\tilde{\chi} x^2 + \tilde{\delta} xy) \bar{q}_t N_t + \phi x_e \nu \bar{q}_t N_t + \check{\phi} (\check{\chi} y^2 + \check{\delta} xy) \bar{q}_t N_t + T_t N_t \\ = \tau (\pi \bar{q}_t N_t - \hat{\chi} z^{\hat{\psi}} \bar{q}_t N_t - (\tilde{\chi} x^2 + \tilde{\delta} xy) \bar{q}_t N_t - (\check{\chi} y^2 + \check{\delta} xy) \bar{q}_t N_t). \end{aligned} \quad (\text{B.31})$$

We assume that the government balances its budget at every time period.

Appendix C Data work

C.1 Match the Nikkei NEEDS with patent data

We start with the names of Nikkei NEEDS firms, which report financials for any year in 1964–2016 (as of the Sep 2017 update). We match the Japanese names of the Nikkei NEEDS firms to the Japanese names of the applicants in the IIP Patent database (as of the Sep 2017 release). For this purpose we use the applicant files in the IIP Patent database along with the raw data on patents, coming from the Japan Patent Office and available at the J-PLAT-PAT platform by Japan’s National Center for Industrial Property Information and Training.

Initial match

We conduct the initial match across the names of the Nikkei NEEDS firms and the name of applicants in the IIP Patent database. First, an automated script calculates similarity scores for *kana* name and *kanji* name between all pairs of firms in Nikkei NEEDS and in the Patent database. The script uses Levenstein’s algorithm to compute the Hamming distance. We keep the firms with either of scores greater than 0.7 and then re-examine them during manual refinement.

Second, we keep full address, provided in the patent database. Matches with the same name and addresses with distance less than 3 are considered identical. We keep only one of such identical matches for further refinement.

Third, most recent patent applications for any discovered match are extracted from the IIP Patent database. We use three such applications in the further refinement.

Refinement

We exploit the original information on patent applications from the J-PLAT-PAT platform and use it for verification. The work may be outlined as follows.

First, we take the most recent patent application, found during the initial match for a Nikkei NEEDS firm. For each such application we requested the information on the J-PLAT-PAT database and the response rate was 99.5 percent. The response on a patent application either contained the English names of applicants or the original (non-processed) scan of patent application. The non-response may be interpreted as the absence of the patent application in the J-PLAT-PAT database.

Second, we use the English name of the applicants to establish a similarity score with the Nikkei NEEDS firm.

Third, if only the patent scan is available, we manually examine it for full information about the applicants. The criteria to eliminate errors of the initial match are firm names and location.

C.2 Match the Nikkei NEEDS with the Orbis database

We start with the full list of 4 million Japanese companies, available in Orbis as of Mar 2018. Using an automated script and manual refinement, we match the Nikkei NEEDS companies by name (both in Japanese and English) and prefecture. Next, we use manual refinement of the automated match and check for the correspondence between the industry names, reported in both databases. Finally, we exclude the matches with a high discrepancy among the value of annual sales (turnover) in Nikkei NEEDS and Orbis. Here we control for the similarity of the consolidation code in Nikkei NEEDS and Orbis.

C.3 Match the NIKKEI NEEDS with the Japan National Innovation Survey

We match two lists of firms: the names of Nikkei NEEDS firms, which report financials for any year in 1964–2016 (as of the Sep 2017 update) and names of the JNIS firms (the disclosed list of 18,000 firms as of the 2016 release).

Our automated script uses Levenstein’s algorithm to compute the Hamming distance between the kana readings of the Nikkei NEEDS and the JNIS firms. The script calculates string similarity scores for three best matches. Additionally, the script computes the scores for the match of prefecture names across the Nikkei NEEDS and the JNIS firms (1 if matched).

The manual part of work starts with filtering the results of the script: we use the entries with the score for first best match above 0.7 or with the identical *kanji* names of the match and the master record. Next, we examine the names of the three matches and chose the number for the true match.

Finally, we add full address from the JNIS, so the overlap contains the *kanji* and *kana* names of the firms in the two surveys, prefecture names, the full address, the English names from Nikkei NEEDS and the English name from JNIS (if present).