Vertical Integration and Foreclosure in Multilateral Relations

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15 September 2012

VERY PRELIMINARY AND INCOMPLETE

Abstract

We develop a model of multilateral relations between upstream manufacturers that produce differentiated goods and downstream retailers that sell these goods on to consumers. Contract offers and acceptance decisions are private information to the contracting parties. We show that vertical integration between a manufacturer and a retailer leads to the foreclosure of rival manufacturers from access to the integrated retailer, at the detriment of consumers. Moreover, we show that firms have an incentive to integrate vertically.

JEL Classification: L13, L42, D43.

Keywords: vertical relations, vertical merger, foreclosure, bilateral contracting.

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1 Introduction

While the potential anticompetitive effects of vertical integration and vertical restraints play an important role in antitrust, existing models of vertically related industries typically involve a rather stylized market structure: Many (if not most) models of vertically related industries consider the case of an upstream (or downstream) monopoly. Among the models with (imperfect) competition both upstream and downstream, many focus on "competing vertical structures," where each upstream firm deals with a distinct set of downstream firms (e.g., franchise networks).[1] Yet many industries are characterized by multiple "interlocking" bilateral relations, in which the same competing upstream firms deal with the same set of competing downstream firms. The few papers that allow for such interlocking vertical relationships have two types of limitations: they often restrict attention to particular types of (public) contracts such as linear tariffs (e.g., Dobson and Waterson, 2007) or two-part tariffs (e.g., Rey and Vergé, 2010), or they assume that the upstream firms produce a homogeneous good (e.g., Hart and Tirole, 1990; de Fontenay and Gans, 2005; Nocke and White, 2007 and 2010).

The standard assumption of publicly observable contracts between upstream and downstream firms is, arguably, not very appealing either. The seminal contribution of Hart and Tirole (1990) – hereafter, HT in short – shows that a Coasian commitment problem arises when contract offers are private information to the contracting parties; see also O'Brien and Shaffer (1992), McAfee and Schwartz (1994).[2] However, HT has not been very influential in actual policy decisions as the economic environment is considered to be too stylized. Specifically, the upstream firms produce the same homogeneous good, and one upstream firm is assumed to be more efficient than its rival(s). Thus, in equilibrium, the efficient upstream firm is the only active producer, independently of whether there is any vertical integration. As a result, the analysis is perceived as applying only to upstream monopolists or quasi-monopolists. The literature is therefore still in the want of an appropriate framework for studying industries with market power both upstream and downstream and interlocking relationships.

In this paper, we propose such a framework of a vertically related industry with interlocking bilateral relations between upstream manufacturers and downstream retailers. Building on HT, we assume that contract offers are private information to the contracting parties, but depart

[1] Papers featuring competing vertical structures include Bonanno and Vickers (1988), Rey and Stiglitz (1988, 1995), Gal-Or (1991), Jullien and Rey (2007), and Piccolo and Miklos-Thal (forthcoming).

[2] See Rey and Tirole (2007) for a survey.

from HT in assuming that upstream firms produce differentiated goods. As a result, and by contrast to HT, all firms are active in equilibrium and each vertical merger has an effect on the market outcome, independently of the initial market structure.

In our baseline model, there are two manufacturers, each of whom produces a unique differentiated good, and two retailers who sell the goods on to consumers and compete in quantities. While there is no intrinsic differentiation between retailers, each retailer may stock both goods, only one of them, or none. Each manufacturer makes secret contract offers to each retailer.[3] As regards the terms of the contracts, we allow for general nonlinear tariffs. We show that, in the absence of vertical integration, the equilibrium contract between each manufacturer and each retailer induces the bilaterally efficient quantity, given the set of other contracts. That is, the quantities sourced are as if manucturers sold the goods at marginal cost. This no longer holds if there is some vertical integration. In the case of pairwise vertical integration, each integrated firm does not deal with the other firm in equilibrium. That is, pairwise vertical integration leads to complete foreclosure at both the upstream and downstream levels.

2 The Model

We consider a vertically related industry with two symmetically differentiated manufacturers, M_A and M_B. Manufacturer M_i, $i = A, B$, produces good i at constant unit cost $c > 0$. The manufacturers distribute their goods through two perfectly substitutable retailers, R_1 and R_2, each of whom faces the same constant unit cost, which for the sake of notational simplicity we set equal to zero. Consumers' inverse demand for good $i = A, B$ is of the form $P(q_{i1} + q_{i2}, q_{j1} + q_{j2})$, $j \neq i \in \{A, B\}$, where q_{ik} is the quantity of good $i \in \{A, B\}$ sold by retailer R_k, $k \in \{1, 2\}$.

Throughout the paper, we impose the following conditions on demand:

(A.0) $P(0, 0) > c$, and $P(Q, 0) < c$ for Q sufficiently large.

(A.1) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$,

$$\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_i, Q_j) < 0.$$

[3] As is standard in the literature following HT, out-of-equilibrium beliefs are assumed to be "passive:" if a retailer receives a deviant contract offer from a manufacturer, the retailer continues to believe that the other retailer received the equilibrium contract offer. This refinement is well justified in our model: the assumption of downstream quantity competition implies that a manufacturer's incentive to deviate vis-à-vis one retailer is independent of the contract in place with the other retailer.

Condition (A.0) is essentially a viability assumption, whereas condition (A.1) simply asserts that goods A and B are (imperfect) substitutes.

A contract between M_i and R_k is a (nonlinear) tariff of the form $\tau_{ik}(\cdot)$, where $\tau_{ik}(q)$ is the payment from R_k to M_i in return for the delivery of q units of good i. We do not impose any further restriction on contracts. Special cases of interest are:

- *Two-part tariff:* $\tau_{ik}(q) = F + wq$, where F is the fixed (or "franchise") fee, and $w \geq 0$ the marginal wholesale price. We will denote such a two-part tariff as (F, w).

- *Forcing contract:*
$$\tau_{ik}(q) = \begin{cases} \hat{T} & \text{if } q = \hat{q}, \\ \infty & \text{otherwise,} \end{cases}$$
where $\hat{q}$ is the "forced" quantity. We will denote such a forcing contract as $(\hat{T}, \hat{q})$.

We allow manufacturers to offer menus of such contracts. The contracting terms between M_i and R_k are private information to the two parties (as is R_k's acceptance decision). If M_i and R_k are vertically integrated, they maximize their joint profits, independently of any internal transfer prices. Thus, when vertically integrated, M_i and R_k behave "as if" they relied on $\tau_{ik}(q) = cq$, and each affiliate's decisions take into consideration the impact on the other affiliate's profit. Moreover, there is "information sharing" between the affiliates of a vertically integrated firm. In particular, when making its acceptance and output decisions, the integrated retailer R_k is informed about the offer that its upstream affiliate M_i has previously made to the rival retailer R_l, $l \neq k$.

The timing is as follows:

Stage 1 Manufacturers simultaneously offer (secret) contracts to retailers.

Stage 2 Retailers simultaneously (and secretly) accept or reject the offers.

Stage 3 Retailers who have accepted (some) contract(s) choose how much to put on the final market; the resulting prices are such that markets clear.

(We will also analyze the alternative timing where stages 2 and 3 occur at the same time.) We will look for a Perfect Bayesian Equilibrium with passive beliefs, in which retailers do not revise their beliefs about the offer made to the other retailer when receiving an out-of-equilibrium offer. As retailers compete downstream in quantities, these passive beliefs also coincide with the "wary beliefs" introduced by McAfee and Schwartz (1994), as the contract signed with a retailer has no impact on a manufacturer's gains from trade with the other retailer.

3 Vertical Separation

In this section, we characterize the equilibrium outcome when no firm is vertically integrated. (Equilibrium variables under vertical separation are indexed by the superscript "$\circ$.") We proceed as follows. First, we define the notion of a "cost-based contract," in which the marginal input price coincides with the marginal cost of production. Second, we show that any unintegrated manufacturer offers cost-based contracts to every retailer and that these contracts are accepted in equilibrium. Third, we state the main characterization result for the case of vertical separation.

Let $(q_{A1}^{\circ}, q_{A2}^{\circ}, q_{B1}^{\circ}, q_{B2}^{\circ})$ denote a vector of equilibrium quantities (assuming an equilibrium exists), and δ_{ik}° the equilibrium acceptance decision of R_k vis-a-vis M_i's contract offer, with the convention that $\delta_{ik}^{\circ} = 1$ if M_i and R_k are vertically integrated and, when they are independent, $\delta_{ik}^{\circ} = 1$ if the offer is accepted and $\delta_{ik}^{\circ} = 0$ if it is not (in which case $q_{ik}^{\circ} = 0$).

Definition 1 *In equilibrium, the contract $\tau_{ik}^{\circ}(\cdot)$ between M_i and R_k is said to be* cost-based *if, when accepted (i.e., $\delta_{ik}^{\circ} = 1$), it induces a quantity*

$$q_{ik}^{\circ} \in R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; \delta_{jk}^{\circ}\right) \equiv \arg\max_{q_{ik}} \left[P\left(q_{ik} + q_{il}^{\circ}, q_{jk}^{\circ} + q_{jl}^{\circ}\right) - c\right] q_{ik} + \delta_{jk}^{\circ} \left[P\left(q_{jk}^{\circ} + q_{jl}^{\circ}, q_{ik} + q_{il}^{\circ}\right) - c\right] q_{jk}^{\circ}.$$

The following lemma partially characterizes the equilibrium outcomes when at least one upstream firm is not vertically integrated:

Lemma 1 *Suppose manufacturer M_i is not vertically integrated (whereas manufacturer M_j, $j \neq i$, may or may not be vertically integrated). Then, the equilibrium outcome is the same as if M_i signed a cost-based contract with each retailer R_k. That is, $q_{ik}^{\circ} \in R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; \delta_{jk}^{\circ}\right)$ for $k = 1, 2$.*

Proof. See Appendix A.1. ■

The intuition is simple: under passive beliefs, a retailer R_k expects its rival R_l to stick to the equilibrium quantities even when receiving a deviant offer from an independent manufacturer M_i. Moreover, such a deviant offer does not affect the profit that M_i makes on its contract with R_l. In equilibrium, the contract between M_i and R_k must therefore maximize the joint bilateral profit of the contracting parties, assuming that R_l sticks to its equilibrium quantities, which is achieved by agreeing to a cost-based contract.

It follows from this lemma that, in equilibrium, M_i offers each retailer R_k a cost-based contract, and this contract gets accepted (i.e., $\delta_{ik}^{\circ} = 1$), whenever it is efficient to trade (that is, whenever $0 < q_{ik}^{\circ} \in R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}\right)$); when trade is inefficient (i.e., when $R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}\right) = \{0\}$), either R_k rejects M_i's offer, or M_i offers a "null" contract leading to $q_{ik}^{\circ} = 0$. We are now in the position to state the main characterization result under vertical separation:

Proposition 1 *Suppose no firm is vertically integrated. Then, the equilibrium outcome is the same as if each manufacturer signed a cost-based contract with each retailer. That is, for all* $i, j \in \{A, B\}$, $i \neq j$, *and* $k, l \in \{1, 2\}$, $k \neq l$:

$$q_{ik}^{\circ} \in R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; 1\right) = \arg\max_{q_{ik}} \left[P\left(q_{ik} + q_{il}^{\circ}, q_{jk}^{\circ} + q_{jl}^{\circ}\right) - c\right] q_{ik} + \left[P\left(q_{jk}^{\circ} + q_{jl}^{\circ}, q_{ik} + q_{il}^{\circ}\right) - c\right] q_{jk}^{\circ}.$$

Proof. This follows directly from Lemma 1 and the definition of cost-based contracts:

- For all $i, j \in \{A, B\}$, $i \neq j$, and $k, l \in \{1, 2\}$, $k \neq l$, we have $q_{ik}^{\circ} \in R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; \delta_{jk}^{\circ}\right)$.
- This moreover implies $\delta_{jk}^{\circ} = 1$ whenever $q_{jk}^{\circ} > 0$; and if $q_{jk}^{\circ} = 0$, then by construction $R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; \delta_{jk}^{\circ}\right) = R\left(q_{il}^{\circ}, q_{jk}^{\circ}, q_{jl}^{\circ}; 1\right)$.

