

Dynamic Network Competition

— VERY PRELIMINARY AND INCOMPLETE —

Hanna Hałaburda

Bruno Jullien

Yaron Yehezkel

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Abstract

This paper considers a dynamic platform competition in a market with network externalities. We ask two research questions. The first one asks how the beliefs advantage carries over in time, and whether a low-quality platform can maintain its focal position along time. We show that for very high and very low discount factors it is possible for the low-quality platform to maintain its focal position indefinitely. But for the intermediate discount factor the higher quality platform wins and keeps the market. The second question asks what drives changes in the market leadership along time (observed in many markets, like smartphones and video-game consoles), and how such changes can be supported as a dynamic equilibrium outcome. We offer two explanations. The first explanation relies on intrinsic equilibrium uncertainty. The second explanation relies on the adoption of technology. One could expect such change in the market leader to be a sign of intense competition between platforms. However, we find that changes in leadership indicate *softer* price competition.

1 Introduction

Platform competition typically involves repeated interaction. In each of the markets for smartphones, tablets, video-game consoles, etc., a small set of firms compete with each other repeatedly over time. Platforms should therefore take into account how their strategies today affect their future profits. How the competition in dynamic setting plays out may have an important effect on platforms' profits. Microsoft's Windows wins the market for computer

operating system over Apple’s OS many generations in a row. It has been often suggested that the Apple’s OS is of a better quality, but Windows wins because Microsoft gained the dominant position in the past, and the network effects allow this advantage to carry over time, despite inferior product. In other markets, like video-game consoles and smartphones, market leaders seem to be changing every few generations. So the platforms in those markets cannot count on the same future advantage from winning the market as in the market for computer operating systems.

In repeated interaction between firms, dynamic considerations play a new role when the firms are “platforms,” because in such markets, the firms operate in environments with network effects. A platform offers users a product which has some stand-alone value, but the value of the product increases if other users also join the same platform. The benefit may come directly from the presence of other users, or through endogenous provision of complementary goods (e.g., apps are more likely to be developed for a popular platform). Thus, the dominant platform can charge higher access prices, as it provides a higher value to users. The difficulty, however, is that users need to form beliefs in advance about which platform will offer most network benefits. In many markets we observe that the platform that became the dominant in the recent past has the advantage of users’ expectations that it will attract other users and/or complementor providers (e.g., app developers). That is, it becomes the *focal* platform. But despite this beliefs advantage, the platform that won in the past not necessarily will win in the future.

This paper considers a dynamic platform competition in a market with network externalities. We ask two research questions. First, in some cases, a platform that benefits from the focal position, can dominate the market even if it offers a product of lower stand-alone quality than a new platform. In such a case, the focal platform uses its focal advantage to overcome its quality disadvantage. In a dynamic environment, however, when platforms have an infinite horizon, it is possible to expect that the platform with the highest quality will have the strongest incentive to compete aggressively in order to gain and then maintain a focal position. We therefore ask whether a low-quality platform can maintain its focal position along time, when facing a higher-quality platform.

Second, in some cases, platforms “take turns” in being the dominant platform. In the market for smartphones, for example, Nokia dominated the early stage, along with RIM, with smartphones based on physical keyboard. Apple then revolutionized the industry by betting on the new touch screen technology and its new operating system. Nokia and RIM stuck to

their physical keyboard technology and operating systems in the subsequent updates of their products, and eventually lost the leadership position to Apple. Few years later, Samsung, managed to gain a substantial market share (though not strict dominance) by betting on smartphones with large screens, while Apple continues to stick to its 3.5 inch screen. Only recently, when it became evident that there is a high demand for smartphones with large screens, Apple decided to increase the screen size for iPhone 5. Today, Nokia is trying to regain its dominant position by betting on the new Windows Phone technology. In a bid to win back their position in the smartphone market, RIM (now simply called BlackBerry) introduced new phones, Q10 and Z10, with new operating system and many innovative features. Both Apple and Samsung choose to remain with their previous operating systems, and only offer periodical upgrades.

Such industry leader changes were also common in the history of the video-game consoles market. Nintendo, Sony and Microsoft alternated in being the market leader—none of them winning more than two generations in a row. While the technology significantly improved with each generation of video-game consoles, some generations were marked by radical innovation, e.g. Nintendo's Wii.¹

We therefore ask: What is the rationale behind the changes in the dominant platform with time in an environment with network effects, and how it can be supported as a dynamic equilibrium outcome?

To investigate these two research questions, we consider two platforms competing for infinite number of periods. In each period, one of the platforms wins the market. In order to focus on the dynamic aspects of the model, we assume homogeneous consumers. Hence the winning platform captures the whole market. The dynamic set-up allows consumers to base their behavior in the current period on the observation of the past outcomes. Specifically, the platform that won the market in the previous period becomes focal in the current period. In such a case, winning the market in one period gives the platform an advantage in the future periods. Hence, a non-focal platform may be willing to sacrifice current profit to gain future market position.

In our base model, we assume that each platform has an exogenously given stand-alone quality, independent from its focal position, and constant for all periods. We show that there is an equilibrium in which the focal platform maintains its focal position even though it offers a lower quality than the non-focal platform. Surprisingly, such an equilibrium is more likely

¹Hagiu and Halaburda (2009)

to hold when either the platforms are very short-sighted (i.e., small discount factor), or when they are very long sighted (i.e., a high discount factor). However, for intermediate values of discount factors, only the highest quality platform can maintain its focal position along time.

