

# A Theory of Patent Holdout\*

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February 3, 2020

Preliminary and Incomplete

## Abstract

This paper provides a framework to analyze patent holdout. We show that when patents are probabilistic, a potential user typically has incentives to shun to pay the price offered by a patent holder to license the technology and risk being brought to court. Litigation benefits the user because of its asymmetric effect, specially in the context of Standard Development Organizations (SDOs). While the user may not pay if the court decides that there has been no infringement, the price of the license will not adjust, accordingly, if the court considers that such an infringement exists. We show that this effect is exacerbated under sequential litigation across different jurisdictions or patents, where the outcome of one trial affects the probability in which a court sides with each of the parties in future ones. The incentives to engage in patent holdout as well as its distortions increase when final competition is accounted for.

**JEL codes:** L15, L24, O31, O34.

**keywords:** Intellectual Property, Standard Setting Organizations, Patent Licensing, R&D Investment, Patent Holdout.

## 1 Introduction

Over the last few years competition authorities in the US and elsewhere have repeatedly warned about the risk of patent hold-up in the licensing of Standard Essential Patents (SEPs). Concerns about such risks were front and center in the recent FTC case against Qualcomm, where the Court ultimately concluded that Qualcomm had used a series

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\*The ideas and opinions in this paper, as well as any errors, are exclusively the authors'. Financial support from Qualcomm is gratefully acknowledged. The first author also acknowledges the support of the Regional Government of Madrid through grant H2019/HUM-5793. Comments should be sent to [llobet@cemfi.es](mailto:llobet@cemfi.es) and [jpadilla@compasslexecon.com](mailto:jpadilla@compasslexecon.com).

of anticompetitive practices to extract unreasonable royalties from implementers. This paper evaluates the evidence for such a risk, as well as the countervailing risk of patent hold-out.

In general, hold up may arise when firms negotiate trading terms after they have made costly, relation-specific investments. Since the costs of these investments are sunk when trading terms are negotiated, they are not factored into the agreed terms. As a result, depending on the relative bargaining power of the firms, the investments made by the weaker party may be undercompensated (Williamson, 1979).

In the context of SEPs, patent hold-up would arise if SEP owners were able to take advantage of the essentiality of their patents to charge excessive royalties to manufacturers of products reading on those patents that made irreversible investments in the standard (see Lemley and Shapiro (2007)). Similarly, in the recent *FTC v. Qualcomm* ruling, trial judge Lucy Koh concluded that firms may also use commercial strategies (in this case, Qualcomm’s “no license, no chips” policy, refusing to deal with certain parties and demanding exclusivity from others) to extract royalties that depart from the Fair, Reasonable and Non-Discriminatory (FRAND) benchmark.

After years of heated debate, however, there is no consensus about whether patent hold-up actually exists. Some argue that there is no evidence of hold-up in practice. If patent hold-up were a significant problem, manufacturers would anticipate that their investments would be expropriated and would thus decide not to invest in the first place. But end-product manufacturers have invested considerable amounts in standardized technologies (Galetovic et al., 2015). Others claim that while investment is indeed observed, actual investment levels are “necessarily” below those that would have been observed in the absence of hold-up. They allege that, since that counterfactual scenario is not observable, it is not surprising that more than fifteen years after the patent hold-up hypothesis was first proposed, empirical evidence of its existence is lacking.

Meanwhile, innovators are concerned about a risk in the opposite direction, the risk

of patent hold-out. As Epstein and Noroozi (2018) explain,

By “patent holdout” we mean the converse problem, i.e., that an implementer refuses to negotiate in good faith with an innovator for a license to valid patent(s) that the implementer infringes, and instead forces the innovator to either undertake significant litigation costs and time delays to extract a licensing payment through court order, or else to simply drop the matter because the licensing game is no longer worth the candle.

Patent hold-out, also known as “efficient infringement,” is especially relevant in the standardization context for two reasons. First, SEP owners are oftentimes required to license their patents under FRAND conditions. Particularly when, as occurs in some jurisdictions, innovators are not allowed to request an injunction, they have little or no leverage in trying to require licensees to accept a licensing deal. Secondly, SEP owners typically possess many complementary patents and, therefore, seek to license their portfolio of SEPs at once, since that minimizes transaction costs. Yet, some manufacturers de facto refuse to negotiate in this way and choose to challenge the validity of the SEP portfolio patent-by-patent and/or jurisdiction-by-jurisdiction. This strategy involves large litigation costs and is therefore inefficient. SEP holders claim that this practice is anticompetitive and it also leads to royalties that are too low.

While the concerns of SEP holders seem to have attracted the attention of the leadership of the US DOJ<sup>1</sup>, some authors have dismissed them as theoretically groundless, empirically immaterial and irrelevant from an antitrust perspective.<sup>2</sup>

In this paper we provide a framework to analyze and understand when patent holdup may exist and its implications. In our model, the patent holder sets a royalty rate to license a probabilistic patent to a downstream producer who might then decide whether to pay or not. If a payment is not made the patent holder can take the firm to court.

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<sup>1</sup>see, for example <https://www.justice.gov/opa/speech/assistant-attorney-general-makan-delrahim-deliver>

<sup>2</sup>See <https://www.competitionpolicyinternational.com/wp-content/uploads/2018/05/DOJ-patent-holdup-letter.pdf>.

This procedure involves legal costs and it involves an assessment of the validity of the patent. If litigated, the downstream producer can decide whether to go to court or settle.

In this context we analyze the effects of considering two jurisdictions where the same patent is litigated and whether the patent holder is brought to court simultaneously or sequentially. In this latter case, the outcome of the first trial might influence the second one. The reason is that if, for example, the patent is found valid (and that it has been infringed) in one jurisdiction, this might also indicate that the probability that a second judge reaches the similar conclusion increases.

We show that sequential litigation typically benefits the potential licensees, unless the value of the technology is very high compared to the legal costs. As a result, and compared to simultaneous litigation, sequential trials may result in under-compensation of the innovation and the dissipation of social surplus when litigation costs are high. One of the mechanisms behind the result is the fact that the patent holder may be limited in the increase in the royalty rate in the second jurisdiction after a first success in court. This limit might be particularly important in the context of Standard Development Organizations (SDOs) where patent holders are bound in their licensing offers to FRAND terms.

Over this basic structure the paper introduces two extensions. The first analyzes the effect of final market competition. We assume that the downstream producer coexists with a competitive fringe that sells a very similar good for which it has an advantage. We show that this competition introduces an additional incentives for the downstream producer to hold out the patent holder and avoid paying the royalty rate. By doing so, the firm obtains an advantage over the competitive fringe and, as a result, it can expand its sales.

The second extension relates to the usage of injunctions. We consider the situation in which the patent holder can (partially) prevent the downstream producer to sell unless an agreement is reached. As expected, injunctions improve the patent holder's profits

as it limits the extent of hold out. However, this occurs both under simultaneous and sequential negotiations. The model shows that it is in this latter case that injunctions have a greater impact.

The model relies on two basic and realistic assumptions. First, in sequential lawsuits, the result of a trial affects the probability that each party wins the following one. That is, if the manufacturer wins the first trial, it has a higher probability of winning the second, as a first victory may uncover information about the validity of other patents that relate to the same type of innovation, which will be less likely to be upheld in court. Second, the impact of a validity challenge on royalty payments is asymmetric: they are reduced to zero if the patent is found to be invalid but are not increased if it is found valid (and infringed).