■

This proposition characterizes candidate equilibrium outcomes for retail prices and quantities. We next show that the outcome is, moreover, unique under the following mild regularity conditions on inverse demand:

(A.2) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, we have

$$2\partial_1 P(Q_i, Q_j) + \partial_{11}^2 P(Q_i, Q_j) Q_i < \partial_2 P(Q_i, Q_j) + \partial_{12}^2 P(Q_i, Q_j) Q_i < 0.$$

(A.3) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, and for any $q_i \in [0, Q_i]$ and any $q_j \in [0, Q_j]$, we have

$$\begin{aligned} & 3\partial_1 P\left(Q_i, Q_j\right) + \partial_{11}^2 P\left(Q_i, Q_j\right) q_i + \partial_{22}^2 P\left(Q_j, Q_i\right) q_j \\ < \; & 2\partial_2 P\left(Q_i, Q_j\right) + \partial_2 P\left(Q_j, Q_i\right) + \partial_{12}^2 P\left(Q_i, Q_j\right) q_i + \partial_{12}^2 P\left(Q_j, Q_i\right) q_j \\ < \; & 0. \end{aligned}$$

Condition (A.2) is a standard condition ensuring that a simple Cournot duopoly, in which one firm sells A and the other B, would have a unique, stable equilibrium. Condition (A.3) further ensures that profits are concave in a modified version of this Cournot duopoly, in which both firms can sell A and B. In the case of linear demand, (A.2) and (A.3) simplify to $2\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_i, Q_j) < 0$ and $3\partial_1 P\left(Q_i, Q_j\right) < 2\partial_2 P\left(Q_i, Q_j\right) < 0$, respectively, and thus hold by (A.1).

We have:

Lemma 2 *Under Assumptions (A.2)-(A.3), the characterization provided by Proposition 1 determines a unique vector of quantities, which is moreover symmetric:* $q_{ik}^{\circ} = q^{\circ}$ *for* $i = A, B$ *and* $k = 1, 2$*, where* q° *is characterized by:*

$$P(2q^{\circ}, 2q^{\circ}) - c + [\partial_1 P(2q^{\circ}, 2q^{\circ}) + \partial_2 P(2q^{\circ}, 2q^{\circ})] q^{\circ} = 0.$$

Proof. See Appendix A.2. ■

Finally, we establish the existence of equilibrium. We first show that there exists an equilibrium in which manufacturers offer a two-part tariff:

Proposition 2 *Suppose no firm is vertically integrated. Then, under Assumptions (A.2)-(A.3), there exists an equilibrium in which each manufacturer signs the cost-based two-part tariff* (F°, c) *with each retailer, where the fixed fee* F° *is:*

$$F^{\circ} = [P(2q^{\circ}, 2q^{\circ}) - c] 2q^{\circ} - \max_{q} [P(q^{\circ} + q, q^{\circ}) - c] q.$$

Proof. See Appendix A.3. ■

There exist other equilibria; however, although all these equilibria must rely on cost-based contracts, they can involve different divisions of profits.[4] The following proposition characterizes a range of symmetric equilibria that rely on menus of forcing contracts:

Proposition 3 *Suppose no firm is vertically integrated. Then, under Assumptions (A.2)-(A.3), there exists a range of symmetric equilibria in which each contract consists of a pair of forcing contracts* $\left\{(T^{\circ}, q^{\circ}), \left(\hat{T}, \hat{q}\right)\right\}$*, where the quantity* $\hat{q}$ *is such that*

$$\hat{q} \equiv \arg\max_{q} \{[P(q + q^{\circ}, q^{\circ}) - c] q\},$$

and the payments T° *and* $\hat{T}$ *are:*

$$T^{\circ} = P(2q^{\circ}, 2q^{\circ}) 2q^{\circ} - P(\hat{q} + q^{\circ}, q^{\circ}) \hat{q} + \Delta, \hat{T} = T^{\circ} + \Delta,$$

where $\Delta \in \left[\hat{\Delta}, c(\hat{q} - q^{\circ})\right]$*, for some* $\hat{\Delta} < c(\hat{q} - q^{\circ})$*.*

Proof. See Appendix A.4. ■

Each menu considered in Proposition 3 consists of two forcing contracts: the cost-based contract (T°, q°), which each retailer accepts along the equilibrium path, and the contract $\left(\hat{T}, \hat{q}\right)$,

[4] Interestingly, there is no equilibrium in which each manufacturer offers a single forcing contract to each retailer.

which is "designed" for exclusivity. In equilibrium, each retailer is indifferent between picking (T°, q°) from both manufacturers' offers and picking $\left(\hat{T}, \hat{q}\right)$ from only one of them. While the quantities involved, q° and $\hat{q}$, are uniquely determined, Proposition 3 shows that the payments, T° and $\hat{T}$, are not. In equilibrium, each manufacturer $i \in \{A, B\}$ makes a profit of $\pi_{ik} = T^\circ - cq^\circ$ on the contract with each retailer $k \in \{1, 2\}$. So, in the class of contracts considered in Proposition 3, the one that leaves most of the rents to the manufacturers has $\pi_{ik} = [P(2q^\circ, 2q^\circ) - c]\, 2q^\circ - [P(\hat{q} + q^\circ, q^\circ) - c]\, \hat{q}$ and is outcome-equivalent to the equilibrium in two-part tariffs; all of the other equilibria are worse for the manufacturers (and better for the retailers).

4 Pairwise Vertical Integration

We now turn to the case where there are two vertically integrated firms, $M_A - R_1$ and $M_B - R_2$. (Equilibrium variables under pairwise vertical integrated are indexed by superscript "**.") We proceed as follows. We first consider an associated duopoly game and introduce some regularity conditions which we derive in the Appendix from conditions on demand. We then show that, under these regularity conditions, there exists a unique equilibrium in which there is no cross-selling: each integrated retailer's access to the rival manufacturer's good is foreclosed (and, equivalently, each integrated manufacturer's access to the rival retail outlet is forclosed).

4.1 The Associated Duopoly Game Γ

Consider the following hypothetical duopoly game, denoted Γ. There are two players, firms 1 and 2, and two goods, A and B. Firm 1's strategy consists in selling quantity $q_{A1} \in [0, \infty)$ of good A at the same time as firm 2 sells quantity $q_{B2} \in [0, \infty)$ of good B. In addition, firm 1 also sells an exogenous quantity $\hat{q}_{B1}$ of good B and firm 2 an exogenous quantity $\hat{q}_{A2}$ of good A, so that the profit functions of firms 1 and 2 are given by

$$\hat{\Pi}_1(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}) \equiv [P(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}) - c]\, q_{A1} + [P(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}) - c]\, \hat{q}_{B1}$$

and

$$\hat{\Pi}_2(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}) \equiv [P(\hat{q}_{B1} + q_{B2}, q_{A1} + \hat{q}_{A2}) - c]\, q_{B2} + [P(q_{A1} + \hat{q}_{A2}, \hat{q}_{B1} + q_{B2}) - c]\, \hat{q}_{A2},$$

respectively. In the special case where $\hat{q}_{A2} = \hat{q}_{B1} = 0$, this game simplifies to a standard differentiated goods Cournot duopoly, where each of the two goods is sold by only one firm.

For the main results in this section, we assume that the equilibrium of game Γ has the following properties:

(P.1) Game Γ has a unique Nash equilibrium $(\widetilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}), \widetilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}))$. In addition, equilibrium aggregate profit is maximized for $\hat{q}_{B1} = \hat{q}_{A2} = 0$; that is,

$$\begin{aligned} & \hat{\Pi}_1(\widetilde{q}_{A1}(0,0), \widetilde{q}_{B2}(0,0); 0,0) + \hat{\Pi}_2(\widetilde{q}_{A1}(0,0), \widetilde{q}_{B2}(0,0); 0,0) \\ > \; & \hat{\Pi}_1(\widetilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}), \widetilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}); \hat{q}_{B1}, \hat{q}_{A2}) + \hat{\Pi}_2(\widetilde{q}_{A1}(\hat{q}_{B1}, \hat{q}_{A2}), \widetilde{q}_{B2}(\hat{q}_{B1}, \hat{q}_{A2}); \hat{q}_{B1}, \hat{q}_{A2}) \end{aligned}$$

for $\hat{q}_{B1} + \hat{q}_{A2} > 0$.

(P.2) Assume that (P.1) holds, and that $\hat{q}_{ik} = 0$ for some $ik \in \{B1, A2\}$. Then, $\widetilde{q}_{jk}$, $j \neq i \in \{A, B\}$ is decreasing in $\hat{q}_{jl}$, $k \neq j \in \{1, 2\}$.

Both properties are satisfied in the case of linear demand. In the Appendix, we provide more general sufficient conditions on demand that ensure that (P.1) and (P.2) do indeed hold.

4.2 The Foreclosure Effects of Vertical Integration

We can now characterize the equilibrium outcome under pairwise vertical integration.

Proposition 4 *Suppose $M_A - R_1$ and $M_B - R_2$ are vertically integrated. Assume (P.1).*

(i) *There exists an equilibrium in which there is no cross-selling, i.e., $(q^{**}_{A1}, q^{**}_{A2}, q^{**}_{B1}, q^{**}_{B2}) = (\overline{Q}, 0, 0, \overline{Q})$, where $\overline{Q} \equiv \arg\max_Q \left[P(Q, \overline{Q}) - c\right] Q$.*

(ii) *If the timing and information sharing is such that each retailer R_k is informed about whether its own upstream affiliate's contract offer to the rival retailer R_l was accepted or not when making its output choice, the equilibrium output vector $(q^{**}_{A1}, q^{**}_{A2}, q^{**}_{B1}, q^{**}_{B2})$ is unique. Otherwise, it is unique if (P.2) holds.*

Proof. *Part (i).* Let

$$\Pi(Q_A, Q_B) \equiv [P(Q_A, Q_B) - c] Q_A + [P(Q_B, Q_A) - c] Q_B$$

denote aggregate output when $q_{A1} + \hat{q}_{A2} = Q_A$ and $\hat{q}_{B1} + q_{B2} = Q_B$. To support the vector $(q^{**}_{A1}, q^{**}_{A2}, q^{**}_{B1}, q^{**}_{B2}) = (\overline{Q}, 0, 0, \overline{Q})$ as an equilibrium outcome, we suppose that the two integrated firms do not offer contrats to each other, i.e., $\tau^{**}_{A2} = \varnothing$ and $\tau^{**}_{B1} = \varnothing$. To show that there is no profitable deviation, suppose that the integrated $M_i - R_h$ deviates from this candidate equilibrium by offering a contract $\widetilde{\tau}_{ik}(.)$ that induces a quantity $\hat{q}_{ik} > 0$ through the channel $M_i - R_k$. By assumption, M_j does not offer any contract to R_h in the candidate equilibrium, and thus we still have $\hat{q}_{jh} = 0$, as in the candidate equilibrium. The resulting quantities $\tilde{q}_{ih}(\hat{q}_{ik}, 0)$ and $\tilde{q}_{jk}(\hat{q}_{ik}, 0)$ are the equilibrium quantities in game Γ when $\hat{q}_{jh} = 0$:

$$\tilde{q}_{ih}(\hat{q}_{ik}, 0) = \arg\max_{q_{ih}} \Pi_h \left(q_{ih}, \tilde{q}_{jk}(\hat{q}_{ik}, 0); \hat{q}_{ik}, 0\right), \tag{1}$$

$$\tilde{q}_{jk}(\hat{q}_{ik}, 0) = \arg\max_{q_{jk}} \Pi_k \left(\tilde{q}_{ih}(\hat{q}_{ik}, 0), q_{jk}; \hat{q}_{ik}, 0\right). \tag{2}$$

Now, note that the integrated $M_j - R_k$ can guarantee itself the candidate equilibrium profit $\pi^{**} = \Pi(\overline{Q}, \overline{Q})/2$ by simply rejecting M_i's deviant offer. Therefore, in order to be profitable, the deviation must increase the profits of *both* integrated firms, and thus their joint profit:

$$\Pi(\tilde{q}_{ih}(\hat{q}_{ik}, 0) + \hat{q}_{ik}, \tilde{q}_{jk}(\hat{q}_{ik}, 0)) > 2\pi^{**} = \Pi\left(\overline{Q}, \overline{Q}\right).$$

But this contradicts (P.1).

