Complementary Monopolies and Bargaining

Daniel F. Spulber*

Northwestern University

June 8, 2016

Abstract

How should complementarities affect antitrust merger policy? I introduce a two-stage strategic model in which complementary input sellers offer supply schedules to producers and then engage in bilateral bargaining with producers. The main result is that there is a unique weakly dominant strategy equilibrium and the equilibrium attains the joint profit maximizing outcome. Output equals that of a bundling monopoly and total input prices are lower than prices with a bundling monopoly. The result holds with perfect competition in the downstream market. The result also holds with oligopoly competition in the downstream market. This implies that the Cournot Effect does not hold when companies negotiate supply contracts rather than using posted prices. The analysis has implications for antitrust policy towards vertical, conglomerate, and horizontal mergers.

Keywords: antitrust, complements, mergers, bargaining, supply schedules, contracts, competition, cooperation, Cournot Effect, monopoly

JEL Codes: C7, D4, L

*Elinor Hobbs Distinguished Professor of International Business, Professor of Strategy, Kellogg School of Management, Northwestern University, 2001 Sheridan Road, Evanston, IL, 60208. E-mail: jems@kellogg.northwestern.edu.

I gratefully acknowledge research grant support from Qualcomm, the Marion Ewing Kauffman Foundation, and the Kellogg School of Management. I thank Pere Arque-Castells, Justus Baron, and Alex Galetovic for helpful comments. Any opinions expressed in this paper are solely those of the author.

I Introduction

Complementarities have been used to seek antitrust approval of conglomerate and vertical mergers, as in the blocked merger of General Electric (GE) and Honeywell. To examine the economic effects of complementarities, I introduce a two-stage bargaining game that provides a more complete description of interaction between complementary monopolists and downstream producers. In the first stage of the game, each complementary monopolist offers an input supply schedule to downstream producers. Then, in the second stage of the game, each complementary monopolist engages in bilateral bargaining with each producer with over input prices. Bilateral bargaining occurs simultaneously and input prices are jointly determined. Given these supply schedules and input prices, producers choose input demands and supply final outputs and the downstream market clears. At the unique weakly dominant strategy equilibrium, the final output attains the joint profit maximum and total input prices are less the bundled monopoly level. The efficiency of the equilibrium outcome has implications for antitrust policy towards vertical, conglomerate, and horizontal mergers.

The two-stage bargaining game describes markets in which firms negotiate supply contracts. For many markets, contract negotiation offers a more accurate description of business transactions than does the basic posted price model. Companies use supply contracts because business transactions often take place over time and require capacity commitments from suppliers and demand commitments from buyers. Industries often use contracts for supply chain management and coordination.[1] There is extensive evidence that suppliers negotiate supply contracts with producers, assemblers, and distributors. According to the Bureau of Labor Statistics (BLS), US companies have over 1,740,000 wholesale and manufacturing sales representatives.[2] BLS data also show that US companies have over 72,000 purchasing managers and

[1] See the research and the literature reviewed by Tsay (1999), Tsay et al. (1999), Cachon and Lariviere, (2005), Li and Wang, (2007), and Arshinder et al. (2011).

[2] Data are for 2015 and include the categories 41-4011 Sales Representatives, Wholesale and Manufacturing, Technical and Scientific Products, see http://www.bls.gov/oes/current/oes414011.htm, and 41-4012 Sales Representatives, Wholesale and Manufacturing, Except Technical and Scientific Products, see http://www.bls.gov/oes/current/oes414012.htm, accessed April 7, 2016.

over 400,000 buyers and purchasing agents who evaluate suppliers, review product quality, and negotiate supply contracts.[3]

The main results of the analysis are as follows. First, I consider the two-stage game when the downstream market is perfectly competitive. I show that the strategic game in supply schedules has a unique weakly dominant strategy equilibrium. I find that at the unique equilibrium of the strategic game, suppliers and producers maximize joint benefits. The final output equals the cooperative level and total input prices are strictly less than the bundled monopoly benchmark. The analysis suggests that complementarity of inputs induces coordination rather than blocking it.

The intuition for the efficiency result is as follows. In the first stage of the game, the weakly dominant strategy for every complementary input supplier is to offer a maximum supply equal to the cooperative quantity. So, strategic input suppliers take into account the potential effects of their supply decisions on the product market. If other input suppliers were to choose maximum quantities above that which maximizes joint benefits, then a supplier would strictly prefer to propose a lower maximum quantity that would maximize joint benefits. If other input suppliers were to choose maximum quantities below that which maximizes joint benefits, then a supplier would not restrict the quantity further and would be indifferent between all maximum quantities above the level that maximizes joint benefits. So, the maximum quantity that maximizes joint benefits is the unique weakly dominant strategy for every supplier. In the second stage of the game, simultaneous bilateral bargaining over the division of economic rents provides incentives for cooperation among input

[3] Data are for 2015. Purchasing managers are in the category 11-3061. Buyers and purchasing agents are in the categories 13-1022 Wholesale and Retail Buyers, Except Farm Products (http://www.bls.gov/oes/current/oes131022.htm), and 13-1023 Purchasing Agents, Except Wholesale, Retail, and Farm Products (http://www.bls.gov/oes/current/oes131023.htm). The BLS states that purchasing managers "Plan, direct, or coordinate the activities of buyers, purchasing officers, and related workers involved in purchasing materials, products, and services. Includes wholesale or retail trade merchandising managers and procurement managers." See http://www.bls.gov/oes/current/oes113061.htm, accessed April 7, 2016. The BLS also states that "Purchasing agents and buyers consider price, quality, availability, reliability, and technical support when choosing suppliers and merchandise. Buyers and purchasing agents buy products and services for organizations to use or resell. They evaluate suppliers, negotiate contracts, and review the quality of products." See http://www.bls.gov/ooh/business-and-financial/buyers-and-purchasing-agents.htm#tab-2, accessed April 7, 2016.

suppliers. Competition in the downstream market generates an output equal to the smallest of the maximum input supply offers.

Second, I show that the outcome of the two-stage game generates greater consumer benefits and producer surplus than the Cournot posted-prices game. According to the Cournot Effect, complementary monopolists choose lower prices by cooperating rather than by competing.[4] With posted prices, competing complementary monopolists behave inefficiently because they do not consider how their prices affect each others' profits, which generates a free-rider effect. Economists have applied the Cournot Effect to many problems including vertical and conglomerate mergers, bilateral monopoly, successive monopoly, labor-management negotiations, international trade, money in decentralized exchange, externalities, joint production, innovation, and coordination in network industries. Despite the wide application of the Cournot Effect, the stark contrast between cooperation and competition may be due to artificially restricting competition to posted prices.

Third, I extend the two-stage game to oligopoly competition among producers in the downstream market. I show that the strategic game in supply schedules has a unique weakly dominant strategy equilibrium. Again, I find that at the unique equilibrium of the strategic game, suppliers and producers maximize joint benefits. The final output of the downstream industry equals the outcome when inputs are supplied by a bundled monopoly and total payments are less than the bundled monopoly benchmark.

Fourth, I explore the implications of the results for antitrust policy in markets with complementary inputs or complementary final products. The main implication of the results are that vertical or conglomerate mergers are not necessary for markets to achieve the cooperative outcome. This means that the Cournot Effect need not justify mergers unless it can be established that firms engage in posted price behavior rather than forming supply contracts. So, vertical and conglomerate mergers need not

[4] According to Cournot "An association of monopolists, working for their own interest, in this instance will also work for the interest of consumers, which is exactly the opposite of what happens with competing producers." Cournot finds that "the composite commodity will always be made more expensive, by reason of separation of interests than by reason of the fusion of monopolies" (1838, p. 103). See also Moore (1906).

improve market outcomes. I consider antitrust policy towards conglomerate mergers as in the blocked GE-Honeywell merger. I also consider antitrust policy towards vertical mergers of successive monopolies. Finally, I consider horizontal mergers of competing suppliers of the same input that would generate bilateral monopoly with a monopolistic downstream firm.

The present analysis suggests that allowing for more general strategic interactions is sufficient to resolve the complementary monopolies question. The Cournot Effect has generated nearly two centuries of controversy involving many distinguished economists.[5] Some economists argue that market outcomes are inefficient as predicted by the Cournot Effect and other economists argue that cooperative bargaining among complementary monopolies would result in an efficient outcome.[6] Schumpeter (1928) suggests that Cournot duopolists (or complementary monopolists) would maximize joint profits through tacit coordination.[7]

There is a long literature on Cournot's complementary monopolies problem and its dual, the quantity competition model.[8] Edgeworth (1925) considers competition

[5]Economists who have considered Cournot's analysis include Fisher (1898), Moore (1906), Marshall (1907), Bowley (1924), Edgeworth (1925), Schumpeter (1928), Zeuthen (1930), Stackelberg (1934), Hicks (1935), Kaldor (1936), and Tintner (1939). Machlup and Taber (1960) provide a valuable overview of the early literature.

[6]See Bowley (1928), Wicksell (1934), Tintner (1939), Henderson (1940), Leontief (1946), and Fellner (1947). For example, Bowley (1928, pp. 656-657) considers a bilateral monopoly where "the manufacturer and supplier of material combine to maximise their joint gain" (p. 656) and points out that the same result is obtained "when the manufacturer uses a number of materials, each the subject of an independent monopoly" (p.). Bowley expresses concern that the bargaining outcome is "unstable" because each side may want a larger share of the total benefit. Machlup and Taber (1960, p. 111) note: "negotiations between separate monopolists would, in the case of intermediate products, necessarily be carried on in terms of both quantity and price, and that the quantity agreed upon between the parties would be the same as that produced by an integrated monopolist."

