Sequel Firm Creation and Moral Hazard in Teams

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Abstract

We argue that when firms compete to devise innovations, the provision of incentives to teams of researchers influences the extent to which inventing firms tolerate the creation of sequel firms. The classic Holmstrom (1982) moral hazard in teams problem leads to an "excessive" equilibrium creation of firms, particularly in starting and innovative industries. The implications are not only in line with a series of empirical observations on the dynamics of firm spawning activity but also on firm focus and firm profitability.

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1 Introduction

The adequate creation of firms which devise and exploit innovations is central to low-growth post-crisis economies[1]. When a firm makes an innovation, a sequel firm (also referred to as "spinoff", "spinout" or "spawned" firm) is often created.[2] This paper argues that, because firms compete to devise innovations, the moral hazard in research teams problem inefficiently influences the dynamics of firm creation. Inventing firms excessively allow the creation of sequel firms. The issue dynamically fades away as the number of firms endogenously increases. The resulting dynamics are consistent with a series of empirical findings.

1.1 Our Argument

Within a sector of activity, firms repeatedly compete with each other to make the next innovation. For a firm to make an innovation, it must produce research. To be more likely successful, each firm tries to produce more research than others in the industry. Producing research however requires employing one or more inventing agents. But simply hiring numerous researchers is not in a firm's interest, the production of research in a firm by a set of agents does not just increase with the number of agents: it suffers from the classic Holmstrom (1982) moral hazard in teams problem.

The central engine of the paper is that the ability to contract research teams separately in a new firm, segments the moral hazard in team problems: Whereas each existing firm does not wish to employ more than a number of researchers, it is worthwhile for any newly created firm to employ further researchers to compete to innovate. Absent the moral hazard in team problems, there is no particular value for a new firm in building a further research team, as in equilibrium, any value from employing researchers is already internalized by existing firms.

This paper shows that the segmentation of the moral hazard in team problems influences

[1]What we refer to as innovations here are primary inventions and products whose competitive advantage is based on superior functional performance, offer high unit profit margin, and may require a reorientation of production facilities as well as corporate goals. These differ from deriving minor product and system improvements, whose value we take as embedded in that of a primary innovation.

[2]Hall (1998) notes that about half of the eighty-five U.S semiconductors companies of the 1980s were direct or indirect spin-outs from one original firm, Fairchild Semiconductors. Similar parental links are documented by Keppler and Sleeper (2005) and Sherer (2006) in the laser industry, by Agarwal, Echambadi, Franco and Sarkar (2004) and Franco and Filson (2006) for disk drives, by Klepper (2007) for cars, by Chatterji (2009) for medical devices and by Buenstorf and Klepper (2009) for tires.

the dynamics of firm creation. To establish this impact, we need a base rationale for firm creation. We simply borrow it from the resource-based-view of the firm: An inventing firm benefits from making an innovation, as it can generate cash-flows from exploitation. The characteristics of an innovation are however uncertain. When these are distant from the characteristics of the inventing firm, the innovation would generate more cash if exploited in a newly created firm, specifically adapted to do it.[3] Essentially, the inventing firm is able to internalize the exploitation value of an innovation, not only inside the firm, but also by an outside newly created firm. This yields a simple benchmark firm creation trade-off: The innovation is implemented inside the inventing firm when the distance between the characteristics of the innovation and the firm is less than a threshold distance. Conversely, the innovation is sold-out and a new firm is created if the distance is larger.

We conduct our argument that the ability of newly created firms to segment the moral hazard in research team problems impacts firm creation, relative to the above trade-off. From the perspective of a firm which makes an innovation, the creation of a competing firm which would result from selling out the innovation, entails dynamically evolving add-on benefits and costs, beyond the single internalization of the exploitation value of the innovation:

• On the positive side, the market value of the innovation also includes the value brought by the ability to contract separately a team of inventing agents. That is, the fraction of the aggregate expected value of further innovations, captured by the sequel firm as it constitutes its' own team of researchers, can be captured by the parent firm.

This drive towards firm creation is dimmed by the fact that the innovating firm does not fully internalize the expected value of future innovations made by the sequel firm. In the equilibrium outcome of the race to innovate, firms provide competitive incentives which actually transfer value to their agents. Then, when selling out an innovation, the parent firm does not capture the value provided by the series of sequel firms expected to result from this sale, to their subsequent teams of agents.

• On the negative side, as the exploration of future innovations is competitive, the emergence of an additional research team imposes a negative externality on the parent firm: Some likelihood of making future innovations is transferred away from existing firms towards the new one.

The strengths of these two conflicting forces depend on the number of firms competing

[3]Because the successful exploitation of one innovation may require a costly reorientation of production facilities as well as corporate goals of the inventing firm, more aggregate profits can often be obtained if a new dedicated firm is created. The inventing firm then benefits from selling out the innovation to an outside financier, relinquishing its exploitation rights and allowing the innovation to be exploited outside.

to innovate, which increases through time, as sequel firms are created. That is, the add-on trade-off exhibits strong endogenous time dynamics. The extent to which innovating firms are willing to accept the creation of sequel firms depends on the number of existing firms, the fraction of innovation value captured by agents and the velocity with which innovations are made. We obtain that an innovation leads more frequently to the creation of a sequel firm, where (1) the number of firms is still limited, hence the industry is young, (2) agents only have to be given a small share of firm value and (3) the (expected) number of innovations per year is high.

When innovations lead more frequently to the creation of a sequel firms, then the focus of firms is higher, and firms realize higher profits per innovation. Our argument is therefore in line with the following series of empirical findings on firms spawning activity and dynamics of firms characteristics:

– the frequency with which firms spawn decreases with age; firms with more numerous high quality patents spawn more frequently (Gompers, Lerner and Scharfstein (2005));

– diversified firms spawn less firms than focused ones (Gompers, Lerner and Scharfstein (2005)); firms become less focused with age (Denis Denis and Sarin (1997)); diversified firms are less profitable (Berger and Ofek (1995), Lang and Stulz (1994));

– more profitable firms are more prolific spawners (Gompers, Lerner and Scharfstein (2005)); firms' profitability decreases with age (Eisenberg, Sundgren and Wells (1998), Majumdar (1997), Loderer and Waechli (2009)).

Technically, we construct a principal-agent model of a developing industry, in which firms have repeatedly the opportunity to compete to innovate and generate cash from their innovations.
(a) The unit period is the expected time between innovations in that industry. A first exogenous parameter, the unit-period discount rate, captures this characteristic of the industry.
(b) One firm has a higher chance of devising the next innovation than another firm, if the research output produced by its team of agents is higher than that of the other firm. A firm's research output is a function of the aggregate effort of its agents, where each agent bears a private cost of effort. A second exogenous parameter, captures the sensitivity of research output to the teams' effort.
(c) If a firm makes an innovation, it can use its expertise to generate cash from it. If the inventing firm's expertise is not appropriate enough to exploit the innovation, the firm can alternatively sell it to a new principal. The buyer can then create a sequel firm with most appropriate expertise to exploit this innovation. Creating a new firm however entails a one-off set-up cost. A third exogenous parameter, the cost of setting up a firm, captures this characteristic of the industry.

In equilibrium, existing firms compete to make the next innovation. The principal of each firm can build a research team and provides them incentives to produce the competitive level of research output. We derive the optimal number of agents she approaches and the optimal compensation contract she proposes. We calculate the optimal effort level each agent exerts. We establish the equilibrium threshold characteristics which lead principals to decide to sell an innovation, instead of exploit it. This threshold determines the extent to which new firms are created.

These equilibrium decisions are not first best and inefficiencies have two sources: First, the equilibrium efforts of agents are excessive, because firms engage in a race to innovate. Second, innovations are sub optimally exploited, because the equilibrium firm creation threshold differs from the first best one. There is predominantly an "excessive" creation of firms: The primary reason is that a firm is more inclined to accept the creation of a new firm than it would if research did not suffer from moral hazard in teams problems. As a firm only decides to sell an innovation once it holds it, more sequel firms are created in equilibrium, than desirable from an ex-ante perspective. The issues discussed gradually vanish as the number of firms in the industry becomes large.

1.2 Related Literature

Several theories have been proposed to rationalize the creation of sequel firms:

A first stream comes from the resource-based view of the firm, developed in Wernerfelt (1984), Dierickx and Cool (1989), Chatterjee and Wernerfelt (1991), Peteraf (1993), and often attributed to Penrose (1959) and Chandler (1962). This view, ascribes high creation of value to particularly competitive and scarce resources and capabilities of the firm, which are not only specialized to a restricted set of tasks and environments but also imperfectly mobile. Then local dominance and high switching costs open-up benefits of firm creation when new products are away from the parent firm dominant area. In Cassiman and Ueda (2006), the commercialization capacity of the innovating firm is limited. The firm rejects the commercialization of the innovations with lowest fit with its internal resources. These are externally commercialized by a sequel firm, if set-up costs are not excessive. In Habib, Hege and Mella-Barral (2013), a sequel firm is similarly created if the fit between the product and its parent firm organization is not adequate. As mentioned, we borrow the basic rationale for firm creation from this view.

A second stream of theories is based on asymmetries of information and private learning. In Anton and Yao (1995), agents generate ideas and do not reveal them to their principals in order to create their own firm. A parent firm exploitation of the invention is the joint profitmaximizing outcome, but sequel firm creation cannot be prevented because the principal and the agent face (i) adverse selection due to private discovery, (ii) limited liability of the agent, and (iii) patents do not provide complete protection for parents. In Agarwal et al. (2004), Franco and Filson (2006), Franco and Mitchell (2008), employees privately learn from their employers and they exploit this knowledge by forming a spinoff.

A third stream of theories is based on imperfect evaluations of opportunities. In Klepper and Sleeper (2005), an agent creates a sequel firm because the parent firm either (i) does not recognize an opportunity but an agent does, or (ii) recognizes the opportunity but considers the probability that one of its' agents also recognizes it to be low enough that it gambles that a sequel firm will not be created. In Keppler (2007) and Keppler and Thompson (2010), a firm's strategy regarding implementation of opportunities is chosen by a team of decision makers who each imperfectly evaluate these opportunities. An individual manager chooses to start his firm when his disagreement with the firm strategy exceeds the cost of setting up a sequel firm.

A fourth related literature is about the strengths and weaknesses of internal versus external capital markets. Although not formulated in this context, the arguments directly extend to sequel firm creation versus internal exploitation of innovations.

In Gertner, Scharfstein and Stein (1994), with external financing (and by extension with the creation of a sequel firm), control rights reside with the manager. They are not given to corporate headquarters. Managers have therefore higher ex ante effort incentives, because they are not vulnerable to ex post opportunistic behaviour by corporate headquarters. However, assets cannot be easily redeployed to related business units, if the project fails. In Amador and Landier (2003), sequel firm creation is attributed to the greater contractual flexibility of external versus internal financing. Implementing an idea inside an existing organization allows the sharing of assets and might therefore be cheaper, but it is harder to reward the manager with the cash-flows generated by his project in the existing firm than in a new firm. Gromb and Scharfstein (2003) focus on the redeployability of people, not assets. Entrepreneurs who fail after creating a firm must seek jobs in an imperfect labor market, whereas they can be redeployed internally, if they remain in the parent firm. Safety being bad for incentives, sequel firm creation provides entrepreneurs with high-powered incentives ex ante. It is then more frequent when the external labor market for managers is deep, hence the value of internal labor market is low.

In our paper, the benefit of sequel firm creation also originates in moral hazard problems. Ours is however a Holmstrom (1982) moral hazard in teams problem. Then the value captured by the parent firm is not an overall increment, but a transfer away from competitors for innovations. This value also comes from a greater contractual flexibility, but across separate teams, not separate cash flows.

In our firm creation trade-off, the cost of creating a firm is not based on a distinct element, such as a higher cost of implementation or a lower redeployability of either people or assets. Our disadvantage of creating a sequel firm comes directly from the same argument. It is based on the repeated nature of the innovation game, through a reduction in probability of innovating, which originates in the same moral hazard in teams problem.

The paper is organized as follows: Section 2 describes the set-up of the model. Section 3 establishes the equilibrium strategy. Section 4 studies the dynamics of firm creation. Section 5 examines the implications on the dynamics of observable firm characteristics. It then discusses empirical support. Section 6 assesses the extent to which the equilibrium behaviour is inefficient. Section 7 discusses extensions. Section 8 concludes.

2 Set-Up

2.1 Firms and Innovations

Consider an industry which initiates at date 0 and evolves in discrete time from date $t = 1$ onwards, for T periods. Period $t \in \{1, \ldots T\}$ begins at date t. At date t, the industry consists of a (finite or infinite) set $\mathcal{F}_t$ of firms, with typical element denoted by $f \in \mathcal{F}_t$. A firm is a specialized organization capable of (a) producing innovations (exploration) and (b) generating cash from these innovations (exploitation).[4] Each firm f has an expertise $x^f \in \mathbb{R}$, selected upon firm creation against a set-up cost $\kappa \in (0, 1)$. A firm has and can only have one expertise. This expertise cannot be changed later.

The expertise of a firm, x^f, determines the characteristics of innovations it is most likely to produce: An innovation i is characterised by the required expertise to exploit it most profitably. We shall refer to this as the characteristic of the innovation and denote it by $x^i \in \mathbb{R}$. We measure the relative proximity between the ideal firm expertise demanded by innovation i and the actual expertise of firm f by

$$w(i, f) \;\equiv\; \exp\left[- \left|x^i - x^f\right|\right] \; . \qquad (1)$$

Consider that x^i follows a distribution located around the expertise of the firm which invents it, x^f. For convenience, suppose that, if firm f makes an innovation i, then x^i follows a

[4]The terminology exploration and exploitation is borrowed from March (1991).

Laplace distribution with probability density function[5]

$$f_{x^f}(x^i) \;=\; \frac{1}{2}\exp\left[-\left|x^i - x^f\right|\right] \; . \qquad (2)$$

The expertise of a firm, x^f, determines the characteristics of innovations it is most capable of generating cash from: If firm f makes an innovation i, it can exploit it. Consider that the exploitation of an innovation i made in period t, by firm f, generates a payoff equal to $w(i, f)$, at the end of the period.

2.2 Innovation Sale and Sequel Firm Creation

The inventing firm f does not however have to exploit itself the innovation i. In some instances the innovation is better exploited by a new firm, f^+, created specifically to do this. Firm f can benefit from tolerating the creation of a new firm: If someone is willing to pay more to do so than $w(i, f)$ to exploit the innovation in a newly created firm, firm f should consider selling the innovation.

The advantage brought by the creation of a new firm f^+, is that it gives the buyer of the innovation the opportunity to choose the expertise of the new firm she creates, *after* the innovation i is made. By setting a firm expertise $x^{f^+} = x^i$, one can insure that the proximity between the newly created firm f^+'s and the innovation i, $w(i, f^+)$ equals 1. The cost of creating a new firm is that the creation of a new firm f^+ entails the same set-up cost κ. Overall, the exploitation of an innovation i made in period t, by a newly created firm f^+, generates an alternative payoff equal to $1 - \kappa$, at the end of the period .

Essentially, exploitation within the inventing firm f can only be imperfect, because its' expertise was chosen before the innovation is made. Firm creation, is a costly way of overcoming this imperfection, because of the set-up cost κ. As a consequence of this trade-off, it is optimal for the innovation i to be implemented (a) in firm f when $w(i, f)$ is sufficiently high and (b) in a newly created firm f^+ when $w(i, f)$ is sufficiently low.[6]

[5]This is chosen to simplify the valuation exercise, as the proximity $w(i, f)$ is then uniformly distributed over $[0, 1]$, i.e. $f(w(i, f) = w) = 1$ for all $w \in [0, 1]$.

[6]Notice that since the payoff from exploitation of innovation i within firm f is $w(i, f)$ and the payoff from exploitation within a new firm is $1 - \kappa$, it follows that if the game were one-shot then the principal of firm f would implement the innovation in firm f if $w(i, f) > 1 - \kappa$, and sell it when $w(i, f) < 1 - \kappa$.

2.3 Participants and Industry Development

There is a countable infinite number of deep-pocketed financiers who can potentially create a firm, becoming its principal, but are unable to innovate. Crafting innovations requires skilled individuals, that henceforth are referred to as agents. These skilled individuals are however penny-less, hence must be given access to a firm's organization (by its' principal), to realize their potential. There is a countable infinite number of agents, equally capable of producing innovation, all with reservation value equal to zero. We denote the set of principals and agents by $\mathcal{N}^p$ and $\mathcal{N}^a$, respectively. All discount future cash flows at the same unit period rate of interest $\rho \in \mathbb{R}_+$.

The game starts at date 0. At 0, no innovation exists. Any financier can create a (research) firm and become its principal. Upon creation of a firm f, its principal p chooses once and for all the expertise of the firm, $x^f \in \mathbb{R}$, against a set-up cost κ. Nothing else happens in this initial period.

The game is then repeated at each date $t \in \{1, \ldots T\}$, for T periods. Denote $\mathcal{F}_t$ the set of firms created up to date t, and let $F = |\mathcal{F}_t|$ be their number.[7] Firms compete to innovate, hence we assume $\mathcal{F}_1$ is not a singleton and $F > 1$. At date t,

1 – The principal p of each existing firm f selects $\mathcal{A}_{f,t} \subset \mathcal{N}^a$, the set of agents she wishes to employ for the coming unit-period t, and makes a take-it-or-leave-it contract offer $b_{f,t}$ to each agent in $\mathcal{A}_{f,t}$ simultaneously (hence all the bargaining power is with the principal). We describe in the next subsection the contents of a unit-period employment contract offer $b_{f,t}$. Denote $A_{f,t} \equiv |\mathcal{A}_{f,t}|$ the number of agents offered employment in firm f at date t.

