First-Party Content, Commitment and Coordination in Two-Sided Markets

Andrei Hagiu*and Daniel Spulber†

June 1, 2011

Abstract

We study the effect of two-sided platforms' ability to invest in first-party content on their optimal pricing strategies. If first-party content and third-party seller participation are complements (substitutes) then: i) a monopoly platform facing favorable expectations invests more (less) in first-party content than a platform facing unfavorable expectations; ii) the platform facing unfavorable expectations is more likely to subsidize sellers (buyers) when its investment in first-party content is higher. These results hold with both simultaneous and sequential entry of the the two sides.

With two competing platforms - an incumbent facing favorable expectations and an entrant facing unfavorable expectations - and singlehoming on one side of the market, the incumbent always invests (weakly) more in first-party content relative to the case in which it is a monopolist.

Keywords: Market Intermediation, Search, Two-Sided Markets, Platform Design.

JEL Classifications: L1, L2, L8

1 Introduction

Two-sided platforms face a challenging coordination problem that consists of attracting both buyers and sellers. Participation by buyers and by sellers each depends on their expectations of participation on the other side of the market. Buyers and sellers derive cross-market benefits from participation on the other side of the market, which may be the result of market thickness, variety and scale effects, and connectivity of communications networks (cf. Spulber, 2010). In order to solve this problem, many firms resort to the provision of "first-party content," which makes participation more attractive to one side (typically, users), independently of the presence of the other side - sellers, which we call "third-party content providers." Examples of first-party content include: objective search results, maps, news, entertainment and weather provided by search engines

*Harvard University, ahagiu@hbs.edu.

†Northwestern University, jems@kellogg.northwestern.edu.

and Internet portals; killer software applications and online marketplaces provided by operating system vendors; killer games, motion-activated controllers and online gaming portals provided by videogame console manufacturers; payment systems, and product information and shipping services provided by e-commerce sites, and so on. The prevalence of first-party content in two-sided markets illustrates its strategic importance.

This paper examines how the ability of two-sided platforms to use first-party content affects the pricing strategies they use to coordinate participation by buyers and by sellers. We show that the incentives for firms to use first-party content depend crucially on the nature of buyers' and sellers' expectations. The basic mechanism is as follows. Buyers and sellers play a participation subgame based on the amount of first-party content and the prices chosen by a two-sided platform, where first-party content raises the participation of buyers. Given the inter-dependence between buyer and seller participation decisions, there are typically multiple Nash equilibria in this subgame. When buyers and sellers coordinate on an equilibrium with high participation rates (favorable expectations) the platform firm solves a straightforward profit maximizaton problem. When buyers and sellers coordinate on an equilibrium with low or zero participation (unfavorable expectations), the platform firm faces a more difficult strategic problem. Its choice of first-party content and prices must satisfy the constraint of generating a single Nash equilibrium.

Our analysis reveals that the strategic use of first-party content by two-sided platforms depends in important ways on the relationship between first-party content and seller participation (or third-party content). The first-party videogames offered by Microsoft for its Xbox console (e.g. Halo) and by Nintendo for Wii (e.g. Super Mario, Wii Sports) are *substitutes* for games provided by third-party publishers like Electronic Arts and Take Two Interactive. The same goes for the apps provided by Apple, HTC, Samsung and other smart phone manufacturers on their respective phones. On the other hand, when Amazon and eBay offer expedited shipping, product information and payment systems, they are enhancing the value of the products sold by affiliated sellers: in these cases, first-party content and third-party seller participation are *complements*. Similarly, Microsoft's Kinnect motion-activated game controller and Xbox Live online gaming system are complements to third-party developers' games for Xbox. In the case of Bing, Google and Yahoo!, objective search results (web pages, images, shopping, scholarly articles, books) and enhanced content offerings (e.g. maps, video, blogs, news) may be either complements or substitutes to sponsored search or display ads. Similarly, Facebook's social networking site provides some first-party content that is complementary to third-party applications (information sharing with friends, ability to comment, group memberships, notifications from fan sites), whereas others are substitutes that compete with or detract from third-party content (news feeds, games, digital gifts, e-mail notifications, and friend suggestions).

Our main results are as follows. First, we consider a monopoly platform that serves a two-sided market with simultaneous participation decisions by buyers and by sellers. We compare the platform's profit-maximizing strategies (investments in first party content and prices on the two sides) when buyers and sellers have unfavorable expectations with the case in which they have favorable expectations. We show that a profit-maximizing platform that faces unfavorable expectations offers more (less) first-party content to buyers than a platform that faces favorable expectations when content and seller participation are substitutes (complements) in demand. This result stems from the following tradeoff: the platform facing unfavorable expectations may need to invest more in first-party content in order to make up for its greater difficulty of attracting sellers, while the platform facing favorable expectations may have stronger incentives to invest in first-party content (which can also be interpreted as an investment in platform quality for buyers) because it is able to capture a larger share of surplus from buyers and sellers. We also show that whether first-party content and third-party seller participation are complements or substitutes determines the pricing strategy chosen by the platform facing unfavorable expectations. Specifically, in the case of complementarity, a higher (lower) investment in first-party content increases (decreases) the benefits derived by buyers from the presence of third-party sellers. This means the buyer side is easier (harder) to attract relatively to the seller side, so the platform finds it optimal to subsidize sellers (buyers) and make most of its profits on buyers (sellers). The mechanism is reversed when first-party content and seller participation are substitutes.