[7]Schumpeter (1928, p. 370) states "we are, first, faced by the fact that they cannot very well fail to realise their situation. But then it follows that they will hit upon, and adhere to, the price which maximises monopoly revenue for both taken together (as, whatever the price is, they would, in the absence of any preference of consumers for either of them, have to share equally what monopoly revenue there is). The case will not differ from the case of conscious combination-in principle-and be just as determinate."

[8]Edgeworth (1925) critiques the stability of the Cournot duopoly models for both substitutes and complements and Fisher (1898, p. 126-128) critiques the dynamic analysis in Cournot's basic duopoly models. Economists who consider the effects of conjectural variations on Cournot duopoly

with imperfect complements and substitutes, and points out that perfect complementarity is a limiting case of complementary goods. Economides and Salop (1992) and Denicolo (2000) consider complementarities in consumption. Singh and Vives (1984) compare quantity and price strategies in a one-stage game with differentiated products that are either imperfect complements or substitutes.[9] It can be shown that as products approach perfect complementarity, the quantity-setting equilibrium with complementarity in Singh and Vives (1984) approaches the monopoly outcome.

The economics literature provides many examples of complementary monopolies including copper and zinc monopolists selling to downstream producers of brass (Cournot, 1838), railroad lines (Ellet, 1839, pp. 77-78), and links in a chain of canals (Edgeworth, 1925, p. 124). Choi (2008) discusses the complementarity between inputs such as jet engines and avionics in aircraft component markets. Denicolo (2000) considers markets with generalist and specialist firms that respectively produce all or some of the complements in the market, including for example color film and photofinishing. Casadesus-Masanell and Yoffie (2007) develop a dynamic pricing version of Cournot's complements model and study competition between Microsoft's Windows operating system and Intel's microprocessors. Laussel (2008) examines Nash bargaining over prices of complementary components in automobiles and aircraft. Laussel and Van Long (2012) extend Laussel (2008) with a dynamic equilibrium analysis of the downtream firm's divestiture of complementary suppliers. Llanes and Poblete (2014) examine ex ante agreements with complementarities and technology standard setting.

On the properties of games with general complementary strategies, see generally Topkis (1998) and Vives (1999, 2005). Legros and Matthews (1993) show there is an efficient Nash equilibrium in a partnership with strictly complementary efforts, although in their setting there is a continuum of Nash equilibria without this property. Hirshleifer (1983, 1985) considers complementary efforts in a public goods model

include Frisch (1951) and Hicks (1935). von Stackelberg (1934) considers Cournot reactions in successive moves.

[9] Singh and Vives (1984, p. 547) observe that "Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. Exchanging prices and quantities, we go from one to the other." See also Vives (1985).

with a continuum of Nash equilibria.

The present two-stage model with supply schedules and bargaining over prices differs from Cournot's one-stage game with posted prices. The present model also differs from Cournot's one-stage quantity-competition model in which products are perfect substitutes. The present model further differs from Bertrand's (1883) one-stage model of price-setting in which goods are perfect substitutes. In Bertrand's model, prices fall to players' marginal costs, whereas in the present model with supply schedules, all players choose maximum quantities equal the monopoly outcome.

The discussion is organized as follows. Section II presents the two-stage model of complementary monopolies and characterizes the equilibrium when the downstream market is perfectly competitive. Section III considers complementary monopolies when there is oligopoly competition with differentiated products in the downstream market. Section IV discusses antitrust policy implications of the analysis including conglomerate mergers, successive monopoly, and bilateral monopoly. Section V concludes the discussion.

II Complementary monopolies with perfect competition in the downstream market

This section introduces a two-stage game with complementary monopolists that supply inputs to perfectly competitive downstream producers. In the first stage, input suppliers choose binding supply offers non-cooperatively and entry of producers determines the demand for inputs. In the second stage, each input supplier bargains with producers over input prices and the input and output markets clear.

II.1 Producers

The downstream market is perfectly competitive with a homogeneous final good as in Cournot's model. The next two sections extend the analysis to downstream oligopoly and monopoly. Let p denote the price of the final good and let q be the output of the downstream industry. Assume that the market inverse demand $p = P(q)$ is strictly decreasing and continuously differentiable, $P'(q) < 0$.

Inputs are strict complements also as in Cournot's model. In the competitive case, each producer has unit capacity.[10] The unit capacity restriction is for ease of discussion in the competitive case and can be relaxed without changing the results. The downstream monopoly and oligopoly settings are presented without requiring unit capacity.

The producer's costs of production are the purchase prices of n inputs and a unit cost c. Each active producer has unit costs c excluding the costs of purchased inputs. Input prices $r_1, r_2, \ldots, r_n$ differ across inputs and are symmetric across producers. When the industry output is q, each producer earns a profit of

$$\Pi(q, r_1, r_2, \ldots, r_n) = P(q) - c - \sum_{i=1}^{n} r_i. \tag{1}$$

Producers are active if and only if $\Pi(q, r_1, r_2, \ldots, r_n) \geq 0$.

II.2 Input suppliers

In the first stage of the game, each input supplier i makes a binding commitment to provide whatever quantity q of their input that producers demand up to a maximum amount y_i. Each input supplier offers a *supply schedule* $Y_i(q)$ given by

$$Y_i(q) = \min\{q, y_i\}, \tag{2}$$

$i = 1, ...n$. To simplify notation, let the maximum levels $y_1, y_2, \ldots, y_n$ represent the supply offers $Y_1(q), Y_2(q), \ldots, Y_n(q)$.

Because inputs are perfect complements, downstream output is bounded by the smallest of the maximum input supply offers, $q \leq y_{\min}$ where $y_{\min} \equiv \min\{y_1, y_2, \ldots, y_n\}$.

[10] With unit capacity, each producer's technology can be represented by a Leontief production function, $x = \min\{\delta_1, \delta_2, \ldots, \delta_n\}$, where x is the producer's output, $\delta_i = 1$ if the producer uses input i, and $\delta_i = 0$ otherwise.

Assume that downstream producers enter the market sequentially so that each producer is able to obtain all of the inputs up to $q \leq y_{\min}$.

Bargaining in the second stage implies that all active producers earn non-negative profits. Entry of downstream producers continues until total demand for inputs equals the minimum of the maximum input supply offers,

$$q = y_{\min}. \tag{3}$$

When choosing their supply schedule offers, input suppliers do not know the supply offers of other input suppliers nor do they know the amount q that will be demanded by producers. We consider weakly dominant strategy equilibria in supply offers.

As in Cournot's complementary monopolies model, input suppliers produce to order rather than producing to stock.[11] Each input supplier i incurs costs $k_i q$, $i = 1, 2, ..., n$, on the basis of the amount of the input that is demanded by producers. Because prices are symmetric and given input demand q, each input supplier i earns profits

$$V_i(q, r_i) = (r_i - k_i)q, \tag{4}$$

$i = 1, 2, ..., n$. Input suppliers are active if and only if $V_i(q, r_i) \geq 0$.

In the second stage of the game, each input supplier i bargains bilaterally with each downstream producer over the input price r_i. Bilateral bargaining follows the Nash cooperative bargaining solution, see Nash (1950, 1953), Harsanyi and Selten (1972), Roth (1979), and Binmore (1987). The cooperative approach simplifies the discussion. It is possible to extend the analysis to allow bilateral noncooperative bargaining as in Rubinstein (1982) and Binmore et al. (1986).

Bilateral bargaining between the supplier-producer pairs occurs simultaneously. Each bargaining pair chooses a price in response to the equilibrium outcomes of other negotiations, as in a Nash noncooperative equilibrium. The equilibrium of the bargaining stage is represented by $r_1^*, r_2^*, \ldots, r_n^*$. Let α_i denote the bargaining power of input supplier i relative to any downstream producer. Assume that $0 < \alpha_i < 1$,

[11] Recall that in Cournot's model, each input supplier offers a price to suppliers and then provides whatever amount is demanded by producers.

$i = 1, 2, ..., n$.

Given the input prices chosen by bargaining between other input suppliers with producers r^*_{-i}, the input price r_i solves the asymmetric Nash cooperative bargaining problem for each $i = 1, ..., n$,

$$\max_{r_i}(P(q) - c - \sum\nolimits_{j \neq i}^{n} r_j^* - r_i)^{1-\alpha_i}(r_i - k_i)^{\alpha_i}.$$

The first-order conditions simplify to

$$\alpha_i(P(q) - c - \sum\nolimits_{j \neq i}^{n} r_j^* - r_i) = (1 - \alpha_i)(r_i - k_i), \tag{5}$$

$i = 1, ..., n$.

II.3 The bundled input monopoly benchmark

As a benchmark for the two-stage game, consider a monopolist that sells a bundle of all of the inputs to the downstream industry. The monopolist posts a price ρ for the bundle of inputs. Downstream producers enter the market until marginal returns equal the input price, $P(q) - c = \rho$. The bundled input monopolist chooses downstream output q to maximize profits $\rho q - \sum_{i=1}^{n} k_i q$. Substituting for the price of the bundle of inputs, the monopolist's problem is

$$q^M \in \arg\max_q \left[(P(q) - c)q - \sum\nolimits_{i=1}^{n} k_i q\right].$$

Assume that there exists an interior solution to the monopoly problem, $q^M > 0$. The monopolist's first-order condition is $P'(q^M)q^M + P(q^M) - c - \sum_{i=1}^{n} k_i = 0$. The monopoly profit is positive, $(P(q^M) - c)q^M - \sum_{i=1}^{n} k_i q^M = -P'(q^M)(q^M)^2 > 0$.