Part (ii). To show uniqueness of equilibrium, suppose instead that there exists another equilibrium $(q_{A1}^{**}, q_{A2}^{**}, q_{B1}^{**}, q_{B2}^{**}) \neq (\overline{Q}, 0, 0, \overline{Q})$. This implies, in particular, that $q_{A2}^{**} > 0$ or $q_{B1}^{**} > 0$. The induced aggregate profit is $\Pi(Q_A^{**}, Q_B^{**})$, where $Q_A^{**} \equiv q_{A1}^{**} + q_{A2}^{**}$ and $Q_B^{**} \equiv q_{B1}^{**} + q_{B2}^{**}$. By (P.1), we have $\Pi(Q_A^{**}, Q_B^{**}) < \Pi\left(\overline{Q}, \overline{Q}\right)$. The equilibrium profit of at least one of the two integrated firms, say $M_A - R_1$, must therefore be strictly less than $\Pi\left(\overline{Q}, \overline{Q}\right)/2$. Consider the following deviation by $M_A - R_1$: it does not offer any contract to the rival retailer R_2 nor does it accept any contract from the rival manufacturer M_B. Consider first the case where retailers set output *after* deciding what contracts to accept and there is full information sharing between vertically integrated affiliates. In this case, following $M_A - R_1$'s deviation, there is common knowledge at the output-setting stage that there is no cross-selling between the two vertically integrated firms. Hence, the equilibrium outcome is that of game Γ when $\hat{q}_{A2} = \hat{q}_{B1} = 0$. The resulting profit for each integrated firm is $\Pi\left(\overline{Q}, \overline{Q}\right)/2$, a contradiction. Consider now the case where retailer R_2 at the output-setting stage does not necessarily know that R_1 rejects any offer from M_B, so that $\tau_{B1} = \varnothing$. (We remain silent about whether $\tau_{B1}^{**} = \varnothing$ or $\tau_{B1}^{**} \neq \varnothing$ along the equilibrium path.) Suppose instead that R_2 believes that R_1 sells quantity $q_{B1} \geq 0$ of good B. That is, R_2 believes that the ensuing equilibrium will be given by that of game Γ when $\hat{q}_{A2} = 0$ and $\hat{q}_{B1} = q_{B1} \geq 0$. Property (P.2) implies that R_2 will sell not more than $\overline{Q}$ units of good B, implying that $M_A - R_1$'s deviation profit is bounded from below by $\Pi\left(\overline{Q}, \overline{Q}\right)/2$, a contradiction. ■

The proposition shows that pairwise vertical integration leads to a strong form of foreclosure: each integrated firm refuses to deal with the other integrated firm. It is straightforward to show that consumers suffer as a result: both prices are higher (and consumer surplus thus lower) than in the equilibrium under vertical separation.

5 Single Vertical Integration

We now turn to the case where a single upstream-downstream pair, $M_A - R_1$ say, is vertically integrated. (Equilibrium variables under pairwise vertical integrated are indexed by superscript "*.")

The following proposition provides a very partial characterization of equilibrium:

Proposition 5 *Suppose $M_A - R_1$ are vertically integrated whereas M_B and R_2 are vertically separated. Then, in equilibrium, the unintegrated manufacturer M_B signs a cost-based contract with each retailer. The vector of equilibrium quantities, $(q^*_{A1}, q^*_{A2}, q^*_{B1}, q^*_{B2})$, is thus such that*

$$q^*_{Bk} = \arg\max_{q_{Bk}} \left[P\left(q_{Bk} + q^*_{Bl}, q^*_{Ak} + q^*_{Al}\right) - c\right] q_{Bk} + P\left(q^*_{Ak} + q^*_{Al}, q_{Bk} + q^*_{Bl}\right) q^*_{Ak}$$

for all $k, l \in \{1, 2\}$, $k \neq l$.

Proof. This is an immediate implication of Lemma 1. ■

The proposition shows, amongst other things, that the market is "more competitive" under single vertical integration than under pairwise vertical integration.

TO DO LIST

- Provide complete characterization of equilibrium under single vertical integration. (Conjecture: The integrated firm does not sell to the unintegrated retailer in equilibrium.)
- Investigate whether and to what extent the division of surplus varies across different contract types.

6 Discussion

The above analysis first shows that the basic insights from the foreclosure literature triggered by HT carries over to situations where several suppliers compete imperfectly in the upstream market: vertical integration by one or more upstream firms affects the market outcome and tends to make it less competitive. In particular:

- in the absence of any vertical integration, all upstream firms are willing to supply all downstream firms on a marginal cost basis (using fixed fees to share the profits, say), which fosters competition in the downstream market;
- by contrast, when all upstream firms are vertically integrated, and downstream firms are perfect substitutes, then in equilibrium each integrated firm stops supplying its downstream rivals, which results in a tighter oligopoly market outcome.

The analysis also confirms that, when upstream firms supply differentiated inputs to the downstream firms, each vertical integration matters; that is, while a first merger will induce the new entity to depart from cost-based supply contracts, a second merger will further contribute to generate market foreclosure and result in an even tighter oligopoly outcome. This suggests that the foreclosure concerns captured by HT may be relevant in a broader range of cases than suggested by the actual case law.

A Appendix

A.1 Proof of Lemma 1

Fix a candidate equilibrium, with associated equilibrium quantities $(q_{ik}^{\circ})_{i=A,B,k=1,2}$ and acceptance decisions $(\delta_{ik}^{\circ})_{i=A,B,k=1,2}$, and consider a deviation where an unintegrated manufacturer M_i offers some retailer R_k a cost-based two-part tariff $(\tilde{F}_{ik}, c)$, where the fixed fee $\tilde{F}_{ik}$ is as follows:

- if R_k is vertically integrated with M_j, then:

$$\begin{aligned}\tilde{F}_{ik} &= \max_{q_{ik}} \left\{ \left[P\left(q_{ik} + q_{il}^{\circ}, q_{jk}^{\circ} + q_{jl}^{\circ}\right) - c \right] q_{ik} \right. \\ &\quad \left. + \left[P\left(q_{jk}^{\circ} + q_{jl}^{\circ}, q_{ik} + q_{il}^{\circ}\right) - c \right] q_{jk}^{\circ} + \delta_{jl}^{\circ} \left[\tau_{jl}^{\circ}\left(q_{jl}^{\circ}\right) - c q_{jl}^{\circ} \right] \right\} - \pi_{j-k}^{\circ}, \qquad (3)\end{aligned}$$

  where π_{j-k}° denotes the profit of the integrated firm $M_j - R_k$ in the candidate equilibrium. The terms in curly brackets represent the profit that the vertically integrated firm $M_j - R_k$ makes when R_k accepts M_i's deviant offer, assuming that R_k maintains the equilibrium quantity q_{jk}° and that R_l keeps selling the equilibrium quantities q_{il}° and q_{jl}°:

  - the first two terms are the profits generated by, respectively, the channels $M_i - R_k$ and $M_j - R_k$,
  - whereas the third term is the profit that M_j generates in equilibrium through the sales to the other, unintegrated retailer R_l.

- if instead retailer R_k is not vertically integrated, then:

$$\begin{aligned}\tilde{F}_{ik} &= \max_{q_{ik}} \left\{ \left[P\left(q_{ik} + q_{il}^{\circ}, \delta_{jk}^{\circ} q_{jk}^{\circ} + q_{jl}^{\circ}\right) - c \right] q_{ik} \right. \\ &\quad \left. + \delta_{jk}^{\circ} \left[P\left(q_{jk}^{\circ} + q_{jl}^{\circ}, q_{ik} + q_{il}^{\circ}\right) q_{jk}^{\circ} - \tau_{jk}^{\circ}\left(q_{jk}^{\circ}\right) \right] \right\} - \pi_{k}^{\circ}, \qquad (4)\end{aligned}$$

  where π_k° denotes the profit that the unintegrated R_k makes in equilibrium. The terms in curly brackets represent the profit that the unintegrated R_k makes when it accepts M_i's deviant offer and sticks to its acceptance decision δ_{jk}° vis-à-vis M_j's contract offer as well as to its equilibrium quantity q_{jk}°, assuming that R_l sells the equilibrium quantities q_{il}° and q_{jl}°:

– the first term is the profit generated by the channel $M_i - R_k$,

– whereas the second term is the profit that R_k makes on its contract with M_j.

We first claim that R_k is willing accept the deviant offer $(\tilde{F}_{ik}, c)$:

1. As contract offers are simultaneous, an unintegrated retailer R_k still receives the equilibrium offer $\tau^\circ_{jk}(\cdot)$ from the nondeviant manufacturer M_j.

2. Having passive beliefs, at the acceptance stage R_k continues to believe that its downstream rival R_l has been offered the equilibrium contracts and will sell the equilibrium quantities q°_{il} and q°_{jl} in the continuation game.

3. In the light of these observations, the deviant contract $(\tilde{F}_{ik}, c)$ is such that, at the acceptance stage, R_k expects that, by accepting M_i's deviant offer, it can make at least the same profit as in the candidate equilibrium: R_k can secure this by sticking to its acceptance decision vis-à-vis M_j's nondeviant offer (in case M_j and R_k are not vertically integrated) and maintaining the quantity q_{jk} at its equilibrium level q°_{jk}, and can do only better by optimizing over these decisions.

4. Conversely, given observations 1 and 2, if R_k were to reject M_i's deviant offer it would expect the same profit as in the continuation game following the rejection of M_i's equilibrium offers; therefore, its profit from rejecting M_i's deviant offer is bounded from above by its equilibrium profit (more precisely, it coincides with this equilibrium profit if R_k rejects τ°_{ik} in the candidate equilibrium, and is at least weakly dominated by the equilibrium profit otherwise).

5. It follows that, at the acceptance stage, R_k believes that it is weakly better off accepting M_i's deviant offer than not.