Second, we consider a monopoly firm that serves a two-sided market with sequential participation decisions: sellers decide first, followed by buyers. The results obtained in the case with simultaneous participation decisions continue to hold. There is however an additional strategic decision which arises in this case: should the platform commit to the price it will charge buyers at the time it sets its price for sellers, or wait until sellers have made their participation decisions? A key result is that the platform facing favorable expectations always prefers to commit (in order to avoid a hold-up problem vis-a-vis sellers), while the platform facing unfavorable expectations may find it profitable not to commit. The latter possibility occurs when the platform's optimal pricing strategy is to subsidize sellers and recoup on buyers. Indeed, with this strategy, commitment would require setting a lower price to buyers than what the platform is able to charge *after* sellers have made their participation decisions. Since the goal of subsidizing sellers is to extract as much surplus from buyers as possible, it is better not to commit.

Third, we consider two competing platforms which choose first-party content strategically. The two platforms offer homogenous platform services and content, but they differ in the nature of expectations that market participants hold: one firm (the incumbent) benefits from favorable expectations while the other (the entrant) encounters unfavorable expectations. We show that the incumbent firm's profit-maximizing strategy deters entry and generates positive profits. Furthermore, when one side single-homes and the other multi-homes, the incumbent always invests more in first-party content relative to the case in which it is an unconstrained monopolist.

Our paper introduces investment in first-party content by two-sided platforms, which, in addition to prices charged to buyers and to sellers, leads to optimization problems involving three variables (Spulber, 2010). The model we use builds upon Caillaud and Jullien (2003) and Hagiu (2006), who study pricing by two-sided platforms with favorable versus unfavorable expectations. The key novelty relative to this work is that we allow for investments in first-party content. Our work also extends Hagiu (2006)'s analysis of commitment by two-sided platforms facing sequential entry: our model and the insights we obtain are richer both because of the presence of first-party content and because platform demand on the buyer/user side is linear (in contrast, both Caillaud and Jullien (2003) and Hagiu (2006) assume members of each side are identical).

Platform firms face a coordination problem when buyers and sellers cannot coordinate with each other. Spulber (2007, 2008a, 2008b) considers two-sided markets where there is direct coordination among buyers and sellers. When there is more than one Nash equilibrium in the participation subgame game, and buyers and sellers cannot coordinate directly, players need not wind up at one of the equilibria because there is no convergence of equilibrium expectations. Players may experience confusion and split moves so that almost every combination of player strategies is possible (Farrell and Klemperer, 2009). Jackson and Wilkie (2005) show that when players choose action-contingent side payments noncooperatively, there is no equilibrium that results in the efficient outcome. When players can communicate and form cooperative agreements, they are likely to choose the Pareto-dominant Nash equilibrium, which corresponds to favorable expectations in the participation subgame. In the participation subgame, the Pareto-inferior Nash equilibrium generally corresponds to unfavorable expectations. Various game-theoretic refinements select the Pareto-dominant Nash equilibrium, Aumann (1959), Bernheim, Peleg, and Whinston (1987), and Xue (2000). Without requiring Pareto dominance, Ambrus and Argenziano (2009) apply coalitional rationalizability (Ambrus, 2006) to coordination in a two-sided market, allowing groups of players to implicitly coordinate on strategies that are mutually beneficial by deleting strategies. Our approach in this paper allows for multiple Nash equilibria in the participation subgame. Platforms provide coordination through first-party content and pricing by eliminating the multiplicity of equilibria when there are unfavorable expectations.

The remainder of the paper is organized as follows. Section 2 introduces the basic model and analyzes the case of a monopoly two-sided platform with simultaneous entry of the two sides. Section 3 extends the analysis of a monopoly platform by allowing sequential participation decisions (sellers choose whether or not to participate before buyers). Section 4 deals with platform competition between an incumbent - which benefits from favorable expectations - and an entrant - which encounters unfavorable expectations from market participants. Section 5 concludes the discussion.

2 Monopoly platform with simultaneous entry

2.1 Model set-up

This section lays out the basic version of our model. A monopoly two-sided platform firm connects a continuum of buyers i uniformly distributed over $[0,1]$, and a continuum of sellers j uniformly distributed over $[0,M]$. The platform charges participation fees p to buyers and w to sellers. In addition, the platform can offer buyers an amount x of first-party content (hereafter content) at cost $C(x)$, where $C(.)$ is increasing and convex. The platform's content encompasses is a measure of information, entertainment, quality of service, ease of use, and other non-pecuniary benefits provided to buyers.

Buyer i's net benefit from joining the platform is:

$$U_i(m,x,p) = u(m,x) - i - p$$

The utility function $u(m,x)$ represents buyers' preferences over combinations of the plaform's content x and seller participation m. Buyer i incurs a personal cost of adopting the platform equal to i, so that buyers have different willingness to pay for participation. Let the buyer's utility function $u(.,.)$ be increasing, twice continuously differentiable and concave or linear in each of its arguments. We assume that buyers obtain no benefits if the platform offers no content and no sellers join the platform, i.e. $u(0,0) = 0$. A buyer joins the platform if and only if her expected net benefit is non-negative.

Content x and seller participation m can be either substitutes or complements in demand. They are said to be substitutes if $\frac{\partial^2 u}{\partial m \partial x} < 0$ and complements if $\frac{\partial^2 u}{\partial m \partial x} > 0$. If both the platform and sellers provide information or entertainment, for example, then content and seller participation are likely to be substitutes. If the platform's content is a service which enhances the value of sellers' products, then content and seller participation are likely to be complements.