The monopolist's output choice need not be unique. If there are multiple solutions, then for ease of notation let q^M denote the smallest output. We show that the main result holds whether or not the monopoly output is unique. The monopolist's price for the bundle of inputs equals the marginal return to producers evaluated at the monopoly output,

$$\rho^M = P(q^M) - c. \tag{6}$$

II.4 Equilibrium of the two-stage game

In the first stage, inputs suppliers choose supply offers represented by y_i^*, $i = 1, ..., n$ and producer demand for inputs equals $q^* = \min\{y_1^*, y_2^*, \dots, y_n^*\}$. In the second stage, the equilibrium bargaining outcome is represented by input prices r_i^*, $i = 1, ..., n$. We solve the model by backward induction.

Given the input prices chosen by bargaining between other input suppliers with producers r_{-i}^*, each input price r_i solves the Nash cooperative bargaining problem. Letting $r_i = r_i^*$, the first-order conditions imply that

$$\frac{\alpha_i}{1-\alpha_i}\left(P(q) - c - \sum\nolimits_{j=1}^{n} r_j^*\right) = r_i^* - k_i, \tag{7}$$

$i = 1, ..., n$.

Summing both sides over i implies that the sum of input prices is a function of the equilibrium output,

$$\sum\nolimits_{j=1}^{n} r_j^* = \frac{\left(\sum_{j=1}^{n} \frac{\alpha_j}{1-\alpha_j}\right)(P(q) - c) + \sum_{j=1}^{n} k_j}{1 + \sum_{j=1}^{n} \frac{\alpha_j}{1-\alpha_j}}. \tag{8}$$

To simplify the expressions, define β_i as

$$\beta_i = \frac{\alpha_i}{1-\alpha_i}\frac{1}{1 + \sum_{j=1}^{n} \frac{\alpha_j}{1-\alpha_j}}, \tag{9}$$

$i = 1, ..., n$. Notice that $0 < \beta_i < 1$ and $0 < \sum_{i=1} \beta_i < 1$ for any α_i, $i = 1, 2, ..., n$.

Substituting from the sum of input prices into the simplified first-order conditions gives the equilibrium input prices

$$r_i^* = \beta_i \left(P(q) - c - \sum\nolimits_{j=1}^{n} k_j\right) + k_i, \tag{10}$$

$i = 1, ..., n$. This establishes that the bargaining equilibrium exists and is unique.

The equilibrium input prices $r_i^* = r_i^*(q)$ are functions of industry demand for inputs. It follows that the equilibrium profit of each input supplier i equals

$$V_i(r_i^*, q) = (r_i^* - k_i)q = \beta_i \left[(P(q) - c)q - \sum\nolimits_{j=1}^{n} k_j q \right], \tag{11}$$

$i = 1, 2, ..., n$. At industry demand for inputs q, each producer earns a profit equal to

$$\Pi(q, r_1^*, r_2^*, \ldots, r_n^*) = \left(1 - \sum\nolimits_{i=1}^{n} \beta_i\right) \left(P(q) - c - \sum\nolimits_{j=1}^{n} k_j\right). \tag{12}$$

Consider now the equilibrium of the two-stage game. Proposition 1 presents the main result of the analysis. The result holds whether or not the profit-maximizing monopoly output q^M is unique.

PROPOSITION 1. *In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the profit-maximizing bundled monopoly output, $y_i^* = q^M$, $i = 1, ..., n$, so that equilibrium industry input demand is $q^* = q^M$. In the second stage, input prices are unique, $r_i^* = r_i^*(q^M)$, and the total of input prices equals*

$$\sum\nolimits_{i=1}^{n} r_i^*(q^M) = \left(\sum\nolimits_{i=1}^{n} \beta_i\right) \left(P(q^M) - c\right) + \left(1 - \sum\nolimits_{i=1}^{n} \beta_i\right) \sum\nolimits_{j=1}^{n} k_j. \tag{13}$$

Total input prices are strictly less than the monopoly price for the bundle of inputs, $\sum_{i=1}^{n} r_i^* < \rho^M$.

The proof is given in the Appendix.

This result establishes that with complementary inputs, the non-cooperative equilibrium with quantity-setting suppliers yields the cooperative outcome. The proposition shows that the weakly dominant strategy equilibrium is unique even if the monopoly outcome is not unique because the equilibrium equals the smallest output that maximizes monopoly profit. The result only depends on the assumptions that demand is downward sloping and inputs are perfect complements. Notice also that the weakly dominant strategy equilibrium with supply schedules is unique even though there are many Nash equilibria with fixed quantities.

The complementarity of inputs serves as a tacit coordination mechanism. A supplier strictly prefers the monopoly outcome to any other outcome. This means that a supplier will choose the quantity of an input that would be offered by a monopolist selling the bundle of complementary inputs regardless of what other suppliers are offering. If other suppliers offer higher quantities of inputs in comparison to the monopoly outcome, a supplier strictly prefers to restrict the equilibrium quantities of inputs by offering fewer inputs. If other suppliers offer lower quantities of inputs in comparison to the monopoly outcome, a supplier strictly prefers not to restrict further the quantities of inputs and is indifferent between offering the monopoly quantity and the restricted quantity.

Because inputs are strict complements, every supplier understands that his offer of an input controls the market outcome under some conditions, so that each supplier will choose to offer the quantity of an input that would be offered by a bundled monopolist. In this way, suppliers coordinate without the need for mergers or formal agreements. Also, notice that bargaining power does not affect the equilibrium output. Regardless of how rents are divided, suppliers have an incentive to choose the optimal output.

Proposition 1 shows that an input supplier has an incentive to choose an upper limit on the quantity supplied. Also, the result shows that an input supplier would not choose a positive minimum amount because the input supplier does not know what other input suppliers are offering. Additionally, the result shows that an inputs supplier would not offer a fixed output rather than a supply schedule because that could result in an offer in excess of the quantity offered by other suppliers and in excess of the amount demanded by downstream producers. Making either a minimum offer or a fixed output offer would risk costly over production.

Consider the effects of the number of complementary input monopolists on the outcome of the two-stage game. To examine the effects of more suppliers without changing total costs, suppose that $\sum_{i=1}^{n} k_i = K$ for all n. It follows that having more suppliers does not affect the equilibrium output q^M. Adding more suppliers shifts total bargaining power toward suppliers so that $\sum_{i=1}^{n} \beta_i \to 1$ as n increases, and total prices tend toward $(P(q^M) - c)q^M$ as n increases. This also can hold if

$\alpha_i = 1/n$, which implies that the bargaining power of individual suppliers diminishes with entry.

The analysis translates into complements in consumption. Suppose that the complementary monopolists sell components used by consumers. A consumer has unit demand for consumption of the set of components $x = \min\{\delta_1, \delta_2, \ldots, \delta_n\}$ with willingness to pay u if $x = 1$ and zero otherwise. Let $G(u)$ denote the cumulative distribution of willingness to pay levels across consumers. Suppose that perfectly competitive distributors with operating costs c resell the components to consumers at price p. Then, aggregate demand for the set of complements is given by $q = 1 - G(p)$. Aggregate demand is decreasing because the cumulative distribution is necessarily increasing in willingness to pay levels. Let $p = P(q)$ denote the inverse demand for the composite good. Then, the two-stage game with perfect competition downstream also applies to complements in consumption. Suppliers of complementary products will offer supply schedules $Y_i(q)$ in the first-stage and bargaining over prices r_i with distributors in the second stage, so that Proposition 1 continues to apply. The two-stage game with monopolistic competition downstream considered in the next section also applies to complements in consumption.[12]

II.5 Comparison with Cournot

Compare the present two-stage game with Cournot's posted price game. In Cournot's model, input suppliers choose per-unit prices r_i, $i = 1, ..., n$ and downstream producers choose how much of the inputs to purchase. The downstream industry is perfectly competitive so that the final output price in the downstream market equals $p = c + \sum_{i=1}^{n} r_i$. To characterize the Cournot posted price game assume that demand $D(p)$ is twice continously differentiable and log concave, $\frac{d^2 \ln D(p)}{dp^2} \leq 0$

[12] The analysis of complements in consumption would change when there is competition from firms supplying substitute products for particular components. The analysis also would change when there are imperfect complements so that consumers can purchase subsets of the products.

Input prices in Cournot's non-cooperative equilibrium r_i^C, $i = 1, ..., n$ solve

$$r_i^C = \arg\max_{r_i}(r_i - k_i)D(c + r_i + \textstyle\sum_{j\neq i}^n r_j^C) \tag{14}$$

In equilibrium, the first-order conditions in Cournot's model are

$$[r_i^C - k_i]D'(c + \textstyle\sum_{j\neq i}^n r_j^C + r_i^C) + D(c + \textstyle\sum_{j\neq i}^n r_j^C) = 0. \tag{15}$$

Summing over i implies that

$$\textstyle\sum_{i=1}^n r_i^C - \textstyle\sum_{i=1}^n k_i = -n\dfrac{D(c + \sum_{j\neq i}^n r_j^C)}{D'(c + \sum_{j\neq i}^n r_j^C)}. \tag{16}$$

At the bundled monopoly price, we have

$$\rho^M - \textstyle\sum_{i=1}^n k_i = -\dfrac{D(c + \rho^M)}{D'(c + \rho^M)} < -n\dfrac{D(c + \rho^M)}{D'(c + \rho^M)}. \tag{17}$$

Because demand is log concave, $\frac{d^2 \ln D(p)}{dp^2} \leq 0$, the Cournot Effect holds, $\rho^M < \sum_{j\neq i}^n r_j^C$.

Compare the present two-stage model with the Cournot model. First, note that output is greater in the two-stage model than in the Cournot model because $q^M = D(\rho^M) > D(c + \sum_{j\neq i}^n r_j^C) = q^C$. The downstream price is lower in the two-stage model than in the Cournot model, $P(q^M) < P(q^C)$. Define social welfare as the sum of consumers' and producers' surplus $W(p) = CS(p) + PS(p)$, where consumers' surplus is $CS(p) = \int_p^\infty D(z)dz$ and total producers' surplus is $PS(p) = [p - c - \sum_{i=1}^n k_i]\, D(p)$. This gives the following result.

