

Online Advertising and Privacy*

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Abstract

We study a model in which an online platform makes a profit by auctioning an advertising slot that appears whenever a consumer visits its website. Several firms compete in the auction, and consumers differ in their preferences. Prior to the auction, the platform gathers data which is statistically correlated with consumers' tastes. We study whether it is profitable for the publisher to allow potential advertisers to access the data about consumers' characteristics before they bid. On top of the familiar trade-off between rent extraction and efficiency, we identify a new trade-off, namely that the disclosure of information leads to a better matching between firms and consumers, but results in a higher equilibrium price on the product market. We find that the equilibrium price is an increasing function of the number of firms. As the number of firms becomes large, it is always optimal for the platform to disclose the information, but this need not be efficient, because of the distortion caused by the higher prices. When the quality of the match represents vertical shifts in the demand function, we provide conditions under which disclosure is optimal.

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1 Introduction

The online advertising industry has been growing rapidly in the last decade, thanks, on the one hand, to the growth of the number of Internet users, and, on the other hand, to technological advances.¹ These advances have concerned the ability of firms to gather, stock and analyze considerable amount of data, but also their ability to use this data at a very high speed, making it possible to customize every interaction.

With 33% of total revenue, display is the second most important type of online advertising, behind search (43%) and before classifieds (14%). Let us describe briefly the actors involved in online display advertising: at opposite ends of the spectrum are the advertisers and the consumers, the former trying to reach the latter. Consumers visit various websites (publishers), such as nytimes.com or ESPN.go.com. Publishers typically have advertising space that they wish to sell, either directly with their sales team, or through an intermediary, for instance an advertising network (Google's Double Click, or Yahoo!Network). These networks aggregate supply (the publishers' side) and demand (the advertisers' side) for advertising space and play the role of match makers. Sometimes the advertising network is integrated with the publisher, Facebook or Google being the most prominent examples. The functioning of online advertising intermediated by an ad network is described in Figure 7. In this figure, user 2 visits publisher 1, user 3 visits publisher 2, and user 1 visits both publishers. Publishers register with the advertising network, which is in charge of filling the ad space on their websites. Advertisers 1 to n submit bids which may depend on users' as well as publishers' characteristics. In the example, users 1 and 2 see a different advertisement when they visit publisher 1. User 3 and 1 see the same advertisement when they visit publisher 2, but this advertisement is different from the one that user 1 sees on publisher 1's website.

Because of the technology advances mentioned above, the match making activity has experienced enormous improvements: not only is it possible to match every consumer to a different advertisement using real time auctions, but the accuracy of such a matching may be enhanced by the considerable amount of data that publishers and advertising networks have about consumers. The match may be based for instance on the correspondence between the publisher's

¹See Evans (2008) and Evans (2009) for insightful discussions about this industry.

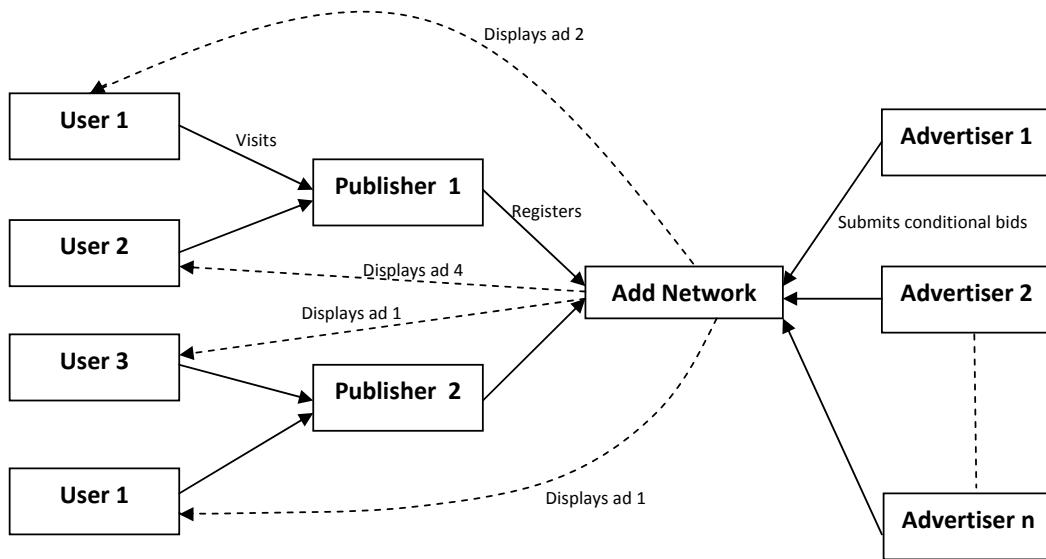


Figure 1: Non-integrated publishers and advertising network

website content and the advertisement, but also on data about the location of the consumer (obtained through the IP address), his past browsing history (obtained through cookies) or whatever information he gave to the publisher or its partners (through subscription questionnaires for instance, or any information left on his Facebook wall). These new opportunities may give firms additional incentives to acquire and use personal information about consumers, which has led regulators and consumers to express worries, or at least to acknowledge some potential pitfalls. Among these are privacy breaches or fraudulent use of personal information, but also practices of behavioral targeting and pricing.

In this paper, we look at the incentives of intermediaries such as advertising networks or integrated publishers to use the information they have gathered about consumers in order to increase their revenue from advertising. Do such practices have social value? Who benefits most from them?

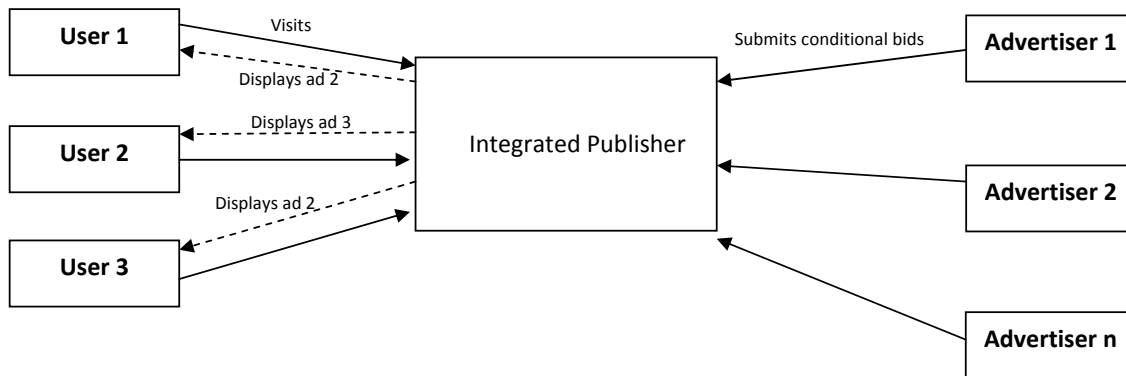


Figure 2: Integrated publisher

More specifically, the situation that we have in mind is the following (see Figure 2): a large number of web users (consumers) visit a website (a publisher), which makes profit by selling

a single advertising slot through an auction. The website is integrated with an advertising network as is the case for Facebook, which allocates the slot to an advertiser. Users are heterogeneous in the sense that they do not derive the same value from consuming advertisers' products. Thanks to its technology, the publisher gathers, for each consumer, information correlated to the consumer's willingness to pay for any product. The publisher does not know how to interpret the information in terms of implied willingness to pay for different products, but advertisers are able to do it. For instance, the publisher knows that the consumer is a young male living in a metropolitan area, but it is not able to infer his willingness to pay for good A or B. On the other hand, firm A knows that young males living in a metropolitan area are especially likely to have a high willingness to pay for its product, whereas firm B offers a product which is less likely to be a good match for such consumers.

In the economic literature, privacy has been defined as connoting "the restriction of the collection or use about a person or a corporation" ². In our set-up, we define privacy as the right of consumers that their personal data be not disclosed to advertisers. In order to make the analysis as transparent as possible, we assume that consumers do not exhibit intrinsic preferences over their privacy, but care about it insofar as it has indirect effects.

We are thus concerned with two main questions: (1) what are the effects of the disclosure policy on market outcomes, that is on the interactions between consumers and advertisers?, and (2) when does the platform provide the efficient amount of privacy?

Regarding (1), we show that disclosing the information has both positive and negative consequences. On the positive side, when advertisers can condition their bids on information about consumers, in equilibrium, the highest bidder is the firm that is the best match, which is efficient. However, when good matches correspond to higher marginal revenues for advertisers, we show that disclosure of personal information leads to higher prices. Although reminiscent of results by Taylor (2004), Acquisti and Varian (2005), Hermalin and Katz (2006), Calzolari and Pavan (2006), the latter effect stems from a different logic. In the aforementioned papers, the structure of the model is the following : (i) the seller observes a signal about the consumer's type, either by previous experimentation or by buying information from another firm, (ii) the seller uses the signal to determine the price (or the menu of contracts) that he offers to the con-

²See Png and Hui (2006) for a survey of the economics of privacy.

sumer. Thus, in all these papers, personal information is used by sellers to price-discriminate between buyers. Such a mechanism is plausible in settings in which the identity of the buyer is verifiable and there are no possibilities of arbitrage among consumers. However, electronic commerce is characterized by the ease with which consumers can hide their identity or erase any existing information, by deleting cookies or by using a different computer. Firms may also be reluctant to overtly engage in price-discrimination, following the public relations backlash to Amazon's price-discrimination experiment in 2000.