## **1.1 Literature Review**

This paper is related to several strands of the literature. It is part of the debate on the distortions in licensing that arise in Standard Setting Organizations. Papers like Lemley and Shapiro (2007) have emphasized the potential effects of patent holdup as well as the risk of royalty stacking. The ensuing discussion has given rise to an active literature on how patent licensing should take place (see, for example, Lerner and Tirole (2015) or Leonard and Lopez (2014)).

Patent litigation has also been incorporated in many papers. The uncertainty about the outcome in court implies that patents are, in practice, probabilistic (see Llobet (2003) or Farrell and Shapiro (2008)). To this literature we add the efficient infringement decision by downstream producers and how this feeds back into the equilibrium royalty rates. These rates are also affected by the legal environment.

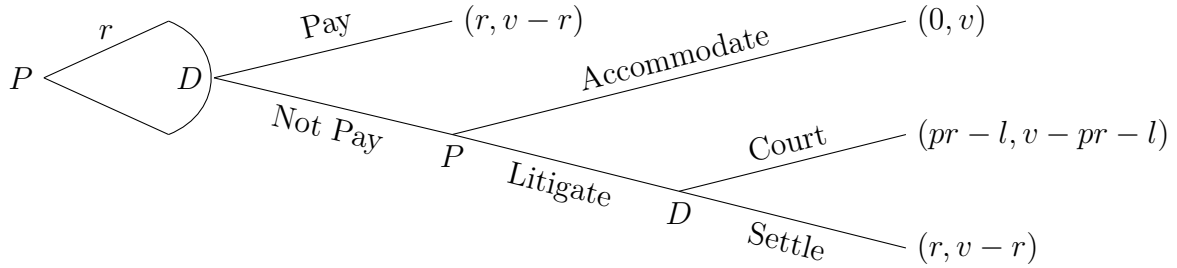
## 2 The Model

Consider a market in which a firm owns a patent (or a portfolio of patents) protecting an innovation, required to sell a good in the final market. This patent holder, that we denote as firm  $P$ , licenses its intellectual property to a downstream monopolist producer, firm  $D$ .

We assume that firm  $P$  has obtained the patent in two different jurisdictions. In each of them, a continuum of buyers of mass 1 are willing to pay an amount  $v$  for a unit of the final good produced by  $D$ . We assume that the marginal cost of production of the good is 0 and that the only cost incurred by the downstream producer is the per-unit royalty required to license the intellectual property and that we denote as  $r$ .

In each jurisdiction, the downstream producer can either accept the royalty  $r$  or refuse to pay it. In the latter case,  $P$  might decide to take  $D$  to court. Litigation implies a legal cost  $l > 0$  for each of the parties. We assume that the ex-ante probability that the patent holder wins in court is known and equal to  $p \in (0, 1)$ . Importantly, depending on the institutional arrangement the patent holder might take the infringer to court simultaneously or sequentially in the different jurisdictions.

Sequential trials imply that the unconditional probability of success of the patent holder,  $p$ , may change once information about the first trial emerges. In particular, we assume that the probability that the patent holder wins the trial in the second jurisdiction contingent on losing in the first one is  $q < p$ , while this probability increases to  $q + \delta$ , with  $\delta > 0$  in case of a first success. That is, if the patent is considered valid in one jurisdiction the probability that the court in the other jurisdiction also finds it valid will increase. The opposite is true if the patent is invalidated in one jurisdiction. The extent of the informational complementarities across trials and jurisdictions is, therefore, measured by  $\delta$ . We assume that  $p(q + \delta) + (1 - p)q = p$  so that sequential litigation does not have any impact on the unconditional probability of success in court.



**Figure 1:** Structure of the One Jurisdiction Game.

We start by analyzing the litigation decisions in one jurisdiction, which constitutes the stage game of the model. We describe the decisions that the patent holder and the downstream producer take and we characterize the equilibrium when only one jurisdiction exists. We then move to the two-jurisdiction case and endogenize the decision of whether patents will be litigated simultaneously or sequentially. Finally, we characterize the optimal royalty for the patent holder.

## 2.1 Litigation in One Jurisdiction

The downstream producer charges a final price to consumers equal to their valuation  $v$  as long as the per-unit royalty that will end up being paid is lower than  $v$ . For this reason, we restrict the royalty rate to be lower or equal than  $v$  as, otherwise, the downstream producer would refuse to produce. This royalty arises endogenously from the stand-alone patent litigation game that we introduce next.

The structure of the game is illustrated in Figure 1. We assume the following timing. First, the patent holder sets the royalty rate  $r$ . Second, the downstream producer decides whether to pay for the use of the innovation covered by the patent owned by  $P$  or not. Third, if no payment is made, the patent holder can decide whether to pursue the infringement in court (litigate) or not (accommodate). Fourth, if the patent holder has decided to litigate, the downstream producer can either settle and pay the royalty rate  $r$  or go to court. If no settlement agreement is reached a court decides that the patents have been infringed according to the probability  $p$  at a legal cost  $l$  for each of the parties.

The payoffs are constructed as follows. If the patent holder accommodates, no royalty

is paid by the downstream producer and the total surplus  $v$  is accrued to this firm. If the patent holder decides to litigate the infringement and a settlement is reached, a royalty rate  $r$  is transferred to this agent, while the downstream producer obtains a net surplus  $v - r$ . Finally, if the case reaches the court, the patent is upheld with probability  $p$  and the patent holder obtains an expected profit of  $pr - l$ . The expected profit of the downstream producer is, consequently,  $v - pr - l$ .

Notice that the previous structure implies that the downstream producer can always guarantee a maximum payment of  $r$  if it settles after being litigated by the downstream producer. As a result, the royalty  $r$  will never be paid upfront and, for this reason, in the remaining of the paper we will assume without loss of generality that the downstream producer always decide not to pay unless it is litigated.

Solving the game by backwards induction we can see that the downstream firm will prefer to go to court, instead of settling, if the royalty rate is sufficiently high,  $r > \frac{l}{1-p}$ .<sup>3</sup> The patent holder prefers to litigate (as opposed to accommodate) if either  $D$  is expected to settle or if a court trial ensues and  $r < \frac{l}{p}$ .

The combination of these two thresholds spawns the three different outcomes for the game, which are taken into account in the optimal choice of  $r$ . The next proposition characterizes the royalty rate that maximizes profits for the patent holder as well as the range of parameters under which each equilibrium outcome emerges.

**Proposition 1.** *In the subgame-perfect equilibrium of the one jurisdiction case the optimal royalty for the patent holder corresponds to*

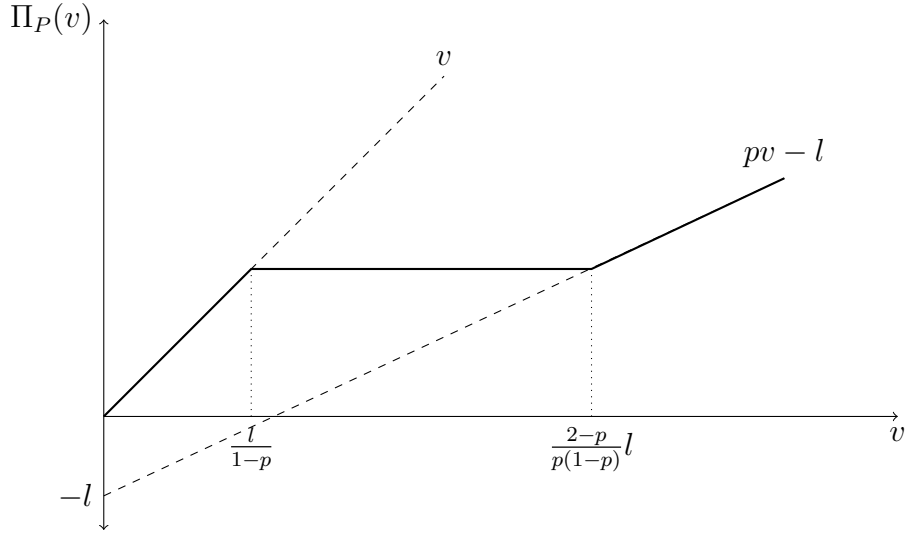
$$r^1 = \begin{cases} v & \text{if } v \leq \frac{l}{1-p}, \\ \frac{l}{1-p} & \text{if } v \in \left( \frac{l}{1-p}, \frac{2-p}{p(1-p)}l \right], \\ v & \text{if } v > \frac{2-p}{p(1-p)}l. \end{cases}$$

*Settlement arises in equilibrium when  $v \leq \frac{2-p}{p(1-p)}l$ . Firms go to court otherwise.*

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<sup>3</sup>We assume that if indifferent the downstream producer prefers to settle. Similarly, if indifferent the patent holder prefers to accommodate.