As R_k is willing to accept this deviant offer (and can be induced to do so, if needed, by reducing the fixed fee $\tilde{F}_{ik}$ by an arbitrarily small amount), and the deviant offer gives M_i a profit from R_k equal to $\tilde{F}_{ik}$, for this deviation to be unprofitable requires that $\tilde{F}_{ik} \leq \pi^\circ_{ik}$, where

$$\pi^\circ_{ik} = \delta^\circ_{ik} \left[\tau^\circ_{ik}\left(q^\circ_{ik}\right) - c q^\circ_{ik}\right]$$

is the equilibrium profit that M_i makes from selling through retailer R_k. But then:

- If R_k is vertically integrated with M_j (implying $\delta_{ik}^{\circ}=1$), we can rewrite π_{ik}° as follows:

$$\begin{aligned}\pi_{ik}^{\circ} &= \left\{\left[P\left(q_{ik}^{\circ}+q_{il}^{\circ},q_{jk}^{\circ}+q_{jl}^{\circ}\right)-c\right]q_{ik}^{\circ}\right.\\ &\quad \left.+\left[P\left(q_{jk}^{\circ}+q_{jl}^{\circ},q_{ik}^{\circ}+q_{il}^{\circ}\right)-c\right]q_{jk}^{\circ}+\delta_{jl}^{\circ}\left[\tau_{jl}^{\circ}\left(q_{jl}^{\circ}\right)-cq_{jl}^{\circ}\right]\right\}-\pi_{j-k}^{\circ}.\end{aligned}$$

  It follows from (3) that $\tilde{F}_{ik}\leq\pi_{ik}^{\circ}$ implies that q_{ik}° maximizes

$$\left[P\left(q_{ik}+q_{il}^{\circ},q_{jk}+q_{jl}^{\circ}\right)-c\right]q_{ik}+\left[P\left(q_{jk}+q_{jl}^{\circ},q_{ik}+q_{il}^{\circ}\right)-c\right]q_{jk},$$

  that is, $q_{ik}^{\circ}\in R\left(q_{il}^{\circ},q_{jk}^{\circ},q_{jl}^{\circ};\delta_{jk}^{\circ}=1\right)$.

- If instead R_k is unintegrated, we can rewrite π_{ik}° as follows:

$$\begin{aligned}\pi_{ik}^{\circ} &= \left\{\left[P\left(q_{ik}^{\circ}+q_{il}^{\circ},\delta_{jk}^{\circ}q_{jk}^{\circ}+q_{jl}^{\circ}\right)-c\right]q_{ik}^{\circ}\right.\\ &\quad \left.+\delta_{jk}^{\circ}\left[P\left(q_{jk}^{\circ}+q_{jl}^{\circ},q_{ik}^{\circ}+q_{il}^{\circ}\right)q_{jk}^{\circ}-\tau_{jk}^{\circ}\left(q_{jk}^{\circ}\right)\right]\right\}-\pi_{k}^{\circ}.\end{aligned}$$

  Thus, from (4), $\tilde{F}_{ik}\leq\pi_{ik}^{\circ}$ implies $q_{ik}^{\circ}\in R\left(q_{il}^{\circ},q_{jk}^{\circ},q_{jl}^{\circ};\delta_{jk}^{\circ}\right)$.

A.2 Proof of Lemma 2

From Proposition 1, we know that the equilibrium quantities, $(q_{ik}^{\circ})_{i=A,B,k=1,2}$, satisfy, for all $i\neq j\in\{A,B\}$ and $k\neq l\in\{1,2\}$:

$$q_{ik}^{\circ}\in\arg\max_{q_{ik}}\left[P\left(q_{ik}+q_{il}^{\circ},q_{jk}^{\circ}+q_{jl}^{\circ}\right)-c\right]q_{ik}+\left[P\left(q_{jk}^{\circ}+q_{jl}^{\circ},q_{ik}+q_{il}^{\circ}\right)-c\right]q_{jk}^{\circ}.$$

We first show that Assumption A then ensures that all quantities are positive; we then rely on first-order conditions to characterize the unique, symmetric equilibrium outcome.

A.2.1 Quantities are all positive

To see this, suppose that q_{B2}°, say, is zero.

Step 1: $q_{B1}^{\circ}>0$. Suppose otherwise that $q_{B1}^{\circ}=0$. By construction, we then have:

$$q_{Ak}^{\circ}=\arg\max_{q_{ik}}\left[P\left(q_{Ak}+q_{Al}^{\circ},0\right)-c\right]q_{Ak}.$$

Note that $q_{A1}^{\circ}=0$ would imply $p_A=P\left(q_{A2}^{\circ},0\right)\leq c$, and thus $q_{A2}^{\circ}=0$ as well (otherwise, a slight reduction in q_{A2}° would increase R_2's profit, since then $\partial_{q_{A2}}\left(\pi_2\right)=p_A-c+q_{A2}\partial_1P\left(0,0\right)<p_A-c\leq0$); this would therefore require $P\left(0,0\right)\leq c$, contradicting the viability condition (A.0). Thus, we can assume that q_{A1}° is positive, and thus satisfies R_1's first-order condition:

$$\partial_{q_{A1}}\pi_1=P\left(Q_A^{\circ},0\right)-c+\partial_1P\left(Q_A^{\circ},0\right)q_{A1}^{\circ}=0.$$

But then, a small increase in q_{B1} would increase R_1's profit:

$$\begin{aligned}\partial_{q_{B1}}\pi_1 &= P\left(0,Q_A^\circ\right)-c+\partial_2 P\left(Q_A^\circ,0\right)q_{A1}^\circ \\ &> P\left(Q_A^\circ,0\right)-c+\partial_1 P\left(Q_A^\circ,0\right)q_{A1}^\circ=0,\end{aligned}$$

where the inequality stems from condition (A.1) ($\partial_2 P>\partial_1 P$, which also implies $P\left(0,Q_A^\circ\right)>P\left(Q_A^\circ,0\right)$). Hence, q_{B1}° is stritly positive and satisfies the first-order condition

$$\partial_{q_{B1}}\pi_1=P\left(Q_B^\circ,Q_A^\circ\right)-c+\partial_1 P\left(Q_B^\circ,Q_A^\circ\right)q_{B1}^\circ+\partial_2 P\left(Q_A^\circ,Q_B^\circ\right)q_{A1}^\circ=0. \tag{5}$$

By (A.1), we have $\partial_1 P_B\leq 0$ and $\partial_2 P_B\leq 0$ (with a strict inequality if $P_B>0$), implying that $P\left(Q_B^\circ,Q_A^\circ\right)\geq c>0$.

Step 2: $q_{A2}^\circ>q_{A1}^\circ$. The first-order condition for $q_{B2}^\circ=0$ yields:

$$\partial_{q_{B2}}\pi_2=P\left(Q_B^\circ,Q_A^\circ\right)-c+\partial_2 P\left(Q_A^\circ,Q_B^\circ\right)q_{A2}^\circ\leq 0. \tag{6}$$

As $P\left(Q_B^\circ,Q_A^\circ\right)>0$ by Step 1, and using (A.1), it follows that $q_{A2}^\circ>0$. It follows that $P\left(Q_A^\circ,Q_B^\circ\right)\geq c>0$ (as otherwise R_2 could increase its profit by reducing q_{A2}). As both prices are positive, the strict inequalities contained in (A.1) and (A.2) must therefore hold.

Substracting (5) from (6) yields:

$$\partial_2 P\left(Q_A^\circ,Q_B^\circ\right)\left(q_{A2}^\circ-q_{A1}^\circ\right)\leq\partial_1 P\left(Q_B^\circ,Q_A^\circ\right)q_{B1}^\circ,$$

and thus (as $\partial_1 P_A<0$ and $\partial_2 P_B<0$ by (A.1), and $q_{B1}^\circ>0$ by Step 1) $q_{A2}^\circ>q_{A1}^\circ$.

Step 3: $q_{A1}^\circ>0$. Suppose otherwise that $q_{A1}^\circ=0$. In that case, $Q_A^\circ=q_{A2}^\circ$ and $Q_B^\circ=q_{B1}^\circ$ satisfy $Q_B^\circ=\hat{R}\left(Q_A^\circ\right)$ and $Q_A^\circ=\hat{R}\left(Q_B^\circ\right)$, where the best response function

$$\hat{R}\left(Q\right)\equiv\arg\max_{\hat{Q}}\left\{\left[P\left(\hat{Q},Q\right)-c\right]\hat{Q}\right\}$$

is characterized by the first-order condition:

$$P\left(\hat{R}\left(Q\right),Q\right)-c+\partial_1 P\left(\hat{R}\left(Q\right),Q\right)\hat{R}\left(Q\right)=0.$$

Assumption (A.2) ensures that this response function satisfies

$$-1<\hat{R}'\left(Q\right)<0.$$

Therefore, we must have $Q_A^\circ=Q_B^\circ=\hat{Q}^\circ$, where $\hat{Q}^\circ$ is characterized by the first-order condition:

$$P\left(\hat{Q}^\circ,\hat{Q}^\circ\right)-c+\partial_1 P\left(\hat{Q}^\circ,\hat{Q}^\circ\right)\hat{Q}^\circ=0.$$

But then, each retailer would want to sell the other brand as well:

$$\partial_{q_{A1}}\pi_1 = \partial_{q_{B2}}(\pi_2) = P\left(\hat{Q}^\circ, \hat{Q}^\circ\right) - c + \partial_2 P\left(\hat{Q}^\circ, \hat{Q}^\circ\right)\hat{Q}^\circ > 0$$

as $\partial_1 P < \partial_2 P < 0$ by (A.1). Hence, $q_{A1}^\circ > 0$.

Step 4. It follows from the previous steps that q_{A2}°, q_{A1}° and q_{B1}° must all be positive; they thus satisfy the first-order conditions:

$$\partial_{q_{A1}}\pi_1 = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ)\, q_{A1}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ = 0, \quad (7)$$

$$\partial_{q_{B1}}\pi_1 = P(Q_B^\circ, Q_A^\circ) - c + \partial_1 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ + \partial_2 P(Q_A^\circ, Q_B^\circ)\, q_{A1}^\circ = 0, \quad (8)$$

$$\partial_{q_{A2}}\pi_2 = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ)\, q_{A2}^\circ = 0, \quad (9)$$

whereas the first-order condition for $q_{B2}^\circ = 0$ yields:

$$\partial_{q_{B2}}\pi_2 = P(Q_B^\circ, Q_A^\circ) - c + \partial_2 P(Q_A^\circ, Q_B^\circ)\, q_{A2}^\circ \leq 0. \quad (10)$$

Substracting (9) from (7) and (8) from (10) yields:

$$-\partial_1 P(Q_A^\circ, Q_B^\circ)(q_{A2}^\circ - q_{A1}^\circ) = -\partial_2 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ,$$

$$-\partial_2 P(Q_A^\circ, Q_B^\circ)(q_{A2}^\circ - q_{A1}^\circ) \geq -\partial_1 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ,$$

or:

$$\frac{-\partial_1 P(Q_A^\circ, Q_B^\circ)}{-\partial_2 P\left(Q_B^\circ, Q_A^\circ\right)} = \frac{q_{B1}^\circ}{q_{A2}^\circ - q_{A1}^\circ} \leq \frac{-\partial_2 P(Q_A^\circ, Q_B^\circ)}{-\partial_1 P\left(Q_B^\circ, Q_A^\circ\right)}.$$

This, in turn, implies

$$\partial_1 P(Q_A^\circ, Q_B^\circ)\, \partial_1 P(Q_B^\circ, Q_A^\circ) \leq \partial_2 P(Q_B^\circ, Q_A^\circ)\, \partial_2 P(Q_A^\circ, Q_B^\circ),$$

a contradiction as $\partial_1 P < \partial_2 P < 0$ by (A.1). Hence, there is no equilibrum in which $q_{B2}^\circ = 0$.