In our model, we rule out price-discrimination and we exhibit another channel through which disclosure can lead to higher prices. More precisely, we assume that firms choose their prices before learning the information. Once they learn the information, they submit a bid and the winner of the auction has its advertisement displayed to the consumer. The mechanism through which the price increase occurs is a change in the composition of the demand. By allowing firms to condition their bid on consumers' characteristics, the disclosure of information leads to a situation in which firms expect their ads to reach only the consumers with a low price-elasticity of demand. Firms then rationally set higher prices. The magnitude of the price increase depends on the number of bidders and on the number of advertising slots. With a large number of bidders (or a low number of slots) winning the auction is more informative than with fewer bidders, so that the price will be higher when there are many bidders.

The model can also be interpreted as a model of targeted advertising: by disclosing information about the consumer, the platform ensures that a consumer will see the most relevant advertisement, whereas when no information is disclosed (the privacy case) ads are displayed randomly. Some papers in the literature on targeted advertising also find that targeting leads to higher prices, but for different reasons: in Roy (2000), Iyer, Soberman, and Villas-Boas (2005), or ?, targeting allows firms to segment the market, thereby softening price competition. In Esteban, Gil, and Hernandez (2001), targeting leads to higher prices for cost-related reasons. On the other hand, de Cornière (2011) shows that when consumers actively search for products, targeting leads to more intense competition.

Regarding our second question, namely whether the platform provides the right amount of information from a social welfare perspective, our analysis partially relies on insights formulated by Ganuza (2004) and Ganuza and Penalva (2010). As in these papers, disclosing information

increases the total profits of the industry (platform and advertisers) but comes at the price, for the platform, of leaving an informational rent to the winning bidder. We show that when the number of firms is large, the platform always prefers to disclose the information about consumers. Indeed, in that case the rent left to the winning bidder vanishes. However, and in contrast to Ganuza (2004), such a policy is not necessarily efficient, because some consumers are excluded from the market following the increase in the equilibrium price of the goods. Following the approach of Cowan (2007), we give conditions under which privacy or disclosure is optimal when the quality of the match determines a vertical shift of the demand function. In section 2 we present the model in a rather general way. In section 3 we characterize symmetric equilibria under privacy and disclosure, and analyse the main implications of either regime. In section ??, we put more restrictions on the model and conduct a normative analysis. Section ?? presents some extensions of the basic model.

2 Model

The market we model is the following: there are n advertisers, who compete for a single slot on a publisher's website. There is a continuum of consumers who visit the website. A consumer's type is a vector $\Theta = (\theta_1, \dots, \theta_n)$. The θ_i are independent and identically distributed according to a continuous cdf F over an interval set $[0, \bar{\theta}]$. The probability distribution function of θ_i is f . If a consumer of type Θ is matched with firm i , which sets a price p_i , firm i 's profit is $\pi(p_i, \theta_i) = (p_i - c)D(p_i, \theta_i)$ (firms are assumed to be symmetric).³ Welfare and consumer's indirect utility, if consumer Θ is matched with firm i , are noted respectively $W(p_i, \theta_i)$ and $V(p_i, \theta_i)$. We make the following set of assumptions:

Assumption 1 *The demand function D is twice continuously differentiable in both arguments. There exists \bar{p} such that for all θ_i , and for all $p_i \geq \bar{p}$, $D(p_i, \theta_i) = 0$*

Assumption 2 *π is strictly concave in p_i over $[0; \bar{p}]$. For every θ_i , there exists $p^*(\theta_i) \in [0; \bar{p}]$ such that $\frac{\partial \pi(p^*(\theta_i), \theta_i)}{\partial p_i} = 0$.*

³The assumption of a constant marginal cost is not essential but simplifies the notations.

Assumptions 1 and 2 are made for analytical simplicity. In particular, they ensure that for any ϕ , the function $\int_0^{\bar{p}} \pi(p, \theta) \phi(\theta) d\theta$ is concave in p , which will allow us to only look at the first-order conditions of the profit-maximization problem.

The following assumptions bear more economic significance.

Assumption 3 *For every price $p < \bar{p}$, $D(p, \theta_i) \geq D(p, \theta'_i)$ if and only if $\theta_i \geq \theta'_i$*

Under Assumption 3, we restrict the analysis to cases in which a better match corresponds to an upward move of the demand function. In particular, we rule out situations in which an increase in θ_i corresponds to a rotation of the demand function with an interior rotation point (see Johnson and Myatt (2006)). In section ??, we will make the stronger assumption that the effect of an increase in θ_i on the demand function is independent of the price p_i , but for the time being we only impose the following condition:

Assumption 4 *The profit function exhibits increasing differences : $\frac{\partial^2 \pi}{\partial p_i \partial \theta_i} \geq 0$*

We thus assume that for any price p_i , the marginal revenue of firm i is larger for high values of θ_i . The following parametrizations of demand functions satisfy Assumptions 1-4: (i) $D(p, \theta) = \theta + q(p)$ if $p \leq \bar{p}$, and zero otherwise, with $2q'(p) + pq''(p) \leq 0$ for all p . (ii) $D(p, \theta) = 1 - p^\theta$.

It is immediate to see that Assumptions 1-4 imply

$$\frac{\partial W}{\partial p_i} \leq 0, \quad \frac{\partial W}{\partial \theta_i} \geq 0, \quad \frac{\partial V}{\partial p_i} \leq 0, \quad \frac{\partial V}{\partial \theta_i} \geq 0$$

Note that throughout the paper we assume that if a firm is matched with a consumer, it is in a monopoly situation with respect to that consumer. This strong assumption is made for expositional purpose, since our model can accommodate some downstream competition among firms. However, it is essential that advertising attracts some consumers that would not have considered firm i without being exposed to an advertisement.

It is also not essential whether profit is realized immediately or later on. Indeed we know that display is less efficient than search advertising at generating clicks or immediate sales. Rather, display is often used as a brand-building device (See Manchanda, Dube, Goh, and

K.Chintagunta (2006)).⁴ Under an assumption of delayed sales, our analysis would carry through, provided that firms do not set a different price for consumers who click on an ad and for consumers who visit the website later on.

An important corollary of Assumptions 2 and 4 is the following:

Lemma 1 $p^*(\theta_i)$ is non-decreasing in θ_i .

Proof : The proof is a classical result of monotone comparative statics (see Vives (2001) for instance). Let $\theta_i > \theta'_i$, and $p' > p^*(\theta_i)$. From Assumption 4 we have $\pi_i(p', \theta_i) - \pi_i(p^*(\theta_i), \theta_i) \geq \pi_i(p', \theta'_i) - \pi_i(p^*(\theta_i), \theta'_i)$. But, by Assumption 2, $\pi_i(p', \theta_i) - \pi_i(p^*(\theta_i), \theta_i) < 0$. Therefore p' cannot maximize $\pi_i(p, \theta'_i)$. \square

We assume that the realization of Θ is consumer's private information, but the platform observes a signal about Θ . The platform does not know the mapping from the signal to the actual value of Θ . It can choose to publicly reveal the value of the signal to advertisers. In that case, each firm i privately learns the value of θ_i . One can imagine that θ_i is the score that firm i would affect to consumer θ . The publisher knows the age, gender, address of the consumer, as well as some other information related to his valuations for the different goods, but is not able to compute the score, because it lacks some information about the firm. Still, if the publisher reveals these characteristics to advertisers, they are able to compute the score. If the publisher decides to reveal the information, we say that it follows a *disclosure* policy. If not, we say that it follows a *privacy* policy. Anytime a consumer visits the website, the publisher runs a second price auction in order to determine which firm will appear on the consumers' screen. For simplicity, we assume that the publisher cannot set a reserve price for the auction.

The timing of the game is the following:

1. The publisher commits to a policy $\sigma \in \{\mathcal{D}, \mathcal{P}\}$, where \mathcal{D} stands for *Disclosure* and \mathcal{P} for *Privacy*.
2. Firms choose independently and simultaneously their prices p_i .

⁴See Lewis and Reiley (2009), Chatterjee, Hoffman, and Novak (2003), Drèze and Hushner (2003), Rutz and Bucklin (2009) for more on the links between online advertising and sales.

3. Under *Disclosure*, each firm i learns θ_i . Under *Privacy*, firms do not learn θ_i .
4. Under *Disclosure*, firms can submit bids which depend on the realization of θ_i : $b_i^{\mathcal{P}}(\theta_i, p_i)$. Under *Privacy*, they submit a single bid $b_i^{\mathcal{P}}(p_i)$. The auction is a second price auction with no reserve price.
5. The consumer is matched the winning firm, say firm j . Total welfare, consumer's surplus and firm j 's profit are given by $W(p_j, \theta_j)$, $S(p_j, \theta_j)$, and $\pi_j(p_j, \theta_j)$. The platform's revenue R is given by the highest losing bid.

In the auction we only consider equilibria in undominated strategies, i.e in which firms bid truthfully.

3 Equilibrium under privacy and disclosure - the general case

The case of privacy Suppose that the platform chooses not to let firms learn anything. Let $P \equiv (p_1, \dots, p_n)$ be the vector of prices, and P_{-i} be the vector of prices of firms other than i . If it sets a price p_i , firm i 's profit is

$$E[\pi_i^{\mathcal{P}}(p_i, P_{-i})] = \max\left\{\int_0^{\bar{\theta}} \pi(p_i, \theta_i) f(\theta_i) d\theta_i - T_i(P_{-i}), 0\right\}$$

where $T_i(P_{-i}) = \max_{j \in N-i} \int_0^{\bar{\theta}} \pi(p_j, \theta_j) f(\theta_j) d\theta_j$ is firm i 's payment if it wins the auction. Notice that this payment does not depend on the realization of Θ , because firms do not learn Θ before they bid. Maximizing this profit with respect to p_i leads to the following proposition:

Proposition 1 *When the platform chooses to implement a privacy policy, a symmetric equilibrium is such that the price $p^{\mathcal{P}}$ verifies:*

$$\int_0^{\bar{\theta}} \frac{\partial \pi(p^{\mathcal{P}}, \theta_i)}{\partial p_i} f(\theta_i) d\theta_i = 0 \tag{1}$$

Given that firms cannot infer anything from the fact that they win the auction, they set a price equal to the monopoly case when they have no information about consumers.