Sellers are assumed to be *identical* and each seller's net benefit from joining the platform is:

$$V(n,w) = \pi n - w - \phi$$

where $\pi > 0$ is the profit per buyer made by each seller and $\phi > 0$ is a seller's fixed cost of "porting" his product to the platform. A seller chooses to join if expected benefits are non-negative. Let π and ϕ be exogenous parameters. We assume that ϕ is small enough so that: i) no platform finds it profitable to be simply a content provider to buyers without any sellers, and ii) platforms are viable (i.e. make non-negative profits) under all types of expectations.[1]

Buyer i's platform participation decision is represented by a strategy b_i that takes two values, $b_i = 1$ if the buyer joins the platform and $b_i = 0$ if she does not. Similarly, seller j's platform participation decision is represented by $s_j = 1$ if the seller joins the platform and $s_j = 0$ if the seller does not join. The participation outcome is represented by $(n, m) \in [0, 1] \times [0, M]$, where $n \equiv \int_0^1 b_i di$ is the number of buyers and $m \equiv \int_0^1 s_j dj$ is the number of sellers who join the platform.

Some observations regarding this set-up are in order. First, we only allow the platform to charge fixed participation fees on both sides. This turns out to be the richest scenario in our model. If the platform were to charge variable fees (royalties) to sellers instead or in addition to fixed access fees, the set of its optimal pricing strategies would be identical or strictly smaller, resulting in identical or strictly lower profits (see note 4 below). Second, we have assumed that π is independent of m and x. Introducing competition among sellers (i.e. allowing π to be decreasing in m) would make the derivation of the various equilibria (favorable and unfavorable expectations) more difficult but their nature would remain unchanged.[2] Allowing π to depend on x (either positively or negatively) would change the specific expressions of the various first-order conditions derived below, but not the comparison between them, which is what we are ultimately interested in. Thus, the substance of our analysis would be unchanged.

2.2 Monopoly platform with simultaneous entry

The game with simultaneous entry of the two sides - buyers and sellers - has two stages. In the first stage, the profit-maximizing platform chooses the amount of first-party content x, as well as participation prices p and w for buyers and sellers respectively: (p, w, x) are publicly announced and observed by all players. In the second stage, individual buyers and sellers simultaneously choose whether or not to join the platform, based on their individual benefits from participation, the content and prices chosen by the firm, and their expectations about market participation by others. There is no communication or coordination among buyers or sellers, which means that buyers and sellers play a Nash non-cooperative game in participation decisions.

The equilibrium of the full game consists of the first-stage choices of the platform, (p^*, w^*, x^*), and the Nash equilibrium vector of buyer participation decisions and seller participation decisions

[1] Based on the analysis that follows, the precise form of this assumption is easily shown to be:

$$\max_x \left[\frac{u(0,x)^2}{4} - C(x) \right] \leq \min \left\{ \max_x \left[\frac{u(M,x)^2}{4} - C(x) \right], \max_x \left[\frac{(u(M,x) - M\pi)^2}{4} + M\pi u(0,x) - C(x) \right] \right\} - M\phi$$

.

[2] Two-sided platform pricing with competition among members on at least one side has been studied at length elsewhere (cf. Belleflamme and Toulemonde (2009), Hagiu (2009), Nocke et al. (2007)).

in the second-stage subgame, (b^*, s^*). Each buyer's and each seller's participation decision depends on their respective expectations about participation on the other side of the market. Because buyers' and sellers' actions only depend on the total participation on the other side of the market, expectations can be represented in terms of total buyer participation and total seller participation, (n^e_j, m^e_i), with $i \in [0, 1]$ and $j \in [0, M]$.

We assume that expectations are consistent across buyers and across sellers and fulfilled in equilibrium. As usual in the context of network effects, there can be multiple participation equilibria even with consistent and fulfilled expectations. Indeed, the equilibrium buyer participation $n = n(x, p, w)$ and the equilibrium seller participation $m = m(x, p, w)$ solve the following two equations:

$$n = \max\{u(m, x) - p, 0\}$$

$$m = M \times H(\pi n - w - \phi) = \begin{cases} M & \text{if} \quad \pi n - w - \phi \geq 0 \\ \\ 0 & \text{if} \quad \pi n - w - \phi < 0 \end{cases}$$

where $H(x)$ is a Heaviside step function, with $H(x) = 1$ for $x \geq 0$ and $H(x) = 0$ otherwise.

In our model, the concavity of buyer benefits in seller participation and step-function shape of seller benefits in buyer participation narrow the number of possible stable equilibria to two: a low-participation equilibrium and a high-participation equilibrium. For any price pair (p, w) and content x, the configuration of buyer participation and seller participation typically looks like Figure 1.

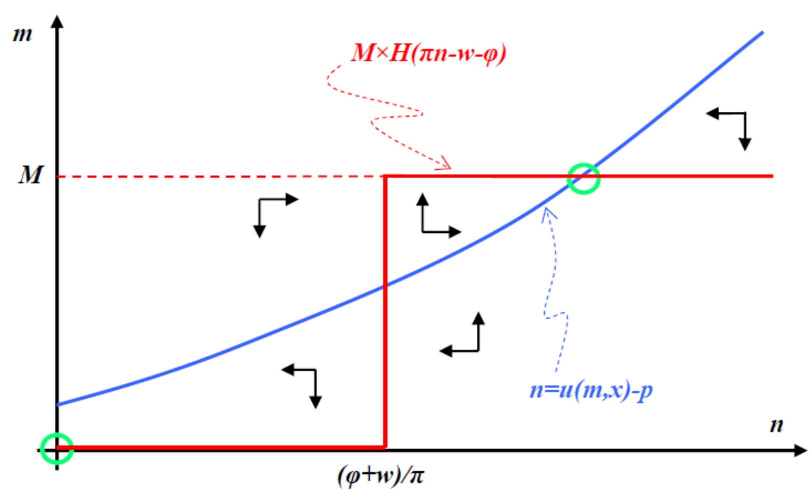

Figure 1

The circles identify the stable equilibria, with arrows indicating dynamic adjustment. Denote the low-participation Nash equilibrium (0 in this case) as the "unfavorable expectations" equilibrium and the high-participation equilibrium as the "favorable expectations" equilibrium - given prices and content (p, w, x).