**Figure 2:** Equilibrium profits of the patent holder,  $\Pi_P(v)$ , as a function of  $v$  in the one-jurisdiction case.

The equilibrium payoff of the patent holder resulting from  $r^1$  in the previous proposition as a function of the value  $v$  is displayed in Figure 2. The intuition for the equilibrium is as follows. If  $v$  takes a low value, even if the patent holder charges the highest possible royalty,  $r^1 = v$ , the downstream producer will always settle, as the probability of success and invalidation of the patent will not compensate for the legal costs involved. As  $v$  increases, however, charging a high royalty may induce the downstream producer to go to court. For this reason, the patent holder has to decide whether to engage in some sort of limit pricing (charge the highest possible royalty for which settlement is preferred by the downstream producer) or charge a royalty  $r^1 = v$  and go to court. The second option dominates when  $v$  is high, while the limit price  $r^1 = \frac{l}{1-p}$  is preferred for lower values of  $v$ .

It is also important to notice that inefficiencies will only arise when  $v$  is high since it is in this situation where legal costs are incurred. For lower values of  $v$  the royalty rate only affects the ex-post division of surplus from the innovation.<sup>4</sup>

This proposition also indicates that the threshold on the value above which litigation

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<sup>4</sup>Of course, from an ex-ante point of view welfare will also depend on the allocation of this surplus. Innovation incentives will be shaped, among other things, by the returns from the investment of  $P$  and  $D$ .

will emerge in equilibrium depends on  $p$  in a non-monotonic way. That is, for a given value of  $v$  litigation is less likely to arise when  $p$  is either very large or very low. This is due to the cost  $l$  of going to court. The patent holder will propose a very low  $r$  when  $p$  is small in order to avoid litigation. Similarly, the downstream producer will choose to settle if  $p$  is high and the prospects from going to court are bad.

In the rest of the paper we will focus on the intermediate region so that in the one-jurisdiction case litigation will not arise in equilibrium. For this reason, we make the following assumption.

**Assumption 1.**  $v \in \left( \frac{l}{1-p}, \frac{2-p}{p(1-p)}l \right]$

This is the most relevant case, as the royalty rate and the way the surplus is split between the patent holder and the downstream producer depends on the probability of success. In contrast, when  $v$  is very low, the royalty rate does not affect the incentives for the downstream firm to litigate and, therefore, the existence of sequential trials will not change the outcome in a meaningful way. Similarly, when  $v$  is large, litigation will occur in any case.

## 2.2 Simultaneous Litigation in Two Jurisdictions

In the next section we will discuss the effect of the sequential litigation in two jurisdictions. For this reason it is useful to briefly discuss first, as a benchmark case, the situation in which the patent portfolio of the patent holder is litigated simultaneously in both jurisdictions. As in that case there would be no interaction between both legal procedures we can conclude that when  $v$  takes the intermediate value assumed in the previous section, the royalty rate will be  $r^1 = \frac{l}{1-p}$  in both jurisdictions and no litigation will take place. Total profits will be  $\Pi_D(v) = 2(v - r^1)$  for the downstream producer and  $\Pi_P(v) = 2r^1$  for the patent holder.

### 3 Sequential Litigation in Two Jurisdictions

We start this section by showing that the royalty rate characterized in Proposition 1 will not arise as part of the equilibrium when litigation takes place sequentially.

**Lemma 2.** *A royalty rate  $r^1 = \frac{l}{1-p}$  cannot be part of an equilibrium under sequential litigation.*

This result arises from the fact that in the stage game limit pricing works for the patent holder because it makes the downstream producer indifferent between settling and litigating. Once we introduce a second trial where the probability of success depends on the outcome of the first one, litigation has an additional value for the downstream producer. If the patent holder succeeds in the first trial the royalty rate would still be  $r^1$ , leading to profits of  $v - r^1$ , as in the case in which there had been no litigation. In contrast, if the patent is invalidated in the first jurisdiction, the profits of the downstream producer of going to court again in the second jurisdiction increase, as the expected payment is lower than  $r^1$ , since

$$v - r^1 = v - pr^1 - l < v - qr^1 - l.$$

To have access to this option value, the downstream producer finds worthwhile to go to court in the first jurisdiction in circumstances where in the stand-alone case settlement would have been preferred.

It is easy to see that the patent holder will be worse off under sequential litigation if, as a result of the previous effect, and the increasing interest of the downstream producer to litigate in the first jurisdiction the royalty rate was reduced. That is, the amount the patent holder will receive in each jurisdiction will always be lower, regardless of whether litigation arises in equilibrium or not.

Alternatively, the patent holder might raise the royalty rate above  $r^1 = \frac{l}{1-p}$ . Doing so implies a trade-off. On the one hand, a high royalty fosters litigation in the first jurisdiction leading to legal costs  $l$ . On the other hand, the patent holder benefits from

the potential initial success. In that case, the expected profits in the second jurisdiction increase, since the probability of success raises from  $p$  to  $q + \delta$ . The reason is that the royalty rate that the downstream producer is willing to accept after the patent has been upheld in the first jurisdiction to avert further litigation increases. However, the next result shows that the room to increase this royalty rate is limited and fostering litigation in the second jurisdiction even after a first victory would never be interest of the patent holder.

**Lemma 3.** *Any royalty rate  $r > \frac{l}{1-q-\delta} = \frac{2-p}{p(1-p)}l$  leads to lower profits for the patent holder than in the simultaneous litigation case.*

The intuition for the previous result is as follows. Assumption 1 implies that in the one jurisdiction case litigation always leads to lower profits than a lower rate that promotes an acceptable settlement by the downstream producer. If in the sequential litigation case  $r$  is very high, litigation will not only arise in the first jurisdiction. It will also lead to litigation in the second one regardless of the initial outcome. To the extent that the unconditional probability of success in the second jurisdiction is also  $p$ , profits must be lower in that case.

This lemma also implies that the highest royalty that might be profit maximizing for the patent holder has to be  $r \leq \bar{r} = \min \left\{ v, \frac{l}{1-q-\delta} \right\}$ . In situations in which  $v < \frac{l}{1-q-\delta}$  it will be optimal for the downstream producer to accept a royalty  $r = v$  in the second jurisdiction after the patent has been upheld in the first one. Notice that in that case choosing any royalty higher than  $r^1$  but below  $v$  would never be profit maximizing since it would still entice litigation in the first jurisdiction but it would lead to lower profits in the second.

The next result shows that the positive effect in the second jurisdiction will be small compared to the higher initial legal costs when  $\delta$  is relatively small and the probabilities in the second trial are close to  $p$ . This condition corresponds to situations where the linkage between jurisdictions is weak and  $q$  is large.