Proposition 2 *Under privacy, the platform extracts all the profits of the industry:*

$$R^{\mathcal{P}} = E[\pi_i^{\mathcal{P}}(p^{\mathcal{P}}, \theta)]$$

Proof: Since firms are symmetric, they all set the same price, and thus bid the same amount for every consumer. \square

The case of disclosure Now we assume that firms privately learn the consumer's type before bidding (but after having chosen their price). We look for a symmetric equilibrium, in which firms charge a price $p_n^{\mathcal{D}}$ and bid truthfully for every realization of Θ .

Since firms bid truthfully, firm i 's bid is $\pi(p_i, \theta_i)$. Suppose that all the firms other than i set a price $p_n^{\mathcal{D}}$. Let $\hat{\theta}_{-i}$ be the highest realization of θ_j for $j \in N - i$. Let j_0 be the identity of the corresponding firm. By Assumption 3, i will win the auction if it bids more than firm j_0 , since j_0 outbids all the other firms. Let $\phi(\hat{\theta}_{-i}, p_i, p_n^{\mathcal{D}})$ be the smallest value of θ_i such that i wins the auction. Notice that by Assumption 3, $\phi(\hat{\theta}_{-i}, p, p) = \hat{\theta}_{-i}$ for every p . Firm i 's expected profit is therefore

$$E[\pi_i^{\mathcal{D}}(p_i, p_n^{\mathcal{D}})] = \int_{\hat{\theta}_{-i} \in [0, \bar{\theta}]} \int_{\theta_i \in [\phi(\hat{\theta}_{-i}, p_i, p_n^{\mathcal{D}}), \bar{\theta}]} \left[\pi(p_i, \theta_i) - \pi(p_n^{\mathcal{D}}, \hat{\theta}_{-i}) \right] f_{n-1:n-1}(\hat{\theta}_{-i}) f(\theta_i) d\theta_i$$

where $f_{k:m}$ is the probability distribution function of the k th order statistic of θ_j among m .⁵ At a symmetric equilibrium, we must have $\frac{\partial E[\pi_i^{\mathcal{D}}(p_i, p_n^{\mathcal{D}})]}{\partial p_i} \Big|_{p_i=p_n^{\mathcal{D}}} = 0$, by concavity of the profit function. This first order condition rewrites as

$$\int_{\hat{\theta}_{-i} \in [0, \bar{\theta}]} \left\{ \int_{\theta_i \in [\hat{\theta}_{-i}, \bar{\theta}]} \frac{\partial \pi_i(p_n^{\mathcal{D}}, \theta_i)}{\partial p_i} f(\theta_i) d\theta_i - \frac{\partial \phi(\hat{\theta}_{-i}, p_i, p_n^{\mathcal{D}})}{\partial p_i} \left(\pi(p_n^{\mathcal{D}}, \hat{\theta}_{-i}) - \pi(p_n^{\mathcal{D}}, \hat{\theta}_{-i}) \right) \right\} f_{n-1:n-1}(\hat{\theta}_{-i}) d\hat{\theta}_{-i} = 0$$

After some extra manipulations, we get :

Proposition 3 *Under disclosure, a symmetric equilibrium price is given by*

⁵ $f_{m:m}$ corresponds to the highest realization, $f_{m-1:m}$ to the second highest, and so on.

$$\int_0^{\bar{\theta}} \frac{\partial \pi(p_n^{\mathcal{D}}, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i = 0 \quad (2)$$

The difference between (1) and (2) comes from the term $F^{n-1}(\theta_i)$ in the integrand. Under privacy, winning the auction for a consumer does not bring any information about the consumer's type. Under disclosure, on the other hand, firm i wins the auction only when all the θ_j 's are smaller than θ_i , which occurs with probability $F^{n-1}(\theta_i)$. As we show in the next proposition, the equilibrium price is then higher under disclosure than under privacy.

Proposition 4 (i) *For every n , the equilibrium price under disclosure is larger than the equilibrium price under privacy: $p_n^{\mathcal{D}} \geq p^{\mathcal{P}}$. (ii) Under disclosure, the equilibrium price increases with the number of firms.*

The intuition for proposition 4 is straightforward: under disclosure, conditional on winning the auction, firm i expects to face consumers with a higher θ_i than under privacy, and therefore the optimal strategy is to charge a higher price. This effect is all the more important as the number of firms is large, because being the winner among a large set of bidders is a stronger signal that the value of θ_i is high.

The following proposition summarizes the results, and describes the effects of the disclosure policy on profits.

Proposition 5 (i) *The adoption of a disclosure policy by the platform leads to a better expected match between advertisers and consumers, but also to a higher price of the advertised good. (ii) The profits of the industry (advertisers and platform) are higher under disclosure. (iii) Advertisers' share of the total profits is also higher under disclosure.*

Proof: (i) The expected value of the match is $E[\theta_{n:n}]$ under disclosure, against $E[\theta]$ under privacy. The fact that disclosure leads to higher prices has been shown in Proposition 4. (ii) The industry's profit under disclosure is $E[\pi(p_n^{\mathcal{D}}, \theta_{n:n})]$. Since $p_n^{\mathcal{D}}$ is the price that maximizes $E[\pi(p, \theta_{n:n})]$, we have $E[\pi(p_n^{\mathcal{D}}, \theta_{n:n})] \geq E[\pi(p^{\mathcal{P}}, \theta_{n:n})]$. Using the fact that, for every p and every $\theta > \theta'$, $\pi(p, \theta) \geq \pi(p, \theta')$, we get $E[\pi(p^{\mathcal{P}}, \theta_{n:n})] \geq E[\pi(p^{\mathcal{P}}, \theta)]$, the last term being the industry's profit under privacy. (iii) The expected profit of an advertiser is $\frac{1}{n} (E[\pi(p_n^{\mathcal{D}}, \theta_{n:n})] - E[\pi(p_n^{\mathcal{D}}, \theta_{n-1:n})]) > 0$ under disclosure, whereas it is zero under privacy. \square

The fact that disclosure leads to better matches is intuitive, and in line with empirical findings (see, e.g. Goldfarb and Tucker (2011a) and Goldfarb and Tucker (2011b)). Proposition ?? also reveals that advertisers have strong incentives to lobby for more disclosure by intermediaries who possess some information about consumers. We saw that under privacy, the platform extracts all the industry profits, while this is not the case under disclosure, because of the informational rent left to the winning bidder.

From a positive point of view, one would like to know under which conditions the platform is likely to adopt a disclosure policy. While we cannot predict whether the platform prefers privacy or disclosure for any n without imposing further restrictions on the demand function, the following polar case will prove useful in our normative analysis:

Proposition 6 *(i) When n goes to infinity, the platform's optimal policy is disclosure. (ii) The equilibrium price for the good tends to $p^*(\bar{\theta})$.*

The proof is in the appendix. Part (i) relies on the observation that as n goes to infinity, the informational rent of the winning bidder goes to zero, which implies that the platform captures the whole industry profit. The intuition for (ii) is that when the number of firms is very large, firm i knows that it will win the auction only when θ_i is very close to $\bar{\theta}$.

4 Normative analysis

So far, the analysis does not allow to determine in which cases privacy must be favored over disclosure. In order to gain further insight, we specify the model by putting more structure on the family of demand functions $D(p, \theta)$.

To facilitate the analysis, we assume that $n = \infty$.⁶ Although we believe that the case with a large number of firms is the relevant one when one thinks about advertising platforms such as Google or Facebook, let us stress the implications of this assumption. First, as shown in Proposition 6, the platform captures the whole industry profits, and always prefers to implement disclosure. Thus, we assume away cases in which the platform discloses strictly less information than what would be optimal. Second, when $n = \infty$ firms charge a price $p^*(\bar{\theta})$ under disclosure.

⁶The reader is referred to Appendix B for the analysis in the case in which $n < \infty$.

Therefore, our results will depend on the distribution F of types θ only through its mean m and the upper-bound of its support $\bar{\theta}$.

Following Cowan (2007), let us assume that for every $p \leq \bar{p}$, $D(p, \theta) = \theta + q(p)$, with $q' < 0$ and $D(p, \theta) = q(p) = 0$ for all $p > \bar{p}$. Let $P(x, \theta)$ be the inverse demand, defined as $D(P(x, \theta), \theta) = x$, or $P(D(p, \theta), \theta) = p$. Such a formulation corresponds to a situation in which a consumer of type θ buys θ units of the product as long as the price is lower than the reservation price \bar{p} , and, in addition to these units, buys $q(p)$ units.

The elasticity of demand is $\eta(p, \theta) = -pq'(p)/(\theta + q(p))$, and the elasticity of the slope of the demand is $\alpha(p, \theta) = -pq''(p)/q'(p)$. Notice that the latter expression does not depend on θ , so that we can drop θ from the arguments of α .

For any θ , the optimal price $p^*(\theta)$ is given by the Lerner formula

$$\frac{p^*(\theta) - c}{p^*(\theta)} = \frac{1}{\eta(p^*(\theta), \theta)} \quad (3)$$

The second-order condition is $(p-c)q''(p) + 2q'(p) \leq 0$ at $p = p^*(\theta)$ (i.e. $2\eta(p^*(\theta), \theta) > \alpha(p^*(\theta))$).⁷ Since the type θ enters linearly in the demand function, one can see that the optimal price under privacy is $p^{\mathcal{P}} = p^*(m)$. Under privacy, firms behave as if they faced the average type. Thanks to Proposition 6, we also know that the price under disclosure is $p^*(\bar{\theta})$.