In reference to our second research question, we offer two explanations for how the changes in market leadership can be supported as a dynamic equilibrium outcome. The first explanation relies on intrinsic equilibrium uncertainty captured by assuming a public correlated equilibrium. Although being focal provides an advantage, it is limited in the sense that for a small value adjusted price differential the non-focal platform wins with positive probability. We find that while there may still be equilibria where the focal platform maintains forever its dominant position, existence conditions are more restrictive. Moreover there will be equilibria where alternation occurs in any period with positive probability.

Our second explanation relies on the adoption of technology. We extend our basic model by assuming that in every period there is a preliminary stage in which each of the two platforms chooses a technology for its product; and then the platforms compete in prices, while the technologies are set. One technology is safe and provides a certain quality to consumers for sure. The other one is a radical technology that can be superior to the safe technology with some probability, but otherwise fail. The “radical technology” may be interpreted as including new features, where the demand for those features is unknown (e.g., touch screen vs keyboard in smartphones); or investing in R&D of a new operating system which will succeed only with some probability. We assume that a platform can invest in one technology only and must invest to stay active. We find conditions for an equilibrium in which the focal platform chooses the incremental technology while the non-focal chooses the radical technology. If the radical technology fails, the non-focal platform sets a soft price, letting the focal platform win another round. If the radical technology succeeds, the non-focal platform competes aggressively and gains the focal position in the market. Then, in the next round, the new focal platform will now choose the incremental technology, while the new non-focal platform will now chose the radical technology. As in the mixed strategy case, each platform internalizes that a shift in the focal position can occur with some probability in every round.

The intuition behind both of our explanations is qualitatively the same. Platforms will not want to deviate from such equilibrium because otherwise they expect intense price competition. The focal platform knows that if it will not let the non-focal platform to have

some probability of becoming focal in future rounds, the non-focal platform will compete aggressively in the current round. Likewise, for the non-focal platform, it becomes easier to gain a focal position when the old focal platform knows that the shift in focal position is only temporary, so the old focal platform will prefer to give up on its focal position and wait for several rounds, other than competing aggressively to maintain its focal position in the current round. Therefore, changes in market leadership indicates *softer* price competition, as opposed to the situation where always the same platform dominates the market and keeps the competitor(s) from winning.²

Most theoretical analyses of platform competition focus on static games. Caillaud and Jullien (2001, 2003) consider competition between undifferentiated platforms, where one of them benefits from favorable beliefs. Hagiu (2006) considers undifferentiated platform competition in a setting where sellers join the platform first, and only then buyers. Lopez and Rey (2009) consider competition between two telecommunication networks when one of them benefits from customers' inertia," such that in the case of multiple responses to the networks' prices, consumers choose a response which favors one of the networks. Halaburda and Yehezkel (2012) consider competition between platforms when one of them has only partial beliefs advantage. While all those papers acknowledge the dynamic nature of the platform competition, they aim at approximating the characteristics of the market in static models. Halaburda and Yehezkel (2012) explore how platform's strategies affect their future profits in a simple multi-period setup where the beliefs advantage depends on the history of the market. Markovich (2008) analyzes hardware standardization in a dynamic market where software firms invest in new product innovation. But the dynamics of platform competition is still underexplored. Cabral (2012) develops a dynamic model of competition with forward looking consumers but where only one consumer chooses at a time avoiding the coordination issue we focus on. Our paper is related to ongoing work by Biglaiser and Cremer (2012) trying to define a notion of consumer inertia creating an history dependency. We do not try to model how history dependency emerges but its implications for competition.

²We assume there always exist a non-focal platform ready to compete, e.g. potential entrant.

2 The Model

Consider an homogeneous population of size 1 and two competing networks with the same cost normalized to 0. Time goes from $t = 0$ to infinity. In the benchmark model the value of each network is fixed over time, with value v_i at network i . Consumers also benefit from network effects, the value other consumers joining same network is β .³ We assume that network effects are sufficiently strong that it is not possible to predict for sure which network will emerge:

Assumption: $\beta > |v_A - v_B|$

At any date the network i sets a price p_{it} , and consumers decide which network to join for the current period. In what follows prices can be negative, interpreted as price below cost.⁴

The issue with competition in an environment with network effects is that there is a multiplicity of equilibria. Indeed consider the allocation of consumers that emerges for given prices. If $v_i - p_{it} > v_j - p_{jt} + \beta$, then all consumers would join network i . But if

$$|v_A - v_B + p_{Bt} - p_{At}| < \beta, \tag{1}$$

there are two possible allocations, all consumers join A or all join B. This multiplicity creates a major difficulty to discussing dynamic competition in environments with network effects, and several solutions have been proposed to address this issue. In this paper we rely on the idea of pessimistic beliefs and focal platform as developed in Caillaud-Jullien (2003), Hagiu (2006) and Jullien (2000). We say that network i is *focal* if under condition (1), the consumers join network i . Then it is straightforward to see that if network i is focal, the equilibrium of a one period game is

$$p_i = v_i - v_j + \beta, p_j = 0$$

and consumers join the focal network.

Let us now turn to the dynamic model. The general idea is that consumers will base their behavior at date t on the observation of past history. To capture this phenomenon we rely on pessimistic beliefs and focal platform:

³Since the consumers are homogeneous they all join the same platform.