PROPOSITION 2. *Consumers' surplus, total producers' surplus, and social welfare are greater in the two-stage non-cooperative game with supply schedules than with Cournot's price-setting suppliers.*

The result holds because $CS(P(q^M)) > CS(P(q^C))$ and joint profit maximization implies $PS(P(q^M)) > PS(P(q^C))$. The result suggests that the Cournot Effect is due to the restriction of competition to posted prices.

There is another interesting difference between the present model and Cournot's model. In the two-stage model, holding total costs constant, the number of complementary input suppliers does not affect the weakly dominant strategy equilibrium output. So, in the two-stage game holding total costs constant, entry of additional input suppliers does not affect social welfare. In Cournot's pricing model, an increase in the number of complementary inputs increases the sum of input prices when demand is log-concave. This is because a greater number of suppliers worsens the free-rider effects of non-cooperative competition. This means that in Cournot's model, a greater number of input suppliers reduces both equilibrium output and social welfare.

III Complementary monopolies with oligopoly competition in the downstream market

This section considers complementary monopolies with oligopoly competition in the downstream market. In the first stage, each input supplier i chooses a supply schedule $Y_i(q)$ represented by y_i^*, $i = 1, ..., n$ and total producer demand for inputs equals $q^* = \min\{y_1^*, y_2^*, \dots, y_n^*\}$. In the second stage, each input supplier bargains bilaterally with each producer over two-part tariffs r_i^*, R_i^*, $i = 1, ..., n$.

III.1 Producers

There are m downstream producers each offering a differentiated product x_h, $h = 1, 2, ..., m$. Each of the downstream producers sells multiple units of output. Each producer has a Leontief production function, $x_h = \min\{\zeta_1, \zeta_2, \dots, \zeta_n\}$ where ζ_i is the amount of input i. Let $q = y_{\min}$ be the minimum of the maximum input supplies and assume that all active producers obtain the same amount of the inputs. Then, each input supplier faces the constraint

$$x_h \leq \frac{q}{m}.$$

Market demand for each producer j is $x_h = D(p_h, p_{-h}; m)$, $h = 1, 2, ..., m$. Assume that demand per producer with symmetric prices $x(p; m) = D(p_h, p_{-h}; m)$ is strictly decreasing in the market price and let $P(x; m)$ be the inverse of demand per producer $x(p; m)$. The slope of each producer's demand with symmetric prices is $z(p; m) = \frac{\partial D(p_j, p_{-j}; m)}{\partial p_j} < 0$. Assume that products are substitutes so that the market price effect on each producer's demand is greater than the own-price effect on demand, $z(p, m) < x_p(p, m)$.[13]

Producers engage in Bertrand-Nash price competition with differentiated products. Producers have unit costs c excluding the costs of purchased inputs. Assume that market equilibrium prices are symmetric and the producer price strategy $p^* = p^*(\sum_{i=1}^n r_i + c; m)$ is increasing in per unit costs $\sum_{i=1}^n r_i + c$. These properties can be derived from standard assumptions on market demand.[14]

When producers do not face input constraints, each producer's first-order condition for the symmetric equilibrium price p^* can be written as

$$\left(p^* - c - \sum\nolimits_{i=1}^n r_i\right) z(p^*; m) + x(p^*; m) = 0. \tag{18}$$

Without capacity constraints, the equilibrium net returns for each producer are

$$\Pi(q, r_1, r_3, ..., r_n, R_1, R_2, ..., R_n) = \left[p^* - c - \sum\nolimits_{i=1}^n r_i\right] x(p^*; m) - \sum\nolimits_{i=1}^n R_i, \tag{19}$$

where $p^* = p^*(\sum_{i=1}^n r_i + c; m)$. Each producer demands a quantity $x(p^*(\sum_{i=1}^n r_i + c; m); m)$ of each input.

If producers face binding input constraints, that is $x(p^*(\sum_{i=1}^n r_i + c; m); m) \geq \frac{q}{m}$, each producer demands inputs $\frac{q}{m}$. The market equilibrium prices solve $x(p; m) = \frac{q}{m}$, so that $p = P(\frac{q}{m}; m)$. So, with capacity constraints, we can write the equilibrium net returns for each producer as

$$\Pi(q, r_1, r_3, ..., r_n, R_1, R_2, ..., R_n) = \left[P(\frac{q}{m}; m) - c - \sum\nolimits_{i=1}^n r_i\right] \frac{q}{m} - \sum\nolimits_{i=1}^n R_i. \tag{20}$$

[13]The reduced-form model of oligopoly competition among producers follows Vives (2005, 2008). Demand per producer is decreasing in the market price, $x_p(p, m) < 0$ (Vives, 1999, 2008).

[14]See Vives (2008) and Spulber (2013).

III.2 The bundled input monopoly benchmark

As a benchmark, consider a monopolist that sells the bundle of inputs to the downstream industry using a per-unit tariff ρ and a lump-sum tariff Γ. The monopolist input supplier will increase the lump-sum tariff until it equals operating profits for each producer,

$$\Gamma = [p^*(\rho + c; m) - c - \rho]\, x(p^*(\rho + c; m); m). \tag{21}$$

The monopolist's profit is then

$$\left(\rho - \sum\nolimits_{i=1}^{n} k_i\right) mx(p^*; m) + m\Gamma = \left[p^* - c - \sum\nolimits_{i=1}^{n} k_i\right] mx(p^*; m), \tag{22}$$

where $p^* = p^*(\rho + c; m)$.

The monopolist problem can be recast in terms of total input demand q, where the per-unit input tariff ρ solves $x(p^*(\rho + c; m); m) = \frac{q}{m}$ and the output price is $p = P(\frac{q}{m}; m)$. The monopolist's profits equal

$$\left(\rho - \sum\nolimits_{i=1}^{n} k_i\right) q + m\Gamma = \left[P(\frac{q}{m}; m) - c - \sum\nolimits_{i=1}^{n} k_i\right] q. \tag{23}$$

The first-order condition for the monopolist's problem is

$$P(\frac{q}{m}; m) - c - \sum\nolimits_{i=1}^{n} k_i + P'(\frac{q}{m}; m)\frac{q}{m} = 0. \tag{24}$$

As before the solution need not be unique. Let $q^M > 0$ be the smallest profit-maximizing input demand level, again for ease of notation.

Then, the equilibrium output price is $p^M = P(\frac{q^M}{m}; m)$. The per-unit tariff ρ^M solves $x(p^*(\rho^M + c; m); m) = \frac{q^M}{m}$. The monopolist's lump-sum tariff for the bundle of inputs equals

$$\Gamma^M = \left[P(\frac{q^M}{m}; m) - c - \rho^M\right] \frac{q^M}{m}. \tag{25}$$

From the producers' first-order conditions, per-unit tariff for the bundle of inputs is

$$\rho^M = p^* - c + \frac{x(p^*; m)}{z(p^*; m)}. \tag{26}$$

From the bundled monopolist's first-order condition and $q = mx(p^*; m)$, the per-unit tariff equals

$$\rho^M = \sum\nolimits_{i=1}^{n} k_i - P'(\frac{q}{m}; m)\frac{q}{m} + \frac{1}{z(p^*; m)}\frac{q}{m}. \tag{27}$$

Because products are substitutes, $-P'(\frac{q}{m}; m) = -\frac{1}{x_p(p^*, m)} > -\frac{1}{z(p^*; m)}$. This implies that the monopolist's per-unit tariff is greater than total marginal cost, $\rho^M > \sum_{i=1}^{n} k_i$, so there is some double marginalization. Applying the monopolist's first-order condition, the monopolist's lump-sum tariff equals

$$\Gamma^M = \left[\sum\nolimits_{i=1}^{n} k_i - P'(\frac{q}{m}; m)\frac{q}{m} - \rho^M\right]\frac{q^M}{m} = -\frac{1}{z(p^*; m)}\frac{q}{m} > 0. \tag{28}$$

The monopolist's lump-sum tariff is positive, $\Gamma^M > 0$, because the slope of each producer's demand is negative. Two-part tariffs reduces the per-unit tariff on the bundle of inputs, which reduces double marginalization.

III.3 Equilibrium of the two-stage game

At the first stage, input suppliers choose supply schedules $Y_1(q), Y_2(q), \ldots, Y_n(q)$ to maximize net benefits

$$V_i(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) = r_i q + mR_i - C(q), \tag{29}$$

where $q = y_{\min}$. Input suppliers will participate only if they receive non-negative net benefits, $V_i(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) \geq 0$.

At the second stage, each input supplier bargains bilaterally with each producer. All of the bilateral bargaining occurs simultaneously and each bargaining pair takes into account the equilibrium outcome of other bargains. There are mn bargaining pairs and as in the Cournot model, an input supplier receives the same payment from every producer. The equilibrium of the bargaining stage is represented by $r_1^*, r_2^*, \ldots, r_n^*, R_1^*, R_2^*, \ldots, R_n^*$.

Denote the total transfer from a producer to an input supplier by $t_i = r_i \frac{q}{m} + R_i$.

Then, suppliers have net benefits

$$V_i(q, r_1, r_2, \ldots, r_n, R_1, R_2, \ldots, R_n) = mt_i - k_i q, \tag{30}$$

$i = 1, ..., n$. The equilibrium net returns for each producer are

$$\Pi(q, r_1, r_3, ..., r_n, R_1, R_2, ..., R_n) = \left(P(\frac{q}{m}; m) - c\right)\frac{q}{m} - \sum\nolimits_{j \neq i}^{n} t_i. \tag{31}$$

Given the transfers chosen by bargaining between other input suppliers with producers t^*_{-i}, each transfer t_i solves the Nash cooperative bargaining problem,

$$\max_{t_i}\left[\left(P(\frac{q}{m}; m) - c\right)\frac{q}{m} - \sum\nolimits_{j \neq i}^{n} t^*_j - t_i\right]^{1-\alpha_i}\left(t_i - k_i\frac{q}{m}\right)^{\alpha_i},$$

$i = 1, ..., n$.