Let us introduce the following function:

$$W(\theta) = \int_0^{D(p^*(\theta), \theta)} P(x, \theta) dx$$

The function W measures the social welfare when a firm faces a consumer of type θ and charges the price $p^*(\theta)$. Given the previous observations, we can see that disclosure is preferable to privacy from a social welfare point of view when $W(\bar{\theta}) > W(m)$.

We are interested in sufficient conditions over q such that privacy or disclosure is better. To do so, let us consider the derivative of W :

$$W'(\theta) = p^*(\theta)[1 + p'^*(\theta)q'(p^*(\theta))] + \int_0^{D(p^*(\theta), \theta)} \frac{\partial P(x, \theta)}{\partial \theta} dx \quad (4)$$

⁷Notice that we no longer require the profit to be concave everywhere.

Notice that $P(x, \theta) = q^{-1}(x - \theta)$, so that $\frac{\partial P(x, \theta)}{\partial \theta} = -\frac{\partial P(x, \theta)}{\partial x}$. Therefore, (4) simplifies to

$$W'(\theta) = \underbrace{p^*(\theta)p^{*'}(\theta)q'(p^*(\theta))}_{\text{price effect}} + \underbrace{\bar{p}}_{\text{match effect}} \quad (5)$$

The right-hand side of (5) is made of two terms. The first term, the “price effect”, measures the loss in utility due to a higher price: an increase in θ leads to an increase in the price, and thus a lower quantity from the elastic part of the demand $q(p)$. The second term, the “match effect” corresponds to the shift in the inverse demand that results from the increase in θ .

At this point it may be useful to discuss the relationship between Cowan (2007) and the present paper. Cowan (2007) uses the parametrization $D(p, \theta) = \theta + q(p)$ to study whether third degree price discrimination is desirable. He assumes that there are two subpopulations, one with demand $q(p)$ (the weak market) and one with demand $\theta + q(p)$ (the strong market). However, contrary to what we do, he assumes that the utility of a consumer in the strong market is $U(q(p))$, so that θ does not directly enter the utility function. This remark sheds some light on the connection between our approach and the well-known analysis of price-discrimination. The negative effect of disclosure is akin to the negative effect of price discrimination: it leads to an increase in the price for consumers with a strong demand (the price effect). However, while the positive effect of price discrimination in Cowan’s paper is that consumers in the weak market will buy more units, the positive effect of disclosure in our paper is that it allows a better matching, so that all consumers have a strong demand (the match effect), and they directly benefit from it.

A sufficient condition for disclosure (privacy) to be socially optimal is for $W'(\theta)$ to be positive (negative) for all θ . This leads to the following proposition, in which $\eta \equiv \eta(p^*(\theta), \theta)$ and $\alpha \equiv \alpha(p^*(\theta))$.

Proposition 7 *Disclosure (resp. Privacy) is optimal if, for every θ , $\frac{\bar{p}}{p^*(\theta)} \geq$ (resp. \leq) $\frac{\eta}{2\eta - \alpha}$*

Proof: For notational simplicity, we drop the arguments of the functions. We have

$$W' \geq 0 \iff \frac{\bar{p}}{p^*} \geq -p^{*'}pq' \quad (6)$$

p^{*} may be obtained from the profit maximizing condition:

$$(p^*(\theta) - c)q'(p^*(\theta)) + \theta + q(p^*(\theta)) = 0$$

Differentiating with respect to θ gives

$$p^{*'} = -\frac{1}{2q' + (p^* - c)q''}$$

Plugging this expression into (6), and rearranging terms, one gets the desired condition. \square

Thanks to proposition 7, we can determine whether disclosure or privacy is optimal for some classes of demand functions $q(\cdot)$.

Corollary 1 *If $\alpha < \eta$ for every θ , then disclosure is socially optimal. In particular, disclosure is optimal when q is concave or linear.*

Proof: Notice first that the ratio $\bar{p}/p^*(\theta)$ is bounded below by 1. If $\alpha < \eta$, then $\frac{\eta}{2\eta - \alpha} < 1$, so that we always have $\frac{\bar{p}}{p^*(\theta)} > \frac{\eta}{2\eta - \alpha}$. The sign of α is the same as that of q'' , which, along with the fact that $\eta > 0$, proves the second point of the proposition. \square

Let us interpret $D(p, \theta)$ as the probability that a consumer of type θ buys at price p . Let v be the willingness to pay of a consumer, and let $H(p) = P[v \leq p | v \neq \bar{p}]$. We have $P[v = \bar{p}] = \theta$ and $q(p) = 1 - H(p)$. A concave demand function q corresponds to an increasing density of willingness to pay h . Corollary 1 indicates that when h is increasing, the price effect always dominates the match effect.

Given Proposition 7, we would like to know which characteristics of the demand function $D(p, \theta)$ make it more desirable to disclose the information. The first one such characteristic is the reservation price \bar{p} : loosely speaking, an increase in \bar{p} leads to more disclosure, as disclosure leads to a higher θ . Proposition 7 also suggests that a higher price elasticity speaks in favor of more disclosure. The intuition is that with a high price elasticity of demand, the price effect is small, because firms do not want to increase their price by much for fear of losing too many consumers. Therefore the dominant effect of disclosure is the match effect, which is positive for welfare. However, given the existence of indirect effects, such a claim cannot be proved in

general, but numerical simulations with demand functions exhibiting constant price elasticity confirm this intuition.

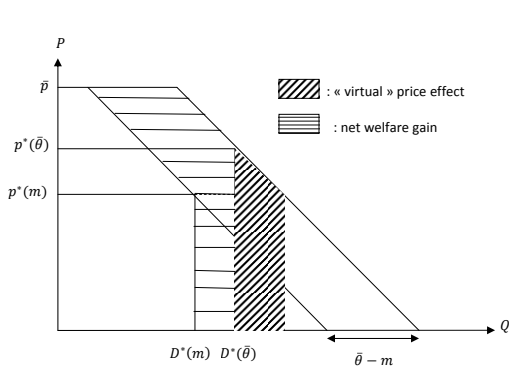


Figure 3: Linear demand

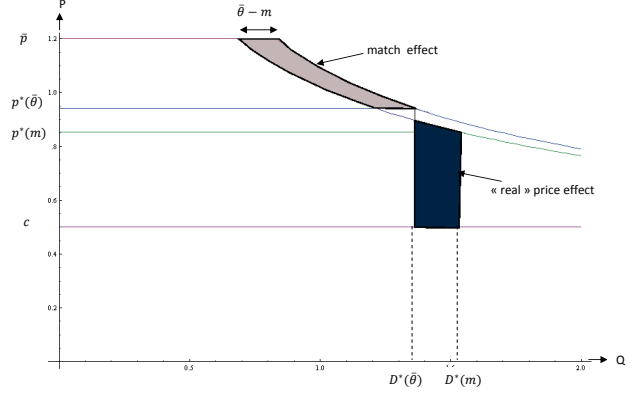


Figure 4: Constant elasticity

Examples In figure 3, we represent the case of a linear demand $q(p) = a - bp$. Switching from privacy to disclosure leads to a vertical shift of $\bar{\theta} - m$. Notice that here the quantity consumed under disclosure $D^*(\bar{\theta})$ is greater than that under privacy ($D^*(m)$). This is why we speak of a “virtual” price effect: under disclosure, consumers buy a smaller quantity than if the price remained the same as under privacy. As we see, the welfare gain is always positive, as implied by Corollary 1. Figure 4 represents the inverse demand function when q has a constant price elasticity: $q(p) = p^{-\epsilon}$ if $p \leq \bar{p}$.⁸ Here we see that disclosure leads to a lower quantity (the price effect) that is partially compensated by the increase in the utility generated by the infra-marginal units (the match effect). In this specific example, privacy leads to a higher welfare than disclosure.

5 Extensions

In this section we provide some extensions to our basic model. We first discuss the mechanism through which the platform allocates the advertising slot(s), and then introduce the possibility of partial disclosure.

⁸We take $\bar{\theta} = 0.2, m = 0.05, \bar{p} = 1.2, \epsilon = 2.5$ and $c = 0.5$.

Optimal mechanism with one slot. In our model we focus on the platform's choice of information revelation rather than on the optimal mechanism. Here the optimal mechanism is straightforward to design. The platform should sell the access to information at a price T equal to the expected net profit of a firm, and forbid firms to participate if they do not pay T . This way, the platform can extract the expected profit of the whole industry $E[\pi(p_n^D, \theta_{n:n})]$, which is maximized under disclosure. However for legal reason or reputation concerns the platform may not be able to sell detailed information on his customers. However it may be authorized to share this information with its commercial partners. In such a case information revelation to bidders may then be seen a means for the platform to still monetize his information without selling it.

Uniform multi-unit auctions On many websites users can see several advertisements on each page, yet in the model we only allow for one advertisement to be displayed. What can we say if the platform was allowed to choose the number of ads? Suppose that the platform has K slots up for sale, and that it allocates them through a uniform price auction, in which the price paid by all the firms whose ads are displayed is equal to the highest losing bid.

Suppose also that firms do not compete on the product market, but only compete for consumer attention. That is, we assume that the demand for product i is independent from the demand for product j .

Whether the platform prefers privacy or disclosure still depends on the trade-off between increasing the value of trade (with disclosure) and eliminating the informational rent of the winner (privacy). However, we show that the equilibrium price charged by advertisers will decrease as the number of slots increases, *even though* we explicitly rule out competition on the product market.