2 – The solicited agents receive the offers made by the principals. We assume for simplicity that if more than one firm proposes to the same agent, the agent receives only one of the offers and the probability of receiving any of them is positive.

3 – Each agent individually responds to the offer he receives. At the time the agent receives an offer from f at date t, the agent knows, in addition to past history before date t, the proposal of firm f and the set of agents, $\mathcal{A}_{f,t}$ who receive offers from firm f at t; but he does not know the contracts offered by other firms and the set of agents to which other firms make an offer to at date t.[8]

We shall denote the response of the agent a to firm f proposal at date t by $r^a_{f,t}$ where,

[7] F could be finite or infinite. However, due to fixed cost of setting up a firm κ, F will be finite in equilibrium.

[8] The details of the extensive form information are not important for the results. We have assumed these partly for realism and partly to simplify the exposition.

$r^a_{f,t} = 1$ refers to a accepting the offer and $r^a_{f,t} = 0$ refers to rejecting the offer. Denote $\mathcal{A}^*_{f,t} = \{a' \in \mathcal{A}_{f,t} \mid r^a_{f,t} = 1\}$ the set of agents who accept the offer from f at date t (so $\mathcal{A}^*_{f,t} \subseteq \mathcal{A}_{f,t}$).

We will assume that the principal p of a firm f which employed a set of agents $\mathcal{A}^*_{f,t-1}$ in the previous period $t-1$, has a biais for offering to employ again these agents in period t.[9] Hence in step **1** above, $\mathcal{A}_{f,t} \subset \mathcal{A}^*_{f,t-1}$ if $A_{f,t} \leq |\mathcal{A}^*_{f,t-1}|$, and $\mathcal{A}_{f,t} \supset \mathcal{A}^*_{f,t-1}$ if $A_{f,t} \geq |\mathcal{A}^*_{f,t-1}|$. Thus, if the principal p wants to expand the number of employees, all the firm's exiting employees receive an offer, and if she wishes to shrink the number of agents, it only makes offer to existing employees.

4 – Each accepting agent $a \in \mathcal{A}^*_{f,t}$ chooses $e^a_{f,t} \in \mathbb{R}_+$, the effort he is willing to exert in this unit-period, bearing a private cost $e^a_{f,t}$. The efforts of the agents employed in firm f determine the research output of the firm in this unit-period, $n_{f,t}$, according to the following concave and increasing function:

$$n_{f,t} = \left(\sum_{a \in \mathcal{A}^*_{f,t}} e^a_{f,t} \right)^\alpha, \tag{3}$$

where $\alpha \in (0,1)$. The parameter α being the rate of transformation of effort in research output, it captures the importance of agents in exploration (research output).[10] Notice that the firm's research output, $n_{f,t}$, is a sub-additive function of its' team of agents' efforts, which captures the feature that work in group entails coordination problems.[11]

5 – Research output results in an innovation. In this paper, we assume that one innovation happens at the end of each period. Denote i the innovation made at the end of unit-period t. The probability, $q_{f,t}$, firm f makes the innovation i depends on its research output relative to the total:

$$q_{f,t} = \frac{n_{f,t}}{\sum_{f' \in \mathcal{F}_t} n_{f',t}}, \tag{4}$$

if there exists $f' \in \mathcal{F}_t$ such that $n_{f',t} \neq 0$. This is a race model, hence if no effort is exerted by anyone (i.e. $n_{f',t} = 0$ for all $f' \in \mathcal{F}_t$), all firms have an equal probability $q_{f,t} = 1/F$ of making the innovation. The characteristic, x^i, of innovation i is chosen according to the distribution $f_{x^f}(x^i)$ in (2).

[9] A natural justification for this assumption is that switching to an non experimented agent typically involves adjustment costs. This is more natural than assuming that any agent can only be employed for a single period, or assuming some random matching at each period.

[10] It is less than one, otherwise agents would not exert finite levels of effort.

[11] The research output of one agent a is $\left(e^a_{f,t}\right)^\alpha$. The firm's research output, $n_{f,t}$ in (3), is here a sub-additive function of the efforts of its agents, in that $n_{f,t} < \sum_{a \in \mathcal{A}^*_{f,t}} \left(e^a_{f,t}\right)^\alpha$ (given that $\alpha \in (0,1)$).

6 – Payments to employed agents according to contracts, $b_{f,t}$, are made at the end of the period. Inventing agents relinquish all control rights on the innovation to the principal who employed them.

7 – The principal of the successful firm (the firm that innovates) f decides at the end of the period to implement the innovation in firm f or to sell it to a financier p^+. This financier cannot be the principal of a non-inventing existing firm.[12] At the time she makes this decision at the end of period t, she knows, in addition to the previous history of play before period t and the identity of the innovation i, the set of agents who have accepted her offer at date t, but she does not know the contracts offered by other firms and the set of agents to which other firms make an offer to at date t. We shall denote the denote the decision of the principal of successful firm by $d_{f,t}$ where, $d_{f,t} = 1$ refers to implementing the innovation in the successful firm f and $d_{f,t} = 0$ refers to selling the innovation to a new financier p^+. Clearly, for any innovation i, $d_{f,t}$ depends on $w(i, f)$, the relative proximity of i from the successful firm f.

The financier p^+ who just bought the innovation i creates a new (sequel) firm and becomes its principal. Upon creation of firm f^+ at the end of period t, its principal p^+ chooses once and for all the expertise of the firm, $x^{f^+} \in \mathbb{R}$, against a set-up cost κ. Already created firms continue existing unchanged without incurring again set-up costs.[13]

Since we assume there is a countable infinite number of deep pocketed financiers, the price at which any innovation is sold is set to be the competitive price such that the principal p^+ of the new firm obtains zero payoff from creating a firm f^+.

When moving to the next period, $\mathcal{F}_{t+1}$, the set of firms created up to date $t+1$, is equal to $\mathcal{F}_t$, if $d_{f,t} = 1$ and the innovation i is implemented in the successful firm f. Conversely, $\mathcal{F}_{t+1}$ is equal to $\mathcal{F}_t \cup f^+$, if $d_{f,t} = 0$ and the innovation is sold to a financier p^+ who creates a new firm f^+.

The timeline repeated at period $t \in \{1, \ldots T\}$ is illustrated in Figure 1. At the end of period T, the continuation value of all principals and agents in $\mathcal{N}^p$ and $\mathcal{N}^a$ is equal to zero.

[12] The setup above is fairly general as nothing precludes the principal p^+ from being the principal p of the inventing firm f (in which case p simply creates a new firm without selling the innovation). However, to simplify the analysis, we do not allow an innovation to be sold to a non-inventing existing firm, $f' \in \mathcal{F}_t \setminus \{f\}$, whose expertise is closer to the innovation. In Section 7.2 we discuss how to capture this alternative and the extent to which it affects our results.

[13] To simplify the set-up, we do not allow entrance of a financier who does not hold any innovation at the end of each period $t > 0$, in the same way it is allowed at date 0. This is justified when (i) the value of a firm created without innovation decreases with the number of existing firms and/or (ii) when the costs of setting up such a firm increases with the number of existing firms.

2.4 Employment Contracts

The principal p of firm f selects the unit-period contract, $b_{f,t}$, she offers each agent at the beginning of each period t, in order to incentivise agents to exert efforts.

We assume that contracts are incomplete in that an agent's effort level, $e^a_{f,t}$, and a firm's research output, $n_{f,t}$, are not contractible. Contracts can be written on any observable outcome at the time of payment (stage 6 of the game), which includes (i) whether the firm is successful in making the innovation and (ii) the characteristics of the innovation. Since a firm agent's effort level influence the likelihood of devising the innovation, $q_{f,t}$, but not the characteristics of the innovation itself, x^i, the optimal incentive contract belongs to the set of contracts which involve two mutually exclusive fixed compensations: One if the firm's team of agents is successful, and one if the firm's team of agents is unsuccessful.

We assume that contracts cannot impose penalties on the agents, hence contracted ex-post payoffs are non-negative. This directly implies that in any optimal contract, the payment promised to an agent, in case the firm's team of agents is unsuccessful in innovating, will be set to zero: The principal has no reason to give any reward for failing.

The optimal contract, $b_{f,t}$, is therefore characterized by a single payment, as follows: If the agents are successful in making the innovation i, each agent $a \in \mathcal{A}^*_{f,t}$ employed by firm f receives a fixed compensation $b_{f,t} \in \mathbb{R}_+$. After receiving this payment, all control rights on the innovation are relinquished by the inventing agents to the firm's principal.

If an innovation is made, the p principal therefore pays either $A_{f,t}\, b_{f,t}$ to her agents, and receives the remaining value of the innovation. In case the p principal then decides to implement the innovation within firm f, the innovation value is the value of the proceeds from exploitation. In case the innovation is implemented in a newly created firm f^+, the innovation value is the amount any principal, p^+, is competitively willing to pay to create such a firm with full rights to exploit this innovation.

3 Equilibrium Behaviour

The equilibrium concept that we consider is perfect Bayesian equilibrium (PBE).[14] In general, the decision of each agent at any date in such an equilibrium may depend on the entire

[14] We employ this concept because the game is a multi-stage game, in which in each stage/date, some players do not know the information other players have. Here, at any date t, the agents that receive a contract offer from firm f do not know the contract offered to the agents receiving offers from other firms $f' \in \mathcal{F}_t \setminus \{f\}$.

history of the past. Here, we limit ourselves to Markov perfect Bayesian equilibria in which the decision at each date depends on payoff relevant states.

The payoff relevant state at any date t in our set-up is the set of existing firms, $\mathcal{F}_t$, and the set of agents employed by the different firms in the previous period, $\mathcal{A}^*_t \equiv \{\mathcal{A}^*_{f,t-1}\}_{f\in\mathcal{F}_{t-1}}$. Thus, in describing such an equilibrium we write the decision problem of each player in terms of a state variable $s = (\mathcal{F}, \mathcal{A}^*)$, where $\mathcal{F}$ is the set of existing firms at the beginning of the period, and $\mathcal{A}^*$ is the set of agents employed by the different firms in the previous period. Denote the set of all such state variables $s = (\mathcal{F}, \mathcal{A}^*)$ by $\mathcal{S}'$.

We can then formally describe a Markov PBE as $\hat{E} = \{\ \hat{\mathcal{A}}_{f,s,t};\ \hat{b}_{f,s,t};\ r^a_{f,s,t} : 2^{\mathcal{N}^a} \times \mathbb{R}^2_+ \to \{0,1\};\ e^a_{f,s,t} : 2^{\mathcal{N}^a} \times \mathbb{R}^2_+ \to \mathbb{R};\ d_{f,s,t} : 2^{\mathcal{N}^a} \times \mathbb{R}^2_+ \to \{0,1\}\ \}_{a\in\{\hat{\mathcal{A}}_{f,s}\}, f\in\mathcal{F}, s\in\mathcal{S},\text{ and } t\in\{1,\ldots T\}}$, where, for any $s = (\mathcal{F}, \mathcal{A}^*)$ and t,

– $\hat{\mathcal{A}}_{f,s,t} \in \mathcal{N}^a$ is the set of agents firm f proposes to in period t;

– $\hat{b}_{f,s,t} \in \mathbb{R}^2_+$ is the unit period contract proposal by firm f in period t;

– $r^a_{f,s,t}(\mathcal{A}, b) \in \{0,1\}$ is the response of agent a to a proposal by firm f of a contract b made to a set of agents $\mathcal{A}$ in period t;

– $e^a_{f,s,t}(\mathcal{A}, b) \in \mathbb{R}_+$ is the effort level of agent a in period t when employed in firm f, given that this firm employs a set of agents $\mathcal{A}$ with contract b;

– $d_{f,s,t}(\mathcal{A}, b, w) \in [0,1]$ is the implementation decision at the end of period t of the principal p of an inventing firm f, given that the set of agents employed by the firm $\mathcal{A}$ have contract b, and that the relative proximity between the innovation and the inventing firm f is w.

A subset of Markov PBEs are those and that are symmetric in the sense that all principals choose the same contract and the same number of agents to propose to, all agents respond the same way to any offer and all principals have the same policy regarding implementation of innovation.[15] Formally, a Markov PBE $\hat{E} = \{\ \hat{\mathcal{A}}_{f,s,t};\ \hat{b}_{f,s,t};\ r^a_{f,s,t}(.);\ e^a_{f,s,t}(.);\ d_{f,s,t}(.)\}_{a\in\{\hat{\mathcal{A}}_{f,s,t}\}, f\in\mathcal{F}, s\in\mathcal{S},\text{ and } t\in\{1,\ldots T\}}$, is said to be symmetric if $\hat{\mathcal{A}}_{f,s,t} = \hat{\mathcal{A}}_{f',s,t}$, $\hat{b}_{f,s,t} = \hat{b}_{f',s,t}$, $r^a_{f,s,t}(\mathcal{A}, b) = r^{a'}_{f',s,t}(\mathcal{A}, b)$, $e^a_{f,s,t}(\mathcal{A}, b) = e^{a'}_{f',s,t}(\mathcal{A}, b)$ and $d_{f,s,t}(\mathcal{A}, b, w) = d_{f',s,t}(\mathcal{A}, b, w)$ for all f, f', a, a',and w. Hence, for any symmetric Markov PBE $\hat{E} = \{\ \hat{\mathcal{A}}_{f,s,t};\ \hat{b}_{f,s,t};\ r^a_{f,s,t}(.);\ e^a_{f,s,t}(.);\ d_{f,s,t}(.)\ \}_{a\in\{\hat{\mathcal{A}}_{f,s,t}\}, f\in\mathcal{F}, s\in\mathcal{S},\text{ and } t\in\{1,\ldots T\}}$, to simplify notation, we denote respectively $\hat{\mathcal{A}}_{f,s,t}$, $\hat{b}_{f,s,t}$, $r^a_{f,s,t}(.)$, $e^a_{f,s,t}(.)$ and $d_{f,s,t}(.)$ by $\hat{\mathcal{A}}_{s,t}$, $\hat{b}_{s,t}$, $r_{s,t}(.)$, $e_{s,t}(.)$ and $d_{s,t}(.)$ for all f, a, s and t and describe the equilibrium by $\hat{E} = \{\ \hat{\mathcal{A}}_{s,t};\ \hat{b}_{s,t};\ r_{s,t}(.);\ e_{s,t}(.);\ d_{s,t}(.)\}_{s\in\mathcal{S}\text{ and } t\in\{1,\ldots T\}}$.

In the appendix, we provide (Theorem 1) the symmetric Markov PBE strategy $\hat{E}$ that

[15] Note that symmetry here requires that all principles choose the same contract and the same number of agents; it does not, however require that all principals make offers to the same set of agents. In fact given our assumptions, it is trivial to show that in any equilibrium no two principles make offers to the same agent

induces the continuation payoffs and outcome path given below in Proposition 1. We show that the decisions of any principal and any agent in equilibrium strategy $\hat{E}$ at any date with any state $s = (\mathcal{F}, t, \mathcal{A}^*)$ have the following properties:

– First, the set of agents employed in the previous period $\mathcal{A}^*$ affects no decision other than the choice of agents the prinicipal chooses to make offer to. More precisely, if the principal employed k agents in the previous period then the equilibrium strategy require the principal to select each of these agents with probability $\frac{1}{k}$all decisions.

– Second, the decisions of any principal and any agent in depend only on the number of existing firms with $|\mathcal{F}|$ and not on their identities.

– Third, for any offer $b \in \mathbb{R}_+$ to any set of agents $\mathcal{A}$ the response of each one is to accept the offer.[16]

– Fourth, for any set of agents $\mathcal{A}$ employed by a principal and any offer $b \in \mathbb{R}_+$, the effort level of any agent as well as the implementation decision of the principal depend on the number of agents $|\mathcal{A}|$ employed by the firm and not the identity of these agents.

The second, third and the fourth properties described above make the equilibrium strategy $\hat{E}$ anonymous with respect to the identity of agents and firms. As our purpose is to analyse the equilibrium behaviour, the body of the text now focuses on the induced outcome path.