We now characterize the equilibrium of the two-stage game with competing complementary input suppliers when there is oligopoly competition in the downstream market.

PROPOSITION 3. *In the first stage, the weakly dominant strategy equilibrium in supply schedules is unique and equivalent to the smallest profit-maximizing bundled monopoly output,* $y^*_i = q^M$, $i = 1, ..., n$. *In the second stage, transfers are unique,* $t^*_i = t^*_i(q^M)$, *and the total of transfers per producer equals*

$$\sum\nolimits_{i=1}^{n} t^*_i(q^M) = \left(\sum\nolimits_{i=1}^{n}\beta_i\right)\left(P(\frac{q^M}{m}; m) - c\right)\frac{q^M}{m} + \left(1 - \sum\nolimits_{i=1}^{n}\beta_i\right)\sum\nolimits_{j=1}^{n} k_j\frac{q^M}{m}. \tag{32}$$

The total of transfers is less than the total bundled monopoly tariff,

$$m\sum\nolimits_{i=1}^{n} t^*_i(q^M) < \rho^M q^M + m\Gamma^M.$$

The proof is given in the Appendix.

With oligopoly competition downstream, complementary monopolists achieve the bundled monopoly output, which is the cooperative outcome. Bargaining between input suppliers and producers reduces total transfers in comparison to bundled monopoly.

Because total transfers are strictly less than monopoly profits, $m\sum_{i=1}^{n} t_i^* < \rho^M q^M + m\Gamma$, there is sufficient demand for inputs such that the quantity constraint is binding, $x_h = \frac{q}{m}$. It is possible to construct two-part tariffs that ration inputs by price, $\sum_{i=1}^{n} r_i^* = \rho^M$ and $\sum_{i=1}^{n} R_i^* \leq \Gamma^M$, only if total transfers exceed total per-unit payments for the monopoly bundle, $m\sum_{i=1}^{n} t_i^* \geq \rho^M q^M$. Otherwise, inputs are allocated by quantity rationing.

IV Discussion: antitrust and the Cournot Effect

This section considers some antitrust policy implications of the two-stage model of complementary monopolies with bargaining. First, we consider antitrust policy towards conglomerate mergers. Second, we examine the problem of successive monopoly and vertical mergers. Finally, we discuss bilateral monopoly.

IV.1 Conglomerate mergers and bundling

The results obtained here are useful in formulating antitrust policy towards conglomerate mergers. The analysis shows that the presence of complementarities in production or in consumption need not justify conglomerate mergers. Competing complementary input monopolists can achieve the cooperative outcome by offering supply schedules to producers and bargaining over prices. The resulting output will equal the joint monopoly outcome and total input prices will be less than the monopoly outcome. This means that a merger of complementary monopolists need not generate any benefits that would result from bundling.

In contrast, the Cournot effect suggests that when firms offering complementary goods merge, they may increase social welfare. The merged firms can reduce prices by bundling complementary goods, which would eliminate non-cooperative posted prices that existed before the merger. According to the OECD (2001), the Cournot Effect would justify a merger of firms offering complementary goods if pre-merger prices were above competitive levels and the merged firm would have a significant market share or would engage in tying or bundling of the complementary goods.[15]

The Cournot Effect relies on particular assumptions about the conduct of complementary monopolists. It depends on the assumption that complementary monopolists rely on posted prices when selling inputs to producers and the assumption that suppliers choose prices non-cooperatively. As a consequence, inputs suppliers do not take into account the effects of their prices on the profits of other complementary monopolists, leading to total input prices above the bundled monopoly level.

The dependence of the hypothetical Cournot Effect on specific competitive conduct limits its use as a justification for mergers. The effect cannot be a defense of conglomerate mergers unless it can also be established that before the merger companies indeed engage in non-cooperative price setting. A conglomerate merger need not generate benefits from product bundling.

The absence of a Cournot Effect does not in itself rule out such mergers. In practice, conglomerate mergers may offer various cost economies associated with consolidation of production or transactions. However, conglomerate mergers may also create problems resulting from reduced competition. The DOJ's non-horizontal merger guidelines identifies some of these issues.[16] The policy implication of the present analysis is that antitrust policy should focus on how the merger would affect costs, prices, and competitive behavior, without necessarily relying on the presence of complementarities.

The Cournot Effect played a significant role in antitrust policy towards the pro-

[15] According to the OECD (2012), "In addition to efficiency effects there is a less obvious reason why a merger uniting complements could lead to lower prices. Such a merger could also internalise the effects of lowering the price of one complement on sales and profits earned on another. This Cournot effect will not exist or be significant unless pre-merger prices were above competitive levels in at least one of the complements. Another necessary condition is that the merged entity will either have a significant market share in at least one of the complements in which there were pre-merger supracompetitive pricing, or will engage in some form of tying, bundling or analogous practice having the effect of internalising a pricing externality in complementary products."

[16] The DOJ identifies various challenges to non-horizontal mergers. See Non-Horizontal Merger Guidelines (Originally issued as part of "U.S. Department of Justice Merger Guidelines, June 14, 1984." https://www.justice.gov/atr/non-horizontal-merger-guidelines, accessed April 9, 2016.

posed merger between GE and Honeywell.[17] Both GE and Honeywell supplied complementary inputs such as engines and avionics to aircraft producers. There are many reasons to suppose that GE and Honeywell did not rely on non-cooperative posted prices as a means of selling components to aircraft producers. It is more likely that these companies engaged in bilateral contract negotiations with specification of supply schedules and demand orders, as well as bargaining over prices and other contract terms. The companies would be more likely to rely on bargaining because of the small number of companies involved, the high cost of inputs, the need to establish production schedules, and the need to develop delivery schedules. In addition, companies would rely on contracts because of investments needed to manufacture engines and other components and the investment needed to produce final outputs. Additionally, contracts would be necessary to address the complex technological issues associated with product quality, interoperability of components, and allocation of intellectual property.

Although the U.S. Department of Justice approved the proposed $43 billion merger, the European Commission (EC) rejected the merger. The EC decision directly addressed the Cournot Effect (EC, 2001, pp. 91-92). The companies seeking to merge argued that aircraft engines and components such as avionics were complements and that the merger would facilitate bundling, which would lower final prices. The EC (2001, p. 92) stated: "Therefore, even if the demand for aircraft at the industry level were inelastic, i.e., even in the face of a price reduction by all entities for the product bundle, it did not increase sufficiently to render price reduction profitable[;] the Commission's investigation has indicated that a price reduction of the bundled system by the merged entity is likely to shift customers' demand away from competitors to the merged entity's bundled product." The EC expressed concerns that the merger would increase the market power of the merging companies in jet engines for commercial, regional and corporate jets as well as for components such as avionics.

The EC considered the Cournot Effect without performing sufficient theoretical or empirical analysis to determine whether that effect was applicable to the market in question. The European Court of First Instance reviewed the EC decision and

[17] For additional discussion of the case, see also Choi (2001), and Vives and Staffiero (2009).

various presentations by economists, noting that "the question as to whether the Cournot effect would have given the merged entity an incentive to engage in mixed bundling in the present case is a matter of controversy." (Court of First Instance, 2005, p. II-5740).

According to the Court of First Instance (2005, p. II – 5733), the EC argued "it follows from well-established economic theories, particularly the 'Cournot effect' ... that the merged entity would have an economic incentive to engage in the practices foreseen by the Commission and that there was no need to rely on a specific economic model in that regard." The Court of First Instance (2005, p. II-5742) found that "by merely describing the economic conditions which would in its view exist on the market after the merger, the Commission did not succeed in demonstrating, with a sufficient degree of probability, that the merged entity would have engaged in mixed bundling after the merger."

Manufacturers of aircraft engines and components and assemblers of aircraft would be likely to specify input supply and demand commitments and to bargain over input prices. So, the present analysis suggests that even with strict complements and complementary monopolies, the Cournot Effect need not be observed. This suggests that evaluating the competitive effects of the GE-Honeywell mergers would require additional economic analysis.

The effects of mergers when there are complements in consumption is affected by the structure of consumer preferences and the presence of competitors. Choi (2008) extends the analysis of the Cournot Effect to include "mixed bundling", which involves the merged firm selling complementary components both separately and as a bundle. Choi (2008) finds that mergers can have positive or negative effects on social welfare depending on consumer preferences and how the merger affects competitors. The present analysis suggests that consideration of competitive interaction with supply schedules and bargaining should be extended to markets with mixed bundling.

IV.2 Successive monopoly and vertical mergers

The present two-stage model suggests that successive monopoly need not lead to welfare losses from double or multiple marginalization. The upstream and downstream monopolists can coordinate through non-cooperative supply schedules and cooperative or non-cooperative bargaining over prices. This suggests that a merger of successive monopolies need not reduce final prices. Conversely, a breakup of a vertical integrated firm need not increase final prices.