The platform firm's profits are:

$$\Pi(p, w, x) = pn(x, p, w) + wm(x, p, w) - C(x)$$

The firm chooses content x and prices p and w to maximize profits given the effects of its actions on the outcome of the participation subgame played by buyers and sellers. Note that content x changes the *shape* of each buyer's net benefit curve $u(m, x) - p$, while the price p simply shifts the net benefit curve up or down without altering its shape.

In theory, even with at most two possible participation equilibria for any given prices and content vector (p, w, x), there can be an infinity of distinct, two-sided demand correspondences $(p, w, x) \rightarrow (m, n)$. To simplify things however, we will focus our analysis on two polar cases corresponding to two specific demand correspondences. In the first case, buyers and sellers *always* (i.e. for any (p, w, x)) coordinate on the equilibrium with the lowest possible levels of participation on both sides - we say that buyers and sellers hold unfavorable expectations for the platform. In the second case, buyers and sellers *always* coordinate on the equilibrium with the highest possible levels of participation on both sides - we say that they hold favorable expectations for the platform. The unfavorable expectations outcome describes the problems faced by entrepreneurial entrants without established brands and name recognition. In contrast, favorable expectations describes the situation of established firms with well-known brands that are extending existing businesses through diversification. Transaction costs suggest another interpretation of market expectations. Unfavorable expectations describe markets with high transaction costs that make it difficult for buyers and sellers to engage in pre-play communication, while favorable expectations describe markets with low transaction costs in which buyers and sellers can coordinate through some pre-play communication.

Favorable expectations

Suppose first that the platform benefits from favorable expectations. The platform maximizes profits given buyer participation subject to the sellers' individual rationality constraint,

$$\max_{p,w,x} \{p \times [u(M, x) - p] + w \times M - C(x)\}$$

$$\text{subject to } \pi [u(M, x) - p] - w - \phi \geq 0.$$

The seller's participation condition is binding, so the platform's profit maximization problem is equivalent to:[3]

$$\max_{p,x}\left\{\left(p+M\pi\right)\left[u\left(M,x\right)-p\right]-C\left(x\right)-M\phi\right\}. \tag{1}$$

The first-order conditions for the platform's choices of first-party content and buyer price are:

$$\begin{aligned} p_f &= \frac{u\left(M,x_f\right)-M\pi}{2} \\ \frac{u_x\left(M,x_f\right)\left[u\left(M,x_f\right)+M\pi\right]}{2} &= C'\left(x_f\right) \end{aligned} \tag{2}$$

The price to sellers is given by the sellers' participation condition: the platform extracts the entire seller surplus. Notice that the profit-maximizing buyer price equals the ratio of the marginal cost to the marginal benefit of first party content, discounted by the benefits created by the participation of an additional buyer on the seller side:

$$p_f=\frac{C'\left(x_f\right)}{u_x\left(M,x_f\right)}-M\pi$$

This shows the platform's tradeoff between inducing buyer participation through price reductions and through first-party content.

Unfavorable expectations

Suppose now that the platform anticipates that it will face unfavorable expectations, i.e. that buyers and sellers will always coordinate on the equilibrium with the lowest levels of adoption. Then, to have a chance of making positive profits, the platform must set its prices so as to eliminate the unfavorable expectations equilibrium. The corresponding seller participation condition is:

$$\pi\max\left[u\left(0,x\right)-p,0\right]-w-\phi\geq 0,$$

which means that prices must be such that an individual seller finds it profitable to join even when (s)he expects the platform will atract no other sellers. The resulting platform profits are:

$$\max_{p,x}\left\{p\left[u\left(M,x\right)-p\right]+M\pi\max\left[u\left(0,x\right)-p,0\right]-M\phi-C\left(x\right)\right\} \tag{3}$$

The optimal buyer price and first-party content $\left(p_{uf},x_{uf}\right)$ cannot be such that $u\left(0,x_{uf}\right)=p_{uf}$. Indeed, if this were the case, the optimality of p_{uf} given x_{uf} would require $u\left(0,x_{uf}\right)\geq u\left(M,x_{uf}\right)/2$ and $u\left(0,x_{uf}\right)+M\pi/2\leq u\left(M,x_{uf}\right)/2$, which cannot be satisfied simultaneously. Thus, the buyer's benefit from first-party content without seller participation, $u\left(0,x_{uf}\right)$, is either strictly lower than or strictly greater than the buyer price, p_{uf}. This implies that there are only two *possible* solutions

[3]The price offered to sellers equals $w=\pi\left[u\left(M,x\right)-p\right]-\phi$.

to the platform's optimization problem. Depending on parameter values, only one or both of these solutions might be feasible. When both are feasible, the platform chooses the one yielding higher profits.

The first possible solution, (p_{uf1}, x_{uf1}), is defined by:

$$\begin{aligned} p_{uf1} &= \frac{u\left(M, x_{uf1}\right)}{2} \\ \frac{u_x\left(M, x_{uf1}\right) u\left(M, x_{uf1}\right)}{2} &= C'\left(x_{uf1}\right) \end{aligned} \tag{4}$$

and must satisfy $p_{uf1} > u\left(0, x_{uf1}\right)$ to be viable. This implies that the seller price fully subsidizes the sellers' fixed costs, i.e. $w_{uf1} = -\phi$. The platform therefore generates all of its profits from the buyer side of the market. Note that the buyer price equals $p_{uf1} = C'\left(x_{uf1}\right)/u_x\left(M, x_{uf1}\right)$, i.e. the ratio of marginal cost to marginal benefit of first-party content: there is no discount here because the platform subsidizes sellers in order to extract as much surplus as possible from buyers. We refer to this solution as the Seller Subsidy Strategy and illustrate it in Figure 2 below. Comparing Figure 2 to Figure 1, it is apparent that the Seller Subsidy Strategy eliminates the low participation equilibrium by moving the seller participation curve, $m = M \times H\left(\pi n - w - \phi\right)$, to the left.