To see this, consider a symmetric configuration in which all firms (except i) charge a price $p_{K,n}$ and bid their profit $\pi(p_{K,n}, \theta_j)$ after learning the consumer's type $\Theta = (\theta_1, \dots, \theta_n)$.⁹

Let $\hat{\theta}_k$ be the k -th highest value of θ_j , for $j \neq i$. Then firm i wins a slot in the auction if and only if $\pi(p_i, \theta_i) \geq \pi(p_{K,n}, \hat{\theta}_K)$.

Let $\phi(\hat{\theta}_K, p_i, p_{K,n})$ be the smallest value of θ_i such that i wins a slot. Notice that in equilibrium

⁹Such a strategy is weakly dominating in the uniform auction with unit-demand.

$$\phi(\hat{\theta}_K, p_{K,n}, p_{K,n}) = \hat{\theta}_K.$$

Firm i 's profit is

$$\pi_i(p_i, p_{K,n}) = \int_{\hat{\theta}_K \in [0, \bar{\theta}]} \int_{\theta_i \in [\phi(\hat{\theta}_K, p_i, p_{K,n}); \bar{\theta}]} \left(\pi(p_i, \theta_i) - \pi(p_{K,n}, \hat{\theta}_K) \right) f_{n-K:n-1}(\hat{\theta}_K) f(\theta_i) d\hat{\theta}_K d\theta_i$$

The first order condition at a symmetric equilibrium is

$$\int_{\theta \in [0, \bar{\theta}]} \frac{\partial \pi(p_{K,n}, p_{K,n})}{\partial p_i} F_{n-K:n-1}(\theta) dF(\theta) = 0 \quad (7)$$

Then there is a new trade-off, between the quantity sold (the number of slots) and the per-unit price (the highest losing bid). Interestingly, by choosing the number of slots the platform is able to fine tune the extent to which firms use the information under disclosure. To see this clearly, suppose that there is a large number of firms on the market. If the platform only sells one spot, firms expect that if they win the auction they will face a consumer with a high θ , and they charge a high price accordingly. Now, if the platform sells a large number of slots, winning the auction is less informative with respect to the consumer's type. Firms have an incentive to charge a lower price. We see then that selling more slots leads to a decrease in the equilibrium price of the goods, for reasons that have nothing to do with downstream competition on the product market.

Partial disclosure So far we have only considered two types of information revelation policy for the platform. Under the disclosure policy, the platform reveals all the available information about consumers prior the auction, whereas under the privacy policy it reveals nothing. One can imagine that it could be better to adopt an information revelation policy between these two extremes. The question of the optimal provision of information by a principal has been investigated in different contexts in the literature. Most related to this work are the papers that study this question in the context of auctions (e.g Ganuza and Penalva (2010)), and in the context of a monopolistic seller (e.g Lewis and Sappington (1994), Johnson and Myatt (2006), Anderson and Renault (2006)).

Hereafter, we focus on the *truth or noise* technologies of information revelation a la Lewis and

Sappington (1994) with a simple demand function $D(p, \theta) = \theta \mathbf{1}_{p \leq v_H} + \mathbf{1}_{p \leq v_L}$, with θ uniformly distributed in $[0, 1]$, and an infinite number of bidders. According to this demand specification, as θ increases the size of the population with a high valuation for the good increases. Let us also assume that $c = 0$.

We assume that the ad-sponsored platform may design its information revelation policy so that firms learn the true value of θ with probability γ , and receive a random signal uniformly distributed on $[0; 1]$ with probability $1 - \gamma$. Upon receiving a signal $s_i \in [0; 1]$, the expected value of θ_i is $\beta_i \equiv E[\theta_i | s_i, \gamma] = \gamma s_i + \frac{1-\gamma}{2}$. Ex ante, the β_i 's are uniformly drawn from $[\frac{1-\gamma}{2}, \frac{1+\gamma}{2}]$. Because there is an infinite number of firms, the optimal price p_γ set by firms maximizes $pD(p, \frac{1+\gamma}{2}) = p(\frac{1+\gamma}{2} \mathbf{1}_{p \leq v_H} + \mathbf{1}_{p \leq v_L})$. One gets $p_\gamma = v_H$ if and only if $V \geq \frac{3+\gamma}{1+\gamma}$, where $V = \frac{v_H}{v_L}$. Otherwise, $p_\gamma = v_L$. It is immediate to see that firms' profit is an increasing function of γ , so that the optimal level of information revelation for the platform is $\gamma = 1$, since it captures the entire industry profit. This yields a welfare $W(1) = v_H$.

Now we consider the optimal level of information revelation that would be chosen by a regulator willing to maximize welfare - with advertiser that still chose their product prices. If γ is such that firms charge a low price (i.e $\gamma \leq \frac{3-V}{V-1} \equiv \tilde{\gamma}$), then the social welfare is $W(\gamma) = \frac{1+\gamma}{2}v_H + v_L$. If however, $\gamma > \tilde{\gamma}$, firms charge a price equal to v_H and the units valued at v_L by consumers are not sold, so that $W(\gamma) = \frac{1+\gamma}{2}v_H$. Figures 5 and 6 depict the welfare as a function of γ .

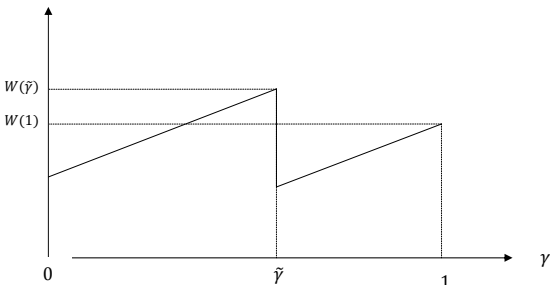


Figure 5: $\operatorname{argmax}_\gamma W(\gamma) = \tilde{\gamma}$

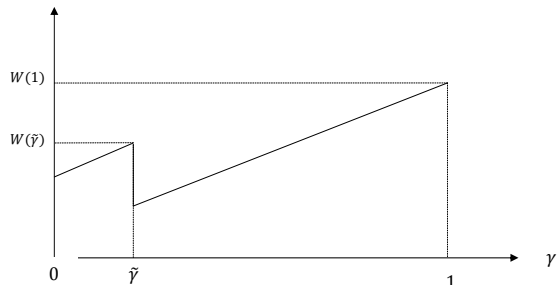


Figure 6: $\operatorname{argmax}_\gamma W(\gamma) = 1$

It is clear that there can be two solutions: $\gamma^* = \tilde{\gamma}$ or $\gamma^* = 1$. Straightforward computations reveal that

$$\gamma^* = \begin{cases} \frac{3-V}{V-1} & \text{if } V \in [2, 3] \\ 1 & \text{otherwise} \end{cases}$$

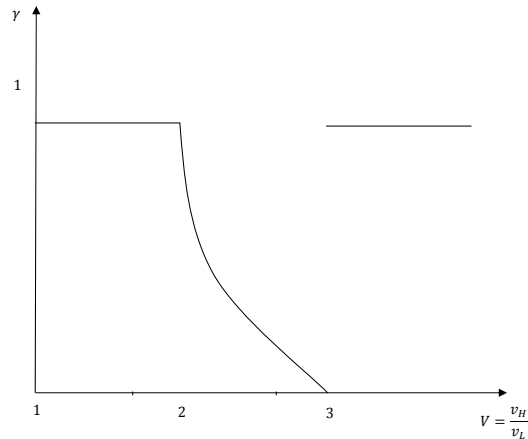


Figure 7: Socially optimal level of information revelation

The bottom line is that partial disclosure can be welfare-maximizing, in which case the platform reveals too much information.

6 Conclusion

In this paper, we study the incentives of an ad-supported platform to disclose information about its users to advertisers prior to an auction. Disclosing the information increases the sum of the platform's and advertisers' profits, but the platform has to leave an informational rent to the winning bidder. As the number of firms grows large, the later effect is dominated so that the platform always prefers disclosure. The disclosure of information by the platform induces a shift in the demand function, which leads to an increase in the equilibrium price of goods. When the shift is uniform along the demand curve, we give sufficient conditions on the elasticity of demand and of its slope for privacy or disclosure to be socially optimal.

Our model ignores several issues, that we discuss below. Although introducing these features would enrich the analysis by making it more realistic, most of the effects that we highlight would not be fundamentally altered.

Competition between platforms One important assumption of the model is that the platform is a monopoly. What would change in a model with several platforms? We think that the main consequence would be that the platforms would tend to internalize to a lesser

extent the impact of their policy on advertisers' pricing strategy, with the implication that they would implement more disclosure. This does not change our analysis of the effects of disclosure on the product market, nor does it affect our normative analysis, since in it we focus on the case in which the platform discloses the information.

Elastic participation Another assumption of the model is that the number of advertisers and the number of consumers is fixed. An interesting avenue for future research would be to study a model in which the platform can influence the number of users. For instance, if consumer participation is determined by the surplus that they expect to obtain through their interactions with advertisers, the platform could be tempted to foster competition by displaying several ads from competing advertisers. On the other hand, in order to attract advertisers the platform would need to give them some market power. These questions are related to the two-sided markets literature (see Armstrong (2006), in particular pp. 686-687, or de Cornière (2011) for an application to the search engine industry). If users exhibit intrinsic privacy concerns, the platform could also use the degree of privacy as an instrument to generate more traffic.

Information acquisition We do not model the way through which the platform obtains the data about consumers. A straightforward way to do this would be to assume that this information can be obtained at a cost, but this would leave most of the analysis unchanged. Another, more interesting way to tackle the problem would be to study a dynamic game in which the platform could learn information over time. We believe that this represents an interesting avenue for future research.