⁴To allow for truly negative prices, we need to assume that agents who collect the subsidy indeed join the platform and provide the benefit to other users.

Assumption: At any date there is a focal network.

The market dynamics in this context is determined by the dynamics of the focal platform. In what follows we discuss the effect of various possibility for the determination of the focal network with the idea that which platform is focal depends on the history. To simplify matters we focus on a one period dynamics.

At every period t , let us summarize the outcome by the focal network $f_t \in \{a, b\}$ and the active network denoted $a_t \in \{a, b\}$.⁵ Based on the observation on past outcome, consumers will form conjectures about the current period network most likely to win. These conjectures are assumed to converge to a single focal platform, and we refer to this process as the formation of market spirit. In $t = 0$ one of the platforms is arbitrarily set as the focal platform. Call this platform A . The dynamics of the market spirit is then given by transition probabilities

$$\Pr(f_t = i \mid a_{t-1}, f_{t-1}).$$

Consider now the game between networks given the dynamics of market spirit. At any date the focal platform f_t is common knowledge and it is the only payoff relevant variable. In what follows we concentrate on *Markov Perfect equilibria* where the state variable is f_t . A (Markov) equilibrium is characterized by equilibrium prices p_i^f , interpreted as the equilibrium price of network i when the focal platform is f . Given these equilibrium prices, we define the value function V_i^f as the expected discounted profit of network i when network f is focal.

The model thus involves an interaction between the dynamics of market spirit on one side and the dynamics of prices and market outcome on the other side. The final market outcome will then depend on this interaction.

In this paper we assume that the rule determining market spirit is deterministic and contrast two rules:⁶

- Stationary spirit: A is always focal.
- Market oriented spirit: the last winner of the market game becomes focal.

⁵In this model there cannot be market sharing in equilibrium: at each date, a single network attracts the whole population.

⁶In future versions we plan to explore more general rules.

3 Pure Strategy Equilibria

3.1 Stationary spirit

Consider the case where A is always focal, i.e. $f_t \equiv A$. Then at any date, network A wins the markets if $p_{At} - p_{Bt} \leq v_A - v_B + \beta$. To build the dynamic equilibrium we use the value functions V_i^f . Since the focal platform remains unchanged, the date t discount profit of network i is δV_i^A if it doesn't sell and $p_{it} + \delta V_i^A$ if it sells. Hence selling at date t generates additional profit p_{it} at date t . There is no dynamic consideration in pricing and the outcome is the same as the outcome of a one period game.

Proposition 1 *With a stationary spirit, network A is active at all dates at price $v_A - v_B + \beta$.*

In this case the only possibility for the entrant to capture the market is to offer a value larger than $v_A + \beta$. Indeed if v_B were larger than $v_A + \beta$, the equilibrium would have network B active at price $v_B - v_A - \beta$.

3.2 Market oriented spirit

Suppose now that the last winner is focal, i.e.

$$\Pr(f_t = a_{t-1} \mid a_{t-1}, f_{t-1}) = 1.$$

The situation is now clearly different as winning period t confers an advantage for period $t + 1$. Hence a non focal platform may be willing to sacrifice profit to gain a future market position. In this case the profit when selling is $p_{it} + \delta V_i^i$ while it is δV_i^j when the competing network sells. The gain from selling is then $p_{it} + \delta (V_i^i - V_i^j)$.

The interaction between current prices and future market spirit creates the possibility of multiple equilibria that we now consider. Typically we can envision two types of equilibria: either the same network wins irrespective of the focal platform, or the focal platform always win.

Consider the first type of situation. Is it possible to have an equilibrium where A always wins. In this case the value functions are $V_B^B = V_B^A = 0$ and network B always sets price $p_{Bt} = 0$. Then A sets a price $p_A^A = v_A - v_B + \beta$ if A is focal and a price $p_A^B = v_A - v_B - \beta$ if B is focal. This implies that the value function solves

$$V_A^A = v_A - v_B + \beta + \delta V_A^A; \quad V_A^B = v_A - v_B - \beta + \delta V_A^A.$$

and incentive compatibility for network A requires that

$$V_A^f \geq \delta V_A^B, \text{ for } f = A, B$$

Lemma 2 *With market oriented spirit, there is an equilibrium where network i wins in all circumstances if $v_i - v_j \geq \beta(1 - 2\delta)$.*

Proof. From the conditions above we obtain

$$V_A^A = \frac{v_A - v_B + \beta}{1 - \delta},$$

$$V_A^B = \frac{v_A - v_B}{1 - \delta} + \beta \frac{2\delta - 1}{1 - \delta}.$$

It is always the case that $V_A^A \geq \delta V_A^B$; This is an equilibrium $V_A^B \geq \delta V_A^B$, hence if $V_A^B \geq 0$ or $v_A - v_B + \beta(2\delta - 1) \geq 0$.