**Proposition 4.** *Under sequential litigation, when  $\delta \leq 1 - \frac{l}{(1-p)v}$  so that informational complementarities are weak, the patent holder is worse off compared to the simultaneous litigation case.*

In order to interpret this result, it is worth to start by analyzing the optimal royalty in this case. For a royalty rate to lead to higher profits it has to be that  $r \geq r^1$ , so that litigation will ensue in the first jurisdiction. If  $D$  wins the first trial it is easy to see that it will always be interested in going to court in the second jurisdiction. Anticipating that, the patent holder will litigate if and only if  $q > \frac{l}{v}$ . If, instead,  $P$  won the first trial, using the result from Lemma 3, we know that settlement will be reached in that case.

When  $\delta$  is sufficiently small as specified in the previous proposition, the increase in the royalty rate that the patent holder expects to get from the settlement after the success in the first trial is small. This small increase does not compensate for the legal costs incurred in the first jurisdiction and, as a result, the patent holder is always worse off.

**Proposition 5.** *Under sequential litigation, when  $\delta > 1 - \frac{l}{(1-p)v}$  and informational complementarities are strong the patent holder is worse off compared to the simultaneous litigation case when  $v < \tilde{v}$ , where*

$$\tilde{v} = \begin{cases} \frac{(1+(1-p)^2)l}{p(1-p)} & \text{if } q \geq \frac{l}{v}, \\ \frac{3-p}{(1-p)2p}l & \text{otherwise.} \end{cases}$$

When  $\delta$  is high sequential litigation might be worthwhile for the patent holder, particularly when  $v$  is high. The reason for this result is that the balance between the two effects discussed earlier changes. It is still the case that the legal costs incurred in the first jurisdiction reduce profits. However, the high value of  $q + \delta$  allows the patent holder to raise the royalty rate and not spur litigation in the second jurisdiction after a first success. The value of the royalty rate that  $P$  can charge is bounded by  $v$  so that, the higher the value of the technology, the bigger the expected gain in the second jurisdiction.

This second effect is compounded by the fact that, as discussed in the one-jurisdiction case, it is only when the probabilities of success are extreme that litigation will not occur,

as one of the parties will take the necessary actions to avoid it. Under the conditions of the previous proposition, litigation in the first jurisdiction reduces welfare but the patent holder can appropriate a larger part of the surplus, which increases profits.

To understand how the two effects play out it is useful to consider the case in which  $\delta = 1$  so that  $q = 0$  and the outcome in the first jurisdiction completely determines the outcome in the second one.

**Example 1.** *Suppose that  $\delta = 1$ . Any royalty  $r \geq r^1$  will lead to litigation in the first jurisdiction. However, litigation in the second jurisdiction will never take place. Hence, contingent on raising the royalty rate compared to the one-jurisdiction case, it is always optimal to choose  $r^* = v$ .*

*As a result, in the second jurisdiction, payoffs for  $P$  and  $D$  are  $v$  and  $0$ , respectively, after  $P$ 's success in the first trial, whereas the opposite payoffs arise when  $D$  succeeds.*

*In the first jurisdiction  $D$  prefers to go to court than to settle, since*

$$\Pi_D = v - pr^* - l + v - pr^* > v - r^* + v - pr^* - l.$$

*Profits for the patent holder become  $\Pi_P = 2pv - l$ , which exceed those accrued from two simultaneous trials,  $\frac{2l}{1-p}$ , if  $v > \frac{3-p}{2p(1-p)}l$ .*

*The downstream producer is better off under sequential litigation if  $v < \frac{1+p}{2p(1-p)}l$ . Interestingly, when  $v$  takes an intermediate value, both the patent holder and the downstream producer are worse off due to the increase in litigation.*

### 3.1 Sequential vs Simultaneous Litigation Choice

We now turn to the effect of litigation decisions being sequential as opposed to situations in which both cases are resolved simultaneously.

Suppose that in an initial stage  $D$  can choose whether to litigate the patents of firm  $P$  at the same time in the two jurisdictions or sequentially, so that the outcome of one trial affects the other. As explained before, when both patents are litigated simultaneously,

firm  $D$  makes profits of

$$\Pi_D = 2 \left( v - \frac{l}{1-p} \right).$$

When both patents are litigated separately, firm  $P$  has two options. One possibility is to choose a low  $r$  and always engage in limit pricing. This option always leads to higher profits for  $D$ . Alternatively, as we have seen in the previous section, if  $\delta$  is high and the result in the second jurisdiction largely depends on the result of the first one, it can choose  $r = v > r^1$ , so that it obtains higher profits in the second trial.

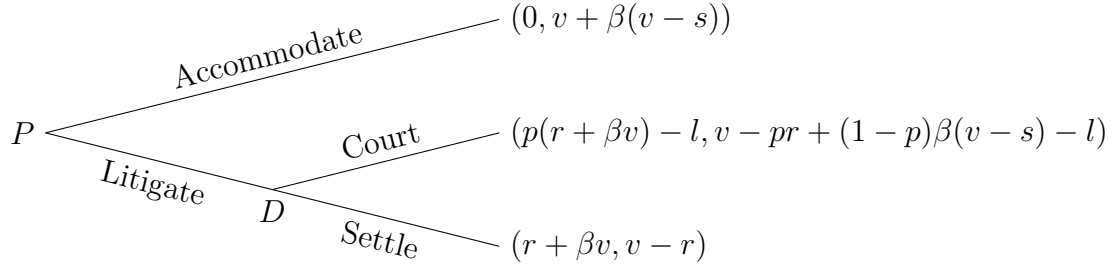
Firm  $D$  will choose a sequential trials unless  $v$  and  $\gamma$  are both large. To the extent that in the second jurisdiction total surplus decreases compared to the simultaneous litigation case where no legal costs are incurred, this implies that  $P$  would be worse off.

## 4 Downstream Competition

We now extend the model to study the implications of sequential litigation for competition and efficiency in the final market. In the benchmark case only  $D$  was present in the final market and litigation created no distortion in consumption because the price was always equal to  $v$ .

We now propose a richer model that accounts for the effects in the final market. In order to do that, we consider the interaction of the downstream producer with a competitive fringe. In particular, we assume that there are two segmented markets. As in the benchmark model, a continuum of consumers of size 1 is exclusively served by the downstream producer  $D$ . There is, however, a second segment of the market of size  $\beta$  that can be served by firm  $D$  or by a continuum of identical firms. On these consumers we assume that the downstream producer has a comparative disadvantage vis-a-vis the firms in the competitive fringe, that we measure by the parameter  $s \in (0, v)$ .

In the licensing negotiation with the patent holder, competition among firms in the fringe means that they pay a per-unit royalty  $r^f = v$  to sell their product in the final market. As a result, the price that these firms charge in the final market is also equal to



**Figure 3:** Structure of the stage game under downstream competition when  $r > \bar{r}$ .

$v$ .