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A Proofs

A.1 Proof of Proposition 4

(i) The proof is based on a comparison of the first order conditions (1) and (2). Let

$$\zeta^1(p) \equiv \int_0^{\bar{\theta}} \frac{\partial \pi(p, \theta_i)}{\partial p_i} f_i(\theta_i) d\theta_i$$

From (1), we have $\zeta^1(p^{\mathcal{P}}) = 0$.

Also, since by Assumption 4 we have $\frac{\partial^2 \pi_i}{\partial p \partial \theta_i} \geq 0$, then, for any increasing function h ,

$$\int_0^{\bar{\theta}} \frac{\partial \pi(p, \theta_i)}{\partial p_i} h(\theta_i) f_i(\theta_i) d\theta_i \geq 0 \quad (8)$$

Now let

$$\zeta^n(p) \equiv \int_{\{\theta_i \in [0, \bar{\theta}]\}} \frac{\partial \pi(p, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f_i(\theta_i) d\theta_i$$

From (2), we have $\zeta^2(p_n^{\mathcal{D}}) = 0$. Using (8) with $h \equiv F^{n-1}$, one gets $\zeta^2(p^{\mathcal{P}}) \geq 0$.

Moreover, ζ^n is non increasing by concavity of the profit function, and so we obtain $p^{\mathcal{P}} \leq p_n^{\mathcal{D}}$.

(ii) The proof of the second point obeys a similar logic. Let

$$\zeta^n(p) \equiv \int_0^{\bar{\theta}} \frac{\partial \pi(p, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f_i(\theta_i) d\theta_i$$

For every n , $p_n^{\mathcal{D}}$ is such that $\zeta^n(p_n^{\mathcal{D}}) = 0$. By choosing $h_n \equiv F^{n-1}/F^{n-2} = F$, which is increasing, we get $p_n^{\mathcal{D}} \geq p_{n-1}^{\mathcal{D}}$. \square

A.2 Proof of Proposition 6

Let $p_n^{\mathcal{D}}$ be the price if the platform chooses to implement a disclosure policy when n firms are on the market.

First, notice that since $(p_n^{\mathcal{D}})_{n \geq 1}$ is non decreasing and bounded (by \bar{p}), it has a limit, that we note $p_{\infty}^{\mathcal{D}}$.

From (2), we have

$$\int_0^{\bar{\theta}} \frac{\partial \pi_i(p_n^D, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i = 0$$

Therefore, for any n and $\epsilon \in (0, \bar{\theta})$,

$$n \left(\int_0^{\bar{\theta}-\epsilon} \frac{\partial \pi_i(p_n^D, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i + \int_{\bar{\theta}-\epsilon}^{\bar{\theta}} \frac{\partial \pi_i(p_n^D, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i \right) = 0$$

Let

$$A_{n,\epsilon} \equiv n \int_0^{\bar{\theta}-\epsilon} \frac{\partial \pi_i(p_n^D, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i$$

and

$$B_{n,\epsilon} \equiv n \int_{\bar{\theta}-\epsilon}^{\bar{\theta}} \frac{\partial \pi_i(p_n^D, \theta_i)}{\partial p_i} F^{n-1}(\theta_i) f(\theta_i) d\theta_i$$

Since $\frac{\partial^2 \pi_i}{\partial p_i \partial \theta_i} \geq 0$, one can write

$$\frac{\partial \pi_i(p_n^D, 0)}{\partial p_i} \int_0^{\bar{\theta}-\epsilon} n F^{n-1}(\theta_i) f(\theta_i) d\theta_i \leq A_{n,\epsilon} \leq \frac{\partial \pi_i(p_n^D, \bar{\theta}-\epsilon)}{\partial p_i} \int_0^{\bar{\theta}-\epsilon} n F^{n-1}(\theta_i) f(\theta_i) d\theta_i$$

The integral on the left side and on the right side is equal to $F^n(\bar{\theta}-\epsilon)$, and goes to zero as n goes to infinity.

Therefore $\lim_{n \rightarrow \infty} A_{n,\epsilon} = 0$

By the same argument, we can provide a lower and an upper bound on $B_{n,\epsilon}$:

$$\frac{\partial \pi_i(p_n^D, \bar{\theta}-\epsilon)}{\partial p_i} [F^n(\bar{\theta}) - F^n(\bar{\theta}-\epsilon)] \leq B_{n,\epsilon} \leq \frac{\partial \pi_i(p_n^D, \bar{\theta})}{\partial p_i} [F^n(\bar{\theta}) - F^n(\bar{\theta}-\epsilon)]$$

Using the fact that $F^n(\bar{\theta}) = 1$, and that $A_{n,\epsilon} + B_{n,\epsilon} = 0$, we obtain

$$A_{n,\epsilon} + \frac{\partial \pi_i(p_n^D, \bar{\theta}-\epsilon)}{\partial p_i} [1 - F^n(\bar{\theta}-\epsilon)] \leq 0 \leq A_{n,\epsilon} + \frac{\partial \pi_i(p_n^D, \bar{\theta})}{\partial p_i} [1 - F^n(\bar{\theta}-\epsilon)]$$

By taking n to infinity, one gets

$$\frac{\partial \pi_i(p_\infty^D, \bar{\theta}-\epsilon)}{\partial p_i} \leq 0 \leq \frac{\partial \pi_i(p_\infty^D, \bar{\theta})}{\partial p_i}$$

If $\epsilon \rightarrow 0$, and by continuity of the derivative of the profit, we finally get

$$\frac{\partial \pi_i(p_\infty^D, \bar{\theta})}{\partial p_i} = 0$$

i. e $p_\infty^D = p^*(\bar{\theta})$.

The platform's profit is

$$E[\Pi_{0,n}^D] = \int_0^{\bar{\theta}} \pi(p_n^D, \theta_i) f_{n-1:n}(\theta_i) d\theta_i = E[\pi(p_n^D, \theta_{n-1:n})]$$

Notice that $\theta_{n-1:n}$ converges almost surely to $\bar{\theta}$. Then, by continuity of π_i , $\pi(p_n^{\mathcal{D}}, \theta_{n-1:n})$ converges to $\pi(p^*(\bar{\theta}), \bar{\theta})$ almost surely.

By the monotone convergence theorem, we can conclude that

$$\lim_{n \rightarrow \infty} E[\Pi_{0,n}^{\mathcal{D}}] = E[\lim_{n \rightarrow \infty} \pi(p_n^{\mathcal{D}}, \theta_{n-1:n})] = \pi(p^*(\bar{\theta}), \bar{\theta})$$

B Finite number of firms

The normative analysis that we carry in the paper focuses on the case in which the number of firms is infinite. For the sake of completeness, we study here a special case of the model that allows us to study how the number of firms affects whether a monopolistic platform reveals too much or too little information.

More specifically, we assume that a consumer of type $\Theta = (\theta_1, \dots, \theta_n)$ has a probability θ_i to have a high willingness to pay for the good i ($v = v_H$), and a probability $1 - \theta_i$ to have a low willingness to pay for the same good ($v = v_L$). We assume that the θ 's are drawn independently from the c.d.f $F(\theta) = \theta^\gamma$ with $\theta \in [0, 1]$ and $\gamma > 0$. A higher value of γ means that the frequency of high values of θ increases. We also normalize v_H to 1 and denote $v_L = v \in [0, 1]$. Let $m \equiv \frac{\gamma}{1+\gamma}$ be the expected value of θ_i .

B.1 Equilibrium under privacy

If the platform chooses not to disclose information to the firms, the equilibrium of the subgame is as follows:

Proposition 8 *If $mv_H \leq v_L$, the equilibrium price is $p^{\mathcal{P}} = v_L$. The platform's revenue is $R^{\mathcal{P}} = v_L$. Consumers' surplus is $V^{\mathcal{P}} = m(v_H - v_L)$, and social welfare is $W^{\mathcal{P}} = (1 - m)v_L + mv_H$.*

If $mv_H > v_L$, the equilibrium price is $p^{\mathcal{P}} = v_H$. The platform's revenue is $R^{\mathcal{P}} = mv_H$. Consumers' surplus is $S^{\mathcal{P}} = 0$, and social welfare is $W^{\mathcal{P}} = mv_H$.

Proof: Under privacy, a firm expects to be matched with a consumer who has a probability m of having a high willingness to pay. If $mv_H \leq v_L$, firms prefer to serve every one rather than charge a high price and serve only the v_H consumers. If $mv_H > v_L$, the opposite is true. The expressions of the platform's profit, consumers' surplus and social welfare are straightforward. \square

B.2 Symmetric equilibrium under disclosure

When the publisher chooses a disclosure policy, firms learn, for each consumer, the probability that he has a high valuation for its product. Firms are thus able to condition their bid on this information. The following lemma allows us to reduce the number of strategy profiles that can constitute an equilibrium.

Lemma 2 *When the publisher discloses the information about consumers to firms, there is no equilibrium in which more than one firm set a price equal to v .*

Proof: Suppose by contradiction that at least two firms (firms 1 and 2) set a price $p = v$. Then they both bid v for every consumer, and thus they never make a positive profit. Now, if instead one of them (say, firm 1) deviates and sets a price 1 and bids θ_i , for every consumer there is a positive probability that it wins the auction and makes a profit equal to $\theta_1 - \max_{i \neq 1} b_i^{\mathcal{D}}(\theta_i, p_i) > 0$. This is therefore a profitable deviation. \square

Lemma 2 tells us that when the publisher discloses the information about consumers, there can exist only two kinds of equilibrium¹⁰: a symmetric one in which all firms choose a high price 1, and an asymmetric one in which one firm chooses the low price equal to v while $n - 1$ firms choose the high price 1. To keep the analysis simple we focus on the symmetric equilibrium. The qualitative results derived when the equilibrium is asymmetric are similar.