A successive monopoly refers to a market in which a single input supplier sells a necessary input to a single producer that uses the input and manufacturing services to provide a good. In models of successive monopoly, the producer usually sells to a competitive downstream market, although there may be multiple levels of successive monopoly. The successive monopoly with fixed proportions is thus identical to the complementary monopolies model where the number of levels corresponds to the number of complementary inputs n. Just as the manufacturer purchases the input, the input supplier can be viewed as purchasing manufacturing services. The input supplier and the manufacturer divide the rents from selling to the downstream market because neither monopolist can transact with the competitive downstream market without transacting with each other.[18] *Alcoa* is classic antitrust example of a case alleging a successive monopoly, because the company produced both aluminum ingots and aluminum sheets.[19]

Just as the Cournot Effect justifies conglomerate mergers, so the successive monopoly model has been applied to justify vertical mergers. Vertical integration avoids the problem of double or multiple marginalization because the vertically-integrated

[18] There has been extensive discussion of the problem of successive monopoly in the economics literature. For example, Machlup and Taber (1960, p. 107) note that "Wicksell's exposition is enlivened by a picturesque illustration, drawn from a reference by Babbage ... to the only existing possessor of the skill of making dolls' eyes who sells to the only manufacturer of dolls."Machlup and Taber, (1960, p. 107) are quoting Wicksell (1927, p. 276) who in turn is quoting Babbage (1832, pp. 199-20). Babbage's (1832, pp. 199-200) example of successive monopoly in doll making predates Cournot's book.

[19] *U.S. v. Aluminum Co. of Am. (Alcoa)*, 148 F.2d 416, 437 (2d Cir. 1945). Alcoa was said to have a monopoly in virgin aluminum ingots although there were foreign suppliers of ingots and recycled aluminum. Alcoa also faced competition from other producers of aluminum sheets. This led to charges of a price squeeze of competitors in aluminum sheets to whom Alcoa supplied ingots.

firm efficiently prices internally-produced inputs at their marginal costs.[20] Alternatively, breaking up a vertically-integrated company would cause welfare losses by leading to double marginalization if there is a monopoly at two or more vertical levels.

When products are complements in demand, companies have an incentive to bundle the products. This often raises antitrust policy concerns about tying. However, McChesney (2015) argues that many cases with complementary products should not be treated as tying because they are more accurately described as successive monopoly. McChesney points out that this applies to the cases of *Microsoft*, *Jefferson Parish*, and *Town of Concord*.[21] In *Microsoft*, the complementary products were the Windows operating system and the Internet Explorer Internet browser. In *Jefferson Parish*, the complementary products were hospital medical services and anesthesia. In *Town of Concord*, Boston Edison both produced and distributed electric power.

Timing issues have complicated the economic analysis of successive monopoly. With simultaneous pricing, the outcome is the same as Cournot's complementary monopolies model, so that the final price exceeds the joint-profit maximizing price due to multiple marginalization. With sequential pricing, the outcome is the standard double marginalization result, which again departs from the joint-profit maximum. The final prices can differ as a consequence of timing differences but in each situation the final price exceeds the joint-monopoly price.

The present two-stage model with non-cooperative choices of supply schedules and bargaining provides a characterization of successive monopoly that yields the cooperative outcome. The upstream firm proposes an input level and the downstream firm proposes a manufacturing activity level. Because the input and the manufacturing activity are in fixed proportions, the final output is given by $q = \min\{y_1, y_2\}$ where

[20] For example, Spengler (1950, p. 352) argues that "vertical integration, if unaccompanied by a competition-suppressing amount of horizontal integration and if conducive to cost and price reduction, should be looked upon with favor by a court interested in lower prices and a better allocation of resources."

[21] See *U.S. v. Microsoft Corp.*, 253 F.3d 34 (D.C. Cir. 2001), *Jefferson Parish Hospital District No. 2 v. Hyde*, 466 U.S. 2 (1984), and *The Town of Concord v. Edison Electric Co.*, 915 F.2d 17 (1st Cir. 1990).

y_1 denotes the upstream firm's maximum supply of the input and y_2 denotes the downstream firm's maximum contribution to production of the final output.

The upstream monopoly and the downstream monopoly each produce to order after the final output is determined. They have production costs $k_1 q$ and $k_2 q$ that depend on the equilibrium output. Market inverse demand for the final good is $P(q)$. Final customers may incur a per-unit transportation or transaction cost c, so that the final customers' willingness to pay for output q is given by $U(q) = [P(q) - c]q$. Joint profit $(P(q) - c - k_1 - k_2)q$ is maximized at q^M. Let α be the bargaining power of the downstream monopoly and let $(1 - \alpha)$ be that of the upstream monopoly, where $0 < \alpha < 1$.

The properties of the two-stage game with supply schedules and bargaining follow from Proposition 1.

COROLLARY 1. *In the first stage, the weakly dominant strategy equilibrium with successive monopoly is unique involves the upstream and downstream firms choosing outputs equal to the smallest joint profit-maximizing monopoly output,* $y_1^* = y_2^* = q^M$*. In the second stage, the upstream monopolist receives* $r^* = (1-\alpha)\left[P(q^M) - c - k_2\right] q^M + \alpha k_1$ *and the final price equals the joint-profit maximizing price* $P(q^M)$*.*

The result shows that vertical integration need not lower prices when there is a successive monopoly. When firms engage in contract negotiation, this suggests that eliminating successive monopoly need not be a justification for vertical mergers.

IV.3 Bilateral monopoly

Bilateral monopoly has been widely examined in antitrust studies.[22] One implication for merger policy is that suppliers should be allowed to merge so as to form a monopoly when faced with a monopsony buyer. The rationale is that the resulting bilateral monopoly would then bargain to reach a joint-profit maximum (Campbell, 2007). The present analysis suggests that the bilateral monopoly indeed would reach the joint-profit maximum.

[22] See for example Friedman (1986) and Blair and DePasquale (2014).

Bilateral monopoly is equivalent to the complementary monopolies problem with $n = 2$, because one seller can be viewed as selling part of the downstream market to the other seller.[23] The present analysis shows how a bilateral monopoly might reach the cooperative outcome.[24]

The bilateral monopoly problem sheds light on bargaining in decentralized market exchange.[25] In bargaining between a buyer and a seller, the quantity purchased is strictly complementary to the quantity sold. If a buyer and a seller propose supply schedules to each other, non-cooperative bargaining generates efficient outcomes as a unique weakly dominant strategy equilibrium. This is consistent with representations of cooperative bilateral exchange that assume efficiency along the contract curve; e.g. Edgeworth (1881) and Pareto (1903, 1927).[26] This result also is consistent with axiomatic game theory, which suggests that cooperative behavior should lead to maximization of joint benefits.[27]

Suppose that the monopsonistic buyer has a willingness to pay for output q given by $(P(q) - c)q$. The monopolistic seller can provide output q at a cost of kq. Let q^M be the smallest output that maximizes joint profit $[P(q) - c - k]q$.

In the first stage, the buyer and seller each make maximum offers of the amount

[23] Machlup and Taber (1960, p. 103) point out that Marshall (1907) noticed this equivalence: "Marshall, for example, mentions Cournot's illustration of the monopolists supplying the copper and zinc needed to make brass, and adds his own illustration of spinners and weavers supplying complementary services in the production of cloth, without examining whether or not the more obviously vertical arrangement in his case makes any essential difference." The connection between complementary monopolies and bilateral monopoly also was noted by Zeuthen (1930), see Machlup and Taber (1960).

[24] Economists who have analyzed the closely-related problems of bilateral monopoly and successive monopoly include Edgeworth (1881), Pareto (1903, 1927), Pigou (1908), Schumpeter (1927), Henderson (1940), Leontief (1946), Fellner (1947), and Morgan (1949).

[25] See Hayek's (1934) discussion of Menger on isolated exchange (1871) and see also Wicksell (1934). Böhm-Bawerk (1891) studies supply and demand in terms of buyer and seller pairs.

[26] Tarascio (1972) considers the origins of the Edgeworth-Bowley box and identifies Pareto's critical initial contribution. Coase (1960) observes that bargaining over externalities should generate efficient outcomes when there are no transaction costs and small numbers of agents.

[27] On unrestricted bargaining in game theory, see Shapley (1952), Aumann (1987), Aumann and Shapley (1974), and Shubik (1982, 1984). The axiomatic approach includes for example Nash's (1950, 1953) bargaining framework, although Rubinstein (1982, p. 98) points out that "It was Nash himself who felt the need to complement the axiomatic approach ... with a non-cooperative game."

to be exchanged equal to y_1 and y_2 respectively. The quantity of output to be exchanged is given by the minimum of the two values, $q = \min\{y_1, y_2\}$. In the second stage, the buyer and seller bargain over the price. Let α be the buyer's bargaining power, $0 < \alpha < 1$. The buyer's profit is $\alpha[P(q) - c - k]q$ and the seller's profit is $(1-\alpha)[P(q) - c - k]q$. Proposition 1 implies that the weakly dominant strategy equilibrium is unique and output is given by $y_1^* = y_2^* = q^M$. The equilibrium input price is $r^* = (1-\alpha)(P(q^M) - c) + \alpha k$.

Alternatively, consider bilateral exchange in which a buyer and seller propose maximum amounts that they wish to purchase or sell respectively. Suppose that there is a numeraire commodity and the buyer has an endowment ω of the numeraire. The buyer's benefit is $B(q) + \omega - rq$ and the seller's benefit is $(r - k)q$, where q is the good produced by the seller. Let q^* be the socially optimal output,

$$B'(q^*) = C'(q^*). \tag{33}$$

Proposition 1 implies that the unique weakly dominant strategy equilibrium is unique and output is given by $y_1^* = y_2^* = q^*$. The equilibrium price is $r = (1-\alpha)(\frac{B(q^*)}{q^*} - c) + \alpha k$.

V Conclusion

Strategic interaction involving a combination of non-cooperative supply offers and bargaining over prices can generate an efficient outcome. Models that arbitrarily limit non-cooperative interaction to posted prices remove degrees of freedom. With competition along one dimension as in Cournot, that is with posted prices, prices will exceed monopoly levels. With competition along multiple dimensions as in the present model, complementary monopolists will maximize joint profits and prices will not exceed monopoly levels. This suggests that the Cournot Effect is due to modeling restrictions on competitive strategies rather complementarities or input monopolies.

