Standard Setting Organizations and Networked Oligopolies in General Equilibrium

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We construct a tractable model of Standard Setting Organizations (SSOs) with Networked Oligopolies in General Equilibrium (NOGE) that analyzes non-trivial implications of SSOs for a general oligopolistic equilibrium. We show that a complete network of SSOs at home increases the extensive margin of domestic exports and decreases the extensive margin of domestic imports. A complete foreign network of SSOs reduces the extensive margin of domestic exports and raises the extensive margin of domestic imports. All else equal, the net effect of home and foreign SSO networks on the extensive margins of trade depends on the relative economy of size of each network.

JEL Classification Code(s): L13, L20, F10, F12

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"The rules and procedures adopted by an SSO can be related to … the network of collaborations"

Baron and Spulber (2018)

1. Introduction

The significance for standard setting organizations (SSOs) is widely recognized across professions as the role of technological standards (i.e. the set of rules and technologies adopted to ensure interoperability between products and services and to ensure that they meet specific industry requirements) has grown tremendously over the recent past.[1] While much of the extant economic literature on technology standards has focused on market standardization, it is important to recognize that SSOs cooperate within a dense network of agreements. As Baron and Spulber (2018) underscore, the rules and procedures adopted by any SSO can be related to its position in the network of collaborations since the composition of SSO membership is analogous to coalition building where firms decide in favor or against joining the same SSO as other firms.[2] Banerjee and Chakrabarti (2019) have analyzed the Searle Center Database (SCD) to identify networks of firms by matching joint SSO membership in a sample of 282,585 firms, that includes quantifiable characteristics of 762,146 standard documents, institutional membership in a sample of 195 SSOs, and the rules of 36 SSOs on standard-essential patents, openness, participation, and standard adoption procedures.

Figure 1. Total number of membership observations in the sample by year in the SCD
Source: Baron and Spulber (2018)

At the same time, a rich and growing literature presents mounting evidence that upholds the global presence of networked oligopolies in a wide range of industries (e.g. cement, food, airlines, healthcare, energy, telecommunications, etc.) but theoretical research on the competition structure among firms in networked oligopolies is still at its infancy.[3] We build what is, to the best of our knowledge, the first tractable model of SSOs and networked oligopolies in general equilibrium (NOGE). Our main contribution is in identifying non-trivial implications of the formation of SSOs as networks for a general oligopolistic equilibrium. We highlight any role that interactions between SSOs, efficiency, and concentration can play in determining the pattern of production as well as the extensive margins of trade. More specifically, we demonstrate that a complete network of SSOs at home increases the extensive margin of domestic exports and decreases the extensive margin of domestic imports while a complete foreign network of SSOs reduces the extensive margin of domestic exports and raises the extensive margin of domestic imports. In doing so, while we draw on a vast and expanding body of literature on networks and oligopolies, our analytical construct is most closely related to Neary (2003, 2007). More specifically, we build on Neary's (2003, 2007) foundations of general oligopolistic equilibrium (GOLE) particularly because it has the advantage of distinguishing between domestic and foreign firms even in the absence of any friction, allowing room for a parsimonious linking of network externalities to oligopolies in general equilibrium. The rest of the paper is organized as follows. In the next section, we present our model and propositions. Our findings are summarized in the concluding section.

2. Empirical Backdrop

Banerjee and Chakrabarti (BC-2019) have analyzed the Searle Center Database (SCD) to identify networks of firms by matching joint SSO membership in a sample of 282,585 firms, that includes quantifiable characteristics of 762,146 standard documents, institutional membership in a sample of 195 SSOs, and the rules of 36 SSOs on standard-essential patents, openness, participation, and standard adoption procedures. To effectuate the proposed join, BC-2019 constructed an algorithm to match the observations SCD: a match of 23,158 (i.e. ~ 58%) was feasible. Next, we looked into the possibility of potential misspecification of SSO membership. In context, we recall having presented our preliminary findings at the Fourth Annual Searle Center Research Roundtable on

Technology Standards (2016) on plausible associations between the nature of M&A deals and whether or not the participating firms were SSO members. We took a cue from the discussions at the Roundtable in recognizing that the SCD of SSOs, while spanning a large sample, does not possibly cover all existing SSOs. This is likely to lead to false negatives – missed memberships when they actually exist. The same corporation could be referred to by different variants in the databases, an example being, 'Alcatel Cable', 'Alcatel' and 'Alcatel Corporation' all referring to the same entity. The algorithms attempting to match phrases or strings that approximately match each other is often referred to as "fuzzy matching" in computer science literature. We use a novel modification of Seller's algorithms, identification of the most likely keyword and approximate distance computation techniques to adapt into a probabilistic matching technique for our purpose. However, this matching technique is not perfect (as is any probabilistic technique) – matches could be incorrectly specified due to incorrectly tagged keywords or missing potential matches, therefore possibly leading to both false positives and false negatives in the matches. We address this issue by adopting a hierarchical formulation. To do so, we consider the following set-up for the basic statistical model for this analysis,

$$y_{ij} = f\left(I_{ij}, X_i^1, X_j^2, X_{ij}^3\right) + \epsilon_{ij}.$$

We use a generic functional form 'f(.)' for the regression model since it is not clear that a standard least squared linear regression model would provide the best fit. Of the covariates included in the model, I_{ij} is the most crucial one for our hypotheses of interest. One can think of at least a couple of reasons for I_{ij} to be imperfectly observed. To account for such potential uncertainty in the main covariate of interest I_{ij}, we propose the following hierarchical formulation. Let T_{ij} be the true unobserved value of the covariate and I_{ij} the deduced value from the probabilistic matching algorithm, possibly incorrectly specified As with I_{ij}, the variable T_{ij} could be in one of four categories, based on the target and acquirer's SSO membership. Consider the vector of conditional probabilities

$$\boldsymbol{\pi_{ij}} = \{\pi_{ij}^{a}, \pi_{ij}^{b}, \pi_{ij}^{c}, \pi_{ij}^{d}\}$$

$$\pi_{ij}^{a} = \Pr\left(T_{ij} \in a \mid I_{ij}\right)$$

and, similarly, for the other elements $\pi_{ij}^{b}, \pi_{ij}^{c}, \pi_{ij}^{d}$. However, the estimated value of the proportion of SSO memberships itself depends on the covariate I_{ij}, rendering estimation of $\boldsymbol{\pi_{ij}}$ difficult. We considered two alternative approaches to circumvent the problems posed by incorrect SSO membership specification: we could use an iterative scheme, where a value $\boldsymbol{\pi_{ij}}$ is estimated from the currently computed I_{ij}, or use a Bayesian formulation, when the U_i, V_j, I_{ij} are encoded into prior parameters for quantity of interest, $\boldsymbol{\pi_{ij}}$**.** Using this value of $\boldsymbol{\pi_{ij}}$ in the probability matching scheme, new values of I_{ij} are computed. This iteration then continues until updates or changes to either set of values is minimal. This scheme is conceptually similar to an expectation-maximization (EM) type algorithm in statistics. The expectation maximization scheme is difficult to implement in practice since the computation of I_{ij} is quite burdensome and doing this at each iteration of algorithm would represent infeasible computation time. We resort to the Bayesian approach instead for the estimation of $\boldsymbol{\pi_{ij}}$'s. Once these conditional probabilities are estimated, they replace their counterparts in the basic model mentioned at the start of this subsection, so that the basic model now becomes:

$$y_{ij} = f\left(\pi_{ij}, X_i^1, X_j^2, X_{ij}^3\right) + \epsilon_{ij}$$

Our preliminary investigations reveal the presence of non-linear patterns as well as higher order interaction terms. We use a Bayesian random forest specification for robust quantification of non-linear patterns in the data. In general, random forests represent a regression technique that work by averaging estimates over a collection of individual regression trees. To investigate how well our proposed algorithm works, we devise a simulation study with mock-up data. This mock-up data investigates how well our algorithm is able to pick up associations with imperfect specification of covariates. In doing so, we consider three degrees of misspecification i.e. Low, where actual misspecification is 5% or less; Moderate, where actual misspecification is about 25%; and High, where actual misspecification is at least 50%. It turns out that our model is fairly robust for low to moderate misspecification of covariates whereas standard regressions are not. It is also worth noting that in case of perfect specification, our proposed model performs at least as well as the standard methods. In instances of high misspecification, when it may be argued that attempting the regression analysis itself may be dubious for the main covariate of interest, since the little information is present, our model is able to beat a standard linear model but remains comparable to a hierarchical linear model taking into account estimated conditional probabilities.