A.2.2 The equilibrium outcome is unique and symmetric

It follows from the above analysis that all equilibrium quantities are positive and thus satisfy the first-order conditions. Adding the conditions for good A, namely:

$$\partial_{q_{A1}}\pi_1 = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ)\, q_{A1}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ)\, q_{B1}^\circ = 0,$$

$$\partial_{q_{A2}}\pi_2 = P(Q_A^\circ, Q_B^\circ) - c + \partial_1 P(Q_A^\circ, Q_B^\circ)\, q_{A2}^\circ + \partial_2 P(Q_B^\circ, Q_A^\circ)\, q_{B2}^\circ = 0,$$

implies that

$$P(Q_A^\circ, Q_B^\circ) \geq c > 0$$

and

$$Q_A^\circ = R\left(Q_B^\circ\right),$$

where $Q_A = \tilde{R}\left(Q_B\right)$ denotes the "best-response" function defined by

$$2\left[P\left(Q_A, Q_B\right) - c\right] + \partial_1 P\left(Q_A, Q_B\right) Q_A + \partial_2 P\left(Q_B, Q_A\right) Q_B = 0.$$

Likewise, adding the first-order conditions for good B yields $P\left(Q_B^\circ, Q_A^\circ\right) \geq c > 0$ and $Q_B^\circ = \tilde{R}\left(Q_A^\circ\right)$. Assumption (A.3) ensures that the reaction function $\tilde{R}\left(.\right)$ is "well-behaved", namely, it is uniquely defined and such that

$$\tilde{R}'\left(Q\right) = -\frac{2\partial_2 P\left(\tilde{R}\left(Q\right), Q\right) + \partial_2 P\left(Q, \tilde{R}\left(Q\right)\right) + \partial_{12} P\left(\tilde{R}\left(Q\right), Q\right)\tilde{R}\left(Q\right) + \partial_{21} P\left(Q, \tilde{R}\left(Q\right)\right) Q}{3\partial_1 P\left(\tilde{R}\left(Q\right), Q\right) + \partial_{11} P\left(\tilde{R}\left(Q\right), Q\right)\tilde{R}\left(Q\right) + \partial_{22} P\left(Q, \tilde{R}\left(Q\right)\right) Q} \in (-1, 0),$$

where the inequalities follow from (A.3). Hence, equilibrium is symmetric; that is, $Q_A^\circ = Q_B^\circ = Q^\circ$. The first-order conditions for R_1's quantity choices then yield:

$$\begin{aligned} -\partial_1 P\left(Q^\circ, Q^\circ\right) q_{A1}^\circ - \partial_2 P\left(Q^\circ, Q^\circ\right) q_{B1}^\circ &= P\left(Q^\circ, Q^\circ\right) - c, \\ -\partial_1 P\left(Q^\circ, Q^\circ\right) q_{B1}^\circ - \partial_2 P\left(Q^\circ, Q^\circ\right) q_{A1}^\circ &= P\left(Q^\circ, Q^\circ\right) - c, \end{aligned}$$

and thus

$$q_{A1}^\circ = q_{B1}^\circ = q^\circ \equiv -\frac{P\left(Q^\circ, Q^\circ\right) - c}{\partial_1 P\left(Q^\circ, Q^\circ\right) + \partial_2 P\left(Q^\circ, Q^\circ\right)}.$$

Likewise, we have $q_{A2}^\circ = q_{B2}^\circ = q^\circ$.

A.3 Proof of Proposition 2

Consider a candidate equilibrium in which the manufacturers offer these cost-based two-part tariffs $\left(F^\circ, c\right)$, which the retailers accept.

We first note that the continuation equilibrium is then such that both retailers sell $\left(q_{Ak}, q_{Bk}\right) = \left(q^\circ, q^\circ\right)$, for $k = 1, 2$. From Lemma 2, this constitutes the unique candidate equilibrium. Conversely, if a retailer anticipates that its rival will sell $\left(q^\circ, q^\circ\right)$, its best response is indeed to sell $\left(q^\circ, q^\circ\right)$ as well, as:

- Each R_k's profit, gross of fixed fees, coincides with

  $$\Pi_k\left(q_{Ak}, q_{Bk}; q^\circ, q^\circ\right) \equiv \left[P\left(q_{Ak} + q^\circ, q_{Bk} + q^\circ\right) - c\right] q_{Ak} + \left[P\left(q_{Bk} + q^\circ, q_{Ak} + q^\circ\right) - c\right] q_{Bk},$$

  implying that it will choose $\left(q_{Ak}, q_{Bk}\right)$ satisfying $q_{ik} = R\left(q^\circ, q_{jk}, , q^\circ; 1\right)$; from the proof of Lemma 2, $\left(q^\circ, q^\circ\right)$ is thus the unique solution to the first-order conditions.

- To verify that each R_k's profit is concave with respect to its own quantities (q_{Ak}, q_{Bk}), note that:

$$\begin{aligned}
\partial^2_{ii}\Pi_k &= 2\partial_1 P(Q_i, Q_j) + \partial^2_{11} P(Q_i, Q_j) q_{ik} + \partial^2_{22} P(Q_j, Q_i) q_{jk}, \\
\partial^2_{AB}\Pi_k &= \partial_2 P(Q_A, Q_B) + \partial^2_{12} P(Q_A, Q_B) q_{Ak} + \partial_2 P(Q_B, Q_A) + \partial^2_{12} P(Q_B, Q_A) q_{Bk}.
\end{aligned}$$

Assumptions (A.1) and (A.3) ensure that $\partial^2_{ii}\Pi_k < 0$ and $H = \partial^2_{AA}\Pi_k \partial^2_{BB}\Pi_k - (\partial_{AB}\Pi_k)^2 > 0$.

Next, we note that each retailer is indeed willing to carry both brands; indeed, the fee F° is designed in such a way that, if its rival to accept both offers and sell (q°, q°), then a retailer:

- obtains the same profit by accepting both manufacturers' offers or only one of them:

$$2\{[P(2q^\circ, 2q^\circ) - c] q^\circ - F^\circ\} = \max_q [P(q + q^\circ, q^\circ) - c] q^\circ - F^\circ.$$

- strictly prefers securing this profit, π°_R, to rejecting both offers:

$$\begin{aligned}
\pi^\circ_R &= 2\{[P(2q^\circ, 2q^\circ) - c] q^\circ - F^\circ\} \\
&= 2\left\{[P(2q^\circ, 2q^\circ) - c] q^\circ - \left[[P(2q^\circ, 2q^\circ) - c] 2q^\circ - \max_q [P(q^\circ + q, q^\circ) - c] q\right]\right\} \\
&= 2\left\{\max_q [P(q^\circ + q, q^\circ) - c] q - [P(2q^\circ, 2q^\circ) - c] q^\circ\right\} \\
&> 2\left\{\max_q [P(q^\circ + q, q^\circ) - c] q - [P(2q^\circ, q^\circ) - c] q^\circ\right\} \\
&\geq 0.
\end{aligned}$$

Thus, if these contracts are accepted, it is a continuation equilibrium for both retailers to accept both manufacturers' offers, and then to sell (q°, q°). We now show that manufacturers cannot profitably deviate from this candidate equilibrium. As the profit that a manufacturer achieves with a retailer is not affected by its relation with the other retailer, without loss of generality we can restrict attention to "one-sided" deviations, in which a manufacturer offers a deviating contract to one of the retailers. Furthermore, the above tariffs are profitable for the manufacturers:

$$\begin{aligned}
F^\circ &= [P(2q^\circ, 2q^\circ) - c] 2q^\circ - \max_q [P(q + q^\circ, q^\circ) - c] q \\
&= \max_{q_A, q_B} \{[P(q_A + q^\circ, q_B + q^\circ) - c] q_A + [P(q_B + q^\circ, q_A + q^\circ) - c] q_B\} - \max_q [P(q + q^\circ, q^\circ) - c] q \\
&> 0,
\end{aligned}$$

where the second equality comes from the definition of q° and the inequality comes from the fact that the second optimization problem is more constrained than the first one. If follows that a deviation cannot be profitable if it is not accepted by the retailer; and since the retailer can secure its equilibrium profit π_R° by accepting only the rival's offer, it must be the case that the deviation increases the joint profit of the manufacturer and of the retailer.

Suppose first that the deviation induces the retailer to keep dealing with the other manufacturer. The joint profit of the manufacturer and of the retailer (gross of the manufacturer's cost of supplying q° to the rival retailer, which is not affected by the deviation) then cannot exceed

$$\max_{q_A, q_B} \left\{ \left[P\left(q_A + q^\circ, q_B + q^\circ\right) - c \right] q_A + \left[P\left(q_B + q^\circ, q_A + q^\circ\right) - c \right] q_B \right\} = \left[P\left(2q^\circ, 2q^\circ\right) - c \right] 2q^\circ,$$

which is precisely what the two parties generate in the candidate equilibrium. Therefore, such a deviation cannot be profitable for the deviating manufacturer.

Consider now a deviation that would induce the retailer to reject the other manufacturer's offer. In that case the joint profit of the manufacturer and of the retailer (again gross of the manufacturer's cost of supplying q° to the rival retailer) cannot exceed

$$\begin{aligned} \max_q \left\{ \left[P\left(q + q^\circ, q^\circ\right) - c \right] q \right\} + F^\circ &= \max_q \left\{ \left[P\left(q + q^\circ, q^\circ\right) - c \right] q \right\} + \left[P\left(2q^\circ, 2q^\circ\right) - c \right] 2q^\circ - \max_q \left[P\left(q + q^\circ, q^\circ\right) - \right. \\ &= \left[P\left(2q^\circ, 2q^\circ\right) - c \right] 2q^\circ, \end{aligned}$$

which is again what they obtain in the candidate equilibrium. Therefore, such deviations cannot be profitable either, which concludes the argument.

A.4 Proof of Proposition 3

Consider a candidate equilibrium in which:

- the manufacturers offer both retailers the pair of forcing contracts $\left\{ \left(T^\circ, q^\circ\right), \left(\hat{T}, \hat{q}\right) \right\}$,
- the retailers accept both manufacturers' offers, and then select the option (T°, q°).

In this candidate equilibrium, each retailer obtains a profit equal to

$$\pi_R^\circ \equiv 2 \left[P\left(2q^\circ, 2q^\circ\right) q^\circ - T^\circ \right], \tag{11}$$

whereas each manufacturer obtains

$$\pi_M^\circ \equiv 2 \left(T^\circ - c q^\circ \right).$$

We further restrict attention to configurations such that, along the equilibrium path, a retailer is indifferent between accepting both manufacturers' offers, or accepting only one of them, selecting in that case the option $\left(\hat{T}, \hat{q}\right)$:

$$2\left[P\left(2q^{\circ}, 2q^{\circ}\right) q^{\circ} - T^{\circ}\right] = P\left(\hat{q} + q^{\circ}, q^{\circ}\right) \hat{q} - \hat{T}. \tag{12}$$

We now provide conditions ensuring that manufacturers do not have incentives to deviate (we will show later on that retailers then prefer accepting both manufacturers' offers, and selecting the tariffs (T°, q°), to any other combination). As the profit that a manufacturer achieves with a retailer is not affected by its relation with the other retailer, without loss of generality we can again restrict attention to "one-sided" deviations, in which a manufacturer offers a deviating contract to one of the retailers. Also, by deviating, the manufacturer cannot reduce the retailer's expected payoff (based on the anticipation that its rival will sell (q°, q°)), which the retailer can secure by opting (on an exclusive dealing basis) for the rival's tariff $\left(\hat{T}, \hat{q}\right)$. As the manufacturer's profit from this relation (of the form "$T - cq$") does not directly depend on the rival retailer's sales, to be profitable, the deviation must increase the joint profit of the manufacturer and the retailer, assuming that the rival keeps selling (q°, q°).