The roles of A and B are reverted if $i = B$. ■

The other case to consider is when the focal network wins in any case. Recall that p_i^f denote the price of network i when f is focal in such an equilibrium. Since the winning platform anticipates it will stay active and focal from the newt period on, we have values function

$$V_i^i = \frac{p_i^f}{1 - \delta}, \quad V_i^j = 0$$

The benefits of selling at a given date is $p_{it} + \delta V_i^i$. It follows that the minimal profit that network i is willing to sacrifice today to gain the market is $-\delta V_i^i$. In such an equilibrium it must be the case that the focal platform sets a price $p_i^i \leq v_i - v_j + \beta - \delta V_j^j$, otherwise the competing network would set a price above $-\delta V_i^i$ and wins the market. Ruling out cases where $p_j < -\delta V_j^j$ because this is weakly dominated for firm j , we obtain equilibrium prices⁷

$$p_i^i = v_i - v_j + \beta - \delta V_j^j, \quad p_j^j = -\delta V_j^j.$$

⁷This is innocuous for existence argument

This leads to values function in such an equilibrium solutions of

$$\begin{aligned}(1 - \delta) V_A^A + \delta V_B^B &= v_A - v_B + \beta \\ (1 - \delta) V_B^B + \delta V_A^A &= v_B - v_A + \beta\end{aligned}$$

yielding

$$V_A^A = \frac{v_A - v_B}{1 - 2\delta} + \beta; \quad V_B^B = \frac{v_B - v_A}{1 - 2\delta} + \beta.$$

We then conclude that:

Lemma 3 *With market oriented spirit, there is an equilibrium where the focal network always wins if $\beta |1 - 2\delta| > |v^B - v^A|$.*

Proof. For this to be an equilibrium it is necessary and sufficient that $V_A^A > 0$ and $V_B^B > 0$. ■

To summarize we have shown that if δ is close enough to $1/2$ that $\beta |1 - 2\delta| < |v^B - v^A|$, there is a unique equilibrium where the most efficient network wins in period 0 and keep the market. Otherwise in any equilibrium A stays active all the time because it is focal in period 0 and stays so. Indeed this is the case if the focal network always win, or if A always win. Thus network B can win the market only if $v_B - v_A > \beta (1 - 2\delta)$.

Proposition 4 *With a market oriented spirit, network B wins the market in any equilibrium if and only if $v_B - v_A > \beta |1 - 2\delta|$, and in one equilibrium if $v_B - v_A \geq \beta (1 - 2\delta)$.*

Proof. This follows from the assumption that A is initially focal. ■

The equilibrium active network is depicted in the Figure 1. The proposition shows that to win the market with certainty, network B must offer a service of superior quality but the quality premium can be smaller than in the case where A is focal. *In particular for $\delta = 1/2$, the unique equilibrium is the efficient equilibrium.*

As we move away from the intermediate level of discount factors, it becomes more difficult for the entrant to compete. Indeed *the remarkable feature is that entry only occurs at intermediate levels of discount factors.* The reason is twofold. For low discount factors, the dynamics is not relevant and we converge to the static framework and no entry. But,

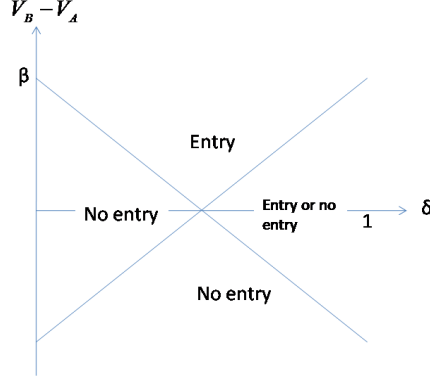


Figure 1: Equilibrium configuration

when network are very patient, the entrant is not able either to leverage a quality differential smaller than β . In this case network A is ready to fight intensively to keep its market position and leverages his current focal position for this purpose. Given that $\beta > v_B - v_A$ this is sufficient to keep the other out of the market.

Observation: Very patient network. Indeed the case where δ converges to 1 is of particular interest. The profit for $\delta = 1$ is bounded and equal to

$$V_A^A = v_B - v_A + \beta; \quad V_B^B = v_A - v_B + \beta.$$

This implies that the per period profit $(1 - \delta)V_A^A$ of platform A converges to zero while the per period consumer surplus is $v_A + \beta$. Thus, platform A sells and stays focal all the time but is forced to set a price close to zero, while the network B set a very low price is $-(v_A - v_B + \beta) < 0$. Thus with very patient firms, network B doesn't succeed to sell by the rivalry for the market is sufficient that almost all profit is passed on to consumers.

4 Generalization

The above notion of a focal platform is extreme in that it wins with probability 1. In order to extend the concept, we assume that consumers coordinate on a public correlated equilibrium. In this equilibrium, under condition 1, consumers coordinate by means of sunspot in such a way that they all join network A with some probability x_A , while they all join network B with complementary probability $x_B = 1 - x_A$. Intuition suggests that the probability to

win the market is mono tonic with the price differential. The idea of a focal platform can then be expressed as the fact that at equal value of the two network, $x_A > x_B$. To apply this approach let us define

Definition 1 *The coordination rule is a non-decreasing right-continuous function $\varphi(x)$ from \mathbb{R} into $[0, 1]$ with $\varphi(x) \geq 1 - \varphi(-x)$, $\varphi(x) = 1$ if $x \geq \beta$ and $\varphi(x) = 0$ if $x < -\beta$.*

Definition 2 *If the platform i is focal, then at prices (p_A, p_B) , consumers all join i with probability $\varphi(v_A - v_B + p_B - p_A)$ and they all join $j \neq i$ with complementary probability*

The advantage of been focal is embedded in $\varphi(x) \geq 1 - \varphi(-x)$. We denote by

$$\bar{\beta} = \min \{x \mid \varphi(x) = 1\} \in [-\beta, \beta]$$

the minimal value differential at the focal platform sells with probability 1. In what follows we assume that φ is linear

$$\varphi(x) = \min \left\{ \frac{x + \beta}{\bar{\beta} + \beta}, 1 \right\} \quad \text{with } \bar{\beta} \in (-\beta, \beta)$$

The model encompasses the previous case as a limit when $\bar{\beta}$ tends to $-\beta$ and a symmetric case with no focal platform when $\bar{\beta} = \beta$. We expect the results generalize to the case where φ is non linear but, on the range $0 < \varphi(x) < 1$, it is such that φ'/φ decreases and $\varphi'/(1 - \varphi)$ increases.