3. A Tale of Two Networked Economies

In what follows, we present a parsimonious model of SSOs and networked oligopolies that has the advantage of providing a framework within which an industry structure can be analyzed, at the same time keeping the general equilibrium analysis tractable. Our construct preserves the key characteristic of a typical GOLE model to the extent that we are looking at a continuum of atomistic industries within each of which firms have market power and interact strategically. Visualize a stylized world containing two open economies populated by a continuum of atomistic industries indexed by $\tilde{z} \in [0, 1]$. Let $\tilde{y}(z)$ and $\tilde{y}^*(z)$ be the industry outputs at home and abroad, respectively. Each industry z contains $(n + n^*)$ firms competing `a la Cournot (1838), partitioned into a set F of firms $\{F_i: i = 1,2, \ldots, n\}$ at home and a set F^* of firms $\{F_i^*: i = 1,2, \ldots, n^*\}$ in the foreign country such that $F \cap F^* = \emptyset$ i.e. no firm operates in both countries. Each country hosts a set of SSO networks: N at home connecting n_1 firms and N^* in the foreign country connecting n_1^* firms; $(n_1 + n_{-1}) = n$ and $(n_1^* + n_{-1}^*) = n^*$ where n_{-1} and n_{-1}^* are the number of firms operating outside any SSO network at home and in the foreign country, respectively. We assume, as in Neary (2003, 2007), that without networking fixed costs are zero and technology is subject to constant returns-to-scale. We limit our analysis to SSO networks that are a) *symmetric* i.e. all firms within a network have the same number of links; and b) *stable* i.e. any firm linked to another within a network has a strict incentive to maintain the link and any two firms that are not linked have no incentive to form a link with each other. We will assume that $N \cap N^* = \emptyset$ i.e. no SSO network links firms across borders. Let

(1a) $$n_{ij} = \begin{cases} 1 & \forall\, F_i, F_j \in N \\ 0 & \text{otherwise} \end{cases}$$

and

(1b) $$n_{ij}^* = \begin{cases} 1 & \forall\, F_i^*, F_j^* \in N^* \\ 0 & \text{otherwise} \end{cases}$$

where network $\eta = \left\{ (n_{ij})_{F_i, F_j \in N} \right\}$ and $\eta^* = \left\{ (n_{ij}^*)_{F_i^*, F_j^* \in N^*} \right\}$ define pair-wise relationships between firms at home and in the foreign country, respectively: $(\eta + n_{ij})$ denotes a network obtained by replacing $n_{ij} = 0$ in network η by $n_{ij} = 1$ and $(\eta - n_{ij})$ by replacing $n_{ij} = 1$ in network η by $n_{ij} = 0$, at home; and $(\eta^* + n_{ij}^*)$ denotes a network obtained by replacing $n_{ij}^* = 0$ in network η^* by $n_{ij}^* = 1$ and $(\eta^* - n_{ij}^*)$ by replacing $n_{ij}^* = 1$ in network η^* by $n_{ij}^* = 0$, in the foreign country. Let $N_i(\eta)$ be the set of firms linked to F_i in η with cardinality $n_i(\eta) = |N_i(\eta)|$; and $N_i^*(\eta^*)$ be the set of firms linked to F_i^* with cardinality $n_i^*(\eta^*) = |N_i^*(\eta^*)\,|$. There is a small but positive cost *f* of linking any pair of firms which is fixed and evenly split between each pair of firms forming a link. Technology exhibits positive network externalities if:

(2a) $$\beta_i(z) = \begin{cases} \beta(z) - \theta n_i(\eta; 1) & \forall\, F_i \in N \\ \beta(z) & \text{otherwise} \end{cases}$$

and/or

(2b) $$\beta_i^*(z) = \begin{cases} \beta^*(z) - \theta^* n_i^*(\eta^*; 1) & \forall F_i^* \in N^* \\ \beta^*(z) & \text{otherwise} \end{cases}$$

where $\beta(z) > 0$ and $\beta^*(z) > 0$ measure the unit labor requirements without any network externality, respectively, at home and in the foreign country; β_i measures the unit labor requirement for F_i; and β_i^* measures the unit labor requirement for F_i^*. We assume that $\beta(z) = \beta^*(1 - z)\ \forall\, z$ and sort the industries, without any loss of generality, so that $\beta(z)$ is increasing in z while $\beta^*(z)$ is decreasing in z.

Demand is characterized by quadratic utility functions

(3a) $$U[\{x(z)\}] = \int_0^1 \left[\rho_1 x(z) - \frac{1}{2}\rho_2 x(z)^2\right] dz$$

(3b) $$U[\{x^*(z)\}] = \int_0^1 \left[\rho_1 x^*(z) - \frac{1}{2}\rho_2 x^*(z)^2\right] dz$$

where $x(z)$ and $x^*(z)$ measure the consumption of good z at the home and in the foreign country, respectively.

A representative consumer in the home country maximizes (3a) subject to the budget constraint

(4a) $$\int_0^1 p(z)x(z)dz \leq I$$

Analogously, a representative consumer in the foreign country maximizes (3b) subject to the budget constraint

(4b) $$\int_0^1 p(z)x^*(z)dz \leq I^*$$

where I and I^* measure the aggregate income at home and in the foreign country, respectively: $(I + I^*)$, which determines demand under the integrated markets assumption, is the sum of wages and profits in both countries and hence is endogenous under general equilibrium. The first and second moments of the distribution of prices are

(5a) $$\mu_1^P \equiv \int_0^1 p(z)dz$$

(5b) $$\mu_2^P \equiv \int_0^1 p(z)^2 dz$$

The inverse demand function for each good, which is linear in its own price conditional on the marginal utility of income (λ), at home is

(6a) $$p(z) = \frac{1}{\lambda}[\rho_1 - \rho_2 x(z)]$$

where $\lambda = \frac{a\mu_1^p - bI}{\mu_2^p}$.

Analogously, in the foreign country, the inverse demand function for the same good is given by

(6b) $$p(z) = \frac{1}{\lambda^*}[\rho_1{}^* - \rho_2 x^*(z)]$$

where$\lambda^* = \frac{a\mu_1^p - bI^*}{\mu_2^p}$. The effects of prices on λ and λ^* are thus summarized by the first and second moments of the distribution of prices.