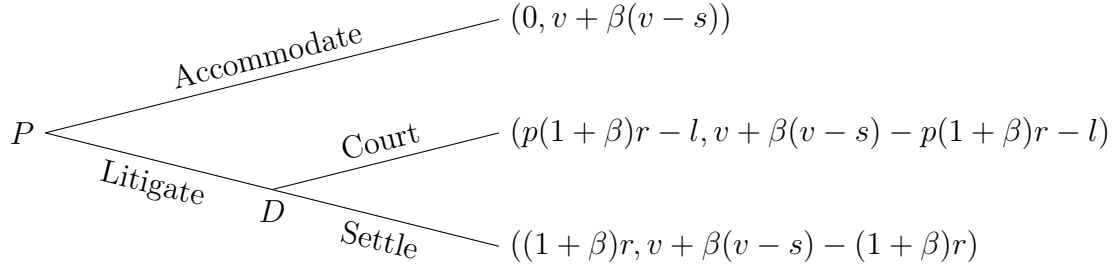
Due to the preference for the product sold by the fringe in the contested segment of the market, the downstream producer  $D$  will only sell if it offers a price that compensates for the disadvantage  $s$ . This means that it can only serve the contested part of the market if it pays an effective royalty rate  $r \leq \bar{r} = v - s$ .<sup>5</sup> It is important to emphasize that this lower royalty can occur for two reasons. The obvious one is that the patent holder might simply set a royalty rate  $r$  below  $\bar{r}$ . However, the royalty might be equal to 0 because the downstream producer invalidated the patent portfolio of the patent holder by going to court. As we will see next, this second mechanism might engender additional incentives to litigate and have important implications for the efficiency in the final market.

We start by studying the case of one jurisdiction. To carry out this analysis it is useful to distinguish between two cases depending on whether the patent holder sets a royalty above or below  $\bar{r}$ .

Suppose first that  $r > \bar{r}$ . The payoffs of the stage game are described in Figure 3. Compared to the payoffs in the benchmark case, when the royalty  $r$  is accepted by the downstream producer the patent holder obtains additional profits arising from the quantity sold by the competitive fringe,  $\beta v$ . When the royalty is not paid either because the patent holder accommodates or because it is taken to court and the patent is invalidated, the downstream producer obtains additional profits of  $\beta(v - s)$ , as now it can also serve the contested segment of the market.

<sup>5</sup>When indifferent we assume that consumers buy from the downstream producer.





**Figure 4:** Structure of the stage game under downstream competition when  $r < \bar{r}$ .

Solving this game by backwards induction we can observe that the downstream producer is now more likely to go to court and will not accept a royalty if  $r > \frac{l}{1-p} - \beta(v - s)$ . The patent holder, in turn, is more likely to accommodate since a court trial would be profitable only if  $r < \frac{l}{p} - \beta v$ . This decrease in the incentives to litigate is due to the fact that by going to court and losing, the patent holder foregoes profits also from the contested part of the market.

Consider now the situation in which  $r \leq \bar{r}$ . The payoffs of the stage game are described in Figure 4. Compared to the previous case, notice that here the downstream producer can always undercut the competitive fringe. As a result, it always obtains an additional profit of  $\beta(v - s)$ , compared to the benchmark case. Because this amount is independent of the decision of firm  $D$  it also means that the thresholds that determine the decision of both the patent holder and the downstream producer to litigate and go to court, respectively, are similar to those in the benchmark model, except for the fact that the total size of the market is now  $1 + \beta$ . In particular,  $D$  will go to court if  $r > \frac{l}{(1-p)(1+\beta)}$ . Firm  $P$  will litigate if  $D$  is expected to settle or if  $r < \frac{l}{p(1+\beta)}$ .

It is obvious that the patent holder is better off when the competitive fringe produces as this leads to an additional revenue stream arising from their higher efficiency in the contested segment of the market. Since a higher  $r$  makes this case more likely and it also increases the licensing revenues from  $D$  (contingent on not losing in court), the patent holder will always prefer  $r > \bar{r}$  when feasible.

**Proposition 6.** *In the subgame-perfect equilibrium of the one jurisdiction case with*

downstream competition the optimal royalty for the patent holder corresponds to

$$r^c = \begin{cases} v & \text{if } v \leq \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s, \\ \frac{l}{1-p} - \beta(v-s) & \text{if } v \in \left( \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s, \tilde{v} \right], \\ v & \text{if } v > \tilde{v}, \end{cases}$$

when  $s \geq \frac{2l}{p(1+\beta)-\beta}$  and  $\tilde{v} \equiv \frac{2-p}{(1-p)p(1+\beta)}l + \frac{\beta s}{p(1+\beta)}$ . Otherwise,

$$r^c = \begin{cases} v & \text{if } v \leq \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s, \\ \frac{l}{1-p} - \beta(v-s) & \text{if } v \in \left( \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s, \frac{l}{(1-p)(1+\beta)} + s \right], \\ \frac{l}{(1-p)(1+\beta)} & \text{if } v \in \left( \frac{l}{(1-p)(1+\beta)} + s, \tilde{v} \right], \\ v & \text{if } v > \tilde{v}. \end{cases}$$

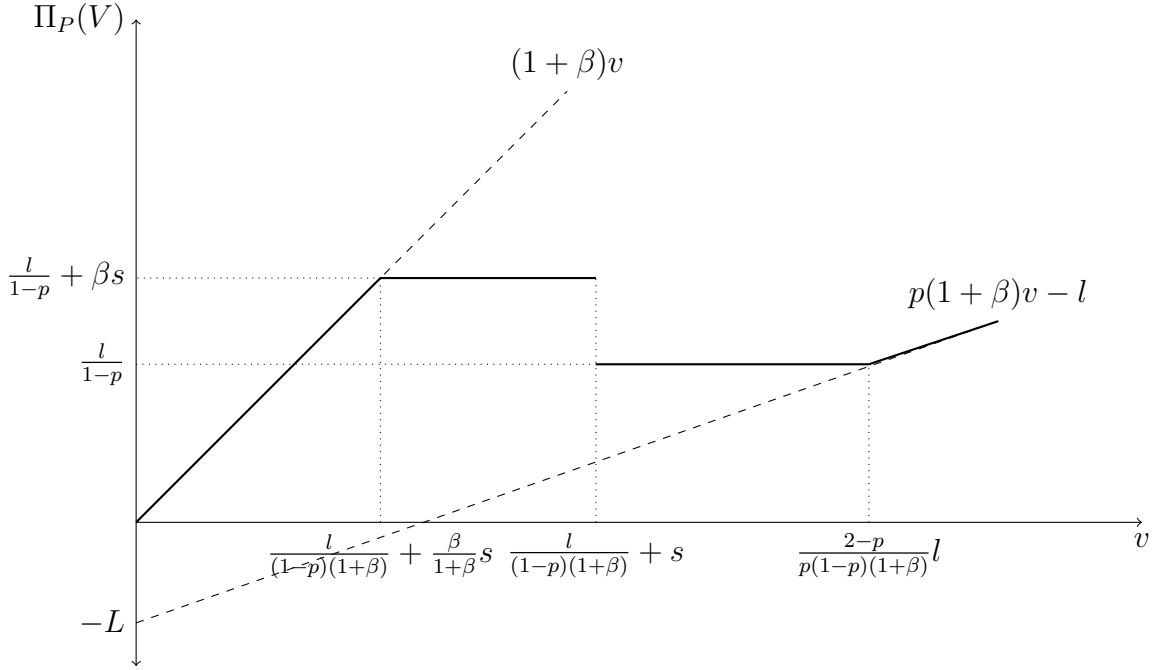
where  $\tilde{v} \equiv \frac{2-p}{p(1-p)(1+\beta)}l$ .

In both cases, settlement arises in equilibrium when  $v \leq \tilde{v}$ . Firms go to court otherwise.

Figure 5 shows how the profits depend on  $v$  when the difference in the valuation that the consumers place in the product of the downstream producer and the competitive fringe, is small,  $s < \frac{2l}{p(1+\beta)-\beta}$ . When  $v$  is low, the figure is similar to the one obtained in the benchmark case. In particular, profits grow linearly with  $v$ . As  $v$  increases the patent holder engages in limit pricing. The royalty rate is set in order to avoid litigation while, at the same time  $r^c > \bar{r}$ , so that the competitive fringe pays a royalty  $r$  for the license. For values of  $v$  above  $\frac{l}{(1-p)(1+\beta)} + s$ , however, it is not possible to engage in this kind of limit pricing while allowing the competitive fringe to produce. In this case, the patent holder needs to lower the royalty further in order to avoid litigation and, as a result, give up the revenue  $\beta s$  which the production of the competitive fringe generated. For even higher values of  $v$ , as in the benchmark model, litigation ensues.