To illustrate the approach, we first discuss some cases without dynamic effects.

4.1 Stationary spirit

In the case of stationary spirit, A is always focal and we can solve as a static game. Short-run profits are $\pi_A = \varphi(v_A - v_B + p_B - p_A)p_A$ and $\pi_B = (1 - \varphi(v_A - v_B + p_B - p_A))p_B$. We have an equilibrium at

$$p_B = 0, \quad p_A = v_A - v_B - \bar{\beta}, \quad x_A = 1$$

if

$$\begin{aligned} \frac{d}{dp_A} \varphi(v_A - v_B - p_A) p_A &\leq 0 \text{ or} \\ v_A - v_B &\geq 2\bar{\beta} + \beta \end{aligned}$$

It is not possible that B wins with probability 1 as this would require $p_A = 0$ and $p_B = v_B - v_A - \beta < 0$. Thus if $v_A - v_B < 2\bar{\beta} + \beta$, the equilibrium involves positive probability that any network wins with:

$$\begin{aligned}\frac{v_A - v_B + p_B + \beta}{2} &= p_A \\ \frac{\bar{\beta} + v_B - v_A + p_A}{2} &= p_B\end{aligned}$$

$$\begin{aligned}\frac{v_A - v_B}{3} + \frac{\bar{\beta}}{3} + \frac{2}{3}\beta &= p_A \\ \frac{v_B - v_A}{3} + \frac{2\bar{\beta}}{3} + \frac{\beta}{3} &= p_B \\ x_A &= \frac{1}{3} \frac{v_A - v_B + 2\beta + \bar{\beta}}{\bar{\beta} + \beta}\end{aligned}$$

4.2 No focal platform

This is the case when $\bar{\beta} = \beta$. Then we can follow the above result and obtain if

$$\begin{aligned}\frac{v_A - v_B}{3} + \beta &= p_A \\ \frac{v_B - v_A}{3} + \beta &= p_B \\ x_A &= \frac{\frac{v_A - v_B}{3} + \beta}{2\beta}\end{aligned}$$

Notice that the profit in this symmetric case is

$$\begin{aligned}\Pi_A &= \frac{\left(\frac{v_A - v_B}{3} + \beta\right)^2}{2\beta(1 - \delta)} \\ \Pi_B &= \frac{\left(\frac{v_B - v_A}{3} + \beta\right)^2}{2\beta(1 - \delta)}\end{aligned}$$

which is increasing in β .

When network effects increase it becomes more difficult to induce a stronger inflexion of the market spirit and to raise the chance to win. As a result, competition is weakened and prices increases.

5 Market oriented spirit with $\bar{\beta} < \beta$

Consider an equilibrium where A always wins. In this case the value functions are $V_B^B = V_B^A = 0$ and network B always sets price $p_{Bt} = 0$. Then A sets a price $p_A^A = v_A - v_B - \bar{\beta}$ if A is focal and a price $p_A^B = v_A - v_B - \beta$ if B is focal. This implies that the value function solves

$$V_A^A = v_A - v_B - \bar{\beta} + \delta V_A^A; \quad V_A^B = v_A - v_B - \beta + \delta V_A^A.$$

and incentive compatibility for network A requires that

$$V_A^f \geq \delta V_A^B, \text{ for } f = A, B$$

Lemma 5 *With $-\beta < \bar{\beta} < \beta$, there is no equilibrium where network i wins in all circumstances if .*

Proof. From the conditions above we obtain

$$V_A^A = \frac{v_A - v_B - \bar{\beta}}{1 - \delta},$$

$$V_A^B = \frac{v_A - v_B}{1 - \delta} - \beta - \frac{\delta}{1 - \delta} \bar{\beta}.$$

It is always the case that $V_A^A \geq \delta V_A^B$ because $-\bar{\beta} > -\beta$; This is an equilibrium if $V_A^B \geq \delta V_A^B$, hence if $V_A^B \geq 0$ or $v_A - v_B - (1 - \delta)\beta - \delta\bar{\beta} \geq 0$.

The firm i should not increase its price which leads to

$$\begin{aligned} \frac{d}{dp_A} \varphi(v_A - v_B^A - p_A^B)(p_A^B) &< 0 \\ \bar{\beta} + \beta &< p_A^B \\ \bar{\beta} + 2\beta &< v_A - v_B \end{aligned}$$

which is not possible as $\bar{\beta} + 2\beta > \beta$.

The roles of A and B are reverted if $i = B$. ■

The incentive to deviate by raising the price when the network is not focal are too strong. As we need

$$\frac{1}{\varphi'(1)} < p_A^B = v_A - v_B - \beta < 0$$

This is quite general.