Proposition 1 *There exists a symmetric Markov perfect Bayesian equilibrium strategy* $\hat{E}$ *that induces the following continuation payoff and outcome path:*
At date 0*, the number of firms created is* F_0 *such that* $V^p_{F_0+1,1} < \kappa \leq V^p_{F_0,1}$.
At any date $t \in \{1, \ldots T\}$*, for any* $s = (\mathcal{F}, \mathcal{A}^*)$ *with* $|\mathcal{F}| = F > 1$*,*

– (i) The continuation equilibrium payoff of any principal p *and an agent* a*,* $V^p_{F,t}$ *and* $V^a_{F,t}$*, respectively, satisfy the backwards recursive equations*

$$V^p_{F,t} = (1 - s_F)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{(1+\rho)\, F}\right] + \frac{(1 - \hat{\omega}_{F,t})\, V^p_{F,t+1} + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{1+\rho}\ , \tag{5}$$

$$V^a_{F,t} = s_F\left(1 - \alpha\left(1 - \frac{1}{F}\right)\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{(1+\rho)\, F}\right] + \frac{(1 - \hat{\omega}_{F,t})\, V^a_{F,t+1} + \hat{\omega}_{F,t}\, V^a_{F+1,t+1}}{1+\rho}, \tag{6}$$

$$\textit{where} \quad s_F \equiv \alpha\left(1 - \frac{1+\alpha}{F+\alpha}\right)\ , \tag{7}$$

$$\bar{\pi}(\omega) \equiv (1-\kappa)\,\omega + \frac{1-\omega^2}{2}\ , \tag{8}$$

$$\hat{\omega}_{F,t} \equiv \max\{0\,, \min\{\omega^*_{F,t}\,, 1\}\}\ , \quad \textit{with} \quad \omega^*_{F,t} \equiv 1 - \kappa + 2\, V^p_{F+1,t+1} - V^p_{F,t+1}\ , \tag{9}$$

[16]This follows because $\mathbf{b} = (b, b^+) \in \mathbb{R}^2_+$, agents could always exert zero effort and the outside option of each agent is zero.

and $V^a_{F,T+1} = V^p_{F,T+1} = 0$, *for any* F*;*

– *(ii) if firm* f *is set up at date* t *then* f *proposes to one agent at* t *and continues making an offer to the same agent at any subsequent date;*

– *(iii) the offer that firm* f *offers to the agent if he makes an innovation a compensation*

$$\hat{b}_{F,t} \equiv s_F \left(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t} \, V^p_{F+1,t+1} \right) \; ; \tag{10}$$

– *(iv) the agent always accepts the offer* $\hat{b}_{F,t}$*;*

– *(v) the agent exerts an effort;*

$$\hat{e}_{F,t} = \alpha \, \frac{\hat{b}_{F,t}}{(1+\rho)\, F} \, (1 - 1/F) \; ; \tag{11}$$

– *(vi) The principal of the firm that innovates* f *decides to implement the innovation in the firm if and only if* $w \geq \hat{\omega}_{F,t}$ *and to sell it to a financier if* $w < \hat{\omega}_{F,t}$*.*

The intuition behind the expressions in Proposition 1 is as follows: The continuation payoffs of a principal p and an agent a at date t, $V^p_{F,t}$ and $V^a_{F,t}$ in (5) and (6) have several contributions to values.

To start with, $\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t} \; V^p_{F+1,t+1}$ is the expected proceeds to firm f from the innovation to be made at the end of the period, conditional on making the innovation. That is, the expected value of (i) either the value $w(i,f)$ of the cash flows firm f generates from exploiting itself the innovation, when the proximity $w(i,f) \geq \hat{\omega}_{F,t}$ is close enough, or (ii) the price $1 - \kappa + V^p_{F+1,t+1}$ a financier is willing to pay for an innovation, when $w(i,f) < \hat{\omega}_{F,t}$ and the innovation is sold out. Given that the equilibrium probability firm f makes this next innovation is $1/F$, and the discount rate is ρ, the square bracket in the first term on the RHS of (5) (similarly for (6)) is the expected value at date t of the next innovation.

It is optimal for any principal p to propose to one agent a a unit-period compensation $\hat{b}_{F,t}$, conditional on success, which is about equal to a fraction α of the expected proceeds from the next innovation, $\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t} \; V^p_{F+1,t+1}$.

The firm's research output, $n_{f,t}$ in (3), is a sub-additive function of its' team of agents' efforts. Then it is not just that, because of the moral hazard in team problem, the principal does not benefit from employing more agents. The principal actually prefers a research team with minimum number of agents, to minimize coordination problems. For parsimony, the model does not consider counter benefits from having more than one agent, such as knowledge aggregation across employed agents. The desired minimum number of agents is then determined and equal to one.

The amount promised to the agent is not precisely a fraction α, but a a fraction s_F, for the following reason: The agent a knows that in (4) any extra effort (i) not only increases the probability firm f makes the next innovation, but also (ii) reduces the probability another firm makes it. Absent the second effect, it would be optimal for the principal to simply give a compensation where $s_F = \alpha$. The second effect however enhances the agent's willingness to exert effort, hence reduces the need for the principal to compensate him ex-ante for doing so. Notice that this second effect diminishes and vanishes as the number of firms F increases, which explains that s_F in (7) increases in F and $\lim_{F\to\infty} s_F = \alpha$. The remaining fraction, $1 - s_F$, of the expected proceeds from the innovation accrues to the principal, p.

The agent a is willing to incur private costs about equal to a fraction α of his expected compensation $\frac{\hat{b}_{F,t}}{(1+\rho)\,F}$. The agents's private cost of effort differs from $\alpha\,\frac{\hat{b}_{F,t}}{(1+\rho)\,F}$ for the two following reasons: First, the willingness of agent a to incur a private cost of effort is determined by the influence effort $e^a_{f,t}$ on the probability firm f makes the innovation, $q_{f,t}$, as captured by (4). Here, $e^a_{f,t}$ not only increases the research output of firm f (the numerator $n_{f,t}$), but also increases the overall research output made by all firms (the denominator $\sum_{f'\in\mathcal{F}_t} n_{f',t}$). This latter effect reduces agent a'se willingness to exert private effort by a factor $\frac{1}{F}$. Notice that here also, this effect reduces and vanishes as the number of firms F increases.

The expressions of $V^p_{F,t}$ and $V^a_{F,t}$ in (5) and (6) also involve a second term on the RHS. This is the date t expected continuation value at date $t+1$, the beginning of the next period. With probability $1-\hat{\omega}_{F,t}$, the number of firms will not be altered by then. With probability $\hat{\omega}_{F,t}$, the number of firms will then be increased by one.

The threshold, $\hat{\omega}_{F,t}$ in (9) characterizes the implementation of the innovation to be devised in period t. The implementation decision is be taken by the principal p of the firm which makes the innovation. When deciding, the principal p knows she holds an innovation and knows the innovation characteristics. Having paid $\hat{b}_{F,t}$ to the inventing agent just before the end of the period t, she holds all property rights on the innovation and examines the two options available to her at that point.

- If the innovation is implemented in the inventing firm f, the principal's payoff is the exploitation value of the innovation value itself after compensating for the agent, $w(i, f)$, plus her continuation payoff at the beginning of the next period, which is $V^p_{F,t+1}$ given that there would still be F firms.

- If she sells the innovation, she is able to sells it at the competitive price such that the new principal p^+ obtains zero payoff from creating a firm f^+ at that time. Now, the market value of the innovation is the exploitation value of the innovation itself plus the continuation value of the new firm's principal at the beginning of the next period $V^p_{F,t+1}$, i.e $1-\kappa+V^p_{F+1,t+1}$. Here, the parent firm is only able to internalize the willingness to pay of the new principal, but is unable to internalize the continuation value of the new firm's agent. This is because the new principal knows she will in turn give equilibrium optimal incentives to the new firm's agent, and the value of these optimal incentive contracts is precisely the continuation value of the new firm's agent. The principal's payoff is then be the proceeds from the sale, $1-\kappa+V^p_{F+1,t+1}$, plus her continuation payoff at the beginning of the next period, which is $V^p_{F+1,t+1}$ given that with the new firm set up, there would then be $F+1$ firms.

The principal's indifference threshold between inside implementation (LHS) and selling out (RHS) is then such that

$$\omega^*_{F,t} \ + \ V^p_{F,t+1} \ = \ 1-\kappa \ + \ V^p_{F+1,t+1} \ + \ V^p_{F+1,t+1} \ . \tag{12}$$

The threshold, $\hat{\omega}_{F,t} = \max\{0, \min\{\omega^*_{F,t}, 1\}\}$ in (9) is simply a transformation of $\omega^*_{F,t}$ which ensures that $\hat{\omega}_{F,t} \in [0,1]$. The inventing firm implements the innovation inside if the proximity w is greater than $\hat{\omega}_{F,t}$ in (9) and sells out to a financier otherwise.

Turning to the initial date $t=0$ the industry begins: A financier finds it worthwhile to create a firm without an innovation as long as a principal's continuation value is higher than the firm set-up cost κ. Now, a principal's continuation value, $V^p_{F,1}$, is decreasing in the number of participating firms, F. Therefore, creating a firm, without an innovation in hand, is worthwhile as long as F is less than a lower bound F_0. At date $t=0$, the number of firms created without an innovation, F_0, is then the highest integer such that the continuation value of a principal $V^p_{F_0,1}$ is greater or equal to κ.[17]

4 Dynamics of Firm Creation

The equilibrium firm creation threshold in (9), $\hat{\omega}_{F,t} = \max\{0, \min\{\omega^*_{F,t}, 1\}\}$ where $\omega^*_{F,t} = 1-\kappa + 2\,V^p_{F+1,t+1} - V^p_{F,t+1}$, differs from $1-\kappa$. $1-\kappa$ is the exploitation value of the innovation in a newly created firm, net of firm set-up cost. It is the competitive amount an outside financier is willing to pay, just to exploit the innovation. Two forces push $\hat{\omega}_{F,t}$ away from $1-\kappa$:

1. An outside financier is willing to pay more than $1-\kappa$ for the innovation. As she will create a new firm, she is also willing to pay for the continuation value becoming

[17]Notice that, as $|\mathcal{F}_t| \geq F_0$ for all $t \geq 0$, no firm without innovation in hand would be worthwhile creating later.

a principal provides her. The creation of a firm therefore allows the inventing firm's principal to also internalize this additional willingness to pay, whose value is equal to $V^p_{F+1,t+1}$. This pushes an inventing firm to create firms more frequently.

2. The creation of a new firm however reduces the continuation value of any existing principal. This because an additional firm will be competing to devise further innovations. The continuation value of the inventing firm's principal is then reduced by $V^p_{F,t+1} - V^p_{F+1,t+1}$. This pushes an inventing firm to create firms less frequently.

These two forces are only the result of problems of moral hazard in research teams. Absent these problems, an outside financier would not be willing to pay more for the innovation than the exploitation value, because all exploration value would already be fully internalized by existing firms.[18] Becoming the principal of a new firm would not provide positive continuation value. There would consequently be no reduction in continuation value of existing principals from the creation of an additional exploitation capacity. No force would push the equilibrium firm creation threshold away from $1 - \kappa$.

These forces are dynamic in that they are only present in a competition to innovate when later there will be further competition to innovate. Notice how they vanish at a date when there will no subsequent future innovation: When firms compete to make the last innovation at date $t = T$, the game is to finish at the end of period. Then, continuation values at date $t + 1$ are equal to zero, $V^p_{F,T+1} = V^p_{F+1,T+1} = 0$, and we have $\hat{\omega}_{F,T} = 1 - \kappa$.

Consider the following Property:

Property 1 *The equilibrium firm creation threshold,* $\hat{\omega}_{F,t}$*,*

- *(i) exceeds* $1 - \kappa$*;*
- *(ii) is decreasing in* F *and* t*;*
- *(iii) is decreasing in* α*,* ρ *and* κ*.*
- *(iv) tends to* $1 - \kappa$*, when* F*,* α *or* ρ *is very large.*

[18]Suppose research in a group was different, and success in devising an innovation could be completely attributed to one single agent, without any contribution to it by any other agent in the team. Contracts could be written on a more precise observable outcome: Not just that the firm's team of agents is successful (as here), but more specifically that one given agent has full paternity of the innovation and that other team members have no claim on this success. When employing several agents in a team, the principal could offer each one of them incentive contracts consisting of an exclusive compensation to the one successful agent. There would be no moral hazard in teams problem: The sum across the agents in a team, of the efforts exerted by each agent, would be larger when the team involves more agents. In the race to innovate, each existing firm principal would offer this type of contract to as many agents as possible.

We now establish that Property 1 holds in most circumstances, but that there however exist extreme instances where is does not hold.

4.1 Model with Two Periods, $T = 2$

We begin examining the simplest possible version of the model, rich enough to capture the dynamic forces which come from the repeated nature of innovations. Consider the case where there is only two innovations to be devised, hence the model with two periods, $T = 2$. Here, we can establish and understand well which of the two opposite dynamic forces above dominates.

The two dynamic forces we seek to analyze are present two periods before the end of the game. Hence, at date $t = 1$. At date $t = 1$, firms compete to innovate knowing there will be a further competition to innovate (one second round). The expression of the date $t = 1$ firm creation threshold $\hat{\omega}_{F,1}$ is established as follows:

- In period 2, firms only compete to innovate for one last period (as $T = 2$). There will be no subsequent innovation, hence the continuation value of a principal at the end of the period is equal to zero, i.e. $V^p_{F,3} = 0$ for any F. From (9), the date 2 firm creation threshold is then simply the static $\hat{\omega}_{F,2} = 1 - \kappa$.

- Backwards in time, from (5), the continuation value of a principal at the end of period 1 is then equal to $V^p_{F,2} = \frac{1-s_F}{F} \frac{\bar{\pi}(1-\kappa)}{1+\rho}$, for any F. At date $t = 1$, from (9), it follows that the firm creation threshold is

$$\begin{aligned}
\hat{\omega}_{F,1} &= \max\{0, \min\{\omega^*_{F,1}\,, 1\}\}\ , \quad \text{where} \\
\omega^*_{F,1} &= 1 - \kappa + \left[2\left(\frac{1 - s_{F+1}}{F+1}\right) - \frac{1 - s_F}{F}\right] \frac{\breve{\pi}}{1+\rho}\ ,
\end{aligned} \tag{13}$$

with s_F is defined in (7) and $\breve{\pi} \equiv \bar{\pi}(1-\kappa)$, hence from (8), $\breve{\pi} = [1 + (1-\kappa)^2]/2$.[19]

[19] A sufficient condition for $\hat{\omega}_{F,1} < 1$ for all F, is that $\kappa > 7 - \sqrt{47}$. Intuitively, if setting up a firm has only a marginal cost, then whatever the innovation, it is always beneficiary to set up a new facility to exploit it in perfectly adapted fashion. The sufficient condition then reversely stipulates that if the firm set-up cost, κ, is not marginal (if it is more than $7 - \sqrt{47} \simeq 0.15$), the creation of a new firm is not simply always desired.

In the Appendix, we show that

$$\begin{cases} \text{(i)} & \hat\omega_{F,1} \ > \ 1-\kappa \ ; \\ \text{(ii)} & \hat\omega_{F,1} \ > \ \hat\omega_{F+1,1} \ \ (\text{unless } \ \hat\omega_{F,1} = \hat\omega_{F+1,1} = 1) \ , \\ & \text{but } \ \hat\omega_{2,1} \ < \ \hat\omega_{3,1} \ \ (\text{unless } \ \hat\omega_{2,1} = \hat\omega_{3,1} = 1); \\ \text{(iii)} & \frac{\partial\,\hat\omega_{F,1}}{\partial\,\alpha} \ < \ 0 \ , \ \ \frac{\partial\,\hat\omega_{F,1}}{\partial\,\rho} < 0 \ \text{ and } \ \frac{\partial\,\hat\omega_{F,1}}{\partial\,\kappa} \ < \ 0 \ \ (\text{unless } \ \hat\omega_{F,1} = 1) \ ; \\ \text{(iv)} & \lim_{F\to+\infty} \hat\omega_{F,t} = 1-\kappa \ , \ \ \lim_{\alpha\to 1} \hat\omega_{F,t} = 1-\kappa \ \text{ and } \ \lim_{\rho\to+\infty} \hat\omega_{F,t} = 1-\kappa \ . \end{cases} \tag{14}$$

The comparative statics of the firm creation threshold in (14) can be directly understood because the effect of each parameter on $\hat\omega_{F,1}$ in (13) is separable: the factor $2\left(\frac{1-s_{F+1}}{F+1}\right) - \frac{1-s_F}{F}$ only depends on F and α, the factor $\frac{1}{1+\rho}$ only depends on ρ, the factor $\frac{1+(1-\kappa)^2}{2}$ only depends on κ.

The intuition is simplest to develop starting from the corner case where $\alpha \to 0$. In this case, the factor $2\left(\frac{1-s_{F+1}}{F+1}\right) - \frac{1-s_F}{F}$ is simply equal to $\frac{2}{F+1} - \frac{1}{F}$. Then, (1) the equilibrium threshold $\hat\omega_{F,1} > 1-\kappa$ when $\frac{2}{F+1} - \frac{1}{F} > 0$ and (2) $\hat\omega_{F,1} > \hat\omega_{F+1,1}$ when $\frac{2}{F+1} - \frac{1}{F} > \frac{2}{F+2} - \frac{1}{F+1}$.

Start with property (i) in (14). Suppose you own one of $F = 10$ shares of an unlevered firm, so you own a fraction $\frac{1}{10}$ of the firm. You would accept to be given for free a newly created share, when no other shares are being created: as you would then own a fraction $\frac{2}{11}$ of the firm instead. Intuitively, $\frac{2}{F+1} - \frac{1}{F} > 0$ when "accepting a free newly created share" is desirable. This is true for all values of $F > 1$.

Then, the intuition behind property (ii) in (14) is that accepting a free newly created share is (in absolute terms) less desirable when there is a large number of outstanding shares. Essentially, $\frac{2}{101} - \frac{1}{100}$ is much less than $\frac{2}{11} - \frac{1}{10}$. However, $\frac{2}{F+1} - \frac{1}{F} > \frac{2}{F+2} - \frac{1}{F+1}$ is not universally true: notice that $\frac{2}{(2+1)} - \frac{1}{2} = \frac{2}{(3+1)} - \frac{1}{3}$. This intuition holds, but for $F > 2$.

Moving to property 1 (iii) in (14), that the desirability of creating a firm decreases with firm set-up cost, $\frac{\partial\,\hat\omega_{F,1}}{\partial\,\kappa} < 0$, is very natural; that it decreases with the discount rate, $\frac{\partial\,\hat\omega_{F,1}}{\partial\,\rho} < 0$, is also very intuitive, given that benefits are obtained at the end on the period.

Consider now the influence of α. Given that in equilibrium, the principals give incentives to their agent which are increasing in the agents influence on success, α, the value of principals decrease in α. That is, both $V^p_{F,2}$ and $V^p_{F+1,2}$ decrease in α. It is then intuitive that the overall benefit $2\,V^p_{F,2} - V^p_{F+1,2}$ decreases in α. That is, the magnitude of the wedge between $\hat\omega_{F,1}$ and $1-\kappa$ decreases in α. As by property 1 (i), $\hat\omega_{F,1} > 1-\kappa$, we have that this wedge is positive, hence that $\frac{\partial\,\hat\omega_{F,1}}{\partial\,\alpha} < 0$.