Therefore, the world inverse demand curve for each good is

(7) $$p(z) = a_0 - b\bar{x}(z)$$

where $a = \frac{\rho_1 + \rho_1{}^*}{\lambda + \lambda^*}$, $b \equiv \frac{\rho_2}{\bar{\lambda}}$, and $\bar{x} = (x + x^*)$ with ρ_1 and $\rho_1{}^*$ being the intercepts and ρ_2 the common slope for home demand $(x(z))$ and foreign demand $(x^*(z))$ respectively. $\bar{\lambda} = (\lambda + \lambda^*)$ is the world marginal utility of income which we choose as the *numeraire*. We will, hereinafter, normalize $b = 1$, $W = \bar{\lambda}w$ and $W^* = \bar{\lambda}w^*$ where w and w^* are the hourly nominal wages at home and in the foreign country, respectively.[4]

Wages are determined by conditions of full employment in each country

(8a) $$L = \int_{\tilde{z}}^{\tilde{z}^*} \left[\beta_i(z) n_1 y_1(W, W^*, z, n_1, n_{-1}, n_1^*, n_{-1}^*) + \beta(z) n_{-1} y_{-1}(W, W^*, z, n_1, n_{-1}, n_1^*, n_{-1}^*)\right] dz + \int_0^{\tilde{z}^*} \left[\beta_i(z) n_1 y_1(W, W^*, z, n_1, n_{-1}, 0, 0) + \beta(z) n_{-1} y_{-1}(W, W^*, z, n_1, n_{-1}, 0, 0)\right] dz$$

(8b) $$L^* = \int_{\tilde{z}^*}^{\tilde{z}} [\beta_i^*(z) n_1^* y_1^*(W, W^*, z, n_1, n_{-1}, n_1^*, n_{-1}^*) + \beta^*(z) n_{-1}^* y_{-1}^*(W, W^*, z, n_1, n_{-1}, n_1^*, n_{-1}^*)] dz + \int_{\tilde{z}^*}^{1} [\beta_i^*(z) n_1^* y_1^*(W, W^*, z, 0, 0, n_1^*, n_{-1}^*) + \beta^*(z) n_{-1}^* y_{-1}^*(W, W^*, z, 0, 0, n_1^*, n_{-1}^*)] dz$$

where L and L^* denote the supply of labor; y_1 and y_1^* measure the output of each networked firm at home and in the foreign country, respectively; y_{-1} and y_{-1}^* measure the output of each firm operating outside any network at home and in the foreign country, respectively; and $\tilde{z}$ and $\tilde{z}^*$ are the threshold sectors for the extensive margins of trade, at home and abroad respectively. In the home country's labor market, full employment ensures that home labor supply matches the sum of labor demands from sectors $z \in [0, \tilde{z}^*]$ in which home firms face no foreign competition (i.e. $n^* = 0$) and from the sectors $z \in [\tilde{z}, \tilde{z}^*]$ in which both home and foreign firms operate. Analogously, in the foreign country's labor market, full employment ensures that foreign labor supply matches the sum of labor demands from sectors $z \in [\tilde{z}, 1]$ in which foreign firms face no foreign competition (i.e. $n = 0$) and from the sectors $z \in [\tilde{z}^*, \tilde{z}]$ in which both home and foreign firms operate.

Definition. For $i \neq j$, a home SSO network is *empty* if $n_{ij} = 0 \; \forall \, F_i, F_j \in F$ and a foreign SSO network is *empty* if $n_{ij}^* = 0 \; \forall \, F_i^*, F_j^* \in F^*$; and a home SSO network is *complete* if $n_{ij} = 1 \; \forall \, F_i, F_j \in F$ and a foreign SSO network is *complete* if $n_{ij}^* = 1 \; \forall \, F_i^*, F_j^* \in F^*$. A home SSO network is *stable* if

(9a) $$\begin{cases} \text{for } n_{ij} = 1, \Pi_i(\eta) > \Pi_i(\eta - n_{ij}) \text{ and } \Pi_j(\eta) > \Pi_j(\eta - n_{ij}) \\ \text{for } n_{ij} = 0, \Pi_i(\eta + n_{ij}) > \Pi_i(\eta) => \Pi_j(\eta + n_{ij}) \leq \Pi_j(\eta) \end{cases}$$

and a foreign SSO network is *stable* if

$$\text{(9b)} \qquad \begin{cases} \text{for } n_{ij}^* = 1, \Pi_i(\eta^*) > \Pi_i(\eta^* - n_{ij}^*) \text{ and } \Pi_j(\eta^*) > \Pi_j(\eta^* - n_{ij}^*) \\ \text{for } n_{ij}^* = 0, \Pi_i(\eta^* + n_{ij}^*) > \Pi_i(\eta^*) => \Pi_j(\eta^* + n_{ij}^*) \leq \Pi_j(\eta^*) \end{cases}$$

Lemma 1. An *empty* home (foreign) SSO network and/or a *complete* home (foreign) SSO network are the only *symmetric* networks that are *stable*.

Proof. Follows directly from Jackson and Wolinsky (1996).

Case 1. $n_{ij} = 0 \ \forall \ F_i, F_j \in F$ and $n_{ij}^* = 0 \ \forall \ F_i^*, F_j^* \in F^*$

In this case of empty SSO networks, each domestic firm would

$$\underset{\{y_i\}}{Maximize}: \qquad \Pi_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w\beta(z)\big)y_i(z) \qquad (i = 1, 2, \dots, n)$$

Each foreign firm would

$$\underset{\{y_i^*\}}{Maximize}: \qquad \Pi_i^* = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w^*\beta^*(z)\big)y_i^*(z) \qquad (i = 1, 2, \dots, n^*)$$

For $z \in [\tilde{z}, z^*]$ i.e. the subset of sectors where all firms are active in equilibrium

$$\text{(10a)} \quad y_i(n, n^*)|_{n_{ij}=0\,;\,n_{ij}^*=0} = \left[\frac{a-(n^*+1)w\beta(z)+n^*w^*\beta^*(z)}{n+n^*+1}\right] \qquad \forall \ i = 1, 2, \dots, n$$

$$\text{(10b)} \quad y_i^*(n, n^*)|_{n_{ij}=0\,;\,n_{ij}^*=0} = \left[\frac{a-(n+1)w^*\beta^*(z)+nw\beta(z)}{n+n^*+1}\right] \qquad \forall \ i = 1, 2, \dots, n^*$$

Therefore, a necessary and sufficient condition for any domestic firm to produce is

$$\text{(11a)} \qquad c \leq \xi a + (1 - \xi)c^*$$

This follows directly from (10a) and $y_i(n,n^*)|_{n_{ij}=0\,;\,n_{ij}^*=0} \geq 0$, where $c = w\beta(z)$; $c^* = w^*\beta^*(z)$; and $\xi = \left(\frac{1}{n^*+1}\right) \in (0,1)$ i.e. it will be profitable for a domestic firm to produce if and only if its unit cost does not exceed a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms. Analogously, a necessary and sufficient condition for any foreign firm to produce is

(11b) $$c^* \leq \xi^* a + (1-\xi^*)c$$

This follows directly from (10b) and $y_i^*(n,n^*)|_{n_{ij}=0\,;\,n_{ij}^*=0} \geq 0$, where $\xi^* = \left(\frac{1}{n+1}\right) \in (0,1)$ i.e. it will be profitable for a foreign firm to produce if and only if its unit cost does not exceed a weighted average of the demand intercept and the unit cost of domestic firms, where the weight attached to the former is decreasing in the number of domestic firms. The threshold sectors pinning down the extensive margins of trade are determined by

(12a) $$W\beta(\tilde{z}) - \xi a - (1-\xi)W^*\beta^*(\tilde{z}^*) = 0$$

(12b) $$W^*\beta^*(\tilde{z}^*) - \xi^* a - (1-\xi^*)W\beta(\tilde{z}) = 0$$

Figure 2 below helps visualize the pattern of production and trade with empty networks at home and in the foreign country. No firm (home or foreign) operate in region *O*, where the unit cost of production exceeds a. Only home firms can compete in region *H* while only foreign firms can compete in region *F*. Both home and foreign firms can co-exist in region *HF* which can be construed as a cone of diversification (in terms of the goods' origin). The *ZZ* schedule shows how costs of production vary across sectors when the networks at home and abroad are empty. The downward slope of *ZZ* is due to the assumption that $\beta(z)$ is increasing in z while $\beta^*(z)$ is decreasing in z. It follows directly that, given wages, $c = w\beta(z)$ falls (rises) and $c^* = w^*\beta^*(z)$ rises (falls) as z decreases (increases). While this explains a movement along *ZZ*, any change in wages would cause a shift in *ZZ*. Since marginal costs vary systematically across countries creating comparative advantage in each sector, as in Neary (2003, 2007), a country could be a net importer of a good in which it has a comparative advantage when the number of firms in that industry is small relative to that in the other country.