Firms

Suppose that all firms choose the high price equal to 1. Their bidding function is therefore $b_i^{\mathcal{D}}(\theta_i, 1) = \theta_i$. Indeed, upon learning that a consumer is a good match with probability θ_i , firm i 's expected profit conditional on being matched with the consumer is θ_i . The winner of the auction is thus the firm j such that $\theta_j = \max_l \theta_l$. The expected profit of firm i is therefore

$$\pi^{\mathcal{D}}(1) = \int_0^1 \int_0^{\theta_i} (\theta_i - \hat{\theta}_{-i}) \gamma (n-1) \hat{\theta}_{-i}^{\gamma(n-1)-1} \gamma \theta_i^{\gamma-1} d\hat{\theta}_{-i} d\theta_i = \frac{\gamma}{(1 + \gamma(n-1))(1 + \gamma n)} \quad (9)$$

Indeed, firm i only wins the auction when θ_i is larger than all the θ_j s, and firm i then pays the second highest bid. The term $\gamma(n-1)\hat{\theta}_{-i}^{\gamma(n-1)-1}$ is the pdf of the random variable equal to the highest realization of θ_j among $n - 1$, while $\gamma\theta_i^{\gamma-1}$ is the pdf for θ_i .

It remains to check that there is no profitable deviation. The only possible deviation is to charge a low price v , which leads to a revenue of v per consumer who sees the ad. Firm i wins the auction as long as $v > \theta_j$ for all $j \neq i$. Its net profit is then:

$$\pi^{\mathcal{D}}(v) = \int_0^v (v - \hat{\theta}_{-i}) \gamma (n-1) \hat{\theta}_{-i}^{\gamma(n-1)-1} d\hat{\theta}_{-i} = \frac{v^{\gamma(n-1)+1}}{1 + \gamma(n-1)} \quad (10)$$

The deviation is not profitable if and only if

$$v < \left(\frac{\gamma}{1 + \gamma n}\right)^{1/(1+\gamma(n-1))} \quad (11)$$

Below we provide the computation of welfare and platform's revenue when (11) holds. When it is not the case, firms play an asymmetric equilibrium.¹¹

¹⁰This result depends on our normalization of $\bar{\theta}$ and of v_H to 1. If we had $v_L > \bar{\theta}v_H$, the only equilibrium would be for all the firms to charge v_L .

¹¹See appendix B.4 for the computations.

Publisher's revenue

The publisher's revenue equals to the second highest bid:

$$R^D = E[\theta_{n-1:n}] = \int_0^1 \theta dF_{n-1:n}(\theta) = \frac{n(n-1)\gamma^2}{(1+\gamma n)(1+\gamma(n-1))} \quad (12)$$

where $F_{n-1:n}(\theta) = \theta^{\gamma n} + n(1-\theta^\gamma)\theta^{\gamma(n-1)}$ is the cdf of the second highest realization among all the θ_i 's.

Consumers surplus and welfare

The expected surplus of consumers is zero: $V^D = 0$. Indeed, the price is set so as to extract the whole surplus of buyers with a high willingness to pay, while other consumers do not buy.

The expected total welfare is the sum of the publisher and firms' profits. It is equal to the expectation of the highest realization of θ_i :

$$W^D = \int_0^1 \theta \gamma(n-1) \theta^{\gamma(n-1)-1} d\theta = \frac{\gamma(n-1)}{\gamma(n-1)+1} \quad (13)$$

The welfare is an increasing function of the number of firms, as a larger number of firms increases the probability for a consumer to be well matched. It converges to one when the number of firms tends to infinity.

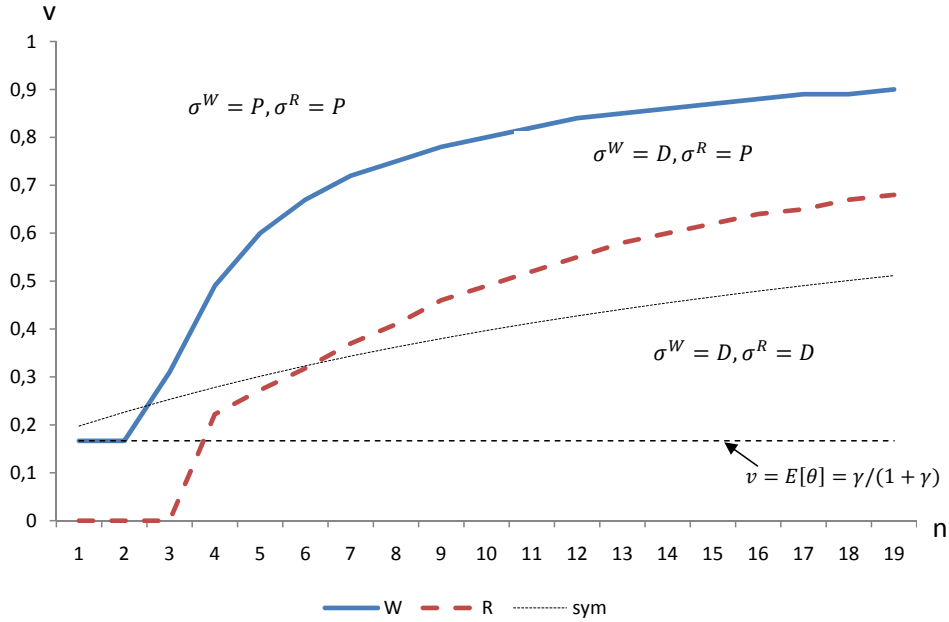


Figure 8: Socially vs privately optimal policy ($\gamma = 0.2$).

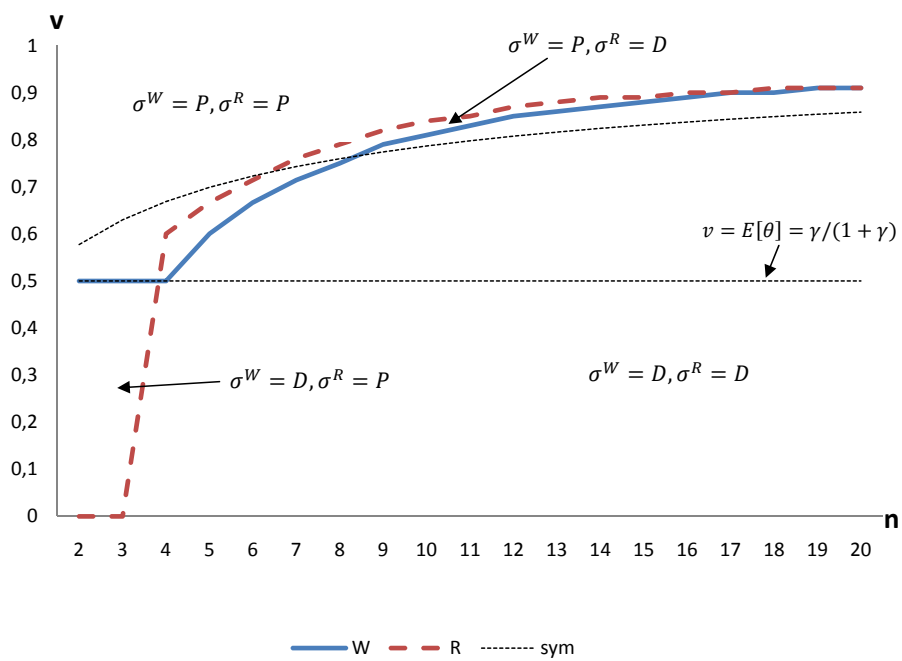


Figure 9: Socially vs privately optimal policy ($\gamma = 1$).

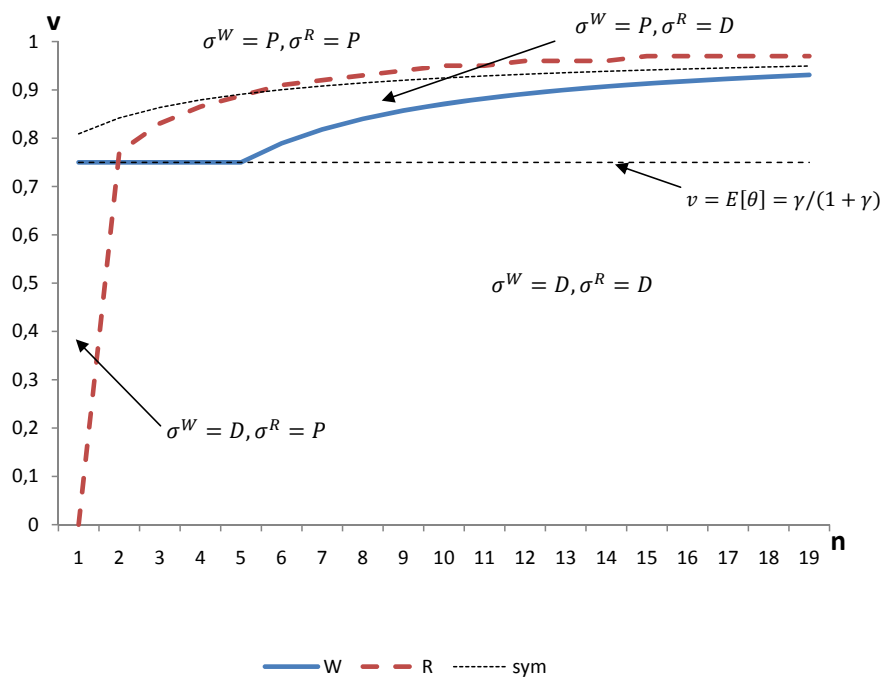


Figure 10: Socially vs privately optimal policy ($\gamma = 3$).