The comparative statics (14) establish the following:

Proposition 2 *In the model with two-periods ($T = 2$), Property 1 holds for all $F > 2$.*

4.2 Model with a Large Number of Periods

What applies in the model with two-periods also applies in the last two periods of the general case model, when the number of periods $T > 2$. (1) The firm creation threshold in the last period, $\hat{\omega}_{F,T}$, is still equal to $1 - \kappa$. (2) Two periods away from the end, $\hat{\omega}_{F,T-1}$ is such that $\omega^*_{F,T-1}$ is still equal to the RHS of (13), for the following two reasons: (i) two period ahead continuation values, $V^p_{F,t+2}$, are all equal to zero; (ii) following period firm creation thresholds, $\hat{\omega}_{F,t+1}$, are all equal to $1 - \kappa$.

At dates t earlier than $T-1$, the expression of the firm creation threshold $\hat{\omega}_{F,t}$ increasingly deviates from the RHS of (13), as one works backwards in time the model. The deviations from the expression at date $t = T - 1$ have the following two intuitive origins: (i) two period ahead continuation values, $V^p_{F,t+2}$, become strictly positive; (ii) following period firm creation thresholds, $\hat{\omega}_{F,t+1}$, are increasingly different from $1 - \kappa$. Essentially, future period continuation values ($V^p_{F,t+i}$, for all F and $i \in \{2, \ldots T - t\}$) and differences in future period firm creation thresholds ($\hat{\omega}_{F,t+j}$, for all F and $j \in \{1, \ldots T - t\}$) have compound feedback effects on the current period threshold $\hat{\omega}_{F,t}$.

It is helpful is to consider the limit case where the number of periods T is pushed to be very large. We refer to this case as the long horizon model. We carry out a numerical implementation of this case, for the following reasons:

(a) In the long horizon model, there is no end of game horizon effect on decision thresholds, as the horizon is pushed to be very distant. Continuation values and firm creation thresholds at date t become time independent (as $T - t \to \infty$);

(b) Apart from arguably being the most natural one, the long horizon model has the strong benefit of giving a full sense of the potential overall magnitude of dynamic effects;

(c) The above mentioned further compound feedback effects are complex to analyse and the comparative statics of the firm creation threshold, $\hat{\omega}_{F,t}$, cannot be established with proof.

(d) Most importantly, we obtain that the intuition developed with the two-period model remains the prime driver of $\hat{\omega}_{F,t}$. The further compound feedback effects do not alter much the dynamics. Property 1 primarily holds, as for the model with two-periods.

Denote $\hat{\omega}_F \equiv \hat{\omega}_{F,t}|_{T-t\to\infty}$ the firm creation threshold at a date t distant from the end of the game, in the long horizon model. Figure 8 shows the impact of α and ρ on the dynamics of the firm creation threshold, around a central case $\{\alpha; \rho; \kappa\} = \{10\%; 10\%; 0.5\}$.[20] In all cases we observe that $\hat{\omega}_F > 1 - \kappa$. $\hat{\omega}_F$ is largest and most decreasing in F when the industry

[20]We take as a central case a firm set-up cost equal to $\kappa = \frac{1}{2}$ and only exhibit variations around α and ρ. This makes equilibrium deviations most understandable, in that, absent the dynamic trade-off this paper is about (and in the reference static first-best), there would be the creation of a firm in 50% of the cases.

is one where innovations are more frequent (low discount rate ρ, hence large laps of time between innovations). $\hat{\omega}_F$ is also largest and most decreasing in F, when the importance of agents transformation of effort in research output is small (α closer to the minimum 0), hence the principals give a small fraction α of the firm value to the agents.

Property 1 holds, but counter examples exist: Pushing both inputs to $\alpha = 95\%$ and $\rho = 100\%$ we obtain $\omega_{F=2} = 0.5062$, $\omega_{F=3} = 0.5233$ and then decreasing values of ω_F tending towards 0.5. As for the two-period model, Property 1 here holds for $F > 2$.

The essential message of our analysis is as follows: Property 1 holds, apart from extremely reduced number of firms in the industry. There is "excessive" creation of firms, particularly in young industries. The issue gradually vanishes as the industry becomes largely developed (as F becomes large).

5 Empirical Predictions and Support

We here examine the empirically observable implications of this equilibrium behaviour on the dynamics of (i) the frequency with which sequel firms are created in the industry, (ii) the focus of firms and (iii) the profitability of firms.

Denote Q_t^+ the probability a sequel firm is created in period t. Sequel firms are created when $w(i,f) \in (0; \omega_{F,t})$, and from (2), the proximity $w(i,f)$ is uniformly distributed over all possible proximities. So Q_t^+ is simply equal to the firm creation threshold, i.e. $Q_t^+ = \text{Prob}\left[w(i,f) < \omega_F\right] = \hat{\omega}_{F,t}$. As the number of firms in the industry, $F = |\mathcal{F}_t|$, can only increase over time, it follows that, when Property 1 holds, $Q_{t+1}^+ \leq Q_t^+$ for all $t > 0$. Hence:

Result 1 *When Property 1 holds, an innovation leads more frequently to the creation of a sequel firm, in (1) young industries, where (2) the expected number of innovations per year is high (low ρ), (3) research agents only have to be given a small fraction of expected firm value (low α) and (4) the cost of setting up a firm is small (low κ).*

The focus of firm f at date t is the similarity of the innovations in the portfolio of innovations exploited by firm f at that date. A direct measure of focus is the average proximity between the expertise demanded by innovations and the expertise of firm the firm which exploits them. That is,

$$\frac{1}{|\mathcal{I}_t^f|} \sum_{i \in \mathcal{I}_t^f} \exp\left[- \left|x^i - x^f\right|\right] , \tag{15}$$

where $\mathcal{I}_t^f$ is the set of innovations firm f has decided to exploit itself, up to date t. The larger is average proximity of innovations implemented in firm f, the more focused is the firm.

Given that, when there are $F = |\mathcal{F}_t|$ firms in the industry, an innovation $i \in \mathcal{I}_t^f$ is exploited by the inventing firm f when $w(i, f) \in [\hat{\omega}_{F,t}, 1)$, the above measure of focus simply increases with $\hat{\omega}_{F,t}$, for all firms $f \in \mathcal{F}_t$. It follows:

Result 2 *When Property 1 holds, the focus of firms is higher, in (1) young industries, where (2) the expected number of innovations per year is high (low ρ), (3) research agents only have to be given a small fraction of expected firm value (low α) and (4) the cost of setting up a firm is small (low κ).*

Similarly, given that the value of an innovations retained by a firm is $w(i, f)$ for $w(i, f) \in (\hat{\omega}_{F,t}, 1)$, and from (2), the proximity $w(i, f)$ is uniformly distributed over all possible proximities, the expected firm profitability of innovations implemented is $(\hat{\omega}_{F,t} + 1)/2$. It follows:

Result 3 *When Property 1 holds, firms realize higher profits from the innovations they implement, in (1) young industries, where (2) the expected number of innovations per year is high (low ρ), (3) research agents only have to be given a small fraction of expected firm value (low α) and (4) the cost of setting up a firm is small (low κ).*

In the long run, given that $\lim_{F \to +\infty} \hat{\omega}_{F,t} = 1 - \kappa$, the frequency with which sequel firms are created, the focus and the profitability of firms become stable.

These results are well in line with a series of empirical facts on firms spawning activity by Gompers, Lerner and Scharfstein (2005).

- Result 1 part (1) is in line with their finding that the frequency with which firms spawn decreases with age.
- Result 1 part (2) is in line with their finding that firms with more numerous high quality patents spawn more frequently.
- The correspondence of high frequency of sequel firm creation and firm focus in Results 1 and 2 is in line with their finding that diversified firms spawn less firms than focused ones. They observe that firms which report just one top 3-digit SIC segment have spawning levels that are 19% higher than those operating in multi segments.

- The correspondence of high frequency of sequel firm creation and firm profitability in Results 1 and 3 is in line with their finding that more profitable firms are more prolific spawners.

These results are also in line with empirical facts on the dynamics of firm focus and profitability.

- The correspondence of firm focus and firm profitability in Results 2 and 3 is in line with Lang and Stulz (1994), who find that firm diversification and Tobin's q are negatively related. They are also in line with Berger and Ofek (1995) who find that operating margin and ROA profitability measures are lower for diversified companies. Lins and Servaes (1999) find similar results in Japan and the United Kingdom. Note that Campa and Kedia (2002) find that the diversification discount is reduced once the endogeneity of the diversification decision is controlled for.
- Result 2 part (1) is in line with Denis Denis and Sarin (1997), who find that firms become less focused with age;
- Result 3 part (1) is in line with Eisenberg, Sundgren and Wells (1998), Majumdar (1997) and Loderer and Waechli (2009), who find that firms' profitability decreases with age.

6 Analysis of Inefficiencies

6.1 Aggregate Value and First-Best

We now study the extent to which the equilibrium outcome is inefficient. This first requires establishing the aggregate value of the industry to all players and determining the benchmark first-best strategy.

Let $W_{F,t}$ denote the continuation value of all players in $\mathcal{N}^p$ and $\mathcal{N}^a$ (the set of all principals and agents available), under the equilibrium outcome path in Proposition 1, at the beginning of a period t such that the number of existing firms is $F = |\mathcal{F}_t|$. $W_{F,t}$ is the aggregate value of (i) the discounted sum of cash flows expected to be generated from the exploitation of all innovations not yet devised, minus (ii) the discounted sum of all costs of efforts expected to be exerted by agents, under the equilibrium outcome path.

In the appendix, we show:

Proposition 3 *Under the outcome path of the symmetric Markov perfect Bayesian equilibrium strategy $\hat{E}$ in Proposition 1, at any date $t \in \{1, \dots T\}$, for any $s = (\mathcal{F}, \mathcal{A}^*)$ with $|\mathcal{F}| = F > 1$, the aggregate continuation value of all players*

$$W_{F,t} \;=\; F\,\left[V^p_{F,t} + V^a_{F,t}\right] + \Sigma^{V^a}_{F,t}\;, \tag{16}$$

where $\Sigma^{V^a}_{F,t}$ satisfies the backwards recursive equation

$$\Sigma^{V^a}_{F,t} \;=\; \frac{\hat{\omega}_{F,t}\;V^a_{F+1,t+1}}{1+\rho} + \frac{(1-\hat{\omega}_{F,t})\,\Sigma^{V^a}_{F,t+1} + \hat{\omega}_{F,t}\,\Sigma^{V^a}_{F+1,t+1}}{1+\rho}\;, \tag{17}$$

and $\Sigma^{V^a}_{F,T} = 0$, for any F.

The aggregate continuation value, $W_{F,t}$, is the sum of (i) the current industry participants' continuation value, $F\,\left[V^p_{F,t} + V^a_{F,t}\right]$ (obtained summing across all existing firms in $\mathcal{F}_t$, the equilibrium continuation values of the principal p and the agent a, in each firm $f \in \mathcal{F}_t$) and (ii) the date-t expected value of all the agents who are not employed yet, but *will* be recruited as firms are created in the future, $\Sigma^{V^a}_{F,t}$: $\frac{V^a_{F+1,t+1}}{1+\rho}$ is the value at date t of the agent which will be recruited at date $t+1$, if the period t innovation leads to the creation of an additional firm. The probability this is occurs being $\hat{\omega}_{F,t}$. Summing across all firms created in the future, $\Sigma^{V^a}_{F,t}$ is the probability discounted sum of future agents' value.

Proposition 3 highlights the following: Current participants do internalize the continuation value of all the *principals* of all firms that will be created in the future. However, they do not internalize the continuation value of agents employed in the future. This is because, in the race to innovate, the principals of newly created firms provide equilibrium competitive incentives to the agents they will recruit whose value is above their reservation value of zero. $\Sigma^{V^a}_F$ is this part of the continuation value of all players, which current industry participants do not internalize.

The equilibrium behaviour is the result of a sequence of private optimizations by different parties with conflicting interests. It therefore most likely does not maximize the aggregate continuation value of all players. We now provide characteristics of a first-best strategy. That is, we establish features which must hold for a strategy to yield the highest aggregate continuation value of all players. This will enable us to examine the extent to which the equilibrium outcome path is inefficient. We obtain:

Proposition 4 *Under a first best strategy, at any date $t \in \{1, \dots T\}$, for any $s = (\mathcal{F}, \mathcal{A}^*)$ with $|\mathcal{F}| = F > 1$, the level of effort exerted by any agent a is*

$$\check{e} \;=\; 0\;. \tag{18}$$

The firm that innovates implements itself the innovation if and only if $w \geq \breve{\omega}$. The innovation is implemented in a new firm if $w < \breve{\omega}$, where the first best firm creation threshold

$$\breve{\omega} \;=\; 1-\kappa\ . \tag{19}$$

The aggregate continuation value of all players is

$$\breve{W}_t \;=\; \frac{\breve{\pi}}{\rho}\left[1 \;-\; \frac{1}{(1+\rho)^{T+1-t}}\right]\ , \tag{20}$$

where $\breve{\pi} \equiv \bar{\pi}(1-\kappa)$, hence from (8), $\breve{\pi} = [1 \,+\, (1-\kappa)^2]/2$.

6.2 Agency-Costs

To assess the significance of equilibrium inefficiencies we examine the extent to which the equilibrium continuation value of the sector, $W_{F,t}$, falls short of its first best value, $\breve{W}_t$. We can break down the difference $\breve{W}_t - W_{F,t}$ as follows:

Proposition 5 *The agency costs which result from the equilibrium strategy*

$$\breve{W}_t \;-\; W_{F,t} \;=\; C^e_{F,t} \;+\; C^\omega_{F,t}\ , \tag{21}$$

where $C^\omega_{F,t}$ are agency costs from inefficient exploitation of innovations and $C^e_{F,t}$ are agency costs from excessive efforts. $C^\omega_{F,t}$ and $C^e_{F,t}$ satisfy the backwards recursive equations

$$C^\omega_{F,t} \;=\; \frac{\breve{\pi} \;-\; \bar{\pi}(\hat{\omega}_{F,t})}{1+\rho} \;+\; \frac{(1-\hat{\omega}_{F,t})\,C^\omega_{F,t+1} + \hat{\omega}_{F,t}\,C^\omega_{F+1,t+1}}{1+\rho}\ , \tag{22}$$

$$C^e_{F,t} \;=\; \alpha\, s_F\,\left(1-\frac{1}{F}\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\ V^p_{F+1,t+1}}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F,t})\,C^e_{F,t+1} + \hat{\omega}_{F,t}\,C^e_{F+1,t+1}}{1+\rho}\ , \tag{23}$$

where $C^\omega_{F,T+1} = C^e_{F,T+1} = 0$, for any F. s_F, $\bar{\pi}(\omega)$, $\hat{\omega}_F$ are given in (7), (8), (9), respectively, and $\breve{\pi} \equiv \bar{\pi}(1-\kappa)$.

There are essentially two sources of inefficiency in the equilibrium strategy:

1. The equilibrium firm creation thresholds, $\hat{\omega}_{F,t}$ in (9) are inefficient. $\bar{\pi}(\hat{\omega}_{F,t})$ is the expected value of the cash flows from exploiting an innovation, under the equilibrium threshold, $\hat{\omega}_{F,t}$. $\breve{\pi}$ is the same, but under the first best firm creation threshold, $\breve{\omega} = 1-\kappa$. The former falls short of the later. Figure 3 illustrates this. The suboptimal equilibrium creation of firms, generates a first agency cost, $C^\omega_{F,t}$, because of the resulting inefficient exploitation of innovations. $C^\omega_{F,t}$ is the aggregate expected value of such agency costs.

2. The equilibrium effort levels, $\hat{e}_{F,t}$, are excessive. Because firms are engaged in a race to innovate, the principal of each firm induces her agent to exert an inefficiently large level of effort, in order not to fall behind others. This generates a second agency cost, $C^e_{F,t}$. $C^e_{F,t}$ is the aggregate expected excessive cost of efforts incurred by agents in the future, because of the race to innovate.

Figures 4 and 5 exhibits the magnitude of these two agency costs as a fraction of first best value, $\frac{C^\omega_{F,t}}{\check{W}_t}$ and $\frac{C^e_{F,t}}{\check{W}_t}$, as the number of firms in the industry progresses, in the long horizon model ($C^\omega_F/\check{W} \equiv C^\omega_{F,t}/\check{W}_t|_{T-t\to\infty}$ and $C^e_F/\check{W} \equiv C^\omega_{F,t}/\check{W}_t|_{T-t\to\infty}$, at a date t distant from the end of the game).

The agency cost from suboptimal exploitation of innovations, $C^\omega_{F,t}$ gradually vanishes, whereas the agency cost from costs of effort, $C^e_{F,t}$, stabilizes to a fraction of first best value. The first issue vanishes when the industry is largely developed. We actually easily show that

$$\lim_{F\to\infty} \frac{C^\omega_{F,t}}{\check{W}_t} = 0 \qquad \text{and} \qquad \lim_{F\to\infty} \frac{C^e_{F,t}}{\check{W}_t} = \alpha^2 \; . \tag{24}$$

7 Extensions

To extend the argument, we here first examine the extent to which a central regulation of contracts can restore efficiency, and second discuss ways to capture the many dimensions of expertise as well as the possibility for the innovative firm to sell its innovation to existing firms.

7.1 Limits of Contract Regulation

The first-best is attained when both (19) and (18) are satisfied. However contracts are incomplete. Neither firm creation thresholds, $\hat{\omega}_{F,t}$, nor agents' efforts, $\hat{e}_{F,t}$, are contractible.