The second possible solution, (p_{uf2}, x_{uf2}), is defined by:

$$\begin{aligned} p_{uf2} &= \frac{u\left(M, x_{uf2}\right) - M\pi}{2} \\ \frac{u_x\left(M, x_{uf2}\right)\left[u\left(M, x_{uf2}\right) - M\pi\right]}{2} + M\pi u_x\left(0, x_{uf2}\right) &= C'\left(x_{uf2}\right) \end{aligned} \tag{5}$$

and must satisfy $p_{uf2} < u\left(0, x_{uf2}\right)$ to be viable, which implies that $w_{uf2} = \pi\left[u\left(0, x_{uf2}\right) - p_{uf2}\right] - \phi > -\phi$. In other words, the price to buyers is relatively low in order to ensure that even when each individual seller expects no sellers to join, there is sufficient buyer demand to make it profitable for that seller to join. The buyer price equals the marginal cost divided by the marginal benefit of content, discounted by a subsidy term proportional to the benefit created by an additional buyer on the seller side:

$$p_{uf2} = \frac{C'\left(x_{uf2}\right)}{u_x\left(M, x_{uf2}\right)} - \frac{M\pi u_x\left(0, x_{uf2}\right)}{u_x\left(M, x_{uf2}\right)}$$

We refer to the second solution as the Buyer Attraction Strategy and illustrate it in Figure 3. Comparing Figure 3 to Figure 1, this strategy eliminates the low participation equilibrium by moving the buyer participation curve $n = u\left(m, x\right) - p$ to the right.

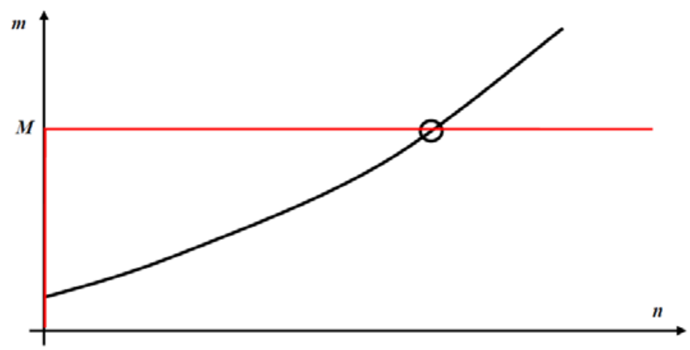

Figure 2

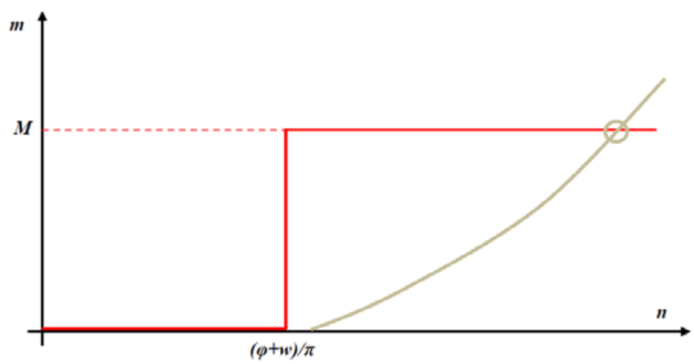

Figure 3

These two solutions correspond to different pricing and content strategies for solving the chicken-and-egg problem by the platform. Both strategies rely on convincing sellers to join despite unfavorable expectations. The first strategy simply subsidizes sellers' fixed costs and then charges a high price to buyers, whereas the second strategy charges a low price to buyers and then charges a higher price to sellers.[4] The following lemma, proven in the appendix, formalizes the platform's optimal choice of pricing strategies.

[4]In our model, if the platform were charging sellers variable fees (royalties) $\rho \in [0,1]$ instead of fixed fees w, only the second pricing strategy would be feasible. Indeed, the seller participation constraint would be $(1-\rho)\,\pi\,[u\,(0,x)-p] \geq \phi$, which would make it impossible to charge $p > u\,(0,x)$. Thus, the platform would have to charge $p \leq u\,(0,x)$ and ρ such that the constraint above is binding, leading to the same platform profits as the solution defined in (5). Therefore, the platform facing unfavorable expectations does weakly better with fixed fees than with royalties.

Meanwhile, the platform facing favorable expectations would obtain the exact same profits with royalties as with fixed fees. Indeed, if it charged royalties ρ, its seller participation constraint would be $(1-\rho)\,\pi\,[u\,(M,x)-p] \geq \phi$, yielding (1).

Lemma 1 *With unfavorable expectations and simultaneous entry, the platform's optimization problem is equivalent to solving* $\max_x \Pi(x)$*, where:*

$$\Pi(x) = \begin{cases} \frac{u(M,x)^2}{4} - C(x) - M\phi & \text{if } \; u(M,x) \geq 2u(0,x) + \frac{M\pi}{2} \\ \\ \frac{[u(M,x)-M\pi]^2}{4} + M\pi u(0,x) - C(x) - M\phi & \text{if } \; u(M,x) \leq 2u(0,x) + \frac{M\pi}{2} \end{cases} \tag{6}$$

The resulting solution is either x_{uf1}*, defined in (4) and corresponding to the Seller Subsidy strategy, or* x_{uf2}*, defined in (5) and corresponding to the Buyer Attraction Strategy.*

An important implication is that the level of investment in first-party content x chosen by a platform facing unfavorable expectations determines its optimal choice of pricing strategy. Whether the Seller Subsidy Strategy or the Buyer Attraction Strategy is optimal for higher x in turn depends on the nature of buyer preferences over first and third party content, that is, it depends on whether m and x are complements or substitutes. Before focusing on this aspect with several examples, it is useful (and straightforward) to compare the levels of investments in first-party content made by platforms facing favorable, respectively unfavorable expectations. The following proposition summarizes this comparison:

Proposition 1 *When the platform facing unfavorable expectations finds it optimal to use the Seller Subsidy Strategy, it always provides less first-party content* x *than the platform benefitting from favorable expectations. When the platform facing unfavorable expectations finds it optimal to use the Buyer Attraction Strategy, it provides more (less) first-party content than the platform benefitting from favorable expectations if content and seller participation are substitutes (complements) for buyers.*

Proof. Assume second-order conditions are verified so that x_f, x_{uf1} and x_{uf2} are uniquely defined. First, comparing (2) with (4), we have $x_f > x_{uf1}$ always. Second, comparing (2) with (5), we have $x_f \leq x_{uf2}$ if and only if $u_x(M, x_f) \leq u_x(0, x_f)$. ■

To interpret this proposition, note that first-party content is both a means of attracting participation and a means of extracting surplus from buyers and sellers. The dual role of first-party content makes it possible for either type of platform to choose a higher level of first-party content. A platform facing unfavorable expectations has a stronger incentive to invest in first-party content to attract participation. Conversely, a platform benefitting from favorable expectations has an easier time attracting the two sides, which may provide it with stronger incentives to invest in first-party content as a means of extracting surplus.

If first-party content and seller participation are complements in buyer demand, then the surplus-extraction effect dominates: a platform benefitting from favorable expectations always invests more in first-party content than a platform facing unfavorable expectations. On the other hand, if first party-content and seller participation are substitutes, then a platform facing unfavorable expectations invests more in first party content if and only if it pursues the Buyer Attraction Strategy. Then, the participation-attraction effect dominates: a platform facing unfavorable expectations has to invest more in first-party content in order to compensate for sellers not showing up. The following table summarizes these scenarios by taking the perspective of the platform facing unfavorable expectations:

	Complements	Substitutes
Buyer Attraction	*underinvest*	*overinvest*
Seller Subsidy	*underinvest*	*underinvest*

The notions of over- and under-investment are relative to the platform facing favorable expectations. Based on these results, we can derive some implications for the evolution of vertical scope of two-sided platforms over time. Thus, in contexts in which first-party content is a substitute for third-party content and a platform subsidizes buyers, one should expect it will start off by being highly integrated in first-party content and then, as it becomes established in the marketplace, rely less and less on first-party relative to third-party content. In all other three contexts, one would expect platforms to start off with relatively low levels of integration in first-party content and then progressively become more integrated as they overcome unfavorable expectations and gain widespread market acceptance.

In order to achieve a better understanding of the effects of first-party content on optimal pricing strategies, it is useful to fully derive the optimal platform strategies with three examples of specific utility formulations.

Example 1 *Let* $u(m,x) = sm + x$ *and* $C(x) = c\frac{x^2}{2}$, *where*[5] $s > 0$ *and* $2c > 1$. *Then:*

- *the platform facing favorable expectations chooses* $x_f = \frac{M(s+\pi)}{2c-1}$.
- *the platform facing unfavorable expectations chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{Ms}{2c-1} < x_f$ *if* $c\left(1 - \frac{\pi}{2s}\right) \geq 1$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{M(s+\pi)}{2c-1} = x_f$ *if* $c\left(1 - \frac{\pi}{2s}\right) \leq 1$.

In this example, x and m are neither substitutes nor complements ($\partial^2 u/\partial m \partial x = 0$). They can be thought of as applications, games, or features offered to buyers by the platform (x) and

[5] The last condition ensures concavity of all profit expressions in (p, x).

by third-party developers (m). Buyers may however place a different weight, s, on a third-party relative to a first-party feature (e.g. Amazon users might prefer buying products sold and shipped by Amazon to buying the same products from affiliated merchants, in which case $s < 1$).

Example 2 *Let* $u(m,x) = u_0 + mx$ *and* $C(x) = c\frac{x^2}{2}$*, where*[6] $2u_0 > M\pi$ *and* $2c > M^2$*. Then:*

- *the platform facing favorable expectations chooses* $x_f = \frac{Mu_0 + M^2\pi}{2c - M^2}$,
- *the platform facing unfavorable expectations chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{Mu_0}{2c-M^2} < x_f$ *if* $c \leq \frac{2u_0M^2}{2u_0+M\pi}$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{Mu_0 - M^2\pi}{2c-M^2} < x_f$ *if* $c \geq \frac{2u_0M^2}{2u_0+M\pi}$.

In this example, u_0 is the (fixed) stand-alone utility offered by the platform to buyers, whereas m and x are complements: x can be thought of as a set of platform features that *enhance* the value of third-party products (e.g. eBay's PayPal payment system which make it easier for sellers to transact with buyers; Microsoft's Xbox Live service which makes it easier for third-party developers to endow their games with online gaming capabilities).

Example 3 *Let* $u(m,x) = m(1-x) + M_0x$ *and* $C(x) = c\frac{x^2}{2}$*, where*[7] $M_0 > M$ *and* $2c > M_0(M_0 - M) + \pi M(M_0 + M) > (M_0 - M)^2$*. Then:*

- *the platform facing favorable expectations chooses* $x_f = \frac{M(M_0-M)(1+\pi)}{2c-(M_0-M)^2}$,
- *the platform facing unfavorable expectations chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{M(M_0-M)}{2c-(M_0-M)^2} < x_f$ *if* $c(2-\pi) \geq 2M_0[M_0 - M + \pi M]$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{M[M_0-M+\pi(M_0+M)]}{2c-(M_0-M)^2} > x_f$ *if* $c(2-\pi) \leq 2M_0[M_0 - M + \pi M]$.