When  $s$  is sufficiently high, the intermediate region where  $r < \bar{r}$  does not exist, meaning that the downstream producer does not produce in the segmented part of the market.

It is important to notice that the presence of the competitive fringe highlights two potential sources of inefficiency in the negotiation between the patent holder and the



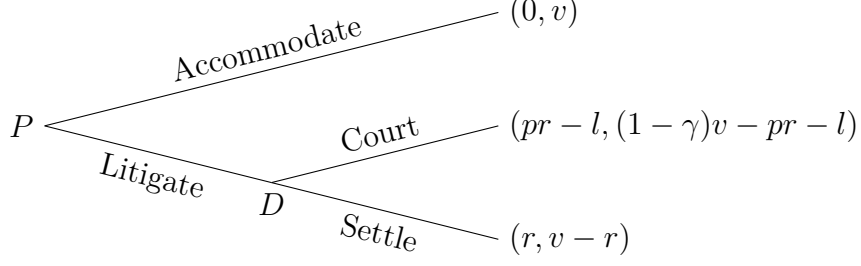
**Figure 5:** Profits when  $s < \frac{2l}{p(1+\beta)-\beta}$ .

downstream producer. The first, which occurs for high values of  $v$ , is the occurrence of litigation in equilibrium. While this also arose in the benchmark model, here it not only implies a legal cost but also a probability  $1 - p$  that the downstream producer sells to the consumers in the segmented part of the market in spite of its lower efficiency.

More interestingly for the purpose of this paper, in the intermediate region, when  $s$  was low we have shown that there is a region in which the patent holder needs to decrease the royalty rate to avoid litigation, below  $\bar{r}$ . By doing so, the competitive fringe is displaced, with the resulting loss in efficiency.

## 5 Patent Injunctions

We now study the effects of the injunctions that the patent holder could obtain. In particular, we assume that  $P$  asks for an injunction before litigation occurs. By doing so, while the trial is not being resolved, there is no production. This means that the value of the patent becomes  $(1 - \gamma)v$ . Figure 6 describe the stage game in that case, given a probability of success  $p$ .



**Figure 6:** Stage game when the patent holder can ask for injunction..

For a given licensing payment  $r$ , in the one jurisdiction case, the downstream producer will prefer to go to court if  $r > \frac{\gamma v + l}{1-p}$ . Compared to the benchmark case, the higher the value of the patent or the delay in production generated by the injunction the more likely it is that litigation occurs. The patent holder will decide to litigate either if  $D$  is expected to settle or if  $r > \frac{l}{p}$ . The next result characterizes the royalty rate that arises in equilibrium in this case.

**Proposition 7.** *In the subgame-perfect equilibrium of one jurisdiction case, the optimal royalty for the patent holder when an injunction is obtained takes a different form depending on the strength of  $\gamma$ .*

- If  $\gamma < p(1-p)$ , then

$$r^I = \begin{cases} v & \text{if } v \leq \frac{l}{1-p-\gamma}, \\ \frac{\gamma v + l}{1-p} & \text{if } v \in \left( \frac{l}{1-p-\gamma}, \frac{2-p}{p(1-p)-\gamma} l \right], \\ v & \text{if } v > \frac{2-p}{p(1-p)-\gamma} l. \end{cases}$$

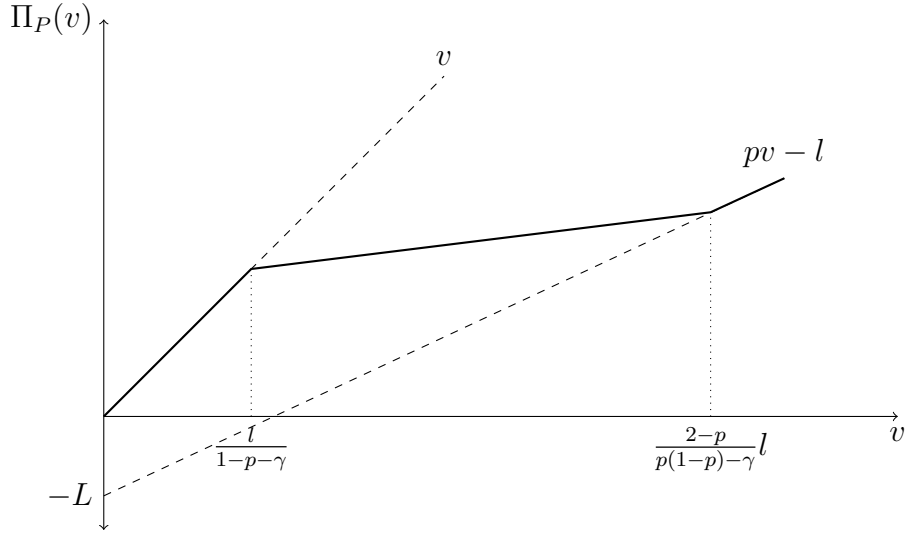
Settlement arises in equilibrium when  $v \leq \frac{2-p}{p(1-p)-\gamma} l$ . Firms go to court otherwise.

- If  $p(1-p) < \gamma \leq 1-p$ , then

$$r^I = \begin{cases} v & \text{if } v \leq \frac{l}{1-p-\gamma}, \\ \frac{\gamma v + l}{1-p} & \text{if } v > \frac{l}{1-p-\gamma}. \end{cases}$$

- If  $\gamma > 1-p$  then  $r^I = v$ .

Figure 7 shows the payoff of the patent holder when a *weak* injunction is used. As expected, profits increase due to three complementary effects. First, the region in which



**Figure 7:** Equilibrium profits of the patent holder in the one-jurisdiction case under an injunction of strength  $\gamma \leq p(1 - p)$ .

a royalty  $v$  can be charged and  $D$  accepts it (when  $v$  is low) expands. Second, the royalty rate that  $P$  offers in the limit-pricing region the rate increases in  $\gamma$ . Finally, and related to the previous effect, as the downstream producer is more willing to accept a high royalty rate, the litigation region contracts.

Interestingly, when  $\gamma$  increases some of the previous regions might disappear. In particular, the proposition indicates that when  $\gamma > p(1 - p)$  litigation never arises in equilibrium. This is due to the fact that when injunctions are sufficiently strong the downstream producer loses more from going to court through the impossibility of selling the product than the potential decrease in the payment associated with a probability of success of  $1 - p$ .

We now briefly turn to the case of two jurisdictions. We will focus on the case in which  $\gamma < p(1 - p)$  so that litigation is a relevant concern. Furthermore, we present our discussion in the context where the outcome of the second trial is completely determined by the first court decision.

**Proposition 8.** *Suppose that  $\delta = 1$  and  $\gamma < p(1 - p)$ . Then, compared to simultaneous litigation,*

- *The lowest value of  $v$  for which the upstream producer prefers sequential litigation,*

$\tilde{v}_P(\gamma)$ , is increasing in  $\gamma$ ,

- The highest value of  $v$  for which the downstream producer prefers sequential litigation,  $\tilde{v}_D(\gamma)$ , is increasing (decreasing) in  $\gamma$  if  $p$  is sufficiently high (low),

where  $\tilde{v}_D < \tilde{v}_P$ .