Suppose a planner can perfectly regulate the compensation contract any principal is allowed to proposes her agents across time, $b_{F,t}$. This planner seeks to maximize, $W_{F,t}$, the aggregate continuation value of the sector. We establish:

Result 4 *An industry-wide regulation of contracts does not permit to obtain the first best outcome.*

Through contract regulation, a planner can induce agents to exert the first-best level of efforts, $\check{e} = 0$ in (18). This requires regulating the agent compensation to be $b_{F,t} = 0$. But then, the firm creation threshold selected by the principal of an innovating firm will however not be equal to the first best threshold, $\check{\omega}_{F,t} = 1 - \kappa$ in (19).

Alternatively, a planner can obtain that principals select the the first best firm creation threshold $\check{\omega}_{F,t} = 1-\kappa$ in (19). But then, the required agent compensation is such that agents are induced to exert level of efforts which are not the first best $\check{e} = 0$ in (18).

7.2 Extending Expertise and Selling-out Options

For simplicity, we represented the expertise of a firm f as a single point on the real line, $x^f \in \mathbb{R}$, and did not allow an innovation to be sold to a non-inventing existing firm. Extending the model to capture the high dimensionality of expertise and allow for the inventing firm f to sell its innovation to a non-inventing existing firm, $f' \in \mathcal{F}_t \setminus \{f\}$, can be done as follows:

Expertise has many facets, more appropriately represented by an N-uple $(x_1^f, x_2^f, \ldots, x_N^f) \in \mathbb{R}^N$, where N is the number of dimensions which characterise expertise. An innovation i is then be represented by the N-uple, $(x_1^i, x_2^i, \ldots, x_N^i) \in \mathbb{R}^N$, which characterises the required expertise to exploit it most profitably. Capturing the possibility to sell-out an innovation to an existing firm can then be done expressing the model in terms of the proximity between the expertise demanded by innovation i and the expertise of the existing firm most capable of exploiting it (instead of just the inventing firm). That is, first redefine $w(i)$ as the relative proximity

$$w(i) \quad \equiv \quad \exp\left[-\, d(i)\right] \ , \tag{25}$$

where $d(i)$ is the Euclidian distance to the existing firm with closest expertise

$$d(i) \quad \equiv \quad \inf_{f \in \mathcal{F}_t} \sqrt{\sum_{j=1}^{j=N} \left(x_j^i \; - \; x_j^f\right)^2} \ . \tag{26}$$

Second, consider that the exploitation of period t's innovation i by the most capable existing firm, generates a stream of cash-flows from date $t+1$ onwards, whose value equals $w(i)$.

There are still benefits of firm creation, as in some instances the innovation is still better exploited by a new firm, f^+, created specifically to do this. Firm f should consider selling the innovation *outside of existing firms*, if a new principal, p^+, is willing to pay more for the innovation than $w(i)$. Creating a new firm, f^+, still gives the principal p^+ the opportunity to choose the new firm's characteristics, *after* the innovation i is made. By choosing expertise characteristics $(x_1^{f^+}, x_2^{f^+}, \ldots, x_N^{f^+}) = (x_1^i, x_2^i, \ldots, x_N^i)$, the distance between the newly created firm f^+'s and the innovation i can be eliminated. The trade-off we studied remains, as the exploitation of the innovation i within a newly created firm f^+ provides an alternative stream of cash-flows, whose date $t+1$ value is $1-\kappa$.

The collective set of existing expertise never becomes universally dominant. It still leaves as a complement, a domain where exploitation by a newly created firm dominates. There are two effects however: On the one hand, this remaining domain now shrinks as sequel firms are created, whereas it did not before. This domain reduction is very limited when the number of firms, F, is small, but becomes more substantial as the industry develops. On the other hand, the complement domain reduction is smaller when the number of dimensions which characterise expertise, N, is larger than one. The reduction essentially vanishes when expertise is complex and multi faceted, so that N is very large.

Accounting for extended sell-out options would therefore reduce the magnitude of equilibrium deviations from the first best, $\breve{\omega}_{F,t}-(1-\kappa)$, hence dim the phenomenon we highlight. Less so however if expertise complexity is also accounted for. Importantly, the time dynamics that the frequency of firm creation, the focus of firms and the profitability of firms, all decreases over time, would only be reinforced.

8 Conclusion

We examined the impact of the moral hazard in research teams problem on the creation of sequel firms. From the perspective of a firm which devises an innovation, the ability of a new firm to segment this problem yields both benefits and costs, which influence the creation of sequel firms: The parent firm can internalize part of the expected value of further innovations captured by the sequel firm, but looses some likelihood of making them.

To expose this, we carefully constructed and solved for a repeated principal-agent model of an industry where firms compete to innovate and the number of firms increases endogenously, as parent firms allow the creation of sequel firms. Comparing the equilibrium behaviour with the first-best strategy, we conclude that the moral hazard in research teams problem leads to an excessive creation of firms, primarily in the early stages of an industry. The resulting dynamics are in line with a series of empirical observations on the frequency with which a firm spawns new firms, the focus of a firm and the profitability of a firm.

One next step for future research, consists of relating the above argument to questions of financing method: Given that excessive sequel firm creation is most significant in the early stages of an industry and that venture capital is the predominant method of financing in starting industries, can the presence of venture capital be related to distinguishing features of venture capitalists which reduce the inefficiencies we discussed?

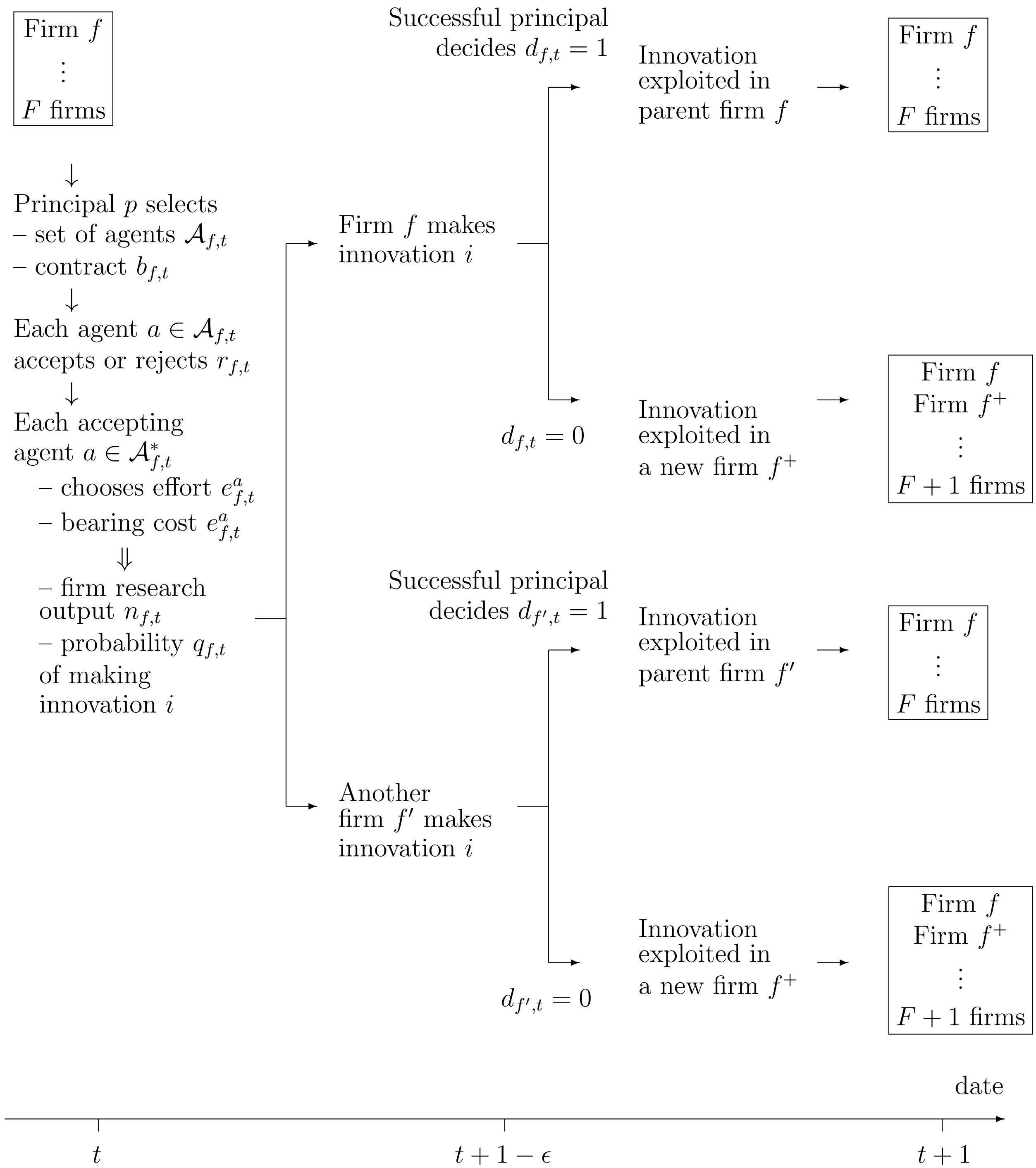
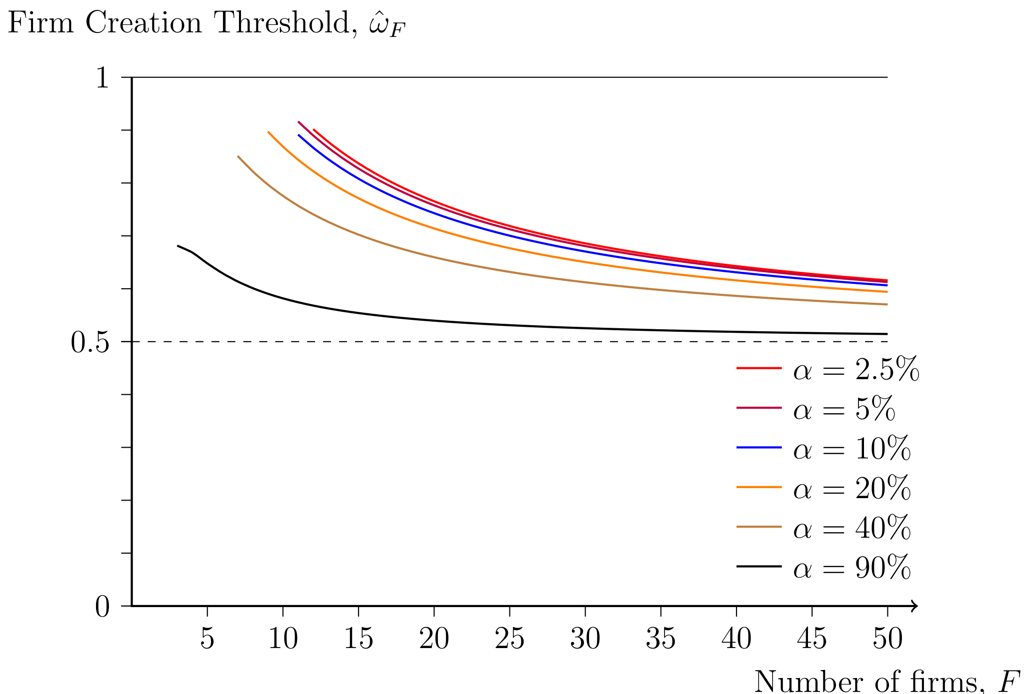
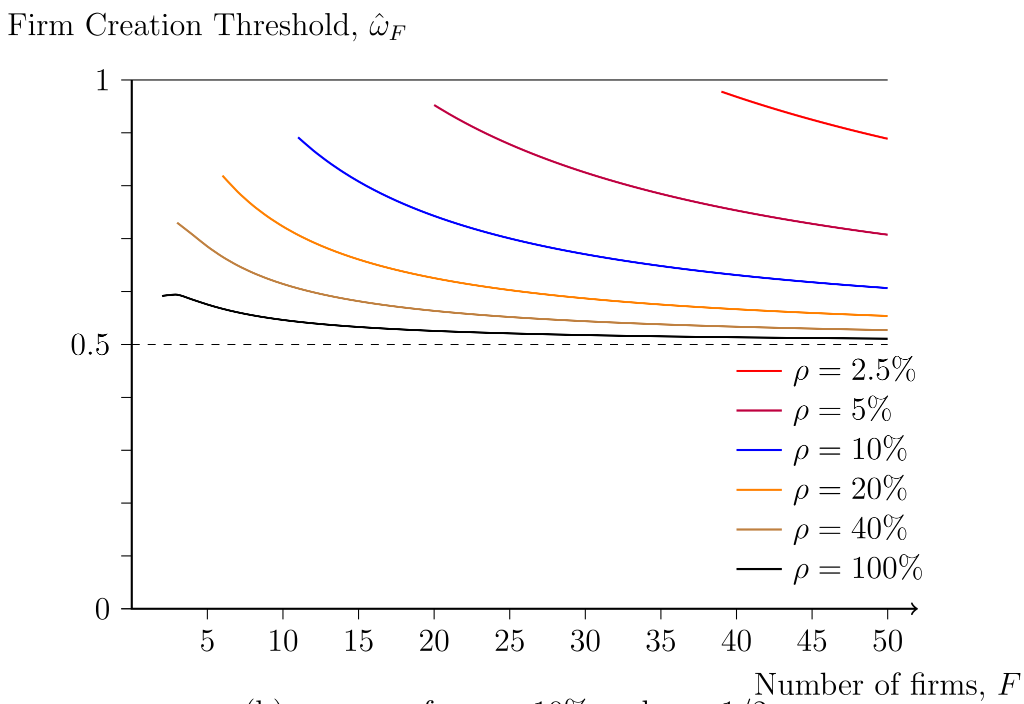


Figure 1: Time Line



(a) across α, for $\rho = 10\%$ and $\kappa = 1/2$.



(b) across ρ, for $\alpha = 10\%$ and $\kappa = 1/2$.

Figure 2: Dynamics of Firm Creation.

$\hat{\omega}_F \equiv \hat{\omega}_{F,t}|_{T-t\to\infty}$, in the long horizon model.

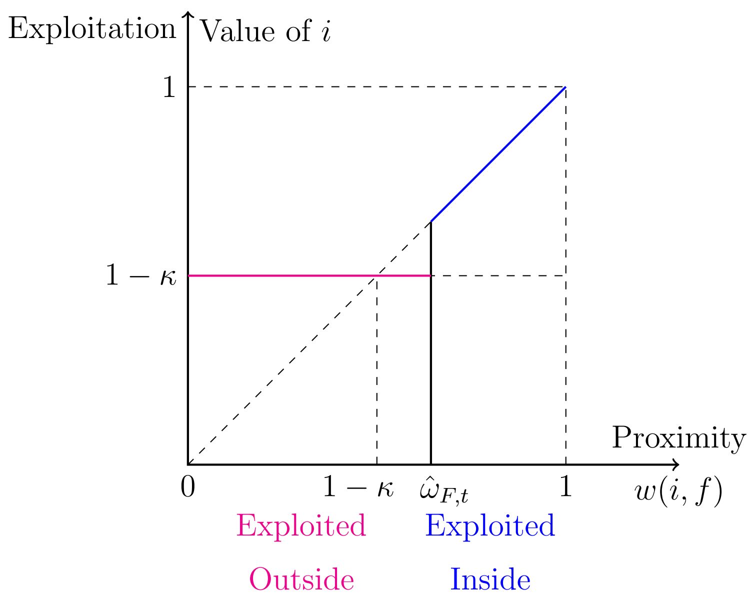
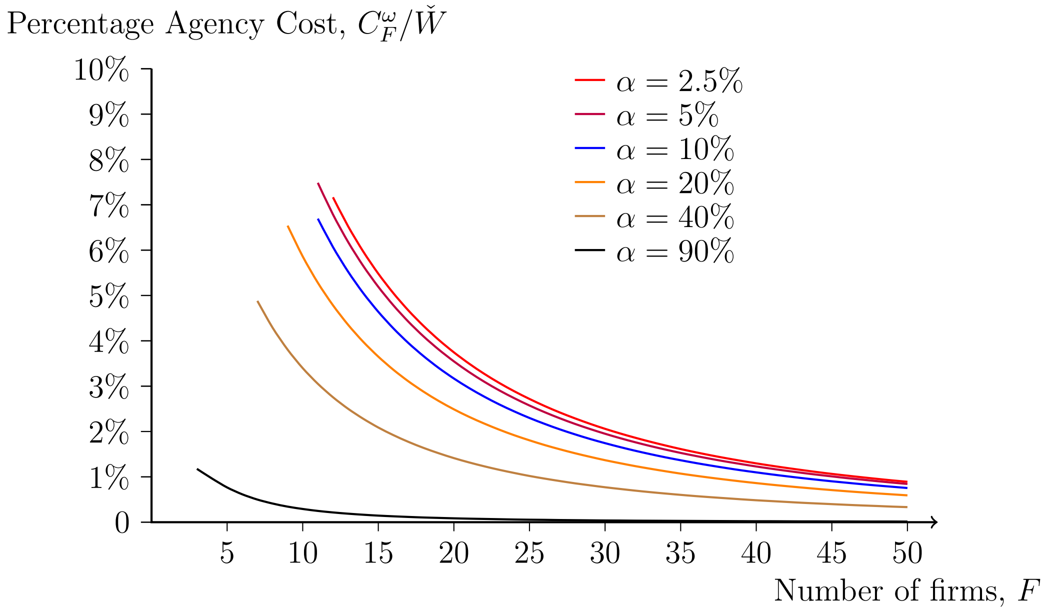
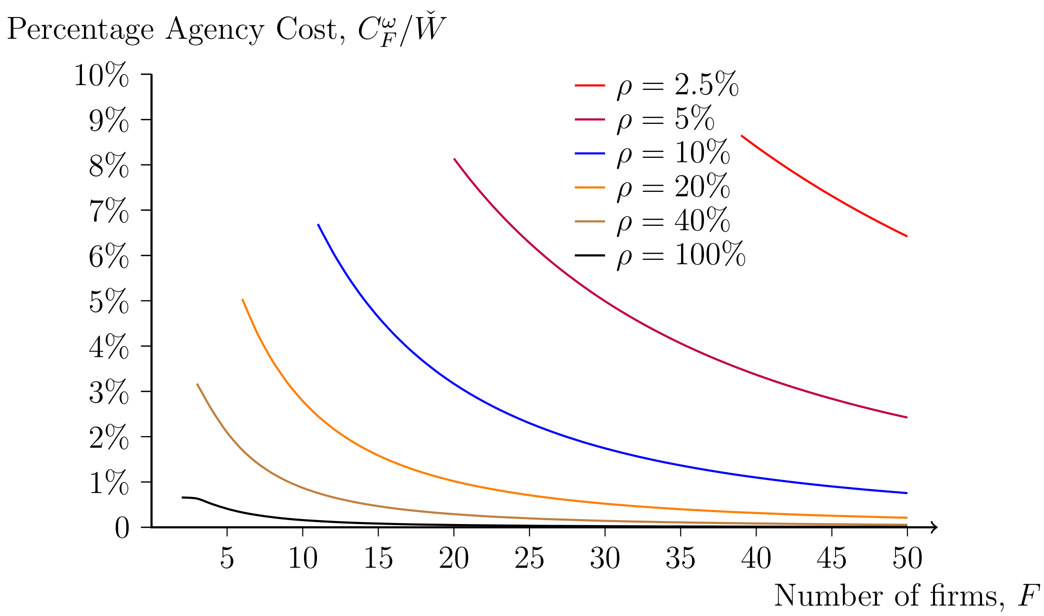


Figure 3: Inefficient Exploitation of Innovations.