In this example, M_0 can be thought of as the number of products (e.g. applications, services, etc.) offered by the platform. First-party and third-party products are substitutes in the eyes of buyers, except for a vertical quality difference: first-party products are of quality $x \leq 1$, whereas third-party products are all of quality 1. In this context, buyers always consume first the m products offered by third-party sellers and then buy $(M_0 - m)$ products of quality x from the platform, obtaining total utility $m + (M_0 - m)x$. Thus, here m and x are substitutes.

Let us now interpret the results obtained with these examples. First, note that they clearly illustrate the general results from Proposition 1: the only case in which the platform facing unfavorable expectations invests more in first-party content is example 3, when first-party content and third-party products are strict substitutes ($x_{uf1} < x_f < x_{uf2}$).

[6] The first condition is needed so that both pricing strategies can be profitable on non-empty intervals in c whereas the second condition ensure concavity of all profit functions in (p, x).

[7] These two conditions ensure that $u(m,x)$ is increasing in both m and x, that all profit functions are concave in (p, x) and that all optimal x's are below 1.

Second, the focus of our analysis is on the way in which the cost c of first-party content impacts the choice of pricing strategy for a platform facing unfavorable expectations. Note that in example 2, the platform chooses the Seller Subsidy Strategy when c is *low*. By contrast, in examples 1 and 3, the platform chooses the Seller Subsidy Strategy when c is *high* (assuming $\pi < 2s$ and $\pi < 2$ respectively, so that both strategies are profitable on non-empty sets of c values). Consider example 2. A lower c increases the platform's incentives to invest in first-party content (x). Because first-party content and third-party products are strict complements from a buyer's perspective, raising x increases the benefits obtained by buyers from the presence of sellers (they are equal to mx) on the platform. On the other hand, seller benefits from the presence of buyers remain unchanged (equal to π). Thus, the *net* effect of an increase in x is to make the buyer side easier to attract *relative to* the seller side. Standard two-sided market logic (cf. Caillaud and Jullien 2003, Hagiu 2009) implies then that the platform is more likely to focus its subsidization efforts on the seller side and extract more profits from the buyer side, i.e. the platform will choose the Seller Subsidy Strategy. This is confirmed by plugging $u(m,x) = u_0 + mx$ in expression (6): in this case, the condition for a platform facing unfavorable expectations to choose the Seller Subsidy Strategy holds for high x.

Example 3 is interpreted in a similar way. Since first-party content and third-party products are strict substitutes, a higher x decreases the benefits obtained by buyers from the presence of sellers (they are equal to $1 - x$) on the platform. Consequently, the *net* effect of a lower c is to make the buyer side *harder* to attract relative to the seller side, which in turn makes the platform more likely to choose the Buyer Attraction Strategy. Note however that the same reasoning does not quite work for example 1 because there x has no impact on the benefits s obtained by buyers from the presence of sellers. For that example, the fact that a lower c makes the platform more likely to choose the Buyer Attraction Strategy is happenstance, determined by the specific structure of our model.

Finally, note that in all three examples the platform facing unfavorable expectations is more likely to choose the Buyer Attraction Strategy over the Seller Subsidy Strategy when π is larger (and vice-versa). This is a general result with a standard intepretation in the two-sided market literature: the Buyer Attraction Strategy involves offering a lower price to the buyer side and extracting more profits from the seller side, therefore it is naturally more attractive when the benefits that sellers obtain from the presence of buyers are larger. Conversely, the Seller Subsidy Strategy should be more attractive when buyers' stand-alone utilities, or the benefits they derive from the presence of sellers, are larger. This is indeed the case in examples 1 and 2: the Seller Subsidy Strategy is more likely to be chosen when s and u_0 are larger. In example 3 however, the Seller Subsidy Strategy is more likely to be chosen for *lower* M_0. This is somewhat surprising given that M_0 can be interpreted as a measure of standalone utility offered to buyers: one would expect that higher M_0 should make buyers easier to attract and therefore make the platform more likely to subsidize sellers. The reason this intuition breaks down is the presence of first-party content, x, which changes the platform's optimal strategy in subtle ways. An increase in M_0 raises the effectiveness of investments in x, which means the platform has stronger incentives to invest in first-party content. But given the substitutability with third-party products, higher x decreases the benefits obtained by buyers from the presence of sellers (they are equal to $1 - x$) on the platform. Since seller benefits from the presence of buyers remain unchanged (equal to π), this means that the *net* effect of an increase in M_0 is to make the buyer side harder to attract *relative to* the seller side. Standard two-sided market logic implies then that the platform is more likely to focus its subsidization efforts on the buyer side, i.e. to choose the Buyer Attraction Strategy.

3 Monopoly platform with sequential participation decisions

In many two-sided markets, one side - typically sellers - "arrives" or has to be secured by the platform before the other side - typically buyers - can make its adoption decisions. This can be either for exogenously given technological reasons (e.g. videogame consoles have to approach independent game publishers 1-2 years before the planned launch of their gaming systems in order to allow enough time for game development) or a strategic choice by the platforms, in an effort to secure participation by one side earlier.

Consequently, in this section we modify the basic model laid out above by assuming sellers make their platform adoption decisions *before* buyers and the latter can observe the sellers' adoption decisions prior to making their own. Since buyers can observe sellers' decisions (i.e. the realization of m), they face no uncertainty and no coordination problem. On the other hand, sellers now face a coordination problem among themselves. Thus, going from simultaneous to sequential adoption by the two sides has transformed a *two-sided* coordination problem with *indirect* network effects into a *one-sided* coordination problem with *direct* network effects (among sellers).