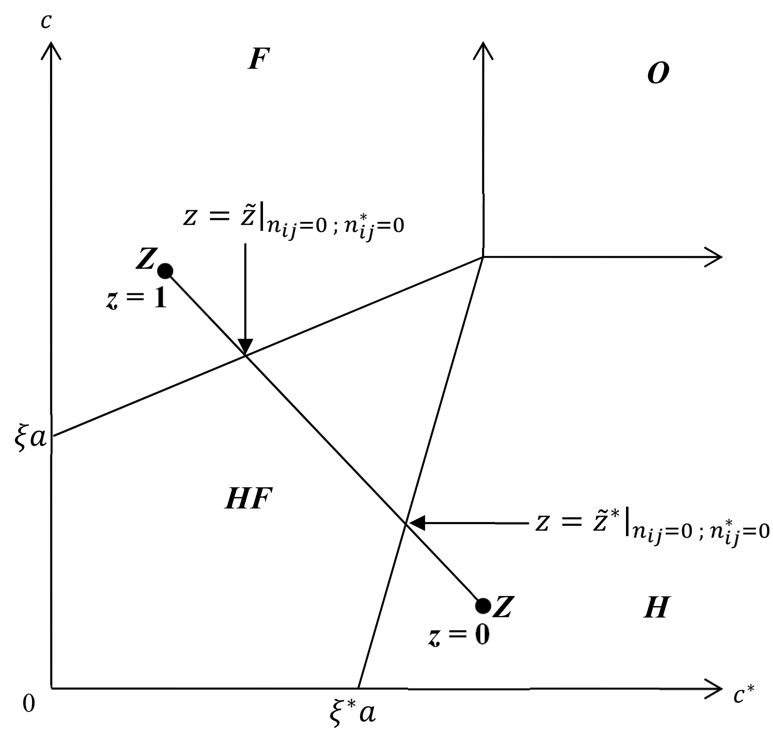

Figure 2. Production and trade with empty SSO networks

To keep our *general equilibrium* analysis tractable, suppose that the countries are identical in size and exhibit symmetric intersectoral differences. In effect, the endowments, preferences, as well as the number of firms are identical across countries. The marginal utilities of income and wages would then be identical across countries and threshold sectors would be symmetric: $\tilde{z} = 1 - \tilde{z}^*$. A competitive outcome would be obtained when $n \to \infty$ and $n^* \to \infty$: *HF* would then collapse to a 45° line as each country specializes completely in line with her comparative advantage replicating the extensive margins of trade that would prevail in a Ricardian world.

Case 2. $n_{ij} = 1 \; \forall \; F_i, F_j \in F$ and $n_{ij}^* = 0 \; \forall \; F_i^*, F_j^* \in F^*$

Each domestic firm would

$$\underset{\{y_i\}}{Maximize}: \Pi_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w(\beta(z) - (n-1)\theta)\big)y_i(z) \qquad (i = 1, 2, \dots, n)$$

Each foreign firm would

$$\underset{\{y_i^*\}}{Maximize}: \Pi_i^* = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w^*\beta^*(z)\big)y_i^*(z) \qquad (i = 1, 2, \dots, n^*)$$

For $z \in [\tilde{z}, z^*]$ i.e. the subset of sectors where all firms are active in equilibrium

(13a) $$y_i(n, n^*)\big|_{n_{ij}=1\,;\,n_{ij}^*=0} = \left[\frac{a-(n^*+1)w(\beta(z)-(n-1)\theta)+n^*w^*\beta^*(z)}{n+n^*+1}\right] \qquad \forall \; i = 1, 2, \dots, n$$

(13b) $$y_i^*(n, n^*)\big|_{n_{ij}=1\,;\,n_{ij}^*=0} = \left[\frac{a-(n+1)w^*\beta^*(z)+nw(\beta(z)-(n-1)\theta)}{n+n^*+1}\right] \qquad \forall \; i = 1, 2, \dots, n^*$$

Therefore, a necessary and sufficient condition for any domestic firm to produce is

(14a) $$c \leq \xi A_1 + (1-\xi)c^*$$

where $A_1 = (a + (n^*+1)(n-1)w\theta)$. This follows directly from (13a) and $y_i(n, n^*)\big|_{n_{ij}=1\,;\,n_{ij}^*=0} \geq 0$.

Analogously, a necessary and sufficient condition for any foreign firm to produce is

(14b) $$c^* \leq \xi^* A_2 + (1-\xi^*)c$$

Where $A_2 = (a - n(n-1)w\theta)$. This follows directly from (13b) and $y_i^*(n,n^*)|_{n_{ij}=1\,;\,n_{ij}^*=0} \geq 0$.

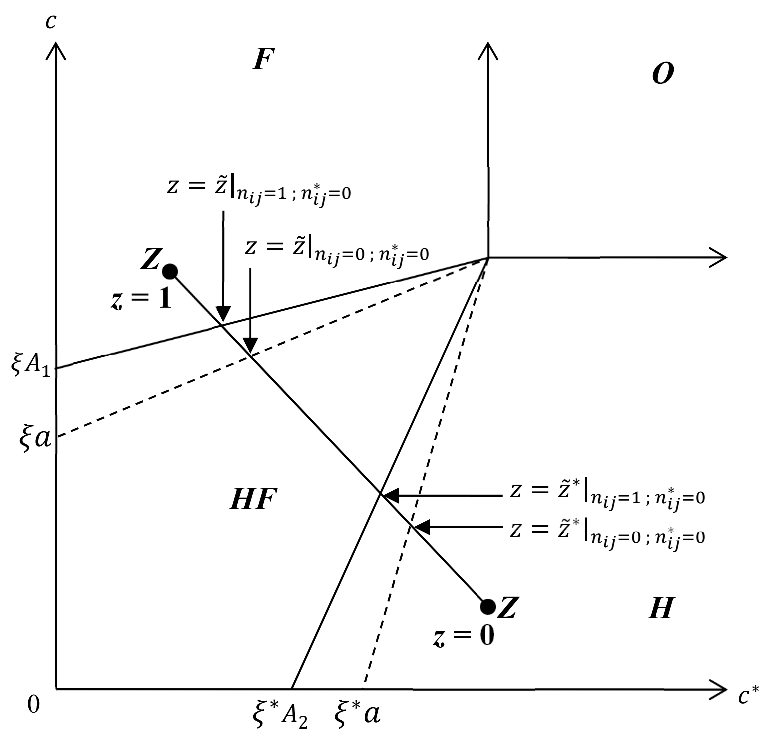

Figure 3. Production and trade in partial equilibrium: Complete SSO network at home

Figure 3 above presents a *partial equilibrium* comparison of patterns of production and trade with and without a complete SSO network at home. When home firms are networked, region *H* (where exclusively home firms can compete) expands and region *F* (where exclusively foreign firms can compete) contracts. This raises the extensive margin of domestic exports and reduces the extensive margin of domestic imports.

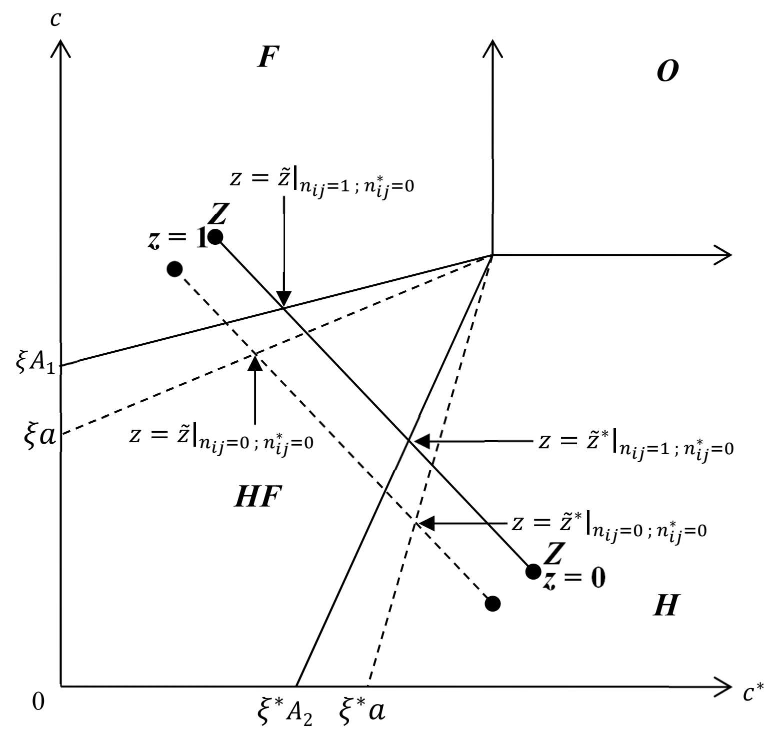

Figure 4. Production and trade in general equilibrium: Complete SSO network at home

Figure 4 above captures the effect of a home network-induced rise in wages causing the *ZZ* locus to shift away from the origin which magnifies the rise in the extensive margin of domestic exports and partially offsets the reduction in the extensive margin of domestic imports.