The other case to consider is when the focal network wins in any case. Recall that p_i^f denote the price of network i when f is focal in such an equilibrium. Since the winning platform anticipates it will stay active and focal from the next period on, we have values function

$$V_i^i = \frac{p_i^i}{1 - \delta}, \quad V_i^j = 0$$

The benefits of selling at a given date is $p_{it} + \delta V_i^i$. It follows that the minimal profit that network i is willing to sacrifice today to gain the market is $-\delta V_i^i$. In such an equilibrium it must be the case that the focal platform sets a price $p_i^i \leq v_i - v_j - \bar{\beta} - \delta V_j^j$, otherwise the competing network would set a price above $-\delta V_i^i$ and wins the market with positive probability. Ruling out cases where $p_j < -\delta V_j^j$ because this is weakly dominated for firm j , we obtain equilibrium prices⁸

$$p_i^i = v_i - v_j - \bar{\beta} - \delta V_j^j, \quad p_j^j = -\delta V_j^j.$$

This leads to values function in such an equilibrium solutions of

$$\begin{aligned} (1 - \delta) V_A^A + \delta V_B^B &= v_A - v_B - \bar{\beta} \\ (1 - \delta) V_B^B + \delta V_A^A &= v_B - v_A - \bar{\beta} \end{aligned}$$

This yields

$$V_A^A = \frac{v_A - v_B}{1 - 2\delta} - \bar{\beta}; \quad V_B^B = \frac{v_B - v_A}{1 - 2\delta} - \bar{\beta}.$$

Value functions must be positive. Moreover a focal network should not be tempted to raise its price slightly. We then conclude that:

Lemma 6 *With $-\beta < \bar{\beta} < \beta$, there is an equilibrium where the focal network always wins if $-(2\bar{\beta} + \beta) |1 - 2\delta| \geq |v^B - v^A|$.*

Proof. For this to be an equilibrium it is necessary and sufficient that $V_A^A > 0$ and $V_B^B > 0$ and

$$\begin{aligned} \frac{d}{dp_A} \varphi (v_A - v_B + p_B^A - p_A^A) (p_A^A + \delta V_A^A) &< 0 \\ \text{or } \bar{\beta} + \beta &< p_A^A + \delta V_A^A = V_A^A \end{aligned}$$

⁸This is innocuous for existence argument

and the symmetric condition for B . So we have that

$$\begin{aligned}\frac{v_A - v_B}{1 - 2\delta} &> \max \{ \bar{\beta}, 2\bar{\beta} + \beta \} = 2\bar{\beta} + \beta; \\ \frac{v_B - v_A}{1 - 2\delta} &> \max \{ \bar{\beta}, 2\bar{\beta} + \beta \} = 2\bar{\beta} + \beta\end{aligned}$$

■

To summarize we have shown that if δ is close enough to $1/2$ that $-(2\bar{\beta} + \beta) |1 - 2\delta| < |v^B - v^A|$, there is no equilibrium where one platform always win, nor the focal platform always wins. A conjecture is then that under this condition, the efficient platform wins with positive probability in any period.

Proposition 7 *With $-\beta < \bar{\beta} < \beta$ and $v_B - v_A > 0$, network B wins the market with positive probability in any equilibrium if and only if $-(2\bar{\beta} + \beta) |1 - 2\delta| < v^B - v^A$.*

TO BE PROVEN

6 Technology

Consider two platforms A and B , a priori the same, interacting in a market for infinite number of periods. Every period, each platform chooses a technology. There are two technologies available to each platform:

- *Incremental* technology. When a platform adopts this technology, it offers its users standalone value of $v_L > 0$ with probability 1.
- *Radical* technology. The radical technology is risky—successful only with probability $\gamma < 1$. When the technology is successful, the platform offers its users $v_H > v_L$ as standalone value; and when the technology fails, the platform offers 0.

Every period each platform first chooses a technology. Then the success or failure realizes if some platform chose the radical technology.⁹ With the technology choice and realization

⁹If both platforms chose the radical technology, their success realization is perfectly correlated. That is either both fail or both succeed.

publicly known, the platforms compete on prices. Finally, customers observe those prices and decide which platform to join.

The platform that won the previous period is focal in current period independently of technology choices and the realization of the radical technology.

We impose the following restrictions on the parameters. First, suppose that $v_H > \beta + v_L$. This assumption reflects the fact that a radical technology is only worth considering when its success provides sufficient high value to the users that the non-focal platform could win the market under some conditions. Specifically, under this assumption the non-focal platform wins the market in a static game if it chooses the radical technology, the focal platform chooses the incremental technology, and the radical technology succeeds. Second, it is possible to distinguish between strong network externalities: $v_L < \beta$, and weak network externalities: $v_L > \beta$. In the latter case, the focal platform does not have a strong advantage over the non-focal, because network effects are weak. As a result, in the static game, the focal platform loses the market as soon as it provides lower stand-alone quality of the product than the non-focal platform.¹⁰ When $v_L < \beta$, the focal platform has a strong advantage over the non-focal, because network effects are strong. As a result, it is possible for the focal platform to keep the market captive even in some cases where it provides lower quality than the non-focal platform. And specifically, the advantage of the strong network effect allows the focal platform to keep the market when it provides 0 stand alone utility and the non-focal platform provides $v_L > 0$. This happens when the non-focal platform chose incremental technology, the focal platform chose radical, and the radical technology fails.¹¹

We focus in our analysis on strong network effects, $\beta > V_L$, as we want to identify how the presence of dynamics, along with strong network effects, determine the platforms' technology choices.