First of all, notice that this result embeds Example 1 as the case with  $\gamma = 0$ . The proposition shows that, as expected, for some intermediate values of  $v$  (between  $\tilde{v}_D$  and  $\tilde{v}_P$ ) sequential litigation is detrimental to the profits of both  $P$  and  $D$ . This is due, of course, to the legal costs incurred under sequential litigation which reduce total surplus. For low values of  $v$  the downstream producer is better off since under simultaneous litigation going to court was not worthwhile. Under sequential litigation the first trial might pay off as it might invalidate the patents in both jurisdictions. For high values of  $v$ , sequential litigation allows the patent holder to charge a higher  $r$  which increases profits.

To the previous discussion, changes in  $\gamma$  introduce an interesting comparative statics exercise. Increases in  $\gamma$  decrease the appeal of sequential litigation for the patent holder. The reason is that, under the extreme parameter values considered, the strength of the injunction does not affect the decision of  $D$  of going to court in the second jurisdiction. Under simultaneous litigation, however, the patent holder can charge a higher royalty rate if the injunction is obtained.

In the case of the downstream producer, the effect is more nuanced and it depends on the value of  $p$ . Both in the sequential and the simultaneous litigation case, the injunction leads to lower profits. While under sequential litigation a higher  $\gamma$  decreases profits linearly, when litigation is simultaneous the effect is higher when  $p$  is smaller. This means that when  $p$  is low increases in  $\gamma$  favor sequential litigation while the opposite is true when  $p$  is large.

## 6 Concluding Remarks

The goal of this paper was to provide a framework to understand the incentives for downstream producers to engage in efficient patent infringement, their implications on the royalty revenues that a patent holder would obtain as a result and the effects of the regulatory framework. The results suggest that sequential litigation typically harms the patent holder due to two related effects. First, it fosters litigation in the first jurisdiction as a way to improve the bargaining power in the second one. Second, this move is typically beneficial to the downstream producer because the patent holder cannot increase the royalty rate as a result of a success in the first jurisdiction.

The rigidity behind this last effect can be the result of many mechanisms. First, FRAND obligations and, particularly, non-discriminatory agreements might reduce the room for offering different contracts. Second, while litigation is sequential, the initial negotiations might not be, and the patent holder could be facing two identical initial situations for which the same royalty rate would be optimal. It is unlikely that courts would allow royalty rates to be adjusted upwards as a result of previous litigation.

There are many related questions that this paper does not address and could be a relevant avenue for future research. As explained before, while litigation may be sequential, negotiations might be simultaneous. It would be worth to explore whether, once we allow for jurisdictions to be different, downstream producers have incentives to go to court first in the ones that are more favorable.

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## A Proofs

The main results of the paper are proved here.

**Proof of Proposition 1:** To characterize the optimal royalty rate we need to distinguish two cases depending on whether  $p$  is greater or smaller than  $1/2$ .

If  $p \geq \frac{1}{2}$  then  $\frac{l}{p} \leq \frac{l}{1-p}$ . As a result, two regions emerge. If  $r \leq \frac{l}{1-p}$ ,  $D$  will find optimal to settle and, in anticipation,  $P$  will always litigate. Hence,  $\Pi_P(v) = r$  and  $\Pi_D(v) = v - r$ . If  $r > \frac{l}{1-p}$ ,  $D$  will prefer to go to court and  $P$  will litigate. Hence,  $\Pi_P(v) = pr - l$  and  $\Pi_D(v) = v - pr - l$ .

If  $p < \frac{1}{2}$  then  $\frac{l}{p} > \frac{l}{1-p}$ . As a result, three regions emerge now. If  $r \leq \frac{l}{1-p}$ ,  $D$  prefers to settle and  $P$  decides to litigate. Hence,  $\Pi_P(v) = r$  and  $\Pi_D(v) = v - r$ . If  $\frac{l}{1-p} < r \leq \frac{l}{p}$ ,  $D$  is expected to go to court and  $l$  accommodates. Hence,  $\Pi_P(v) = 0$  and  $\Pi_D(v) = v$ . Finally, if  $r > \frac{l}{p}$ ,  $D$  goes to court and  $P$  litigates. Hence,  $\Pi_P(v) = pr - l$  and  $\Pi_D(v) = v - pr - l$ .

The previous results, imply that if  $v \leq \frac{l}{1-p}$  it is optimal for the patent holder to choose  $r^1 = v$  so that  $P$  extracts all the surplus. If  $\frac{l}{1-p} < v \leq \frac{2-p}{p(1-p)}l$ , then  $r^1 = \frac{l}{1-p}$  since settlement is preferred to litigation. Finally, if  $v > \frac{2-p}{p(1-p)}l$ , then  $r^1 = v$ .  $\square$

**Proof of Lemma 2:** First notice that the threshold for which the downstream producer will be indifferent in the second jurisdiction between going to court and accepting the settlement is  $\frac{l}{1-q}$  after winning the first trial and  $\frac{l}{1-q-\delta}$  after losing, where  $\frac{l}{1-q} < \frac{l}{1-p} < \frac{l}{1-q-\delta}$ . Hence, it is immediate that under  $r^1$  in the second jurisdiction a settlement will occur if the patent holder won the first trial. Instead, if the downstream producer won, accommodation would occur if  $r < \frac{l}{q}$  or the patent holder would litigate otherwise.

As a result, in the first period, the downstream producer will always prefer to litigate, since

$$2(v - r^1) < v - pr^1 - L + p(v - r^1) + (1 - p)\pi_D(r^1)$$

where  $\pi_D(R^1)$  is the profit associated to second period litigation when  $D$  succeeded in the first trial and it is equal to  $v - qr^1 - l$  when  $r^1 \geq \frac{l}{q}$  and  $v$  otherwise.  $\square$

**Proof of Lemma 3:** In the simultaneous case  $r^1 = \frac{l}{1-p}$  maximizes total surplus as it never induces litigation. Hence, in the sequential case, under a royalty  $r < \frac{l}{1-p}$  the downstream producer can always guarantee total profits higher than under the simultaneous case by settling in both trials. Hence, the patent holder is worse off in this case.

Hence, for the patent holder to be better off under sequential litigation it has to be the

case that  $r > \frac{l}{1-p}$ . Furthermore, notice that a royalty above  $v$  would never be optimal as it would result in no production or in litigation, whereas a royalty rate of  $v$  would guarantee at least the same expected profits.

We can rule out royalty rates  $r > \frac{l}{1-p-\delta} = \frac{lp}{1-p}$ . The reason is that they induce litigation in the second trial regardless of the outcome in the first. In particular, if  $q > \frac{l}{v}$  then patent holder profits become

$$\Pi_P = pr - l + p[(q + \delta)r - l] + (1-p)[qr - l] = 2(pr - l) < 2r^1.$$

When  $q \leq \frac{l}{v}$  then

$$\Pi_P = pr - l + p[(q + \delta)r - l] \leq 2(pr - l) < 2r^1.$$

Hence, the royalty rate that makes the patent holder better off under sequential litigation and maximizes profits has to be  $r^* = \min\left\{\frac{lp}{(1-p)q}, v\right\}$ . The reason is that any royalty between  $r^1$  and  $\frac{lp}{(1-p)q}$  would lead to litigation in the same states of the world it would lead to lower profits. In particular, expected profits for  $P$  are

$$\Pi_P(r^*) = pr^* - l + pr^* + (1-p)\max\{0, qr^* - l\}.$$

□

**Proof of Proposition 4:** Under the conditions of this proposition we have that  $v \geq \frac{lp}{(1-p)q}$  so that  $r^* = \frac{lp}{(1-p)q}$ . Since, by Assumption 1,  $v \leq \frac{2-p}{p(1-p)}l$  we have that  $q \geq \frac{p^2}{2-p}$ . If  $p > \frac{1}{2}$  then  $q\hat{r} > l$  and profits for the patent holder are