Our first proposition follows.

Proposition **I**. A *complete* SSO network at home increases the extensive margin of domestic exports and decreases the extensive margin of domestic imports.

Proof. Follows directly from (12a), (12b), (14a) and (14b).

Intuitively, the productivity of domestic labor is higher with a complete SSO network at home than it is without the network. This increases the number of domestic industries that have a comparative advantage over the foreign industries. At the initial wages, the demand for labor rises as the value marginal product of labor rises due to a rise in productivity. This generates an excess demand for labor and wages rise to clear the labor market. This network-induced rise is wages partially offsets the initial increase in the number of domestic industries that have a comparative advantage over the foreign industries.

Case 3. $n_{ij} = 0 \; \forall \; F_i, F_j \in F$ and $n^*_{ij} = 1 \; \forall \; F^*_i, F^*_j \in F^*$

Each domestic firm would

$$\underset{\{y_i\}}{Maximize}: \Pi_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w\beta(z)\big) y_i(z) \qquad (i = 1, 2, \dots, n)$$

Each foreign firm would

$$\underset{\{y^*_i\}}{Maximize}: \Pi^*_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w^*(\beta^*(z) - (n^* - 1)\theta^*)\big) y^*_i(z) \qquad (i = 1, 2, \dots, n^*)$$

For $z \in [\tilde{z}, z^*]$ i.e. the subset of sectors where all firms are active in equilibrium

(15a) $$y_i(n, n^*)|_{n_{ij}=0\,;\,n^*_{ij}=1} = \left[\frac{a - (n^*+1)w\beta(z) + n^* w^*(\beta^*(z) - (n^*-1)\theta^*)}{n + n^* + 1}\right] \qquad \forall \; i = 1, 2, \dots, n$$

(15b) $$y^*_i(n, n^*)|_{n_{ij}=0;\,n^*_{ij}=1} = \left[\frac{a - (n+1)w^*(\beta^*(z) - (n^*-1)\theta^*) + nw\beta(z)}{n + n^* + 1}\right] \qquad \forall \; i = 1, 2, \dots, n^*$$

Therefore, a necessary and sufficient condition for any domestic firm to produce is

(16a) $$c \le \xi A_3 + (1 - \xi)c^*$$

where $A_3 = (a - n^*(n^* - 1)w^*\theta^*)$. This follows directly from (15a) and $y_i(n, n^*)|_{n_{ij}=0\,;\,n^*_{ij}=1} \ge 0$.

Analogously, a necessary and sufficient condition for any foreign firm to produce is

(16b) $$c^* \leq \xi^* A_4 + (1 - \xi^*)c$$

where $A_4 = (a + (n+1)(n^* - 1)w^*\theta^*)$. This follows directly from (15b) and $y_i^*(n, n^*)|_{n_{ij}=0;\, n_{ij}^*=1} \geq 0$.

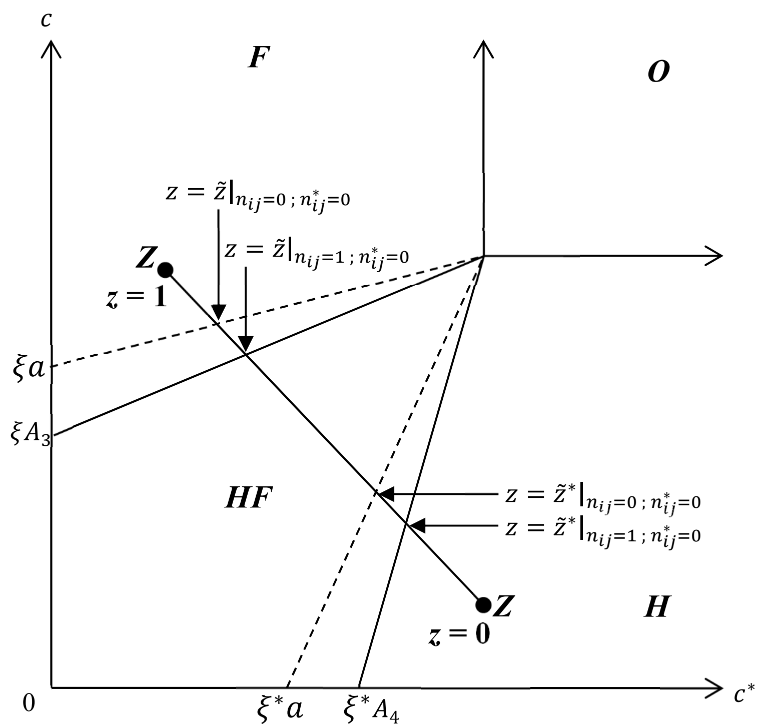

Figure 5. Production and trade in partial equilibrium: Complete network in the foreign country

Figure 5 above presents a *partial equilibrium* comparison of patterns of production and trade with and without a complete foreign network. When foreign firms are networked, region *H* (where exclusively home firms can compete) contracts and region *F* (where exclusively foreign firms can compete) expands. Figure 5 below captures the *general equilibrium* effect of a foreign network-induced rise in wages causing the *ZZ* locus to shift away from the origin which magnifies the initial reduction in the extensive margin of domestic exports and partially offsets the rise in the extensive margin of domestic imports.

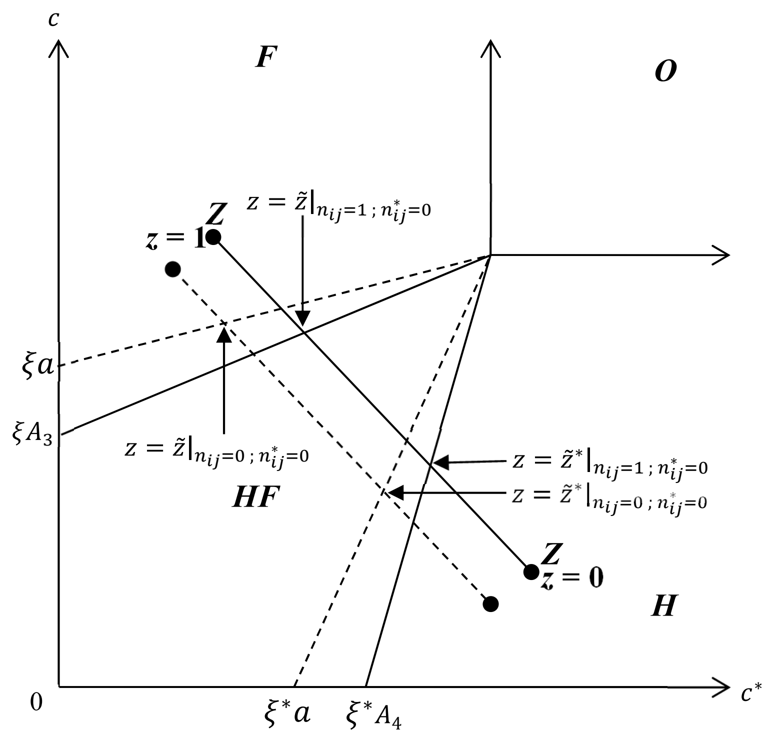

Figure 6. Production and trade in general equilibrium: Complete SSO network in the foreign country

Our next proposition follows.