Consider the following strategy for each platform: whenever it is focal, the platform chooses the incremental technology; and it chooses radical technology whenever it is non-focal (with appropriate prices constituting subgame equilibrium). We will identify conditions under which neither platform has a profitable deviation from this strategy.

¹⁰This happens when the non-focal platform can win the market if it chooses the incremental technology, the focal platform chooses the radical technology, and the radical technology fails.

¹¹Clearly, with positive network effects—whether strong or weak—the non-focal platform cannot win the market if it provides the same stand-alone value.

For the purpose of exposition, suppose that currently platform A is focal and platform B is non-focal. If the radical technology failed, the non-focal platform (platform B) has quality disadvantage. The platform can either set its price to 0—and get the payoff $0 + \delta V_B^A$, where V_B^A is expected future value of being a non-focal platform—or set the price p_B^A that would allow it to win the market and become focal. The payoff then is $p_B^A + \delta V_B^B$. The highest price that the platform B is willing to pay to win when the radial technology failed is such that $p_B^A + \delta V_B^B = \delta V_B^A \iff p_B^A(\text{fail}) = \delta(V_B^A - V_B^B)$. The focal platform (platform A) sets its price such to prevent the non-focal platform from winning and taking over the market: $p_A^A(\text{fail}) = v_L + \beta + p_B^A(\text{fail}) = v_L + \beta + \delta(V_B^A - V_B^B)$.

When the radical technology is successful, the non-focal platform will win the market. The focal platform is willing to pay up to $p_A^A(\text{succ}) = \delta(V_A^B - V_A^A)$ to keep the market. Therefore, platform B sets $p_B^A(\text{succ}) = p_A^A(\text{succ}) + v_H - v_L - \beta = v_H - v_L - \beta + \delta(V_A^B - V_A^A)$.

In order to assess the expected future value of staying focal for platform A , V_A^A , we need to take into account that the radical technology that platform B chooses succeeds with probability γ and fails with probability $1 - \gamma$:

$$V_A^A = \gamma \delta V_B^A + (1 - \gamma) (v_L + \beta - \delta(V_B^B - V_B^A) + \delta V_A^A)$$

Similarly, the expected future value to platform B of staying non-focal is

$$V_B^A = \gamma (v_H - v_L - \beta + \delta(V_A^B - V_A^A) + \delta V_B^B) + (1 - \gamma) \delta V_B^A$$

The analysis is the same if platform B is the focal platform, and platform A is non-focal. This analysis results in

$$V_B^B = \gamma \delta V_A^B + (1 - \gamma) (v_L + \beta - \delta(V_A^A - V_A^B) + \delta V_B^B)$$

and

$$V_A^B = \gamma (v_H - v_L - \beta + \delta(V_B^A - V_B^B) + \delta V_A^A) + (1 - \gamma) \delta V_A^B$$

Together, we have four equations with four unknowns. Solving for V_A^A , V_A^B , V_B^B and V_B^A yields

$$V_i^i = \frac{(1 - \delta)(v_L + \beta) + \gamma(\delta v_H - v_L - \beta)}{1 - \delta} \quad \text{for } i = A, B \quad (2)$$

$$V_i^j = \frac{\gamma(v_H - v_L - \beta)}{1 - \delta} \quad \text{for } i = A, B \text{ and } j \neq i \quad (3)$$

It is straightforward to show that V_i^i and V_i^j are positive, given the conditions on parameters ($v_H > v_L$, $\beta > v_L > 0$ and $v_H - v_L > \beta$).

To make sure that this is an equilibrium, we need to check whether neither of the platforms have incentive to deviate in their strategy choice. It is never profitable for the non-focal platform to deviate and adopt the radical technology, as it would always lose the market and get no profits. But could the focal platform profitably deviate by adopting radical technology instead?

To answer this question, consider the case where given that the non-focal platform chooses radical technology every period, the focal platform also chooses radical technology in every period in the infinite time horizon.¹² The non-focal platform sets the lowest sensible price, $p_j^i = -\delta\tilde{V}_j^j$, where \tilde{V}_j^j is expected continuation value in infinite time horizon given the infinite deviation of the focal platform. The non-focal platform never wins the market, because the focal platform sets $p_i^i = p_j^i + \beta = \beta - \delta\tilde{V}_j^j$, and wins the market. Thus, we obtain $\tilde{V}_i^i = \beta$ and $\tilde{V}_j^i = 0$.

The focal platform does not find it profitable to deviate when $\tilde{V}_i^i \leq V_i^i$, with V_i^i characterized by (2). That is, when

$$\gamma\beta \leq v_L(1 - \delta - \gamma) + \delta\gamma v_H,$$

From this condition, we can find out for which parameter values there exists an equilibrium where the focal platform always chooses the incremental technology, and the non-focal platform always chooses the radical technology.