$$\Pi_P = 2\frac{p^2l}{(1-p)q} - 2(1-p)l < 2r^1.$$

Instead, if  $p \leq \frac{1}{2}$  then  $q\hat{r} < l$  and profits are

$$\Pi_P = 2\frac{p^2l}{(1-p)q} - l < 2r^1.$$

□

**Proof of Proposition 5:** Under the conditions of the proposition  $v < \frac{lp}{(1-p)q}$  and it implies  $R^* = V$ . After  $D$  wins the first trial, in the second one,  $D$  will always go to court. Hence,  $P$  will litigate only if  $qr^* - l > 0$  or  $q > \frac{l}{v}$ . Hence, second period profits for  $P$  and  $D$  are

$$\pi_P(v|q) = \begin{cases} 0 & \text{if } q < \frac{l}{v}, \\ qv - l & \text{otherwise.} \end{cases}$$

$$\pi_D(v|q) = \begin{cases} v & \text{if } q < \frac{l}{v}, \\ v(1-q) - l & \text{otherwise.} \end{cases}$$

Notice that  $\frac{pl}{(1-p)v} > q > \frac{l}{v}$  requires  $p > \frac{1}{2}$ .

In the first trial, after being litigated,  $D$  always goes to court since

$$v - pr^* - l + p(v - r^*) + (1-p)\pi_D(r^*|q) \geq v - r^* + v - pr^* - l.$$

In particular, it implies

$$(1-p)\pi_D(r^*|q) \geq (1-p)(v - r^*) = 0,$$

which is satisfied regardless of whether  $q$  is higher or lower than  $\frac{l}{v}$ .

Anticipating that  $D$  will go to court,  $P$  always prefers to litigate since

$$pr^* - l + pr^* + (1-p)\pi_P(r^*|q) \geq pr^* - l.$$

First period profits for  $P$  can be computed (replacing  $r^* = v$ ) as

- If  $q \geq \frac{l}{v}$ , total patent holder profits become  $\Pi_P = v - l + (1-p)(qv - l) < 2pv - 2(1-p)l$  since  $q < \frac{pl}{(1-p)v}$ . In this case, profits are higher than  $2\frac{l}{1-p}$  if  $V > \frac{(1+(1-p)^2)l}{p(1-p)}$ .
- If  $q < \frac{l}{v}$ ,  $\Pi_P = 2pv - l$ . These profits are higher than  $2\frac{l}{1-p}$  if  $v > \frac{3-p}{(1-p)2p}l$  which is possible, given that this threshold is lower than  $\frac{2-p}{p(1-p)}l$ .

□

**Proof of Proposition 6** We start with the case in which  $v$  is very low. In particular, suppose that  $v \leq \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s$ . In that case, a royalty  $r = v > \bar{r}$  will entice the downstream producer to settle, while it maximizes profits for the patent holder.

If  $v \in \left( \frac{l}{(1-p)(1+\beta)} + \frac{\beta}{1+\beta}s, \frac{l}{(1-p)(1+\beta)} + s \right]$  we have that  $\bar{r} < \frac{l}{1-p} - \beta(v - s)$ .  $P$  has now two options. It can offer a limit royalty rate  $r^c = \frac{l}{1-p} - \beta(v - s)$  so that  $D$  is indifferent between settling and litigating and obtain profits  $\Pi_P = r^c + \beta v$  or set a royalty  $v$ , litigate and obtain profits  $p(1+\beta)v - l$ . The latter is preferred if  $v > \tilde{v} \equiv \frac{2-p}{(1-p)p(1+\beta)}l + \frac{\beta s}{p(1+\beta)}$ . This thresholds leads to two cases depending on whether  $\tilde{v}$  is greater or smaller than  $\frac{l}{(1-p)(1+\beta)} + s$ .

In particular, if  $s \geq \frac{2l}{p(1+\beta)-\beta}$ ,  $\tilde{v} < \frac{l}{(1-p)(1+\beta)} + s$  and we need to consider two regions. If  $v \leq \tilde{v}$  then  $r^c = \frac{l}{1-p} - \beta(v - s)$ . If  $v > \tilde{v}$ , then  $r^c = v$ . Instead, if  $s < \frac{2l}{p(1+\beta)-\beta}$  we now have a third intermediate region. As before,  $v \leq \frac{l}{(1-p)(1+\beta)} + s$  then  $r^c = \frac{l}{1-p} - \beta(v - s)$ . If  $v > \tilde{v}$ , then  $r^c = v$ . In the intermediate region, the previous limit pricing does not avoid  $D$

selling in the two markets. Hence, the highest royalty that  $P$  can set and avoid litigation is now  $r^c = \frac{l}{(1-p)(1+\beta)}$ . The threshold value  $\tilde{v}$  is characterized by comparing the profits in this latter case,  $(1+\beta)(v-s)$  and those that arise from litigation  $p(1+\beta)v-l$ .  $\square$

**Proof of Proposition 7:** When  $\gamma < p(1-p)$  the structure of the proof is identical to the one in Proposition 1. Instead, when  $\gamma > p(1-p)$  it is easy to see that the profits for the downstream patent holder from going to court grow with  $v$  at a lower rate than those from limit pricing. For  $\gamma > 1-p$  the profits from limit pricing are always lower than those from setting  $r = v$ .  $\square$

**Proof of Proposition 8:** Suppose that  $\delta = 1$ . As in the benchmark case,  $r = \frac{l}{1-p-\gamma}$  cannot be an equilibrium in the two-jurisdiction case. For this reason, suppose that  $r^* = v > \frac{\gamma v + l}{1-p}$ . If  $D$  wins the first trial, in second one  $P$  accommodates and second jurisdiction profits are  $\pi_D = v$  and  $\pi_P = 0$ . If  $P$  wins the first trial, in the second one  $D$  always settles and profits are  $\pi_D = 0$  and  $\pi_P = v$ .

In the first stage,  $D$  decides to go to court, since

$$(1-\gamma)v - pr^* - l + v - pr^* > v - r^* - (1-\gamma)v - pr^* - l$$

As a result, total profits for  $P$  are identical to the case without an injunction  $\Pi_P = 2pv - l$ . In the case of  $D$ , profits become  $\pi_D = (2(1-p) - \gamma)v - l$ .

Regarding the preference for sequential litigation, notice that  $P$  will be better off if  $2pv - l > \frac{2}{1-p-\gamma}l$  which occurs for  $v > \tilde{v}_P(\gamma) = \frac{3-p-\gamma}{1-p-\gamma}$  which is increasing in  $\gamma$ . Firm  $D$  will prefer sequential litigation if

$$(2(1-p) - \gamma)v - l > 2v - \frac{2l}{1-p-\gamma} \rightarrow v < \tilde{v}_D(\gamma) \equiv \frac{1+p+\gamma}{(1-p-\gamma)(2p+\gamma)}l.$$

We can compute

$$\frac{\partial \tilde{v}_D}{\partial \gamma} = \frac{p^2 + 2\gamma p + 4p + \gamma^2 + 2\gamma - 1}{(1-p-\gamma)^2(2p+\gamma)^2}l.$$

The numerator is increasing in  $p$ . It can also be shown that  $\left. \frac{\partial \tilde{v}_D}{\partial \gamma} \right|_{p=0} < 0 < \left. \frac{\partial \tilde{v}_D}{\partial \gamma} \right|_{p=1-\gamma}$  which proves the result.  $\square$