Figures 8, 9 and 10 illustrate how the optimal policies vary with v , n and γ . The socially optimal policy σ^W and the platform's optimal policy σ^R can be either Privacy (\mathcal{P}) or Disclosure (\mathcal{D}).

Above the thick dashed line, it is socially optimal to respect privacy ($\sigma^W = \mathcal{P}$) whereas under that line it is better to disclose the information ($\sigma^W = \mathcal{D}$). The platform's decision depends on the position of (n, v) with respect to the thick continuous line. Above it, the platform conceals the information ($\sigma^R = \mathcal{P}$), and it prefers disclosure under the line ($\sigma^R = \mathcal{D}$). The increasing thin dashed line represents the frontier between the symmetric and the asymmetric equilibrium (the symmetric equilibrium is played below the frontier).

The comparative statics with respect to n and v is the same whether we consider social welfare or the platform's revenue. As the number of bidders increases, the improvement in the matching quality under disclosure gets bigger, while the platform leaves a smaller informational rent to the winning bidder. Thus a higher n makes disclosure more desirable.

On the other hand, an increase in v leads to more privacy: as the population of consumers becomes more homogenous, excluding the low valuation consumers by disclosing the information (through a rise in the equilibrium price) is more costly, both socially and for the platform.

Notice that since the lines are not identical, there exists a set of parameters (n, v) such that the platform does not implement the socially optimal policy. It can disclose too much information ($\sigma^W = \mathcal{P}$, $\sigma^R = \mathcal{D}$) when v and n take intermediate values, and γ is not too high ($\gamma \in \{0.2; 1\}$ in the example). The intuition is that the platform only takes v into account insofar as it determines its revenue under privacy. In particular, the platform does not fully internalize the social loss due to the exclusion of low valuation buyers. The number of firms must be large enough to reduce the informational rent, but low enough not to offset the socially negative effect of low valuation consumers' exclusion.

The platform may also disclose too little information ($\sigma^W = \mathcal{D}$, $\sigma^R = \mathcal{P}$), for instance when both n and v are small. Indeed, if v is too small, firms always charge a high price, even under privacy (since $E[\theta] > v$), so that low valuation consumers are always excluded. It is thus clear that disclosure is socially optimal. However, if the number of bidders is small, leaving an informational rent is too costly for the platform, who therefore prefers privacy.

The "under provision" of information by the platform may also happen with larger values of v and n , provided γ is low enough (in our case, $\gamma = 0.2$). In such a situation, the ex ante probability that a match is good is low (0.16 for $\gamma = 0.2$). Switching to disclosure greatly enhances the efficiency of the matching: the probability of a good match would jump to 0.75 with 16 bidders. However, the informational rent left to the winning bidder is large for low values of γ (0.17 with $\gamma = 0.2$ and $n = 16$, i.e 45% of the value created by disclosure). Figure B.2 represents the informational rent as a function of γ , when $n = 16$. It is clear that if γ is small and v is not too small (so that it is not too tempting for the platform to give up a certain revenue of v under privacy), the platform prefers to conceal the information.

The first order effect of an increase in γ is to make disclosure more desirable, both socially and for the platform. However, the private incentives to disclose the information grow more quickly than the public incentives, so

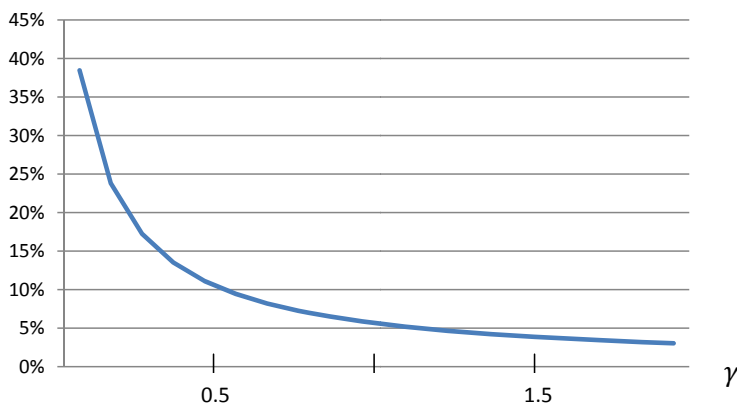


Figure 11: Relative information rent : $E[\frac{\theta_{n:n}-\theta_{n-1:n}}{\theta_{n:n}}]$, with $n = 16$.

that when γ is small there is a bias towards too much privacy provided by the platform, whereas for high values of γ the bias is towards too much disclosure. In the latter case, the improvement in the matching quality following disclosure is relatively modest, since there is a high probability that a consumer is of type v_H absent any information. However,

B.3 The case of disclosure with customized prices

In the basic model we assumed that firms choose their prices before submitting their bids thus preventing any form of price customization based on the specific consumer's type they gain access to. While this assumption is reasonable in many settings, in view of the relative technological ease with which firms that compete online could price discriminate, an important issue is what is the optimal policy disclosure when firms customize their price for each consumer they reach. This issue is not purely academic, as is evidenced by the controversy surrounding Amazon.com's attempts to price discriminate and by public authorities' worries about the possibility of targeted pricing (see Office Of Fair Trading (2010)).

Suppose now that firms are able to condition both their bid and their price on the consumer's type θ , whenever the publisher chooses to reveal it. This means that the second and the third stage of the game in the basic model are inverted. To make the analysis simpler and to focus on an interesting case, we further assume that $\gamma = 1$ and that $v > 1/2$.

Under privacy, the situation is the same than without customized pricing: each firm bids v . Under disclosure, a firm i who learns that a consumer has a probability θ_i to be a good match will bid $b(\theta_i) = \max(\theta_i, v)$. This bid is higher than the bid under privacy, and the bid under disclosure without customized pricing. As a result the publisher always opts for a disclosure policy when firms can customize their prices. In addition, its revenue is always higher than without price discrimination.

As for firms, when customized pricing induces a shift for privacy to disclosure, they become better off. Indeed the expected profit a firm under disclosure and customized pricing is strictly positive. It writes as follows:

$$\pi^{pd} = \int_v^1 \left(\int_0^v (\theta_i - v) \gamma (n-1) \hat{\theta}_{-i}^{\gamma(n-1)-1} d\hat{\theta}_{-i} + \int_v^{\theta_i} (\theta_i - \hat{\theta}_{-i}) (n-1) \hat{\theta}_{-i}^{\gamma(n-2)} d\hat{\theta}_{-i} \right) d\theta_i \quad (14)$$

Firm i only wins the auction when θ_i is larger than all the θ_j s, and firm i then pays the second highest bid. This second bid is equal to v when $\theta_{-i} \leq v$ and θ_{-i} otherwise. We can also deduce from this result that conditioning on being in the disclosure regime, firms are worse off when they engage in customized pricing. This is because the gap between their private valuation for the slot and the expected second highest bid shrinks when customized pricing is allowed. This result therefore delineate a new rationale for the non profitability of price discrimination in competitive environments (See Thisse and Vives (1988), Chen and Zhang (2001), and Chen and Iyer (2002) not to be exhaustive). In our context, firms engaging in price customization are instrumentalized by the platform who further exploit them to extract consumer surplus.

B.4 Asymmetric equilibrium in the binary model with $n < \infty$

If (11) does not hold, the only equilibrium is an asymmetric one, in which one firm charges a low price v while the other firms charge the high price 1. Let us reorder the firms so as to have firm 1 charging the low price and $\theta_i < \theta_j$ for $j > i > 1$. The platform's revenue is the sum of three terms:

$$R_{asym}^{\mathcal{D}} = \int_0^v \theta dF_{n-1:n-1}(\theta) + \int_v^1 \int_0^v v dF_{n-2:n-1}(\tau)(\theta) dF_{n-1:n-1}(\theta) + \int_v^1 \int_v^1 \tau dF_{n-2:n-1}(\tau)(\theta) dF_{n-1:n-1}(\theta)$$

The first term corresponds to situations in which $\theta_{n-1} < \theta_n < v$. The firm who wins in this case is firm 1, and it pays the highest bid among the other firms (here, θ_n). If $\theta_{n-1} < v < \theta_n$, firm n wins the auction and the revenue equals v . Finally, if $v < \theta_{n-1} < \theta_n$, firm n wins and pays θ_{n-1} .

Using the appropriate expressions for $F_{n-1:n-1}$ and $F_{n-2:n-1}$, one gets

$$\begin{aligned} R_{asym}^{\mathcal{D}} &= \frac{\gamma(-2+n)v(1+\gamma(-2+n))}{(1+\gamma(-2+n))} \\ &\quad + v(1+\gamma(-2+n))(1-n+(-2+n)v^\gamma)(-1+v^{\gamma(-2+n)}) \\ &\quad - \frac{(\gamma(-2+n)(-1+n)v^{-2\gamma}(-1+v^{\gamma(-2+n)})(\gamma v^{2\gamma} + v^{1+\gamma n}(-1+\gamma-\gamma n + (1+\gamma(-2+n))v^\gamma))}{((1+\gamma(-2+n))(1+\gamma(-1+n)))} \end{aligned}$$

The social welfare equals

$$W_{asym}^{\mathcal{D}} = \int_0^v v dF_{n-1:n-1}(\theta) + \int_v^1 \theta dF_{n-1:n-1}(\theta)$$

i.e

$$W_{asym}^{\mathcal{D}} = \frac{\gamma(n-2) + v^{1+\gamma(n-2)}}{1+\gamma(n-2)}$$