Consider first a deviation that would induce the retailer to reject the rival manufacturer's offer. By construction, the joint profit of the manufacturer and the retailer are then maximal by trading $q = \hat{q}$. And since the retailer can obtain its equilibrium profit by selecting $\left(\hat{T}, \hat{q}\right)$ on an exclusive dealing basis, the maximal profit that M can obtain by deviating in this way M is $\left(\hat{T} - c\hat{q}\right) - (T^{\circ} - cq^{\circ})$; ruling out such a deviation thus requires:

$$\hat{T} - T^{\circ} \leq c\left(\hat{q} - q^{\circ}\right). \tag{13}$$

Consider now a deviation that would induce the retailer to "combine" the deviant offer with the rival manufacturer's offer. By construction, the joint profits of the manufacturer and the retailer cannot be increased if the retailer were to stick to the rival's tariff (T°, q°). Combining instead a deviant quantity q with the rival's tariff $\left(\hat{T}, \hat{q}\right)$ yields at most a joint profit equal to

$$\left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \left[P\left(\hat{q} + q^{\circ}, q + q^{\circ}\right) \hat{q} - \hat{T}\right] + \left[T^{\circ} - cq^{\circ}\right] = \hat{\pi}\left(q\right) - \left(\hat{T} - T^{\circ}\right),$$

where

$$\hat{\pi}\left(q\right) \equiv \left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \left[P\left(\hat{q} + q^{\circ}, q + q^{\circ}\right) - c\right] \hat{q} + c\left(\hat{q} - q^{\circ}\right).$$

Therefore, ruling out such a deviation requires

$$\max_{q} \hat{\pi}\left(q\right) - \left(\hat{T} - T^{\circ}\right) \leq \pi_M^{\circ} + \pi_R^{\circ} = 2\left[P\left(2q^{\circ}, 2q^{\circ}\right) - c\right] q^{\circ},$$

or:

$$\hat{T} - T^{\circ} \geq \max_{q} \hat{\pi}(q) - 2\left[P\left(2q^{\circ}, 2q^{\circ}\right) - c\right] q^{\circ}. \tag{14}$$

This condition furthermore ensures that manufacturers' profit, π_M°, is non-negative; using (12), we have:

$$\begin{aligned}
T^{\circ} - cq^{\circ} &= \hat{T} - T^{\circ} - cq^{\circ} + 2P\left(2q^{\circ}, 2q^{\circ}\right) q^{\circ} - P\left(\hat{q} + q^{\circ}, q^{\circ}\right) \hat{q} \\
&\geq \max_{q} \left\{\left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \left[P\left(\hat{q} + q^{\circ}, q + q^{\circ}\right) - c\right] \hat{q} + c\left(\hat{q} - q^{\circ}\right)\right\} \\
&\quad -2\left[P\left(2q^{\circ}, 2q^{\circ}\right) - c\right] q^{\circ} - cq^{\circ} + 2P\left(2q^{\circ}, 2q^{\circ}\right) q^{\circ} - P\left(\hat{q} + q^{\circ}, q^{\circ}\right) \hat{q} \\
&= \max_{q} \left\{\left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \left[P\left(\hat{q} + q^{\circ}, q + q^{\circ}\right) - c\right] \hat{q}\right\} - \left[P\left(\hat{q} + q^{\circ}, q^{\circ}\right) - c\right] \hat{q} \\
&\geq 0,
\end{aligned}$$

where the first inequality stems from (13) and the second one follows from setting $q = 0$ in the previous line.

Conditions (14) and (13) thus ensure that manufacturers cannot profitably deviate from a candidate equilibrium where (i) both manufacturers offer both retailers a choice between (T°, q°) and $\left(\hat{T}, \hat{q}\right)$, and (ii) the retailers accept both offers, and select the tariffs (T°, q°). Condition (12) further ensures that, if the manufactures make these offers, then retailers are indifferent between accepting both of them, and selecting the tariffs (T°, q°), or accepting only one of them, and selecting the tariff $\left(\hat{T}, \hat{q}\right)$. These conditions are moreover compatible (that is, the upper bound in (13) exceeds the lower bound in (14)):

$$\begin{aligned}
c\left(\hat{q} - q^{\circ}\right) - \left\{\max_{q} \hat{\pi}(q) - 2\left[P\left(2q^{\circ}, 2q^{\circ}\right) - c\right] q^{\circ}\right\} &= \left\{2\left[P\left(2q^{\circ}, 2q^{\circ}\right) - c\right] q^{\circ}\right\} - \max_{q} \left\{\left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \right. \\
&= \max_{q, \hat{q}} \left\{\left[P\left(q + q^{\circ}, \hat{q} + q^{\circ}\right) - c\right] q + \left[P\left(\hat{q} + q^{\circ}, q + q^{\circ}\right) - c\right] \hat{q} \right. \\
&\geq 0.
\end{aligned}$$

To complete the proof, we now check that, when receiving these equilibrium offers, the retailers are indeed willing to accept both manufacturers' tariffs (T°, q°), which gives them a profit π_R°.

We first note that $\hat{q} > q^{\circ}$. q° is characterized by the first-order condition $\phi(q^{\circ}) = 0$, where

$$\phi(q) \equiv P\left(q + q^{\circ}, 2q^{\circ}\right) - c + \partial_1 P\left(q + q^{\circ}, 2q^{\circ}\right) q + \partial_2 P\left(2q^{\circ}, q + q^{\circ}\right) q^{\circ},$$

whereas $\hat{q}$ is characterized by the first-order condition $\hat{\phi}(q) = 0$, where

$$\hat{\phi}(q) \equiv P\left(q + q^{\circ}, q^{\circ}\right) - c + \partial_1 P\left(q + q^{\circ}, q^{\circ}\right) q.$$

Assumption $(A.2)$ ensures that $\phi(.)$ and $\hat{\phi}(.)$ are both decreasing in q. The conclusion then follows from $\hat{\phi}(.) > \phi(.)$; indeed, using $\partial_2 P \leq 0$, we have:

$$\begin{aligned}\hat{\phi}(q) - \phi(q) &\geq P(q+q^\circ, q^\circ) - P(q+q^\circ, 2q^\circ) + [\partial_1 P(q+q^\circ, q^\circ) - \partial_1 P(q+q^\circ, 2q^\circ)\, q]\, q \\ &= \int_{q^\circ}^{2q^\circ} - [\partial_2 P(q+q^\circ, \tilde{q}) + \partial_{12} P(q+q^\circ, \tilde{q})\, q]\, d\tilde{q} > 0.\end{aligned}$$

We then show that (12) and (14), (13) and (**??**) ensure that manufacturers cannot profitably deviate from a candidate equilibrium where (i) both manufacturers offer both retailers a choice between (T°, q°) and $(\hat{T}, \hat{q})$, and (ii) the retailers accept both offers, and select the tariffs (T°, q°). Condition (12)

- Retailers prefer obtaining this profit π_R° to accepting only one tariff (T°, q°): using (14), we have

$$\begin{aligned}\pi_R^\circ &= P(\hat{q}+q^\circ, q^\circ)\,\hat{q} - \left(\hat{T} - T^\circ\right) - T^\circ \\ &\geq P(\hat{q}+q^\circ, q^\circ)\,\hat{q} - c(\hat{q} - q^\circ) - T^\circ \\ &= \max_q [P(q+q^\circ, q^\circ)\, \_c]\, q - (T^\circ - cq^\circ) \\ &\geq [P(2q^\circ, q^\circ) - c]\, q^\circ - (T^\circ - cq^\circ) \\ &= P(2q^\circ, q^\circ)\, q^\circ - T^\circ, \end{aligned} \tag{15}$$

  where the first inequality stems from (13) and the second one follows from setting $q = q^\circ$ in the previous line.

- Retailers' profit π_R° is non-negative: using (15), we have

$$2[P(2q^\circ, 2q^\circ)\, q^\circ - T^\circ] \geq P(2q^\circ, q^\circ)\, q^\circ - T^\circ,$$

  and thus

$$T^\circ \leq 2P(2q^\circ, 2q^\circ)\, q^\circ - P(2q^\circ, q^\circ)\, q^\circ,$$

  which in turn implies

$$\begin{aligned}\pi_R^\circ &= 2[P(2q^\circ, 2q^\circ)\, q^\circ - T^\circ] \\ &\geq 2P(2q^\circ, 2q^\circ)\, q^\circ - 2[2P(2q^\circ, 2q^\circ)\, q^\circ - P(2q^\circ, q^\circ)\, q^\circ] \\ &= [P(2q^\circ, q^\circ)\, q^\circ - P(2q^\circ, 2q^\circ)\, q^\circ]\, 2q^\circ \\ &\geq 0. \end{aligned} \tag{16}$$

- Retailers prefer obtaining this profit π_R° to accepting both offers and then selecting the option (T°, q°) from one and the option $\left(\hat{T}, \hat{q}\right)$ from the other: using (15), we have

$$\begin{aligned}
& 2\left[P\left(2q^\circ, 2q^\circ\right)q^\circ - T^\circ\right] - \left\{\left[P\left(\hat{q}+q^\circ, 2q^\circ\right) - c\right]\hat{q} - \hat{T} + \left[P\left(2q^\circ, \hat{q}+q^\circ\right) - c\right]q^\circ - T^\circ\right\} \\
\geq \;& 2P\left(2q^\circ, 2q^\circ\right)q^\circ - \left\{\left[P\left(\hat{q}+q^\circ, 2q^\circ\right) - c\right]\hat{q} + \left[P\left(2q^\circ, \hat{q}+q^\circ\right) - c\right]q^\circ\right\} \\
& + \max_q \left\{\left[P\left(q+q^\circ, \hat{q}+q^\circ\right) - c\right]q + \left[P\left(\hat{q}+q^\circ, q+q^\circ\right) - c\right]\hat{q} + c\left(\hat{q}-q^\circ\right)\right\} - 2\left[P\left(2q^\circ, 2q^\circ\right) - c\right]q^\circ \\
\geq \;& 2P\left(2q^\circ, 2q^\circ\right)q^\circ - \left\{\left[P\left(\hat{q}+q^\circ, 2q^\circ\right) - c\right]\hat{q} + \left[P\left(2q^\circ, \hat{q}+q^\circ\right) - c\right]q^\circ\right\} \\
& + \left[P\left(2q^\circ, \hat{q}+q^\circ\right) - c\right]q^\circ + \left[P\left(\hat{q}+q^\circ, 2q^\circ\right) - c\right]\hat{q} + c\left(\hat{q}-q^\circ\right) - 2\left[P\left(2q^\circ, 2q^\circ\right) - c\right]q^\circ \\
= \;& 2P\left(2q^\circ, 2q^\circ\right)q^\circ + c\left(\hat{q}-q^\circ\right) - 2\left[P\left(2q^\circ, 2q^\circ\right) - c\right]q^\circ \\
\geq \;& 0, \qquad (17)
\end{aligned}$$

where the second inequality follows from setting $q = q^\circ$ in the previous line.