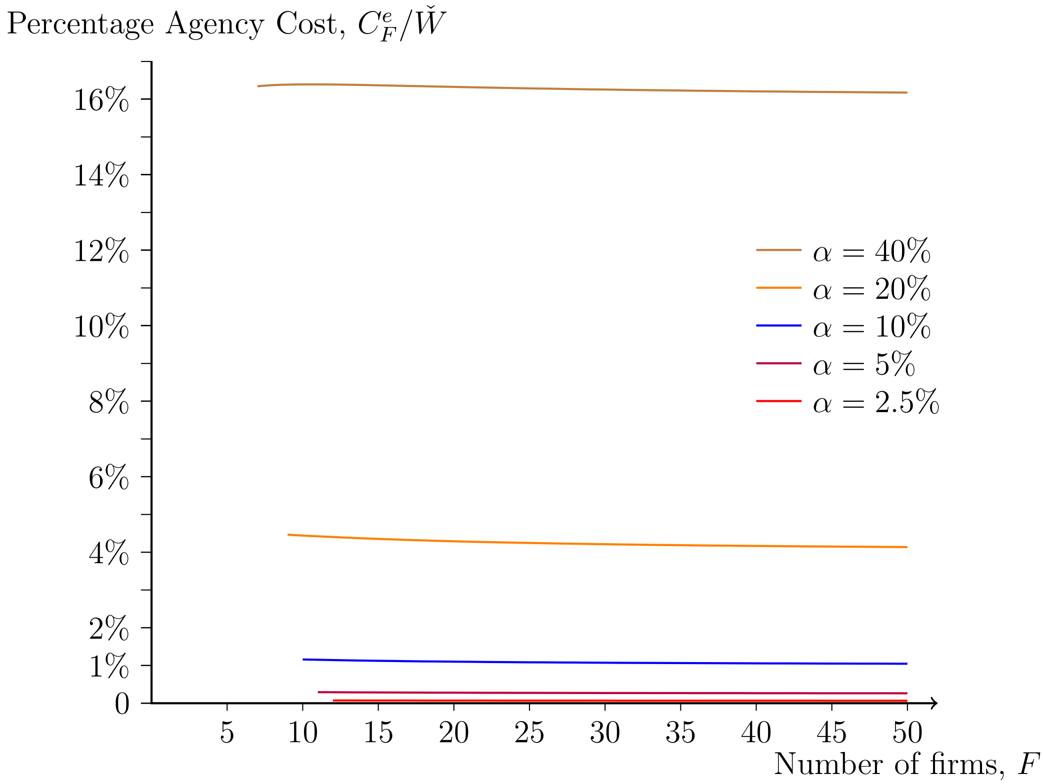


(a) across α, for $\rho = 10\%$ and $\kappa = 1/2$.

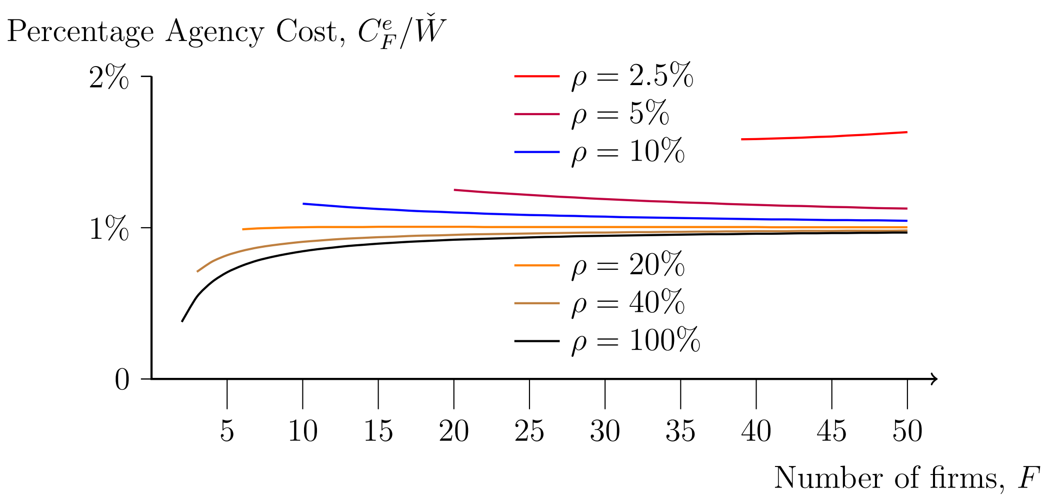


(b) across ρ, for $\alpha = 10\%$ and $\kappa = 1/2$.

Figure 4: Dynamics of Agency Costs: Suboptimal Exploitation of Innovations. $C_F^\omega/\check{W} \equiv C_{F,t}^\omega/\check{W}_t|_{T-t\to\infty}$, in the long horizon model.



(a) across α, for $\rho = 10\%$ and $\kappa = 1/2$.



(b) across ρ, for $\alpha = 10\%$ and $\kappa = 1/2$.

Figure 5: Dynamics of Agency Costs: Excessive Efforts from Race to Innovate. $C_F^e/\check{W} \equiv C_{F,t}^e/\check{W}_t|_{T-t\to\infty}$, in the long horizon model.

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Appendix

To state our main existence result, we first introduce some notation and functions. Denote a proposal by (A, b), whereby a principal offers an individual payment $b \in \mathbb{R}_+$ in case of innovation to $A \in \mathbb{N}_+$ agents. Remember that $F > 1$. At any date $t \in \{1, \dots T\}$, for any $x \in [0, \infty)$, let

$$G_{F,t}(x) \quad \equiv \quad \frac{(1+\rho)\; x^{1-\alpha}\,[x^{\alpha} + (F-1)\,(\hat{e}_{F,t})^{\alpha}]^2}{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}} \; . \tag{27}$$

For any (A, b), let $e^*_{F,t}(A, b)$ be such that

$$G_{F,t}(A\, e^*_F(A, b)) = b \; . \tag{28}$$

Note that $e^*_{F,t}(A, b)$ is well-defined and unique if $b \geq 0$. This is because in this case, $G_{F,t}(x) = 0$, $G_{F,t}(x)$ is increasing and unbounded, and b is independent of x. Furthermore, as $b > 0$, then $G^{-1}_{F,t}(b) > 0$. Hence,

$$e^*_{F,t}(A, b) > 0 \qquad \text{if } b > 0 \text{ and } F > 1 \; . \tag{29}$$

Theorem 1 *There exists a symmetric Markov perfect Bayesian equilibrium strategy $\hat{E}$ such that: At any date $t \in \{1, \dots T\}$, for any $s = (\mathcal{F}, \mathcal{A}^*)$ with $|\mathcal{F}| = F > 1$,*

– (i) any firm f that was set-up at the end of the previous period firm chooses one agent randomly amongst those not employed by other firms in the previous period and any firm that was in operation in the previous period selects one agent amongst its employees in the previous period, with equal probability;

– (ii) The proposal to any agent is $\hat{b}_{F,t}$ in (10);

– (iii) For any $\mathcal{A}$ and any offer $b \in \mathbb{R}_+$, every agent that receives the offer accepts it;

– (iv) For any $\mathcal{A}$ with $|\mathcal{A}| = A$, and $b \in \mathbb{R}_+$, the effort level of any agent is $e_{F,t}(A, b)$ where

$$e_{F,t}(A, b) = \begin{cases} e^*_{F,t}(A, b) & \textit{if } b > 0\,; \\ 0 & \textit{if otherwise}\,. \end{cases} \tag{30}$$

– (v) For any $w \in [0, 1]$, the successful firm that innovates implements the innovation in the firm if and only if $w \geq \hat{\omega}_{F,t}$, where the firm creation threshold $\hat{\omega}_{F,t}$ is given by (9);

– (vi) The continuation equilibrium payoff of any principal p and an agent a at s is respectively given by $V^p_{F,t}$ and $V^a_{F,t}$ as defined in (5) and (6).

Note that $\hat{E}$ is such that at any date with any $s = (\mathcal{F}, \mathcal{A}^*)$: (i) the decisions of any principal and any agent in depend only on the number of existing firms with $|\mathcal{F}|$ and not on their identities and (ii) for any set of agents $\mathcal{A}$ employed by a principal and any offer $b \in \mathbb{R}_+$, the effort level of any agent as well as the implementation decision of the principal depend on the number of agents $|\mathcal{A}|$ employed by the firm and not the identity of these agents.

Proof of Proposition 1: The equilibrium described in Theorem 1 induces the following outcome path. At any date $t \in \{1, \dots T\}$ with $F > 1$ firms in existence, each of the F firms choose one agent and make an offer $\hat{b}_{F,t}$, each selected agent accepts the offer and exerts an effort $e_{F,t}(1, \hat{b}_{F,t})$ and the firm that makes the innovation decides to implement it in the firm if the proximity of the innovation w is no less than $\hat{\omega}_{F,t}$ and sells it to a financier thereby setting up another firm, otherwise. But note that, by substituting for $\hat{b}_{F,t}$ in $e_{F,t}(1, \hat{b}_{F,t})$, we have

$$e_{F,t}(1, \hat{b}_{F,t}) \;=\; \frac{s_F\, \alpha\, \left(\bar{\pi}(\hat{\omega}_{F,t}) \,+\, \hat{\omega}_{F,t}\, V^p_{F+1,t+1}\right)\, (1-1/F)}{(1+\rho)\, F} \;=\; \hat{e}_{F,t}\ . \qquad \square \tag{31}$$

Proof of Theorem 1: We verify below that the symmetric annonymous Markov strategy $\hat{E}$ in Proposition 1 is a perfect Bayesian equilibrium.

Suppose that the continuation payoffs of all principals and agents at any date $t \in \{1, \ldots T\}$, if F is the number of existing firms, is $V^p_{F,t}$ and $V^a_{F,t}$ defined in (5) and (6). The strategy of our proof is to show, working backwards, that the actions described in $\hat{E}$ for each player during each period is indeed optimal given that the players follow the equilibrium strategy from next period. We then complete our proof by showing that $V^p_{F,t}$ and $V^a_{F,t}$ are indeed the continuation payoffs of all principals and agents at the beginning of each period.

Firm creation decision: Fix any date date $t \in \{1, 2\}$. Suppose firm f employs a number $A \in \mathbb{N}_+$ of agents with a contract $b \in \mathbb{R}_+$ and firm f makes an innovation with proximity w. We can state:

Lemma 1 *If the firm makes an innovation with proximity w, then it is optimal for the firm to implement the innovation itself if $w \geq \hat{\omega}_{F,t}$ and to sell it to a financier otherwise, where $\hat{\omega}_{F,t}$ is defined by (9).*

Proof: If the innovation is implemented in firm f, the continuation payoffs of firm f's principal p is the value of the innovation value itself plus her continuation payoff at date $t+1$. That is, $w + \; V^p_{F,t+1}$.
If the innovation is sold and implemented outside firm f, the market value of the innovation is the exploitation value of the innovation itself plus the continuation value of the new firm's principal at date $t+1$, i.e $1 - \kappa + V^p_{F+1,t+1}$. Therefore, the continuation payoffs of firm f's principal p is $1 - \kappa + 2\, V^p_{F+1,t+1}$.
For any innovation with exploitation value w, it is optimal for the principal with the innovation to implement the innovation inside the firm if $w + \; V^p_{F,t+1} \geq 1 - \kappa + 2\, V^p_{F+1,t+1}$ and to sell, otherwise. The claim in the lemma follows from the definition of $\hat{\omega}_{F,t}$ as defined in (9). $\square$

Effort levels: We describe the continuation payoffs at any date t of an agent a employed by a firm f when there are F firms in the industry, the principal of the firm proposes (A, b), the offers are accepted, the agent a exerts effort $e \geq 0$ and before the innovation in that period is made. At such a stage of the game the probability that agent a attaches to firm f making the innovation given that (i) he exerts effort e, (ii) the $A-1$ other agents in firm f's research team respond to the offer (A, b) by each exerting an effort $e_{F,t}(A, b)$, and (iii) other firms employ one agent and each such agent exerts the equilibrium effort $\hat{e}_{F,t}$ is equal to

$$q^*_{F,t}(A, b, e) = \frac{(e + (A-1)e_{F,t}(A, b))^\alpha}{(e + (A-1)e_{F,t}(A, b))^\alpha + (F-1)(\hat{e}_{F,t})^\alpha}\ , \tag{32}$$

if the denominator in the RHS of the above expression is positive.

If firm f makes the innovation, agent a receives a payment b, otherwise he receives nothing.
– When $t \in \{1, \dots T-1\}$, the agent's continuation payoff at date $t+1$ however depends of the whether the innovation has been implement in the inventing firm (whichever it is) or sold out. Conditional on being employed, it is $V^a_{F,t+1}$ or $V^a_{F+1,t+1}$, respectively. Given the equilibrium employment policy of the firm at $t+1$, only one of the current A agents of the firm f will be employed in the next period, $V^a_{F,t+1}$ or $V^a_{F+1,t+1}$ must be multiplied by $\frac{1}{A}$.
– When $t = T$ (the last period of the game), the continuation values at the end of the period are $V^a_{F,T+1} = V^a_{F+1,T+1} = 0$, as no further innovation will be made (and the game ends).
Hence, at date $t \in \{1, \dots T\}$, if $F > 1$ and $\hat{e}_{F,t} > 0$, the continuation payoffs of agent a, $V^a_{F,t}(A, b, e)$, satisfies

$$V^a_{F,t}(A,b,e) \;\; = \;\; -\,e \, + \frac{1}{1+\rho}\Big\{ q^*_{F,t}(A,b,e)\, b + \frac{1}{A}\left[V^a_{F,t+1} \, - \, \hat{\omega}_{F,t}\, (V^a_{F,t+1} - V^a_{F+1,t+1})\right]\Big\}. \quad (33)$$

Next, note that since $s_F \in (0,1)$, $F > 1$, $\hat{\omega}_{F,t} \in [0,1]$ and $\bar{\pi}(\hat{\omega}_{F,t}) > 0$, we have

$$\hat{e}_{F,t} = \frac{s_F\, \alpha\, \left(\bar{\pi}(\hat{\omega}_{F,t}) \, + \, \hat{\omega}_{F,t}\, V^p_{F+1,t+1}\right)\, (1 - 1/F)}{(1+\rho)\, F} > 0\,. \quad (34)$$

We can now state:

Lemma 2 *(i) $\hat{e}_{F,t} > 0$ and (ii) if A agents are employed by a firm f with contracts b, then each employed agent chooses to exert an effort $e_{F,t}(A,b)$ as defined in (30), i.e.*

$$e_{F,t}(A,b) \in \arg\max_e V^a_{F,t}(A,b,e) \;. \quad (35)$$

Proof: $\hat{e}_{F,t} > 0$ implies that $q^*_{F,t}(A,b,e)$ is twice differentiable for every $e \in (0, \infty)$. Hence, we have

$$\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e} = -\,1 \, + \, \frac{\partial\, q^*_{F,t}(A,b,e)}{\partial\, e}\;\frac{b}{1+\rho} \;\; \text{for all } e \, \in \, (0, \infty) \;. \quad (36)$$

There are two cases to consider.