The present discussion shows that competition among complementary monopolists can be consistent with joint profit maximization. Predictions based on the

Cournot Effect need not hold when complementary monopolists engage in general competitive interactions with supply schedules and price negotiation. Antitrust policy makers should not assume that vertical and conglomerate mergers increase economic efficiency by eliminating multiple marginalization. Also, horizontal mergers of suppliers leading to bilateral monopoly need not reduce economic efficiency. Economic performance with complementary monopolies depends on the nature of their strategic interactions.

VI Appendix

PROOF OF PROPOSITION 1. Supplier i's profit in the first stage of the game equals $v_i(y_i, y_{-i}) = \beta_i \left[(P(q) - c)q - \sum_{j=1}^{n} k_j q\right]$ where $q = y_{\min}$ and $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, ..., y_n)$. For any y_{-i}, let $\overline{y} = \min\{y_{-i}\}$. The profit maximizing industry output may or may not be unique. Suppose first that the profit-maximizing monopoly output q^M is unique. Consider first the possibility that $\overline{y} \geq q^M$. Then, because the monopolist selling the bundle of inputs maximizes profits, it follows that $v_i(q^M, y_{-i}) \geq v_i(y_i, y_{-i})$ for all y_i. If $y_i = q$, $v_i(y_i, y_{-i}) = \beta_i[(P(q) - c)q - \sum_{j=1}^{n} k_j q]$. So, if $\overline{y} \geq q^M$, supplier i maximizes profit by choosing the monopoly output, $y_i^* = q^M$. Conversely, if $\overline{y} < q^M$, then because the monopolist selling the bundle of inputs maximizes profits it follows that $v_i(q^M, y_{-i}) \geq v_i(y_i, y_{-i})$ for all y_i and strictly for $y_i < \overline{y}$. Again, supplier i maximizes profit by choosing the monopoly output, $y_i^* = q^M$. This implies that the monopoly output is the weakly dominant strategy for each supplier i, and thus the weakly dominant strategy equilibrium is the monopoly output.

Now suppose that the profit-maximizing monopoly output is not unique and let q' and q'' be monopoly outputs, where $q' < q''$. If $q' < \overline{y} < q''$, then supplier i strictly prefers to offer the lower monopoly output to any other offer, $y_i^* = q'$. If $q'' \leq \overline{y}$, then supplier i is indifferent between the two monopoly outputs. If $\overline{y} \leq q'$, then the supplier is indifferent between q' and $\overline{y}$ and strictly prefers q' to any $y_i < \overline{y}$. Therefore, the smallest profit-maximizing monopoly output q^M is the weakly dominant strategy for each supplier i. Summing input prices evaluated at q^M gives $\sum_{i=1}^n r_i^* = (\sum_{i=1}^n \beta_i)\left(P(q^M) - c\right) + (1 - \sum_{i=1}^n \beta_i)\sum_{j=1}^n k_j$. Because monopoly profit is positive, $(P(q^M) - c)q^M > \sum_{j=1}^n k_j q^M$, so $\sum_{i=1}^n \beta_i < 1$ implies that $\sum_{i=1}^n r_i^* < P(q^M) - c = \rho^M$. $\square$

PROOF OF PROPOSITION 3. The first-order conditions for the Nash cooperative bargaining solution imply

$$t_i^* - k_i \frac{q}{m} = \frac{\alpha_i}{1-\alpha_i}\left(P(\frac{q}{m};m)\frac{q}{m} - c\frac{q}{m} - \sum\nolimits_{j=1}^n t_j^*\right),$$

$i = 1, ..., n$. Summing both sides over i gives

$$\sum\nolimits_{j=1}^n t_i^* = \frac{1}{1+\sum_{j=1}^n \frac{\alpha_i}{1-\alpha_i}}\left[\left(\sum\nolimits_{j=1}^n \frac{\alpha_i}{1-\alpha_i}\right)\left(P(\frac{q}{m};m)\frac{q}{m} - c\frac{q}{m}\right) + \sum\nolimits_{j=1}^n k_j \frac{q}{m}\right].$$

The equilibrium transfers that result from Nash bargaining are

$$t_i^* = \beta_i\left(P(\frac{q}{m};m)\frac{q}{m} - c\frac{q}{m} - \sum\nolimits_{j=1}^n k_j \frac{q}{m}\right) + k_i \frac{q}{m},$$

$i = 1, ..., n$, where the weights β_i are the same as before. The transfers $t_i^* = t_i^*(q)$ are unique functions of industry demand for inputs.

It follows that at industry demand q for inputs, the equilibrium profit of each input supplier i equals

$$V_i(q, t_1^*, t_2^*, \ldots, t_n^*) = \beta_i\left[P(\frac{q}{m};m)q - cq - \sum\nolimits_{j=1}^n k_j q\right],$$

$i = 1, 2, ..., n$. Supplier i's profit in the first stage of the game equals $v_i(y_i, y_{-i}) = \beta_i\left[P(\frac{q}{m};m)q - cq - \sum_{j=1}^n k_j q\right]$ where $q = y_{\min}$ and $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, ..., y_n)$. By arguments similar to those in the proof of Proposition 1, the weakly dominant strategy equilibrium is unique and equivalent to the smallest profit-maximizing monopoly output, $y_i^* = q^M$, $i = 1, ..., n$. Substituting for output gives total transfers as a function of output, $\sum_{i=1}^n t_i^*(q^M)$. By profit maximization, $\left(P(\frac{q}{m};m) - c\right)q^M > \sum_{j=1}^n k_j q^M$. This implies that $m\sum_{i=1}^n t_i^* < \left(P(\frac{q^M}{m};m) - c\right)q^M = \rho^M q^M + m\Gamma^M$. $\square$

References

[1] Arshinder, K., A. Kanda, and S. G. Deshmukh, 2011, "A Review on Supply Chain Coordination: Coordination Mechanisms, Managing Uncertainty and Research Directions," in *Supply Chain Coordination Under Uncertainty*, Berlin: Springer, pp. 39-82.

[2] Aumann, R. J., 1987, "Game Theory," in J. Eatwell, M. Milgate and P. Newman, eds., *The New Palgrave; A Dictionary of Economics*, London: Macmillan, pp. 460-482.

[3] Aumann, R. J. and L. S. Shapley, 1974, *Values of Non-Atomic Games*, Princeton: Princeton University Press.

[4] Babbage, C., 1832, *On the Economy of Machinery and Manufactures*, London, 1832.

[5] Bertrand, J., 1883, "Théorie Mathématique de la Richesse Sociale," *Journal de Savants*, 67, pp. 499–508.

[6] Binmore, K. G., 1987, "Nash Bargaining Theory I-III," in Binmore, K. G. and P. Dasgupta, eds., *The Economics of Bargaining*, Oxford: Blackwell.

[7] Binmore, K., A. Rubinstein, and A. Wolinsky, 1986, "The Nash Bargaining Solution in Economic Modelling," *RAND Journal of Economics*, pp.176-188.

[8] Blair, R. D. and C. DePasquale, 2014, "Bilateral Monopoly: Economic Analysis and Antitrust Policy," *The Oxford Handbook of International Antitrust Economics*, v. 1, R. D. Blair and and D. D. Sokol, eds., Oxford: Oxford University Press.

[9] Böhm-Bawerk, E. von, 1891, *Positive Theory of Capital*, Translated from German by W.E. Smart, London: Macmillan.

[10] Bowley, A. L., 1924, *Mathematical Groundwork of Economics*, Oxford: Oxford University Press.

[11] Bowley, A. L., 1928, "Note on Bilateral Monopoly," *Economic Journal*, 38, December, pp. 651-659.

[12] Cachon, F. P. and M. A. Lariviere, 2005, "Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations," *Management Science* 51(1), pp. 30-44.

[13] Campbell, T., 2007, "Bilateral Monopoly in Mergers," *Antitrust Law Journal*, 74(3), pp. 521–536.

[14] Casadesus-Masanell, R. and D. B. Yoffie, 2007, "Wintel: Cooperation and Conflict," *Management Science*, 53 (4), April, pp. 584–598.

[15] Choi, J.P. 2001, "A Theory of Mixed Bundling Applied to the GE/Honeywell Merger," *Antitrust*, 16, pp. 32-33.

[16] Choi, J.P., 2008, "Mergers with Bundling in Complementary Markets," *Journal of Industrial Economics*, 56(3), pp. 553–577.

[17] Coase, R. H., 1960, "The Problem of Social Cost," *Journal of Law & Economics,* 3, October, pp. 1-69.

[18] Cournot, A. A., 1838 (1897), *Recherches sur les Principes Mathématique de la Théorie des Richesse*, English edition, N. T. Bacon, trans., *Researches into the Mathematical Principles of the Theory of Wealth,* New York: Macmillan.

[19] Court of First Instance, 2005, Judgment of the Court of First Instance (Second Chamber, extended composition) of 14 December 2005, General Electric Company v Commission of the European Communities. Action for annulment - Competition - Commission decision declaring a concentration to be incompatible with the common market - Regulation (EEC) No 4064/89 - Aeronautical markets - Acquisition of Honeywell by General Electric, Case T-210/01.

[20] Denicolo, V., 2000, "Compatibility and Bundling with Generalist and Specialist Firms," *Journal of Industrial Economics*, 48(2), pp. 177–188.

[21] Economides, N. and S. C. Salop, 1992, "Competition and Integration Among Complements, and Network Market Structure," *Journal of Industrial Economics*, 40(1), pp. 105–123.

[22] Edgeworth, F. Y., 1881, *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences,* London: Kegan Paul.

[23] Edgeworth, F. Y., 1925, "The Pure Theory of Monopoly," in *F. Y. Edgeworth, Papers Relating to Political Economy* (Facsim. ed.), Bristol, UK: Thoemmes Press, pp. 111-142, trans. of F. Y. Edgeworth, 1897, "Teoria pura del monopolio," *Giornale degli economisti*, XV, pp. 13-31.