***Proposition* II**. A *complete* foreign network reduces the extensive margin of domestic exports and raises the extensive margin of domestic imports.

Proof. Follows directly from (12a), (12b), (16a) and (16b).

Intuitively, the productivity of foreign labor is higher with a complete network in the foreign country than it is without the network. This increases the number of foreign industries that have a comparative advantage over the domestic industries. At the initial wages, the demand for labor rises as the value marginal product of labor rises due to a rise in productivity. This generates an excess demand for labor and wages rise to clear the labor market. This network-induced rise is wages partially offsets the initial increase in the number of foreign industries that have a comparative advantage over the domestic industries.

Case 4. $n_{ij} = 1 \; \forall \; F_i, F_j \in F$ and $n^*_{ij} = 1 \; \forall \; F^*_i, F^*_j \in F^*$

Each domestic firm would

$$\underset{\{y_i\}}{Maximize}: \Pi_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w(\beta(z) - (n-1)\theta)\big) y_i(z) \qquad (i = 1, 2, \dots, n)$$

Each foreign firm would

$$\underset{\{y^*_i\}}{Maximize}: \Pi^*_i = \big(a - \tilde{y}(z) - \tilde{y}^*(z) - w^*(\beta^*(z) - (\, n^* - 1)\theta^*)\big) y^*_i(z) \qquad (i = 1, 2, \dots, n^*)$$

For $z \in [\tilde{z}, z^*]$ i.e. the subset of sectors where all firms are active in equilibrium

(17a) $y_i(n, n^*)|_{n_{ij}=1\,;\,n^*_{ij}=1} = \left[\frac{a-(n^*+1)w(\beta(z)-(n-1)\theta)+n^*w^*(\beta^*(z)-(\,n^*-1)\theta^*)}{n+n^*+1}\right] \quad \forall \; i = 1, 2, \dots, n$

(17b) $y^*_i(n, n^*)|_{n_{ij}=1;\,n^*_{ij}=1} = \left[\frac{a-(n+1)w^*(\beta^*(z)-(\,n^*-1)\theta^*)+nw(\beta(z)-(n-1)\theta)}{n+n^*+1}\right] \quad \forall \; i = 1, 2, \dots, n^*$

Therefore, a necessary and sufficient condition for any domestic firm to produce is

(18a) $$c \leq \xi A_5 + (1 - \xi)c^*$$

where $A_5 = \Big(a + \big((n^* + 1)(n-1)w\theta - n^*(\,n^* - 1)w^*\theta^*\big)\Big)$. This follows directly from (17a) and $y_i(n, n^*)|_{n_{ij}=1\,;\,n^*_{ij}=1} \geq 0$.

Analogously, a necessary and sufficient condition for any foreign firm to produce is

(18b) $$c^* \leq \xi^* A_6 + (1 - \xi^*)c$$

where $A_6 = \left(a + \left((n+1)(n^*-1)w^*\theta^* - n(n-1)w\theta\right)\right)$. This follows directly from (18b) and $y_i^*(n, n^*)|_{n_{ij}=1;\, n_{ij}^*=1} \geq 0$.

It may be noted that, all else equal, the net effect of the home and foreign networks on the extensive margins of trade depends on the relative economy of size of each network, measured by θ at home and θ^* in the foreign country. To fix ideas, let us simplify further exposition by setting $n = n^*$ and $w = w^*$.

Case 4A. $\frac{\theta}{\theta^*} > \left(\frac{n+1}{n}\right)$

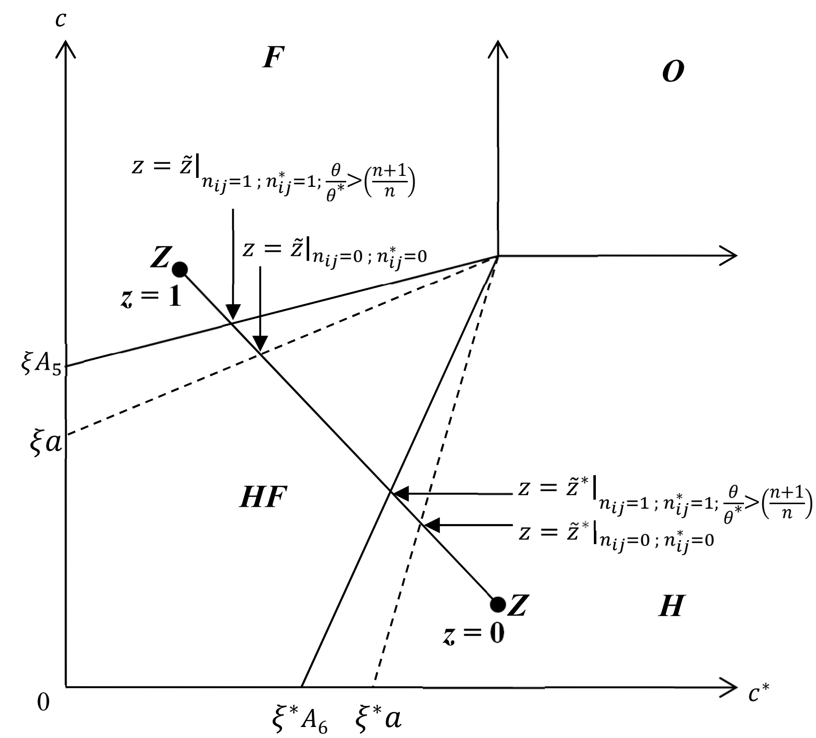

Figure 7. Production and trade in partial equilibrium: Complete SSO network in each country

Figure 7 above presents a *partial equilibrium* comparison of patterns of production and trade with and without a complete SSO network in each country when $\frac{\theta}{\theta^*} > \left(\frac{n+1}{n}\right)$. Region *H* (where exclusively home firms can compete) expands and *F* (where exclusively foreign firms can compete) shrinks. This raises the extensive margin of domestic exports and reduces the extensive margin of domestic imports.

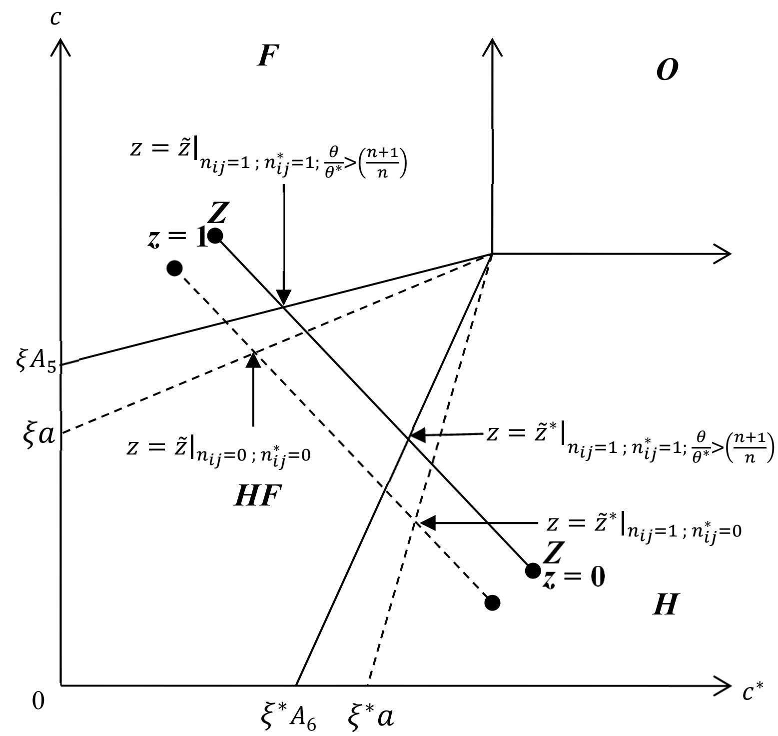

Figure 8. Production and trade in general equilibrium: Complete SSO network in each country

Figure 8 above captures the *general equilibrium* effect of a network-induced rise in wages causing the *ZZ* locus to shift away from the origin which magnifies the initial expansion in the extensive margin of domestic exports and partially offsets the contraction of the extensive margin of foreign exports.