Case 1: $b = 0$. In this case, we need to show that if all other agents employed by firm f exert zero effort and other firms employ one agent and each such agent exerts the equilibrium effort $\hat{e}_{F,t}$, then the best response of the agent is to also exert effort $e = 0$.
Suppose that $e_{F,t}(A,b) = 0$. From (32), we have $q^*_{F,t}(A,b,e) = \frac{e^\alpha}{e^\alpha + (F-1)(\hat{e}_{F,t})^\alpha}$. Let $n_0 = e^\alpha$ and $N_0 = n_0 + (F-1)(\hat{e}_{F,t})^\alpha$. Then $q^*_{F,t}(A,b,e) = \frac{n_0}{N_0}$ and $\frac{\partial\, q^*_{F,t}(A,b,e)}{\partial\, e} = (1 - \frac{n_0}{N_0})\frac{1}{N_0}\frac{\partial\, n_0}{\partial\, e}$. Since $\frac{\partial\, n_0}{\partial\, e} = \alpha e^{\alpha-1} \geq 0$, $\frac{\partial\, q^*_{F,t}(A,b,e)}{\partial\, e} \geq 0$ for all $e \in (0, \infty)$. Hence, by $b \leq 0$ we have $\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e} < 0$. Since $V^a_{F,t}(A,b,e)$ is continuous for all $e \in (0,\infty)$, it then follows that the best response is $e = 0$.

Case 2: $b > 0$. In this case, we need to show that if all other agents employed by firm f exert effort $e^*_{F,t}(A,b)$ and other firms employ one agent and each such agent exerts the equilibrium effort $\hat{e}_{F,t}$, then the best response of the agent is to also exert effort $e^*_{F,t}(A,b)$.

Let $n_e = \left(e + (A-1)e^*_{F,t}(A,b)\right)^\alpha$ and $N_e = n_e + (F-1)(\hat{e}_{F,t})^\alpha$. Then $q^*_{F,t}(A,b,e) = \frac{n_e}{N_e}$. So, $\frac{\partial\, q^*_{F,t}(A,b,e)}{\partial\, e} = (1 - \frac{n_e}{N_e})\frac{1}{N_e}\frac{\partial\, n_e}{\partial\, e}$ and $\frac{\partial\, n_e}{\partial\, e} = \alpha\left(e + (A-1)e^*_{F,t}(A,b)\right)^{\alpha-1}$. Then (36) gives

$$\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e} = -\,1 \, + \, \frac{(F-1)(\hat{e}_{F,t})^\alpha\;\alpha\left(e + (A-1)\, e^*_{F,t}(A,b)\right)^{\alpha-1}}{\left[\left(e + (A-1)\, e^*_{F,t}(A,b)\right)^\alpha + (F-1)(\hat{e}_{F,t})^\alpha\right]^2}\;\frac{b}{1+\rho}\;. \quad (37)$$

By (28), we have $G_{F,t}(A\, e^*_{F,t}(A,b)) = b$. Thus, $e^*_{F,t}(A,b)$ satisfies the following

$$\left(A\ e^*_{F,t}(A,b)\right)^{1-\alpha} = \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}}{[(A\,e^*_{F,t}(A,b))^{\alpha} + (F-1)(\hat{e}_{F,t})^{\alpha}]^2}\ \frac{b}{1+\rho}\ . \tag{38}$$

By setting $e=0$ in (37), using (38) and $b>0$, we have

$$\left.\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e}\right|_{e=0} = -\,1\,+\,\left[\frac{(A\,e^*_{F,t}(A,b))^{\alpha} + (F-1)(\hat{e}_{F,t})^{\alpha}}{\Big((A-1)e^*_{F,t}(A,b)\Big)^{\alpha} + (F-1)(\hat{e}_{F,t})^{\alpha}}\right]^2 \left(\frac{A}{A-1}\right)^{1-\alpha}\ . \tag{39}$$

So, $\left.\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e}\right|_{e=0} > 0$. Hence, the optimal solution to $\arg\max_e V^a_{F,t}(A,b,e)$ must be strictly positive. Thus, to complete to proof it is sufficient to show that $\left.\frac{\partial\, V^a_{F,t}(A,\mathbf{b},e)}{\partial\, e}\right|_{e=e^*_{F,t}(A,b)} = 0$ and the second derivative $\frac{\partial^{\,2} V^a_{F,t}(A,b,e)}{\partial\, e^2}$ is negative at every $e>0$. Replacing $e = e^*_{F,t}(A,b)$ in (37) gives

$$\left.\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e}\right|_{e=e^*_{F,t}(A,b)} = -\,1\,+\left(A\,e^*_{F,t}(A,b)\right)^{\alpha-1}\ \frac{\alpha(F-1)(\hat{e}_{t,F})^{\alpha}}{\left[\Big(A\,e^*_{F,t}(A,b)\Big)^{\alpha} + (F-1)(\hat{e}_{F,t})^{\alpha}\right]^2}\ \frac{b}{1+\rho}\ . \tag{40}$$

Using (38), we then obtain $\left.\frac{\partial\, V^a_{F,t}(A,b,e)}{\partial\, e}\right|_{e=e^*_{F,t}(A,b)} = 0$.

Finally, note that $\frac{\partial^{\,2} V^a_{F,t}(A,b,e)}{\partial\, e^2} = \frac{\partial^{\,2} q^*_{F,t}(A,b,e)}{\partial\, e^2}\ \frac{b}{1+\rho}$. Since $\frac{\partial^{\,2} q^*_{F,t}(A,b,e)}{\partial\, e^2} < 0$ and $b>0$ it follows that $\frac{\partial^{\,2} V^a_{F,t}(A,b,e)}{\partial\, e^2} < 0$ at every $e>0$. □

Agents' responses to offers. Since by accepting an offer and exerting zero effort the agent can guarantee himself zero payoff, accepting any $b \geq 0$ is an optimal response.

Principal's optimal contract and number of employees. We next describe the continuation payoffs at date $t \in \{1,\ldots T\}$ of a principal before efforts are exerted (and the innovation is made) when there are F firms in the industry. Suppose that the principal of firm f proposes (A,b) and the offers are accepted. Then the probability that the principal attaches to firm f making the innovation given that he expects (i) each of his agents to exert effort $e_{F,t}(A,b)$, and (ii) other firms employ one agent and each such agent exerts the equilibrium effort $\hat{e}_{F,t}$ is equal to

$$q^{**}_{F,t}(A,b) = \frac{(A\,e_{F,t}(A,b))^{\alpha}}{(A\,e_{F,t}(A,b))^{\alpha} + (F-1)(\hat{e}_{F,t})^{\alpha}}\ , \tag{41}$$

if the denominator in the RHS of the above expression is positive. Hence, if $F>1$, by (34), $q^{**}_{F,t}(A,b)$ is well-defined and the continuation payoffs of the principal of firm f is

$$\begin{aligned} V^p_{F,t}(A,b) \ &= \ \frac{1}{1+\rho}\Big\{q^{**}_{F,t}(A,b)\Big[\int_{\hat{\omega}_{F,t}}^{1}(w+V^p_{F,t+1})\,dw\,+\,\hat{\omega}_F\,(1-\kappa+2\,V^p_{F+1,t+1})\,-\,b\,A\Big] \\ &\qquad +\,\big(1-q^{**}_{F,t}(A,b)\big)\,\Big[(1\,-\,\hat{\omega}_{F,t})\ V^p_{F,t+1}\,+\,\hat{\omega}_{F,t}\,V^p_{F+1,t+1}\Big]\Big\}. \end{aligned} \tag{42}$$

The continuation payoffs described in (42) can be simplified as follows:

$$V^p_{F,t}(A,b)\ =\ \frac{q^{**}_{F,t}(A,b)\ \pi^p_{F,t}(A,b)\ +\ V^p_{F,t+1}\,-\,\hat{\omega}_{F,t}\,(V^p_{F,t+1}-V^p_{F+1,t+1})}{1+\rho}\ , \tag{43}$$

where $\hat{\omega}_{F,t}$ is given by (9) and

$$\pi^p_{F,t}(A,b) \;\equiv\; \bar{\pi}(\hat{\omega}_{F,t}) \;+\; \hat{\omega}_{F,t}\, V^p_{F+1,t+1} \;-\; b\,A\;, \tag{44}$$
$$\tag{45}$$

and $\bar{\pi}(\omega)$ is given by (8). We now state:

Lemma 3 *It is optimal for any principal to make the equilibrium proposal, which consists of offering one agent a contract $\hat{b}_{F,t}$ as defined in (10), i.e. $(1, \hat{b}_{F,t}) \in \arg\max_{A,b} V^p_{F,t}(A,b)$.*

Proof: To prove Lemma 3, we establish a series of claims.

Claim 1 *Suppose $F > 1$ and $(\tilde{A}, \tilde{b}) \in \arg\max_{(A,b)} V^p_{F,t}(A,b)$. Then $\tilde{b} > 0$.*

Proof: Suppose otherwise; then $\tilde{b} \leq 0$. Then if the principal proposes $\tilde{b}$ to $\tilde{A}$ agents, from Lemma 2, each one of these agents exerts an effort $e_{F,t}(\tilde{A}, \tilde{b}) = 0$. Given that, from (34)) the equilibrium $\hat{e}_{F,t} > 0$ from (41), the probability the firm makes the next innovation $q^{**}_{F,t}(\tilde{A}, \tilde{b}) = 0$. From (43), the principal's payoff is then

$$V^p_{F,t}(\tilde{A}, \tilde{b}) \;=\; \frac{V^p_{F,t+1} - \hat{\omega}_{F,t}\left(V^p_{F,t+1} - V^p_{F+1,t+1}\right)}{1+\rho}\;. \tag{46}$$

Consider next an alternative offer $\hat{b}_{F,t}$ described in (10) to one of these $\tilde{A}$ agents. We have from (10) that $\hat{b}_{F,t} = s_F\left(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}\right) > 0$. Then, from Lemma 2, the agent exerts an effort $e_{F,t}(1, \hat{b}_{F,t}) > 0$ and, from (41), the probability the firm makes the next innovation $q^{**}_{F,t}(1, \hat{b}_{F,t}) = \frac{1}{F}$. Hence, by (43), the principal's payoff is given by

$$V^p_{F,t}(\tilde{A}, \hat{b}_{F,t}) \;=\; \frac{\pi^p_{F,t}(1, \hat{b}_{F,t})}{F\,(1+\rho)} \;+\; \frac{V^p_{F,t+1} - \hat{\omega}_{F,t}\left(V^p_{F,t+1} - V^p_{F+1,t+1}\right)}{1+\rho}\;. \tag{47}$$

From (44), we have $\pi^p_{F,t}(1, \hat{b}_{F,t}) = (1 - s_F)[\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}]$. From (8) and (7), we have $\pi^p_{F,t}(1, \hat{b}_{F,t}) > 0$. Hence, it follows from (46) and (47) that $V^p_{F,t}(1, \hat{b}_{F,t}) > V^p_{F,t}(\tilde{A}, \tilde{b})$; but this is a contradiction. □

Claim 2 *Suppose $(\tilde{A}, \tilde{b}) \in \arg\max_{(A,b)} V^p_{F,t}(A,b)$. Then $\tilde{A} = 1$.*

Proof: Suppose otherwise that $\tilde{A} > 1$. Consider an alternative offer $b' = \tilde{A}\,\tilde{b}/(\tilde{A}-1)$ to $\tilde{A}-1$ agents.

First, from Lemma 2 and Claim 1, for any (A,b), $e_{F,t}(A,b)$ satisfies $G_{F,t}(\,A\,e_{F,t}(A,b)\,) = b$, where $G_{F,t}(x)$ is defined in (27). Now, $G_{F,t}(x)$ is an increasing function of x, and $b' > \tilde{b}$. Therefore, $(\tilde{A}-1)\,e_{F,t}(\tilde{A}-1, b') > \tilde{A}\,e_{F,t}(\tilde{A}, \tilde{b})$. From (41), for any (A,b), $q^{**}_{F,t}(A,b) = 1/\left[1 + (F-1)\left(\frac{\hat{e}_{F,t}}{A\,e_{F,t}(A,b)}\right)^{\alpha}\right]$. It therefore follows that $q^{**}_{F,t}(\tilde{A}-1, b') > q^{**}_{F,t}(\tilde{A}, \tilde{b})$.

Second, from (44), $\pi^p_{F,t}(\tilde{A}-1, b') = \pi^p_{F,t}(\tilde{A}, \tilde{b})$. So, from (43), as $q^{**}_{F,t}(\tilde{A}-1, b') > q^{**}_{F,t}(\tilde{A}, \tilde{b})$ and $\pi^p_{F,t}(\tilde{A}-1, b') = \pi^p_{F,t}(\tilde{A}, \tilde{b})$, we have $V^p_{F,t}(\tilde{A}-1, b') > V^p_{F,t}(\tilde{A}, \tilde{b})$, which is a contradiction. □

Claim 3 $\arg\max_{b \in \mathbb{R}_+} V^p_{F,t}(1,b)$ *is well defined.*

Proof: Since by (34) $\hat{e}_{F,t} > 0$, it follows from (43) that $V^p_{F,t}(1,b)$ is continuous in b. Since $V^p_{F,t}(1,b)$ is bounded, a solution must exist. □

Claim 4 *Suppose* $\tilde{b} \in \arg\max_b V^p_{F,t}(1,b)$. *Then* $\tilde{b} = \hat{b}_{F,t}$, *where* $\hat{b}_{F,t}$ *is defined in (10).*

Proof: Fix any $b > 0$. From (43), we have

$$\frac{\partial V^p_{F,t}(1,b)}{\partial b} = \frac{1}{1+\rho}\left\{- q^{**}_{F,t}(1,b) + \frac{\partial q^{**}_{F,t}(1,b)}{\partial b}\, \pi^p_{F,t}(1,b)\right\} \tag{48}$$

if $\frac{\partial q^{**}_{F,t}(1,b)}{\partial b}$ are well-defined. If $\frac{\partial e_{F,t}(1,b)}{\partial b}$ is well-defined then by (41)

$$\frac{\partial q^{**}_{F,t}(1,b)}{\partial b} = \frac{\alpha\,(F-1)\, e_{F,t}(1,b)^{\alpha-1} (\hat{e}_{F,t})^{\alpha}}{(N_{F,t})^2}\, \frac{\partial e_{F,t}(1,b)}{\partial b}\ , \tag{49}$$

$$\text{where} \quad N_{F,t} \equiv e_{F,t}(1,b)^{\alpha} + (F-1)\,(\hat{e}_{F,t})^{\alpha}\ . \tag{50}$$

As $b > 0$, we have $e_{F,t}(1,b) = e^*_{F,t}(1,b)$. Hence, from (38),

$$\frac{\partial e_{F,t}(1,b)}{\partial b} = \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}}{(1+\rho)\,N_{F,t}}\, \frac{e_{F,t}(1,b)^{\alpha}}{[(1+\alpha)\,e_{F,t}(1,b)^{\alpha} + (1-\alpha)(F-1)\,(\hat{e}_{F})^{\alpha}]}\ . \tag{51}$$

But then

$$\frac{\partial q^{**}_{F,t}(1,b)}{\partial b} = q^{**}_{F,t}(1,b)\ L_{F,t}(b)\ , \quad \text{where} \tag{52}$$

$$L_{F,t}(b) \equiv \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}}{[(1+\alpha)\,e_{F,t}(1,b)^{\alpha} + (1-\alpha)(F-1)\,(\hat{e}_{F,t})^{\alpha}]}\, \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}\ e_{F,t}(1,b)^{\alpha-1}}{(1+\rho)\,(N_{F,t})^2}\ . \tag{53}$$

So by (52), (48) can be written

$$\frac{\partial V^p_{F,t}(1,b)}{\partial b} = \frac{q^{**}_{F,t}(1,b)}{1+\rho}\ \Big(L_{F,t}(b)\ \pi^p_{F,t}(1,b)\ -\ 1\Big). \tag{54}$$

Using (38), $L_{F,t}(b)$ can also be written as follows:

$$L_{F,t}(b) = \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}}{[(1+\alpha)e_{F,t}(1,b)^{\alpha} + (1-\alpha)(F-1)\,(\hat{e}_{F,t})^{\alpha}]\ \ b}\ . \tag{55}$$

Since $\tilde{b} \in \arg\max_b V^p_{F,t}(1,b)$, from the f.o.c. $\frac{\partial V^p_{F,t}(1,\tilde{b})}{\partial b} = 0$, $\tilde{b}$ satisfies $L_{F,t}(\tilde{b})\ \pi^p_{F,t}(1,\tilde{b}) - 1 = 0$, Using (55), $L_{F,t}(\tilde{b})\ \pi^p_{F,t}(1,\tilde{b}) - 1 = 0$ can be written as follows:

$$\tilde{b} = \frac{\alpha\,(F-1)\,(\hat{e}_{F,t})^{\alpha}\ \Big(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\ V^p_{F+1,t+1}\Big)}{(1+\alpha)\,e_{F,t}(1,\tilde{b})^{\alpha} + (F-1)\,(\hat{e}_{F,t})^{\alpha}}\ . \tag{56}$$

From (30), $e_{F,t}(1,\tilde{b})$ is a solution to $G_{F,t}(e_{F,t}(1,\tilde{b})) = \tilde{b}$. Let

$$b(e) \equiv \frac{\alpha\,(F-1)\,(\hat{e}_{F})^{\alpha}\ \Big(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\,V^p_{F+1,t+1}\Big)}{(1+\alpha)\,e^{\alpha} + (F-1)\,(\hat{e}_{F,t})^{\alpha}}\ . \tag{57}$$

Since $\tilde{b} = b(e_{F,t}(1,\tilde{b}))$, we have that $e_{F,t}(1,\tilde{b})$ is the unique solution to $G_{F,t}(e_{F,t}(1,\tilde{b})) = b(e_{F,t}(1,\tilde{b}))$. Therefore $e_{F,t}(1,\tilde{b})$ is unique and equal to $\hat{e}_{F,t}$.