[24] Ellet Jr., C., 1839, [1966], *An Essay on the Laws of Trade in Reference to the Works of Internal Improvement in the United States*, Reprint, New York: Augustus M. Kelley Reprint.

[25] European Commission, 2001, Commission Decision of July 3, 2001 declaring a concentration to be incompatible with the common market and the EEA Agreement Case No COMP/M.2220 - General Electric/Honeywell.

[26] Fellner, W., 1947, "Prices and Wages Under Bilateral Monopoly," *Quarterly Journal of Economics*, 61 (4), August, pp. 503-532.

[27] Fisher, I., 1898, "Cournot and Mathematical Economics," *Quarterly Journal of Economics*, 12, pp. 119–138.

[28] Friedman, R. D., 1986, "Antitrust Analysis and Bilateral Monopoly," *Wisconsin Law Review*, 1986, pp. 873-918.

[29] Frisch, R., 1951, "Monopoly, Polypoly: The Concept of Force in the Economy, *International Economic Papers*, 1, pp. 23-36, translation of Frisch, Ragnar, 1933, "Monopole-Polypole-La Notion de Force dans 1'Economie," *Festschrift für H. Westergaard,Nationalokonomisk Tidsskrift*, Kopenhagen.

[30] Harsanyi, J.C. and Selten, R., 1972. "A Generalized Nash Solution for Two-person Bargaining Games with Incomplete Information, *Management Science*, 18(5-part-2), pp.80-106.

[31] Hayek, F. A., 1934, "Menger," *Economica*,New Series, Vol. 1, No. 4, November, pp. 393-420.

[32] Henderson, A. M., 1940, "A Further Note on the Problem of Bilateral Monopoly," *Journal of Political Economy*, 48 (2), April, pp. 238-43.

[33] Hicks, J. R., 1935, “Annual Survey of Economic Theory: The Theory of Monopoly,” *Econometrica*, 3 (1), January, pp. 1-20.

[34] Hirshleifer, J., 1983, "From Weakest-Link to Best-Shot: The Voluntary Provision of Public Goods," *Public Choice*, 41, pp. 371-386.

[35] Hirshleifer, J., 1985, "From Weakest-Link to Best-Shot: Correction," *Public Choice*, Vol. 46, No. 2, pp. 221-223.

[36] Kaldor, N., 1936, *Economica*, New Series, Vol. 3, No. 10, May, pp. 227-230.

[37] Laussel, D., 2008, "Buying Back Subcontractors: The Strategic Limits of Backward Integration," *Journal of Economics & Management Strategy*, 17(4), pp. 895-911.

[38] Laussel, D. and N. Van Long, 2012, "Vertical Disintegration: A Dynamic Markovian Approach," *Journal of Economics & Management Strategy*, 21(3), pp. 745-771.

[39] Legros, P. and S. A. Matthews, 1993, "Efficient and Nearly-Efficient Partnerships," *Review of Economic Studies*, 60 (3), July, pp. 599-611.

[40] Leontief, W., 1946, "Pure Theory of the Guaranteed Annual Wage Contract," *Journal of Political Economy*, 54 (1), April, pp. 76-79.

[41] Li, X. and Q. Wang, 2007, "Coordination Mechanisms of Supply Chain Systems," *European Journal of Operational Research*, 179(1), pp.1-16.

[42] Llanes, G. and J. Poblete, 2014, "Ex Ante Agreements in Standard Setting and Patent-Pool Formation," *Journal of Economics & Management Strategy*, 23, 1, Spring, pp. 50–67.

[43] Machlup, F. and M. Taber, 1960, "Bilateral Monopoly, Successive Monopoly, and Vertical Integration," *Economica*, New Series, 27 (106), May, pp. 101-119.

[44] Marshall, A., 1907, *Principles of Economics*, 5th edition, London: Macmillan.

[45] Menger, C., 1871, *Grundsätze der Volkswirtschaftslehre*, Viena, Series of Reprints of Scarce Works in Economics and Political Science, no. 17, London, London School of Economics.

[46] Nash, Jr., John F., 1950, "The Bargaining Problem," *Econometrica*, 18 (2), April, pp. 155-162.

[47] Nash, Jr., John F., 1953, "Two-Person Cooperative Games," *Econometrica*, 21 (1), January, pp. 128-140.

[48] McChesney, F. S., 2015, "One Piece at a Time: Successive Monopoly and Tying in Antitrust," *Journal of Competition Law & Economics*, 11 (4), pp. 1013-1032.

[49] Moore, H. L., 1906, "Paradoxes of Competition," *Quarterly Journal of Economics* 20 (2), pp. 211–230.

[50] Morgan, J. N., 1949, "Bilateral Monopoly and the Competitive Output," *Quarterly Journal of Economics*, Vol. 63, No. 3, August, pp. 371-391.

[51] Nash, Jr., J. F., 1950, "The Bargaining Problem," *Econometrica*, 18 (2), April, pp. 155-162.

[52] Nash, Jr., J. F., 1953, "Two-Person Cooperative Games," *Econometrica*, 21 (1), January, pp. 128-140.

[53] OECD (Organization for Economic Cooperation and Develoment), 2001, *Portfolio Effects in Conglomerate Mergers*, Directorate for Financial, Fiscal, and Enterprise Affairs, Compeitition Committee, JT00119854.

[54] Pareto, V., 1903, "Andewendungen der Mathematik auf Nationalokonoomie," in *Encyklopadie der mathematischen Wissenscheften*, I, Heft 7.

[55] Pareto, V., 1927, *Manual of Political Economy*, Ann S. Schwier and Alfred N. Page, trans., *Manuel d'economie politique*, Paris, New York: Augustus M. Kelley, 1971.

[56] Pigou, A. C., 1908, "Equilibrium Under Bilateral Monopoly," *Economic Journal*, 18 (70), June, pp. 205-220.

[57] Roth, A., 1979, *Axiomatic Models of Bargaining,* Berlin: Springer-Verlag.

[58] Rubinstein, A., 1982, "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, pp. 97-109.

[59] Schumpeter, J., 1927, "Zur Einfuihrung der folgenden Arbeit Knut Wicksell's," *Archiv für Sozialwissenschaften und Sozialpolitik*, 58, pp. 238-251.

[60] Schumpeter, J., 1928, "The Instability of Capitalism," *Economic Journal*, 38 (151), September, pp. 361-386.

[61] Shapley, L. S., 1952, "A Value for N-Person Games," Report No. RAND-P-295, Santa Monica, CA: Rand Corporation.

[62] Shapley, L. S. and M. Shubik, 1969, "Price Strategy Oligopoly with Product Differentiation," *Kyklos*, 22, pp. 30–44.

[63] Shubik, M., 1982, *Game Theory in the Social Sciences: Concepts and Solutions*, Cambridge, MA: MIT Press.

[64] Shubik, M., 1984, *A Game-Theoretic Approach to Political Economy*, Cambridge, MA: MIT Press.

[65] Singh, N. and X. Vives, 1984, "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, 15, pp. 546-554.

[66] Spengler, J. J., 1950, "Vertical Integration and Antitrust Policy," *Journal of Political Economy*, 58 (4), pp. 347-352.

[67] Spulber, D. F., 2013, "How Do Competitive Pressures Affect Incentives to Innovate when there is a Market for Inventions?," *Journal of Political Economy*, 121(6), pp. 1007-1054.

[68] Stackelberg, H. von, 1934, *Marktform und Gleichgewicht*, Vienna: Springer.

[69] Tarascio, V. J., 1972, "A Correction: On the Geneology of the So-Called Edgeworth-Bowley Diagram," *Western Economic Journal*; 10 (2), June, pp. 193-197.

[70] Tintner, G., 1939, "The Problem of Bilateral Monopoly: A Note," *Journal of Political Economy*, 47, (2), April, pp. 263-270.

[71] Topkis, D. M., 1998, *Supermodularity and Complementarity*, Princeton, NJ: Princeton University Press.

[72] Tsay, A., 1999, "The Quantity Flexibility Contract and Supplier–Customer Incentives," *Management Science*, 45 (10), pp. 1339–1358.

[73] Tsay, A.A., S. Nahmias, and N. Agrawal, 1999, "Modeling Supply Chain Contracts: A Review," in S. Tayur et al., eds., *Quantitative Models for Supply Chain Management*, New York: Springer, pp. 299-336.

[74] Vives, X., 1985, "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation," *Journal of Economic Theory*, 36, pp. 166–175.

[75] Vives, X., 1999, *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, MA: MIT Press.

[76] Vives, X. , 2005, "Complementarities and Games: New Developments," *Journal of Economic Literature*, 43, June, pp. 437–479.

[77] Vives, X., 2008, "Innovation and Competitive Pressure," *Journal of Industrial Economics*, 56 (3), September, pp. 419-469.

[78] Vives, X. and G. Staffiero, 2009, “The GE-Honeywell Merger in the EU”, in B. Lyons (ed.), *Cases in European Competition Policy: the Economic Analysis*, Cambridge: Cambridge University Press.

[79] Wicksell, K., 1927, "Mathematische Nationalokonomie," *Archiv fur Sozialwissenschaften und Sozialpolitik*, Band 58, 1927, pp. 252-281, translation of Knut Wicksell, 1925, "Matematisk Nationalekonomi," *Ekonomisk Tidsskrift*, 27, pp. 103-125.

[80] Wicksell, K., 1934, *Lectures on Political Economy v. 1: General Theory*, London: Routledge, E. Classen, trans., republished, Auburn, AL: Ludwig von Mises Institute, 2007.

[81] Zeuthen, F., 1930, *Problems of Monopoly and Economic Warfare*, London: Routledge.