Lemma 2. A *complete* network in both countries raises the extensive margin of domestic exports and reduces the extensive margin of domestic imports if $\frac{\theta}{\theta^*} > \left(\frac{n+1}{n}\right)$.

Proof. Follows directly from (12a), (12b), (18a) and (18b).

Case 4B. $\frac{\theta}{\theta^*} < \left(\frac{n}{n+1}\right)$

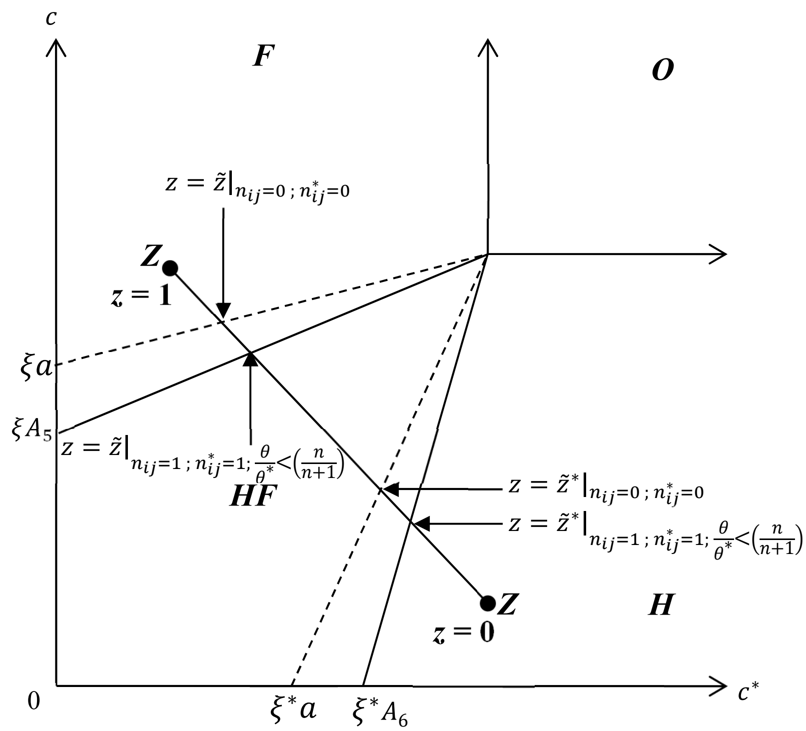

Figure 9. Production and trade in partial equilibrium: Complete SSO network in each country

Figure 9 above presents a *partial equilibrium* comparison of patterns of production and trade with and without a complete SSO network in each country when $\frac{\theta}{\theta^*} < \left(\frac{n}{n+1}\right)$. Region *H* (where exclusively home firms can compete) contracts and region *F* (where exclusively foreign firms can compete) expands. This lowers the extensive margin of domestic exports and raises the extensive margin of domestic imports.

Figure 10 below captures the *general equilibrium* effect of a network-induced rise in wages causing the *ZZ* locus to shift away from the origin which magnifies the initial reduction in the extensive margin of domestic exports and partially offsets the rise in the extensive margin of domestic imports.

Lemma 3. A *complete* SSO network in both countries lowers the extensive margin of domestic exports and raises the extensive margin of domestic imports if $\frac{\theta}{\theta^*} < \left(\frac{n}{n+1}\right)$.

Proof. Follows directly from (12a), (12b), (18a) and (18b).

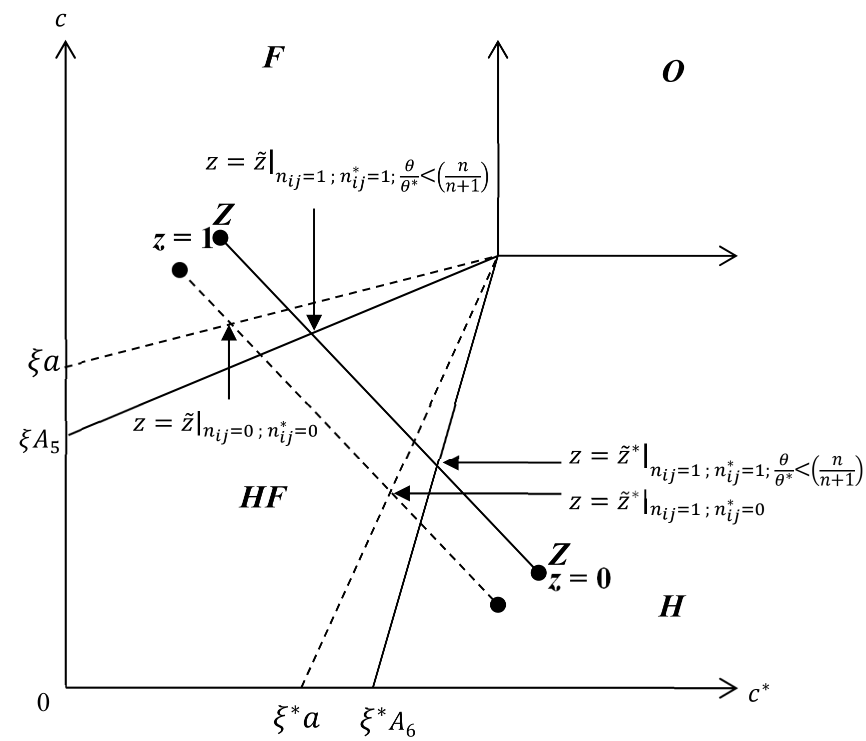

Figure 10. Production and trade in general equilibrium: Complete SSO network in each country

Case 4C. $\left(\frac{n}{n+1}\right) < \frac{\theta}{\theta^*} < \left(\frac{n+1}{n}\right)$

Figure 11 below presents a *partial equilibrium* comparison of patterns of production and trade with and without a complete SSO network in each country when $\left(\frac{n}{n+1}\right) < \frac{\theta}{\theta^*} < \left(\frac{n+1}{n}\right)$.

Region *HF* (where both home and foreign firms can co-exist) expands while regions *H* (where exclusively home firms can compete) and *F* (where exclusively foreign firms can compete) shrink.

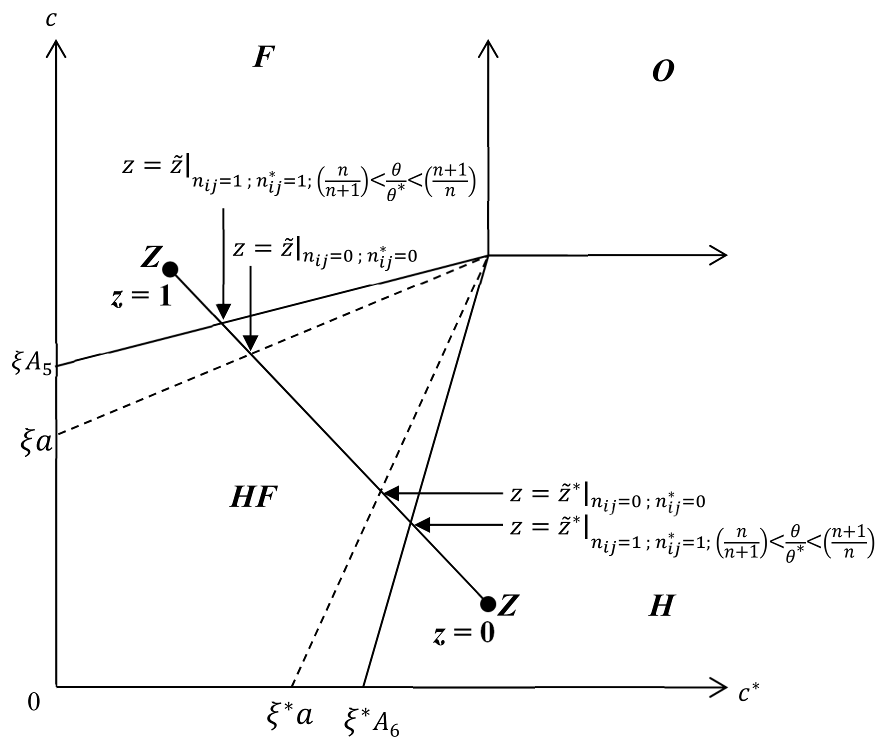

Figure 11. Production and trade in partial equilibrium: Complete SSO network in each country