Replacing $e_{F,t}(1,\tilde{b}) = \hat{e}_{F,t}$ in (56) gives $\tilde{b} = s_F\left(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}\right)$. But from the definition of $\hat{b}_{F,t}$ in (10), $\tilde{b} = \hat{b}_{F,t}$. □
This completes the proof of Lemma 3. □

We finally establish that the continuation values of an agent and a principal are as supposed:

Lemma 4 *The continuation payoffs $V^p_{F,t}$ and $V^a_{F,t}$ are as defined in (5) (6) (62) and (63).*

Proof: Since $V^p_{F,t}(1,\hat{b}_{F,t}) = \max_{A,b} V^p_{F,t}(A,b)$, it must be that $V^p_{F,t}(1,\hat{b}_{F,t}) \geq 0$ and $q^{**}_{F,t}(1,\hat{b}_{F,t}) = 1/F$. From (43) the equilibrium payoff of a principal can be written as

$$V^p_{F,t} \;=\; V^p_{F,t}(1,\hat{b}_{F,t}) \;=\; \frac{\pi^p_{F,t}(1,\hat{b}_{F,t})}{(1+\rho)\;F} \;+\; \frac{V^p_{F,t+1} - \hat{\omega}_{F,t}\left(V^p_{F,t+1} - V^p_{F+1,t+1}\right)}{1+\rho}\,, \tag{58}$$

$$= \frac{1-s_F}{F}\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) \;+\; \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F,t})\, V^p_{F,t+1} + \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{1+\rho}\;. \tag{59}$$

The equilibrium effort is $\hat{e}_{F,t}$ given in (11). Then, from (33),

$$V^a_{F,t} \;=\; V^a_{F,t}(1,\hat{b}_{F,t},\hat{e}_{F,t}) \;=\; -\hat{e}_{F,t} \;+\; \frac{\hat{b}_{F,t}}{(1+\rho)\;F} \;+\; \frac{V^a_{F,t+1} - \hat{\omega}_{F,t}\left(V^a_{F,t+1} - V^a_{F+1,t+1}\right)}{1+\rho}\,, \tag{60}$$

$$= \frac{s_F\left(1-\alpha\left(1-\frac{1}{F}\right)\right)}{F}\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) \;+\; \hat{\omega}_{F,t}\, V^p_{F+1,t+1}}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F,t})\, V^a_{F,t+1} + \hat{\omega}_{F,t}\, V^a_{F+1,t+1}}{1+\rho}. \tag{61}$$

Clearly, $V^a_{F,t} \geq 0$.

When the game ends we have $V^a_{F,T+1} = V^a_{F+1,T+1} = V^p_{F,T+1} = V^p_{F+1,T+1} = 0$. Then, when $t = T$ (the last period the game is played), we have $\hat{\omega}_{F,T} = 1-\kappa$. Then $\bar{\pi}_{F,T} = \frac{1+(1-\kappa)^2}{2}$ Equations (59) and (61), written for $t = T$, give the last period values

$$V^p_{F,T} \;=\; \frac{1-s_F}{F}\left[\frac{1+(1-\kappa)^2}{2\,(1+\rho)}\right]\,, \tag{62}$$

$$V^a_{F,T} \;=\; \frac{s_F\left(1-\alpha\left(1-\frac{1}{F}\right)\right)}{F}\left[\frac{1+(1-\kappa)^2}{2\,(1+\rho)}\right]\;. \;\square \tag{63}$$

Proof of (13) and (14): When the game ends, $V^p_{F,3} = 0$. In the previous period, firms compete to make the last innovations, hence from (9), the threshold is just $\hat{\omega}_{F,2} = 1-\kappa$. Then backwards in time, from (5), we have $V^p_{F,2} = (1-s_F)\frac{\bar{\pi}(1-\kappa)}{(1+\rho)\,F}$, where s_F is defined in (7) and $\bar{\pi}(\omega)$ is defined in (8). We then obtain that at date 1, the firm creation threshold is $\hat{\omega}_{F,1} = \max\{0\,,\,\min\{1-\kappa+\frac{\phi_F\,\breve{\pi}}{1+\rho}\,,\,1\}\,\}$, where $\phi_F \equiv 2\left(\frac{1-s_{F+1}}{F+1}\right) - \frac{1-s_F}{F}$ and $\breve{\pi} \equiv \bar{\pi}(1-\kappa) = \frac{1+(1-\kappa)^2}{2}$. From s_F in (7), we can write

$$\phi_F \;=\; \frac{(1-\alpha)\,F\,(F-1)\,(F+1+2\,\alpha) \;+\; \alpha\,(1+\alpha)\,(F-2)\,(F+1)}{F\,(F+1)\,(F+\alpha)\,(F+1+\alpha)}\;. \tag{64}$$

So, $\phi_F > 0$ for $F > 1$. Hence $\hat{\omega}_{F,1} > 1-\kappa$; We can further write

$$\phi_F - \phi_{F+1} \;=\; \frac{(1-\alpha)\,a_F \;+\; \alpha\,(1+\alpha)\,b_F}{F\,(F+1)\,(F+2)\,(F+\alpha)\,(F+1+\alpha)\,(F+2+\alpha)}\;, \tag{65}$$

where $a_F \equiv F(F-2)(F+\alpha)(F+2+\alpha) + (1+\alpha)(F+2)[F(F+2) + \alpha\, F]$ and $b_F \equiv (F-3)[2(F+1)(F+2) + (1-\alpha)(F+\alpha)]$. So, $\phi_F - \phi_{F+1} > 0$ for $F > 2$, but $\phi_2 - \phi_3 = \frac{-\alpha\,(1+\alpha)}{(2+\alpha)\,(3+\alpha)\,(4+\alpha)} < 0$. So, $\hat{\omega}_{F,1} > \hat{\omega}_{F+1,1}$, for all $F > 2$ (or $\hat{\omega}_{F,1} = \hat{\omega}_{F+1,1} = 1$), but $\hat{\omega}_{2,1} < \hat{\omega}_{3,1}$ (or $\hat{\omega}_{2,1} = \hat{\omega}_{3,1} = 1$).

We then have

$$\frac{\partial\,\phi_F}{\partial\,\alpha} \;=\; \frac{-1}{F\,(F+1+\alpha)}\left\{\frac{(F-1)\,(F-\alpha) + 2\,\alpha\,(1+\alpha)\,F}{F+1} + \frac{1-\alpha^2}{F+\alpha} + \frac{\alpha\,(F-1)}{(F+\alpha)^2}\right\} < 0 \tag{66}$$

Also, $\frac{\partial}{\partial\rho}\left[\frac{\phi_F\,\bar{\pi}}{1+\rho}\right] = \frac{-\,\phi_F\,\bar{\pi}}{(1+\rho)^2} < 0$ and $\frac{\partial}{\partial\kappa}\left[\frac{\phi_F\,\bar{\pi}}{1+\rho}\right] = \frac{-\,\phi_F\,(1-\kappa)}{1+\rho} < 0$. Then, $\frac{\partial\,\hat{\omega}_{F,1}}{\partial\,\alpha} < 0$, $\frac{\partial\,\hat{\omega}_{F,1}}{\partial\,\rho} < 0$ and $\frac{\partial\,\hat{\omega}_{F,1}}{\partial\,\kappa} < 0$, for all F (unless $\hat{\omega}_{F,1} = 1$).

A sufficient condition for $\hat{\omega}_{F,1} < 1$ for all F is $\hat{\omega}_{3,1} < 1$. This can be written $\kappa > z - \sqrt{z^2-2}$, where $z \equiv 1 + \frac{1+\rho}{\phi_3}$ and $\phi_3 = \frac{3\,(1-\alpha)\,(2+\alpha) + \alpha\,(1+\alpha)}{3\,(3+\alpha)\,(4+\alpha)}$. Now, $z - \sqrt{z^2-2}$ is decreasing in z. The lowest value of z is for $\rho = 0$, $\alpha = 0$ and $F = 3$, in which case $z = 7$. So $\kappa > 7 - \sqrt{47}$ is a sufficient condition for $\hat{\omega}_{F,1} < 1$ to hold for all α, ρ, κ and F. □

Proof of Proposition 3: We describe the aggregate continuation value at any date t of all players. Suppose there are F firms in the industry, the principals of all firms propose (A, b), the offers are accepted, all agents exerts effort $e \geq 0$, all firms implementation decision involves the firm creation threshold ω, and before the innovation in that period is made. We have

$$W_{F,t}(A,b,e,\omega) \;=\; -F\,A\,e \;+\; \frac{1}{1+\rho}\left[\int_{\omega}^{1}(w \;+\; W_{F,t+1})\,dw \;+\; \omega\,(1-\kappa+W_{F+1,t+1})\right]\;, \tag{67}$$

$$W_{F,t}(A,b,e,\omega) \;=\; -F\,A\,e \;+\; \frac{1}{1+\rho}\Big[\bar{\pi}(\omega) + (1-\omega)\,W_{F,t+1} + \omega\,W_{F+1,t+1}\Big]\;,\quad . \tag{68}$$

The aggregate continuation value under the the symmetric Markov equilibrium outcome $\hat{E}$ is $W_{F,t} = W_{F,t}(\hat{A}_F, \hat{b}_F, \hat{e}_F, \hat{\omega}_F)$. Replacing the characterization of $\hat{E}$ characterized in Proposition 1, we obtain:

$$W_{F,t} \;=\; \frac{\bar{\pi}(\hat{\omega}_{F,t})}{1+\rho} - s_F\,\alpha\left(1-\frac{1}{F}\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\,V^p_{F+1,t+1}}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F,t})W_{F,t+1} + \hat{\omega}_{F,t}W_{F+1,t+1}}{1+\rho}\;. \tag{69}$$

From (5) and (6),

$$\begin{aligned} F\,[V^p_{F,t} + V^a_{F,t}] \;&=\; \left(1 - s_F\,\alpha\left(1-\frac{1}{F}\right)\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\;V^p_{F+1,t+1}}{1+\rho}\right] \\ &\quad + \frac{(1-\hat{\omega}_{F,t})\,F\,[V^p_{F,t+1} + V^a_{F,t+1}] + \hat{\omega}_{F,t}\,F\,[V^p_{F+1,t+1} + V^a_{F+1,t+1}]}{1+\rho}\;. \end{aligned} \tag{70}$$

Let $\Sigma^{V^a}_{F,t} \equiv W_{F,t} - F\,[V^p_{F,t} + V^a_{F,t}]$. From (69) and (70), we then have

$$\begin{aligned} \Sigma^{V^a}_{F,t} \;&=\; \frac{-\hat{\omega}_{F,t}\;V^p_{F+1,t+1}}{1+\rho} + \frac{(1-\hat{\omega}_{F,t})\,\Sigma^{V^a}_{F,t} + \hat{\omega}_{F,t}\,\Big(W_{F+1,t+1} - F\,[V^p_{F+1,t+1} + V^a_{F+1,t+1}]\Big)}{1+\rho}\;, && (71) \\ &=\; \frac{\hat{\omega}_{F,t}\;V^a_{F+1,t+1}}{1+\rho} + \frac{(1-\hat{\omega}_{F,t})\,\Sigma^{V^a}_{F,t} + \hat{\omega}_{F,t}\,\Sigma^{V^a}_{F+1,t+1}}{1+\rho}\;.\;\square && (72) \end{aligned}$$

Proof of Proposition 4: Let $\check{A}_{F,t}$, $\check{b}_{F,t}$, $\check{e}_{F,t}$ and $\check{\omega}_{F,t}$ denote the number of agents employed by each firm, contract offered to each agent, effort exerted by each agent and firm creation threshold, at date t, under a first-best strategy. The first-best aggregate continuation value of all players is then $\check{W}_{F,t} = W_{F,t}(\check{A}_{F,t}, \check{b}_{F,t}, \check{e}_{F,t}, \check{\omega}_{F,t})$, where $W_{F,t}(A, b, e, \omega)$ is given by (68).

– The first best level of effort exerted by each agent $a \in \mathcal{A}_{f,t}$ maximizes $W_{F,t}(A, b, e, \omega)$ in (68). Given that $\frac{\partial W_{F,t}(A,b,e,\omega)}{\partial e} = -F\,A$, the first best level of effort is $\check{e}_{F,t} = 0$.

– The first best innovation implementation decision $\check{\omega}_{F,t}$ maximizes $W_{F,t}(A, b, e, \omega)$ in (68). Given that $\frac{\partial W_{F,t}(A,b,e,\omega)}{\partial \omega} = \frac{1}{1+\rho}\,[1-\kappa-\omega]$, the first best implementation threshold is $\check{\omega}_{F,t} = 1-\kappa$, for all F and at all date t.

– Under a strategy characterized by effort levels $\check{e}_{F,t} = 0$ and firm creation thresholds $\check{\omega}_{F,t} = 1-\kappa$, the number of agents A and the contract b have no impact on the aggregate value $W_F(A, b, \check{e}_{F,t}, \check{\omega}_{F,t})$. Then, (68) gives

$$\check{W}_{F,t} \;=\; \frac{1}{1+\rho}\left[\check{\omega}_{F,t}\,(1-\kappa) \;+\; \frac{1-(\check{\omega}_{F,t})^2}{2}\right] \;+\; \frac{(1-\check{\omega}_{F,t})\,\check{W}_{F,t+1} + \check{\omega}_{F,t}\,\check{W}_{F+1,t+1}}{1+\rho}\;. \tag{73}$$

The first best aggregate continuation value does not depend on the number of firms, F. Using $\check{W}_{F,t+1} = \check{W}_{F+1,t+1}$, (73) gives

$$\check{W}_{F,t} \;=\; \frac{1}{1+\rho}\left[\frac{1+(1-\kappa)^2}{2}\right] \;+\; \frac{\check{W}_{F,t+1}}{1+\rho}\;. \tag{74}$$

Developing (74), using $\check{W}_{F,T+1} = 0$ and the annuity expression, yields (20). □

Proof of Proposition 5: Denote $\Delta_{F,t} \equiv \check{W}_t - W_{F,t}$. From (69) and (74),

$$\Delta_{F,t} = \frac{\check{\pi} - \bar{\pi}(\hat{\omega}_{F,t})}{1+\rho} - \frac{s_F\,\alpha\left(1-\frac{1}{F}\right)\left[\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\,V^p_{F+1,t+1}\right]}{1+\rho} + \frac{(1-\hat{\omega}_{F,t})\Delta_{F,t+1} + \hat{\omega}_{F,t}\Delta_{F+1,t+1}}{1+\rho}\;. \tag{75}$$

$C^{\omega}_{F,t}$ and $C^{e}_{F,t}$ satisfy (22) and (23), hence $\Delta_{F,t} = C^{\omega}_{F,t} + C^{e}_{F,t}$.

Proof of (24): Given that $\lim_{F\to\infty} \hat{\omega}_{F,t} = 1-\kappa$ and $\check{\pi} \equiv \bar{\pi}(1-\kappa)$, it follows that $\lim_{F\to\infty} C^{\omega}_{F,t} = 0$. Given that $\lim_{F\to\infty} s_F = \alpha$, $\lim_{F\to\infty} \bar{\pi}(\hat{\omega}_{F,t}) = \bar{\pi}(1-\kappa) = \check{\pi}$ and $\lim_{F\to\infty} V^p_{F+1,t+1} = 0$, it follows that $\lim_{F\to\infty} \frac{C^{e}_{F,t}}{\check{W}_{F,t}} = \alpha^2$.

Proof of Result 4: With a planner regulating all contracts the game is modified at the earliest stage 1. The planner selects the compensation, $b_{F,t}$, principals can offer agents at stage 1. Subsequent stages are unchanged. Given that the model is worked backwards from the last decision to the first, the last decisions firm creation threshold and agent optimal effort are chosen in the same fashion, for a given (A, b). Calculations are identical to those in the proof of Proposition 1 until equation (40).

From (9), a necessary condition to have $\hat{\omega}_{F,t} = \check{\omega}$ (where $\check{\omega} = 1-\kappa$) for all F and t, is $V^p_{F,t+1} = 2\,V^p_{F+1,t+1}$ for all F and t. From (38), a necessary condition to have $e^*_{F,t}(A,b) = \check{e}$ (where $\check{e} = 0$) for all F, is $b = 0$ for all F and t. If $b = 0$, $e^*_{F,t}(A,b) = 0$ and $\hat{\omega}_{F,t} = 1-\kappa$ for all F and t, from (43), (given that $q^{**}_{F,t}(A,b) = \frac{1}{F}$ and $\pi^p_{F,t}(A,b) = \frac{1+(1-\kappa)^2}{2} + (1-\kappa)\,V^p_{F+1,t+1}$) the principal's payoff is

$$V^p_{F,t} \;=\; \frac{1}{1+\rho}\left\{\frac{1+(1-\kappa)^2}{2\,F} \;+\; \kappa\,V^p_{F,t+1} \;+\; (1-\kappa)\left(1+\frac{1}{F}\right)\,V^p_{F+1,t+1}\right\}\;. \tag{76}$$

Write $2\,V^p_{F+1,t} - V^p_{F,t}$ using twice the recursive expression (76). Using $2\,V^p_{F+1,t} - V^p_{F,t} = 0$, $2\,V^p_{F+1,t+1} - V^p_{F,t+1}$ and $2\,V^p_{F+2,t+1} - V^p_{F+1,t+1}$, gives

$$0 \;=\; \frac{1 + (1-\kappa) + (1-\kappa)^2}{1\,+\,\rho}\left(\frac{2}{F+1} \;-\; \frac{1}{F}\right)\;. \tag{77}$$

Hence $F = 1$, which contradicts $F > 1$. So, we cannot have $b = 0$, $e^*_{F,t}(A,b) = 0$ and $V^p_{F,t+1} = 2\,V^p_{F+1,t+1}$ (hence $\hat{\omega}_{F,t} = \check{\omega}$), for all F and t.