Lemma 8 *There exists $\bar{\gamma}$ such that for every $\gamma \leq \bar{\gamma}$ there exists the equilibrium where the focal platform always chooses the incremental technology, and the non-focal platform chooses the radical technology. The value of $\bar{\gamma}$ is given by*

$$\bar{\gamma} = \begin{cases} \frac{v_L(1-\delta)}{v_L+\beta-\delta v_H} & \text{for } \delta \in (0, \frac{\beta}{v_H-v_L}] \\ 1 & \text{for } \delta \in (\frac{\beta}{v_H-v_L}, 1). \end{cases}$$

¹²The benefit of deviating from the equilibrium technology choice depends on the platforms' beliefs concerning future deviations. We focus on the case where once the non-focal platform observes that the focal platform deviated to the radical technology, the non-focal platform believes that the focal platform will continue choosing the radical technology for all future periods. Moreover, we assume that in this case the focal platform will also expect to choose the radical technology in all future rounds. Another out-of-equilibrium beliefs may be that both platforms expect the deviation to occur for only one period. We plan to investigate alternative beliefs in future versions of the paper.

The intuition for this result is the following. Suppose first that $\delta = 0$. In this case $0 < \bar{\gamma} < 1$. Since the focal platform has an advantage over the non-focal, the focal platform will win the market even if both platforms chose the radical technology. Given that the non-focal platform chose the radical technology, the focal platform will therefore choose the incremental technology if the radical technology is too risky: $\gamma < \bar{\gamma}$, and choose the radical technology if its safe and therefore more attractive such that: $\gamma > \bar{\gamma}$. Now suppose that $\delta > 0$. Now, choosing the incremental technology has an additional advantage for the focal platform. This advantage is that by choosing the incremental technology, the focal platform provides the non-focal platform with some probability of becoming focal in the future. As a result, the non-focal platform will compete less aggressively, in those periods when its radical technology fails. That is, whenever the focal platform chooses the incremental technology, V_i^j is positive. Notice that for the same reason, the focal platform has less to lose in this case from losing the market and becoming non-focal. This is because the platform will always have the probability of regaining its focal position in the future. Consequently, as δ increases, $\bar{\gamma}$ increases, and the equilibrium in which the focal platform chooses the incremental technology becomes more likely. We summarize this result in the following proposition.

Proposition 9 *$\bar{\gamma}$ is decreasing with β and increasing with δ . That is, in the dynamic game, it is more likely that the focal platform will choose the incremental technology than in the static game.*

Consider now a situation where the focal platform always plays the radical technology. The non-focal platform is then indifferent between playing radical or incremental technology, as it will never win the market. From the analysis above, we can see that the situation where both platforms always choose the radical technology is an equilibrium when $\gamma \geq \bar{\gamma}$, where $\bar{\gamma}$ is defined in Lemma 8. It can also be easily verifiable that when $\gamma \geq \frac{v_L}{v_H}$, there exists an equilibrium where the focal platform always chooses the radical technology and the non-focal platform always chooses the incremental.¹³ In both equilibria where the focal platform chooses the radical technology only differ in prices that the focal platform charges to users, but they are the same in that the same platform always wins the market, i.e., there is no leadership change.

¹³Note that $\bar{\gamma} > \frac{v_L}{v_H}$, so for the non-empty interval $\gamma \in [\frac{v_L}{v_H}, \bar{\gamma}]$ two equilibria are possible, where each platform chooses different technology from the other.

For completeness, notice that the situation where both platforms always choose the incremental technology is never an equilibrium. The non-focal platform has incentive to deviate to the radical technology, as the radical technology offers a positive probability of profitably winning the market.

Therefore, there are three equilibria possible, depending on the parameter values. They are illustrated in Figure 2. As we can see, for $\gamma < \frac{v_L}{v_H}$ the only equilibrium is the equilibrium in which the market leader changes with a positive probability.

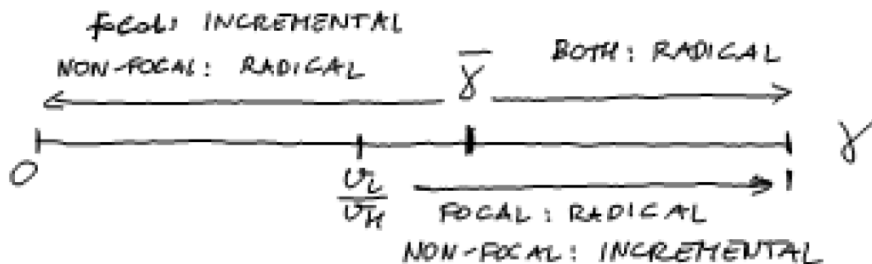


Figure 2: Possible equilibria, depending on parameter values.

The results above have implications for welfare. To maximize welfare, the efficient outcome involves the two platforms choosing different technologies and the platform with the better technology winning the market.

To examine the effects of network externality and dynamics on efficiency, consider first a static game. In the static game where $\delta = 0$, the efficient result holds for all γ if $\beta = 0$, such that $\bar{\gamma} = 1$. In this case, there are no network externalities and therefore no focal advantage, and the two platforms choose in equilibrium different technologies. As β increases, $\bar{\gamma}$ decreases below 1. Now, the focal platform gains an advantage, and therefore the market becomes inefficient for $\bar{\gamma} < \gamma < 1$, as now the focal platform also chooses the radical technology.

The results of Proposition 9 show that dynamic considerations can reduce, and even completely eliminate the inefficient outcome. As δ increases, $\bar{\gamma}$ increases, thus limiting the range of γ in which the inefficient outcome can occur. Moreover, there is a cutoff of δ , such that for higher values of δ , $\bar{\gamma} = 1$, implying that the efficient result is an equilibrium for all γ . The qualitative conclusion of these findings is that if platforms are future looking, we should expect to see more diversity in their choice of technologies. Short-sighted platforms

may choose to adopt similar technologies.

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