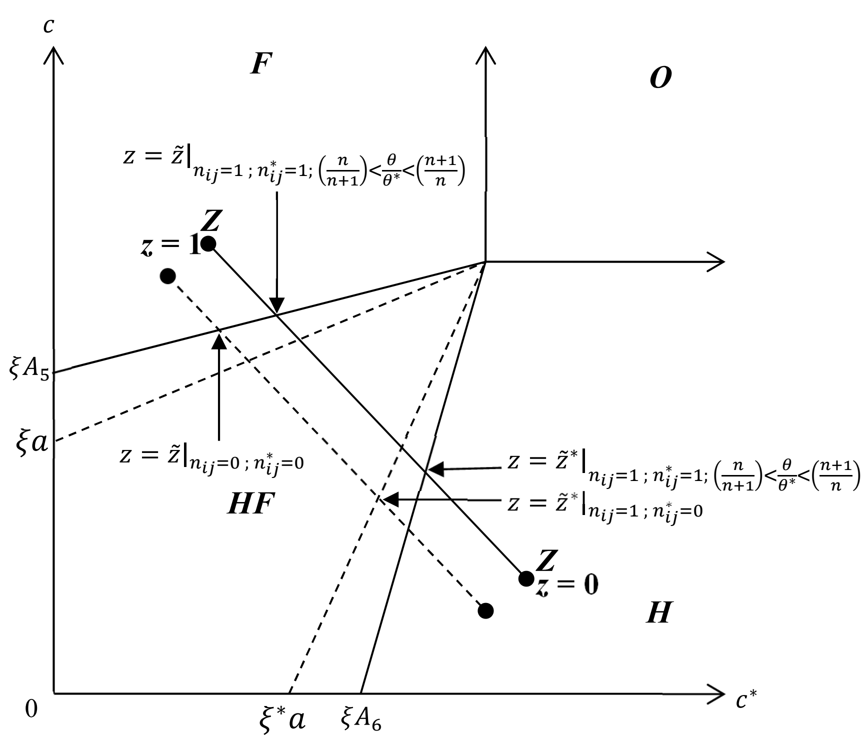

Figure 12. Production and trade in general equilibrium: Complete SSO network in each country

Figure 12 above captures the *general equilibrium* effect of a network-induced rise in wages causing the *ZZ* locus to shift away from the origin which partially offsets the initial effect of networks on the extensive margins of trade.

Lemma 4. A *complete* SSO network in both countries reduces the extensive margins of exports and imports if $\left(\frac{n}{n+1}\right) < \frac{\theta}{\theta^*} < \left(\frac{n+1}{n}\right)$.

Proof. Follows directly from (12a), (12b), (18a) and (18b).

Our final proposition follows.

***Proposition* III**. A *complete* SSO network at home and in the foreign country

a) raises the extensive margin of domestic exports and reduces the extensive margin of domestic imports if $\frac{\theta}{\theta^*} > \left(\frac{n+1}{n}\right)$.

b) reduces the extensive margin of domestic exports and raises the extensive margin of domestic imports if $\frac{\theta}{\theta^*} < \left(\frac{n}{n+1}\right)$.

c) reduces the extensive margins of exports and imports if $\left(\frac{n}{n+1}\right) < \frac{\theta}{\theta^*} < \left(\frac{n+1}{n}\right)$.

Proof. Follows directly from Lemma 2 – 4.

Intuitively, the productivity of domestic labor is higher with a complete SSO network at home than it is without the network and the productivity of foreign labor is higher with a complete SSO network in the foreign country than it is without the network. Consequently, any change in the number of domestic industries that have a comparative advantage over the foreign industries would depend on the relative economy of size of each network. All else equal, if the economy of size of the SSO network at home exceeds that of the foreign SSO network by a margin, the number of domestic industries that have a comparative advantage over the foreign industries would rise. At the initial wages, the demand for labor rises as the value marginal product of labor rises due to a rise in productivity. This generates an excess demand for labor and wages rise to clear the labor market. This network-induced rise is domestic wages partially offsets the initial increase in the number of domestic industries that have a comparative advantage over the foreign industries. If the economy of size of the foreign network far exceeds than of the domestic network by a margin, the number of foreign industries that have a comparative advantage over the domestic industries would rise. At the initial wages, the demand for foreign labor rises as the value marginal product of foreign labor rises due to a rise in productivity. This generates an excess demand for foreign labor and wages rise to clear the labor market. This network-induced rise in foreign wages partially offsets the initial increase in the number of foreign industries that have a comparative advantage over the domestic industries.

4. Conclusion

We have developed a tractable general equilibrium model of oligopolistic competition that allows us to analyze the role of SSOs as networks facilitating specialization according to comparative advantage in the tradition of a typical neoclassical model of international trade. Embedding such networks, domestic and/or foreign, has non-trivial implications for a general oligopolistic equilibrium. More specifically, our model of Networked Oligopolies in General Equilibrium (NOGE) highlights the implications of interactions between the SSO networks, efficiency, and concentration for production and trade. SSO networks affect diversification in production as well as the extensive margins of trade and, consequently, a complete SSO network at home increases the extensive margin of domestic exports and decreases the extensive margin of domestic imports while a complete foreign SSO network reduces the extensive margin of domestic exports and raises the extensive margin of domestic imports. All else equal, the net effect of home and foreign SSO networks on the extensive margins of trade depends on the relative economy of size of each network. A couple of particularly challenging generalization of our model, we are working on, would involve allowing fractional networking and/or uncompensated technology transfers in NOGE. While one-to-one models of networking allow each firm to be linked to one other firm and many-to-many models of networking allow each firm to be linked to multiple firms, fractional SSO networking would allow each firm to be linked to several members of SSOs but only to one firm within an SSO at any given time or to a distribution of many firms within a network of SSOs. This is analogous to a problem of fractional matching: For instance, Alva and Manjunath (2019) have analyzed fractionally matching agents to each other alongside the allotment of a single divisible good. Second, in a setting of Dornbusch, Fischer and Samuelson (DFS)[5] that resembles Neary (2003, 2007) but for the strategic elements of oligopolistic competition, Jones and Ruffin (2008) demonstrate how an adjustment in relative wages may not take place when the country with an advanced technology loses an export sector because of technology transfer without compensation. Not only can that country gain, such a possibility for paradoxical gains must occur in the neighborhood of what Jones and Ruffin (2008) call “turning points” – values of relative country size for which that country in the initial situation would be an incipient producer of an imported commodity, so that if its relative size were to increase, it would become a new producer of that commodity. An identification of such “turning points” for oligopolies with SSO networks is likely to have non-trivial implications.

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Endnotes

[1] See Figure 1.
[2] See Forman and Goldfarb (2006) and Swann (2010).
[3] See Goyal and Joshi (2003) and Jackson (2010).
[4] Following Neary (2003, 2007), we choose the world marginal utility of income as the numeraire: Since the values of real variables are homogeneous of degree zero in three nominal variables (the home and foreign wage rates as well as the inverse of the world marginal utility of income), an arbitrary numeraire or normalization of nominal variables is justified.
[5] See Neary (2016) and Beladi and Chakrabarti (2019) for recent adaptations of the DFS (1977) framework to explore possible links between comparative advantage and intra-firm competition.