- Finally, retailers prefer obtaining this profit π_R° to picking both tariffs $\left(\hat{T}, \hat{q}\right)$: this amounts to

$$2\left[P\left(2q^\circ, 2q^\circ\right)q^\circ - T^\circ\right] \geq 2\left[P\left(\hat{q}+q^\circ, \hat{q}+q^\circ\right)\hat{q} - \hat{T}\right],$$

or

$$\hat{T} - T^\circ \geq P\left(\hat{q}+q^\circ, \hat{q}+q^\circ\right)\hat{q} - P\left(2q^\circ, 2q^\circ\right)q^\circ. \qquad (18)$$

Note that the previous condition amounts to

$$2\left[P\left(2q^\circ, 2q^\circ\right)q^\circ - T^\circ\right] \geq \left[P\left(\hat{q}+q^\circ, 2q^\circ\right)\hat{q} - \hat{T}\right] + \left[P\left(2q^\circ, \hat{q}+q^\circ\right) - q^\circ - T^\circ\right],$$

or

$$\hat{T} - T^\circ \geq P\left(\hat{q}+q^\circ, 2q^\circ\right)\hat{q} + P\left(2q^\circ, \hat{q}+q^\circ\right)q^\circ - 2P\left(2q^\circ, 2q^\circ\right)q^\circ. \qquad (19)$$

If follows that (**??**) derives from (**??**) if:

$$P\left(2q^\circ, \hat{q}+q^\circ\right)q^\circ - P\left(\hat{q}+q^\circ, \hat{q}+q^\circ\right)\hat{q} \geq P\left(2q^\circ, 2q^\circ\right)q^\circ - P\left(\hat{q}+q^\circ, 2q^\circ\right)\hat{q},$$

that is, if

$$\varphi\left(\hat{q}\right) \geq \varphi\left(q^\circ\right),$$

where

$$\varphi\left(q\right) \equiv P\left(2q^\circ, q+q^\circ\right)q^\circ - P\left(\hat{q}+q^\circ, q+q^\circ\right)\hat{q}.$$

The conclusion follows from

$$\begin{aligned}\varphi'(q) &= \partial_2 P(2q^\circ, q+q^\circ)\, q^\circ - \partial_2 P(\hat{q}+q^\circ, q+q^\circ)\, \hat{q} \\ &= -\int_{q^\circ}^{\hat{q}} \left[\partial_2 P(\tilde{q}+q^\circ, q+q^\circ) + \partial_{12}^2 P(\tilde{q}+q^\circ, q+q^\circ)\, \tilde{q}\right] d\tilde{q} \\ &\geq 0.\end{aligned}$$

A.5 Sufficient Conditions for Properties (P.1) and (P.2)

We begin by providing a condition on demand that ensures that the normal-form game Γ has a unique Nash equilibrium. Next, we provide conditions on demand that ensure that an increase in either $\hat{q}_{B1}$ or $\hat{q}_{A2}$ induces an increase in the sum of the outputs of goods A and B. We then show that such an increase in aggregate output beyond some threshold leads to a decrease in aggregate profit, implying (P.1).

To prove existence and uniqueness of equilibrium, we impose some restriction on inverse demand, in addition to (A.0) and (A.1):

(A.4) For any $(Q_i, Q_j) \geq 0$ such that $P(Q_i, Q_j) > 0$, and for any $q_i \in [0, Q_i]$ and any $q_j \in [0, Q_j]$, we have

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + q_j \partial_{22}^2 P(Q_j, Q_i) \\ < \; & \partial_2 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + q_j \partial_{12}^2 P(Q_j, Q_i) \\ < \; & 0.\end{aligned}$$

In the case of linear demand, (A.4) simplifies to $2\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_i, Q_j) < 0$, and thus holds by (A.1).

We obtain the following result on existence and uniqueness:

Proposition 6 *Under Assumption (A.4), the normal-form game* Γ *has a unique Nash equilibrium* $(\widetilde{q}_{A1}, \widetilde{q}_{B2})$.

For the comparative statics, we impose the following condition on demand:

(B.1) For any $Q_i, Q_j \geq 0$ such that $P(Q_i, Q_j) > 0$ and $P(Q_j, Q_i) > 0$, and any $q_i \in [0, Q_i]$ and $q_j \in [0, Q_j]$, we have

$$\begin{aligned} & 2\partial_1 P(Q_i, Q_j) + q_i \partial_{11}^2 P(Q_i, Q_j) + q_j \partial_{22}^2 P(Q_j, Q_i) \qquad \text{(B.1.a)} \\ < \; & \partial_2 P(Q_i, Q_j) + q_i \partial_{12}^2 P(Q_i, Q_j) + q_j \partial_{12}^2 P(Q_j, Q_i) \\ < \; & 0\end{aligned}$$

and

$$-\partial_1 P(Q_i,Q_j)\left[\begin{array}{c} 2\partial_1 P(Q_j,Q_i)-\partial_2 P(Q_j,Q_i) \\ +q_j\left(\partial^2_{11}P(Q_j,Q_i)-\partial^2_{12}P(Q_j,Q_i)\right) \\ +q_i\left(\partial^2_{12}P(Q_i,Q_j)-\partial^2_{11}P(Q_i,Q_j)\right)\end{array}\right] \tag{B.1.b}$$
$$< \; -\partial_2 P(Q_i,Q_j)\left[\begin{array}{c} 2\partial_1 P(Q_i,Q_j)-\partial_2 P(Q_i,Q_j) \\ +[Q_i-q_i]\left(\partial^2_{11}P(Q_i,Q_j)-\partial^2_{12}P(Q_i,Q_j)\right) \\ +[Q_j-q_j]\left(\partial^2_{12}P(Q_j,Q_i)-\partial^2_{11}P(Q_j,Q_i)\right)\end{array}\right].$$

In the case of linear demand (B.1.a) and (B.1.b) simplify to $2\partial_1 P(Q_i,Q_j) < \partial_2 P(Q_i,Q_j) < 0$ and $-\partial_1 P(Q_i,Q_j)[2\partial_1 P(Q_i,Q_j)-\partial_2 P(Q_i,Q_j)] < -\partial_2 P(Q_i,Q_j)[2\partial_1 P(Q_i,Q_j)-\partial_2 P(Q_i,Q_j)]$, respectively, and thus hold by (A.1).

We now show that (B.1) implies that an exogenous increase in either $\hat{q}_{A2}$ or $\hat{q}_{B1}$ generates an increase in the total output of good A, a decrease in the total output of good B, and an increase in the sum of the outputs of goods A and B.

Lemma 3 *Consider game Γ. If (B.1) holds, then an exogenous increase in $\hat{q}_{A2}$ (resp., $\hat{q}_{B1}$) triggers an increase in the equilibrium total output $\tilde{Q}_A$ (resp., $\tilde{Q}_B$) and a decrease in $\tilde{Q}_B$ (resp., $\tilde{Q}_A$). Furthermore, a small exogenous increase in either $\hat{q}_{A2}$ or $\hat{q}_{B1}$ leads to an increase in $\tilde{Q}_A+\tilde{Q}_B$.*

Proof. The two optimality conditions characterizing the two firms' equilibrium output levels $\tilde{q}_{A1}$ and $\tilde{q}_{B2}$ are:

$$\begin{aligned} P(\tilde{q}_{A1}+\hat{q}_{A2},\hat{q}_{B1}+\tilde{q}_{B2})-c+\tilde{q}_{A1}\partial_1 P(\tilde{q}_{A1}+\hat{q}_{A2},\hat{q}_{B1}+\tilde{q}_{B2})+\hat{q}_{B1}\partial_2 P(\hat{q}_{B1}+\tilde{q}_{B2},\tilde{q}_{A1}+\hat{q}_{A2}) &= 0, \\ P(\hat{q}_{B1}+\tilde{q}_{B2},\tilde{q}_{A1}+\hat{q}_{A2})-c+\tilde{q}_{B2}\partial_1 P(\hat{q}_{B1}+\tilde{q}_{B2},\tilde{q}_{A1}+\hat{q}_{A2})+\hat{q}_{A2}\partial_2 P(\tilde{q}_{A1}+\hat{q}_{A2},\hat{q}_{B1}+\tilde{q}_{B2}) &= 0, \end{aligned}$$

or, in terms of total equilibrium outputs $\tilde{Q}_A=\tilde{q}_{A1}+\hat{q}_{A2}$ and $\tilde{Q}_B=\hat{q}_{B1}+\tilde{q}_{B2}$:

$$\begin{aligned} P\left(\tilde{Q}_A,\tilde{Q}_B\right)-c+\tilde{Q}_A\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right) &= \hat{q}_{A2}\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)-\hat{q}_{B1}\partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right), \\ P\left(\tilde{Q}_B,\tilde{Q}_A\right)-c+\tilde{Q}_B\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right) &= \hat{q}_{B1}\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)-\hat{q}_{A2}\partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right). \end{aligned}$$

Differentiating these equations with respect to $\left(\tilde{Q}_A,\tilde{Q}_B\right)$ and $(\hat{q}_{A2},\hat{q}_{B1})$ yields:

$$\begin{aligned} \hat{\lambda}_{AA}d\tilde{Q}_A+\hat{\lambda}_{AB}d\tilde{Q}_B &= \partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)d\hat{q}_{A2}-\partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right)d\hat{q}_{B1}, \\ \hat{\lambda}_{BA}d\tilde{Q}_A+\hat{\lambda}_{BB}d\tilde{Q}_B &= \partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)d\hat{q}_{B1}-\partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right)d\hat{q}_{A2}, \end{aligned}$$

where

$$
\begin{aligned}
\hat{\lambda}_{AA} &= 2\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\tilde{q}_{A1}\partial^2_{11}P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\hat{q}_{B1}\partial^2_{22}P\left(\tilde{Q}_B,\tilde{Q}_A\right),\\
\hat{\lambda}_{AB} &= \partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\tilde{q}_{A1}\partial^2_{12}P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\hat{q}_{B1}\partial^2_{12}P\left(\tilde{Q}_B,\tilde{Q}_A\right),\\
\hat{\lambda}_{BA} &= \partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\tilde{q}_{B2}\partial^2_{12}P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\hat{q}_{A2}\partial^2_{12}P\left(\tilde{Q}_A,\tilde{Q}_B\right),\\
\hat{\lambda}_{BB} &= 2\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\tilde{q}_{B2}\partial^2_{11}P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\hat{q}_{A2}\partial^2_{22}P\left(\tilde{Q}_A,\tilde{Q}_B\right).
\end{aligned}
$$

From (B.1.a), these coefficients satisfy $\hat{\lambda}_{AA} < \hat{\lambda}_{AB} < 0$ and $\hat{\lambda}_{BB} < \hat{\lambda}_{BA} < 0$; the determinant $\hat{D} = \hat{\lambda}_{AA}\hat{\lambda}_{BB} - \hat{\lambda}_{AB}\hat{\lambda}_{BA}$ is therefore positive, and:

$$
\begin{aligned}
\frac{\partial \tilde{Q}_A}{\partial \hat{q}_{A2}} &= \frac{\hat{\lambda}_{BB}\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\hat{\lambda}_{AB}\partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right)}{\hat{D}} > 0,\\
\frac{\partial \tilde{Q}_B}{\partial \hat{q}_{A2}} &= \frac{-\hat{\lambda}_{BA}\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)-\hat{\lambda}_{AA}\partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right)}{\hat{D}} < 0,\\
\frac{\partial \tilde{Q}_A}{\partial \hat{q}_{B1}} &= \frac{-\hat{\lambda}_{AB}\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)-\hat{\lambda}_{BB}\partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right)}{\hat{D}} < 0,\\
\frac{\partial \tilde{Q}_B}{\partial \hat{q}_{B1}} &= \frac{\hat{\lambda}_{AA}\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\hat{\lambda}_{BA}\partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right)}{\hat{D}} > 0.
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
\frac{\partial\left(\tilde{Q}_A+\tilde{Q}_B\right)}{\partial \hat{q}_{A2}} &= \frac{\left(\hat{\lambda}_{BB}-\hat{\lambda}_{BA}\right)\partial_1 P\left(\tilde{Q}_A,\tilde{Q}_B\right)+\left(\hat{\lambda}_{AB}-\hat{\lambda}_{AA}\right)\partial_2 P\left(\tilde{Q}_A,\tilde{Q}_B\right)}{\hat{D}},\\
\frac{\partial\left(\tilde{Q}_A+\tilde{Q}_B\right)}{\partial \hat{q}_{B1}} &= \frac{\left(\hat{\lambda}_{AA}-\hat{\lambda}_{AB}\right)\partial_1 P\left(\tilde{Q}_B,\tilde{Q}_A\right)+\left(\hat{\lambda}_{BA}-\hat{\lambda}_{BB}\right)\partial_2 P\left(\tilde{Q}_B,\tilde{Q}_A\right)}{\hat{D}}.
\end{aligned}
$$

Assumption (B.1.b) implies that these expressions are positive. ■

Let

$$\overline{Q} \equiv \arg\max_{Q}\left[P(Q,\overline{Q})-c\right]Q$$

denote the equilibrium output per good in game Γ if $\hat{q}_{A2} = \hat{q}_{B1} = 0$. (From Proposition 6, we know that $\overline{Q}$ exists and is unique.) We have:

Lemma 4 *Consider a market outcome in game Γ where the total output of each good is equal to Q. Then, aggregate profit $\Pi(Q,Q) = 2[P(Q,Q)-c]Q$ is strictly decreasing in Q for all $Q \geq \overline{Q}$.*

Proof. Taking the derivative of aggregate profit with respect to per-good output Q, yields

$$\frac{d\Pi(Q,Q)}{dQ} = 2\left[P(Q,Q)-c+Q\partial_1 P(Q,Q)\right]+2Q\partial_2 P(Q,Q).$$

By (A.1), the second term on the RHS is strictly negative for all $Q > 0$ such that $P(Q, Q) > 0$. The term in brackets on the RHS is equal to zero at $Q = \overline{Q}$, implying that $d\Pi(\overline{Q}, \overline{Q})/dQ < 0$. Taking the derivative of the term in brackets with respect to Q, we obtain

$$\begin{aligned}\frac{d\left[P(Q,Q) - c + Q\partial_1 P(Q,Q)\right]}{dQ} &= 2\partial_1 P(Q,Q) + Q\partial_{11}^2 P(Q,Q) + \partial_2 P(Q,Q) + Q\partial_{12}^2 P(Q,Q) \\ &< 0,\end{aligned}$$

where the inequality follows as (A.2) implies that both the sum of the first two terms on the RHS and the sum of the last two terms on the RHS are strictly negative for all Q such that $P(Q, Q) > 0$. Hence, we have $d\Pi(Q, Q)/dQ < 0$ for all $Q \geq \overline{Q}$. ■

For the next result, we require the following condition on demand:

(B.2) For any $Q_i, Q_j \geq 0$ such that $P(Q_i, Q_j) > 0$, we have

$$2\left[\partial_1 P(Q_i,Q_j) - \partial_2 P(Q_i,Q_j)\right] + Q_i\left[\partial_{11}^2 P(Q_i,Q_j) - 2\partial_{12}^2 P(Q_i,Q_j) + \partial_{22}^2 P(Q_i,Q_j)\right] < 0.$$

In the case of linear demand, (B.2) simplifies to $\partial_1 P(Q_i, Q_j) < \partial_2 P(Q_i, Q_j)$, and thus holds by (A.1). We obtain:

Lemma 5 *Suppose (B.2) holds. Then, for a fixed level of aggregate output* $Q = Q_i + Q_j$*, aggregate profit* $\Pi(Q_i, Q_j) = [P(Q_i, Q_j) - c]Q_i + [P(Q_j, Q_i) - c]Q_j$ *is strictly decreasing in* Q_i *for* $Q_i \geq Q/2$*.*

Proof. Taking the derivative of aggregate profit with respect to Q_i, holding $Q_i + Q_j = Q$ fixed, yields

$$\left.\frac{d\Pi(Q_i,Q_j)}{dQ_i}\right|_{Q_i+Q_j=Q} = \frac{\partial\Pi(Q_i,Q_j)}{\partial Q_i} - \frac{\partial\Pi(Q_i,Q_j)}{\partial Q_j},$$

which can be expressed as:

$$\left.\frac{d\Pi(Q_i,Q_j)}{dQ_i}\right|_{Q_i+Q_j=Q} = \Psi(Q_i,Q_j) - \Psi(Q_j,Q_i), \tag{20}$$

where

$$\Psi(Q_i,Q_j) \equiv P(Q_i,Q_j) - c + Q_i\left[\partial_1 P(Q_i,Q_j) - \partial_2 P(Q_i,Q_j)\right].$$

Note that if $Q_i = Q_j = Q/2$, then the RHS of (20) is equal to zero. We claim that (20) is negative whenever $Q_i > Q_j$. To show this, we take the derivative of Ψ with respect to Q_i, holding $Q_i + Q_j = Q$ fixed:

$$\begin{aligned}
\left.\frac{d\Psi(Q_i,Q_j)}{dQ_i}\right|_{Q_i+Q_j=Q} &= \frac{\partial\Psi(Q_i,Q_j)}{\partial Q_i} - \frac{\partial\Psi(Q_i,Q_j)}{\partial Q_j} \\
&= 2\left[\partial_1 P(Q_i,Q_j) - \partial_2 P(Q_i,Q_j)\right] \\
&\quad + Q_i\left[\partial_{11}^2 P(Q_i,Q_j) - 2\partial_{12}^2 P(Q_i,Q_j) + \partial_{22}^2 P(Q_i,Q_j)\right] \\
&< 0,
\end{aligned}$$

where the inequality follows from (B.2). Hence, $\Psi(Q_i, Q_j) < \Psi(Q_j, Q_i)$ if $Q_i > Q_j$, implying that (20) is negative if $Q_i > Q_j$. ■

Combining the two lemmas, we obtain the following result:

Lemma 6 *Assume (B.2). Consider any* (Q_i, Q_j) *such that* $Q_i + Q_j > 2\overline{Q}$*. Then,* $\Pi(Q_i, Q_j) < \Pi(\overline{Q}, \overline{Q})$.

Proof. This is an immediate implication of Lemmas 4 and 5. ■

Lemmas 3 and 6 jointly imply:

Proposition 7 *Assume (B.1) and (B.2). Then, game* Γ *has property (P.1).*

Proof. This is an immediate implication of Lemmas 3 and 6. ■

For Property (P.2), we require another condition on demand:

(B.3) For any $Q_i, Q_j \geq 0$ such that $P(Q_i, Q_j) > 0$ and $P(Q_j, Q_i) > 0$, and any $q_j \in [0, Q_j]$, we have

$$\begin{aligned}
&\left(2\partial_1 P(Q_i,Q_j) + Q_i\partial_{11}^2 P(Q_i,Q_j) + q_j\partial_{22}^2 P(Q_j,Q_i)\right)\partial_1 P(Q_j,Q_i) \\
&- \left(\partial_2 P(Q_i,Q_j) + Q_i\partial_{12}^2 P(Q_i,Q_j) + q_j\partial_{12}^2 P(Q_j,Q_i)\right)\partial_1 P(Q_j,Q_i) \\
&+ \left(\partial_2 P(Q_j,Q_i) + (Q_j - q_j)\partial_{12}^2 P(Q_j,Q_i)\right)\partial_2 P(Q_j,Q_i) \\
&-(2\partial_1 P(Q_j,Q_i) + (Q_j - q_j)\partial_{11}^2 P(Q_j,Q_i))\partial_2 P(Q_j,Q_i) \\
< \;& (2\partial_1 P(Q_i,Q_j) + Q_i\partial_{11}^2 P(Q_i,Q_j) + q_j\partial_{22}^2 P(Q_j,Q_i))\left(2\partial_1 P(Q_j,Q_i) + (Q_j - q_j)\partial_{11}^2 P(Q_j,Q_i)\right) \\
&- \left(\partial_2 P(Q_i,Q_j) + Q_i\partial_{12}^2 P(Q_i,Q_j) + q_j\partial_{12}^2 P(Q_j,Q_i)\right)\left(\partial_2 P(Q_j,Q_i) + (Q_j - q_j)\partial_{12}^2 P(Q_j,Q_i)\right).
\end{aligned}$$

In the case of linear demand, (B.3) simplifies to $2(\partial_2 P(Q_i, Q_j))^2 < 2(\partial_1 P(Q_i, Q_j))^2 + 3\partial_1 P(Q_i, Q_j)\,\partial_2 P(Q_i, Q_j)$, which holds by (A.1).

Proposition 8 *Assume (B.1) and (B.3). Then, game* Γ *has property (P.2).*

Proof. Suppose $\hat{q}_{A2} = 0$. We need to show that $\partial\tilde{Q}/\partial\hat{q}_{B1} < 1$. From the proof of Lemma 3, we have

$$\frac{\partial\tilde{Q}_B}{\partial\hat{q}_{B1}} = \frac{\hat{\lambda}_{AA}\partial_1 P\left(\tilde{Q}_B, \tilde{Q}_A\right) + \hat{\lambda}_{BA}\partial_2 P\left(\tilde{Q}_B, \tilde{Q}_A\right)}{\hat{D}}.$$

Condition (B.3) ensures that this expression is less than one. ■

A.6 Proof of Proposition 6

Let

$$r_{A1}(q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}) \in \arg\max_{q_{A1}} \hat{\Pi}_1\left(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}\right)$$

and

$$r_{B2}(q_{A1}; \hat{q}_{B1}, \hat{q}_{A2}) \in \arg\max_{q_{B2}} \hat{\Pi}_1\left(q_{A1}, q_{B2}; \hat{q}_{B1}, \hat{q}_{A2}\right)$$

denote the best-response mappings of firms 1 and 2. An immediate observation is that $r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il}) > 0$ only if $P\left(r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il}) + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) \geq c > 0$. The first-order condition for q_{ik}, $i \in \{A, B\}$, $k \in \{1, 2\}$, is given by

$$\Phi(q_{ik}; q_{lj}; \hat{q}_{jk}, \hat{q}_{il}) \equiv P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) - c + q_{ik}\partial_1 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + \hat{q}_{jk}\partial_2 P\left(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}\right) = 0.$$

Note that

$$\begin{aligned}\frac{\partial\Phi(q_{ik}; q_{lj}; \hat{q}_{jk}, \hat{q}_{il})}{\partial q_{ik}} &= 2\partial_1 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + q_{ik}\partial_{11}^2 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + \hat{q}_{jk}\partial_{22}^2 P\left(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}\right) \\ &< 0,\end{aligned}$$

where the inequality follows by (A.4), provided that $P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) > 0$. Hence, the best response $r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il})$ is unique. Moreover, if $r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il}) > 0$ and thus $P\left(r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il}) + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) > 0$, we have

$$\begin{aligned}\frac{\partial r_{ik}(q_{jl}; \hat{q}_{jk}, \hat{q}_{il})}{\partial q_{jl}} &= -\left.\frac{\partial_2 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + q_{ik}\partial_{12}^2 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + \hat{q}_{jk}\partial_{12}^2 P\left(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}\right)}{2\partial_1 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + q_{ik}\partial_{11}^2 P\left(q_{ik} + \hat{q}_{il}, \hat{q}_{jk} + q_{jl}\right) + \hat{q}_{jk}\partial_{22}^2 P\left(\hat{q}_{jk} + q_{jl}, q_{ik} + \hat{q}_{il}\right)}\right|_{q} \\ &\in (-1, 0),\end{aligned}$$

where the inequalities follow by (A.4). It follows that there exists a unique Nash equilibrium. Moreover, the equilibrium is "stable" in the usual sense.

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