There is however one key difference relative to standard pricing problems in the presence of direct network effects. Since buyers arrive after sellers, the platform has two options: it can either choose to commit (if possible) to the price it will charge buyers at the time it announces its price for sellers or it can wait until after sellers have made their adoption decisions and announce its price to buyers afterwards (which is of course factored in the sellers' decisions). We thus have two possible timings:

With commitment	Without commitment
1. a) Platform sets w, p and x	1. a) Platform sets w and x
1. b) Sellers decide whether or not to adopt	1. b) Sellers decide whether or not to adopt
2. Buyers observe everything and decide whether or not to adopt	2. a) Platform observes sellers' decisions and sets p
	2. b) Buyers observe everything and decide whether or not to adopt

If the platform chooses to commit, then we denote by p^c the buyer price it commits to in stage 1. If it does not commit, then in stage 2 it will choose a buyer price $p(m)$ which is a function of the number m of sellers who have adopted in stage 1. That price maximizes the platform's stage 2 profits, therefore it is given by:

$$p(m) = \arg\max_p \{p[u(m,x) - p]\} = \frac{u(m,x)}{2}$$

In the commitment scenario, the net profits derived by individual sellers when they adopt are:

$$H(m,w) = \pi \max[u(m,x) - p^c, 0] - \phi - w$$

Indeed, note that these profits now depend directly on the number m of sellers who adopt, since that number determines in turn total buyer participation on the platform in the second stage. It is therefore clear that sellers' decisions whether or not to adopt the platform now exhibit *direct* network effects. These network effects are positive since $u(m,x)$ is increasing in m.

In the no-commitment scenario, individual sellers' net profits when they adopt are:

$$H(m,w) = \pi[u(m,x) - p(m)] - \phi - w = \pi\frac{u(m,x)}{2} - \phi - w$$

Again, there are positive, direct network effects between sellers' adoption decisions.

Even though the indirect network effects have now been transformed in direct network effects among sellers, the platform can still face two types of expectations - favorable and unfavorable.

Favorable expectations

If expectations are favorable, then in the first stage all M sellers will coordinate on the fulfilled-expectations equilibrium with highest adoption for the platform. In this case, if the platform commits to p^c then all M sellers adopt if and only if:

$$w \leq \pi[u(M,x) - p^c] - \phi$$

Then the platform's optimization problem is:

$$\max_{p^c,x} \{(\pi M + p^c)[u(M,x) - p^c] - C(x) - M\phi\} \tag{7}$$

which is exactly the same as in the case with simultaneous entry and favorable expectations (cf. (1) above). This means that, with commitment, the platform facing favorable expectations obtains the same profits and chooses the same level of investment in first-party content x_f (cf. (2) above) as in the case with simultaneous entry.

If on the other hand the platform does not commit to p^c then sellers anticipate it will charge $p = u(m,x)/2$ in the second period and therefore adopt if and only if $w \leq \pi u(M,x)/2 - \phi$. Thus, the platform's optimization problem becomes (its profits are the sum of first stage profits from sellers and second stage profits from buyers):

$$\max_{x} \left\{ \left(\pi M + \frac{u(M,x)}{2}\right) \frac{u(M,x)}{2} - C(x) - M\phi \right\} \tag{8}$$

Comparing (7) with (8), it is clear that the platform's profits without commitment are always lower than with commitment since in the latter case the optimization problem has an additional degree of freedom (p^c). This is because without commitment the platform suffers from a time inconsistency (or hold-up) problem: sellers correctly anticipate that in the second stage the platform will choose $p = u(M,x)/2$ to maximize its own second stage profits, whereas the optimal buyer price from the first stage perspective (for the platform since it extracts all seller surplus) is the one that maximizes joint profits, i.e. $p^c = [u(M,x) - \pi M]/2$. Thus, a monopoly platform facing favorable expectations and sequential entry always prefers to commit to the price charged to buyers at the time it sets its price for sellers.

Note also that the level of investment in first-party content chosen by the platform in this case (no commitment) is x_f yet again (indeed, the first order condition of (8) is identical to (2)). Of course, this is not a general result: it is simply a particular feature of our modeling framework. It is however useful, in that it simplifies the analysis of the favorable expectations case, which we treat as a reference point for the analysis of the unfavorable expectations case.

Unfavorable expectations

If expectations are unfavorable, then in the first stage all M sellers will coordinate on the fulfilled-expectations equilibrium with lowest adoption for the platform. In this case, if the platform commits to p^c then sellers adopt if:

$$w \leq \pi \max[u(0,x) - p^c, 0] - \phi$$

Otherwise, no seller adopts. The platform will set w such that this constraint binds, so that its resulting optimization problem is:

$$\max_{p^c,x}\left\{M\pi\max\left[u\left(0,x\right)-p^c,0\right]+p^c\left[u\left(M,x\right)-p^c\right]-C\left(x\right)-M\phi\right\}$$

which is exactly the same as the one for the platform facing unfavorable expectations in the case with simultaneous entry (cf. (3) above). Thus, just like for the platform facing favorable expectations, commitment to the price charged to buyers in the case with sequential entry replicates the outcome of the case with simultaneous entry.

If on the other hand the platform does not commit to p then sellers adopt if and only if $w \leq \pi u\left(0,x\right)/2-\phi$, so that the platform's optimization problem becomes:

$$\max_x \quad \underbrace{M\pi\frac{u\left(0,x\right)}{2}}_{\substack{\text{rents extracted}\\ \text{from sellers}}} + \underbrace{\frac{u\left(M,x\right)^2}{4}}_{\substack{\text{rents extracted}\\ \text{from buyers}}} - C\left(x\right)-M\phi \tag{9}$$

yielding x_{uf3}.

Recall that with commitment (just like in the simultaneous entry case), there are only two possible solutions, corresponding to two distinct pricing strategies: one with $p_{uf} > u\left(0,x_{uf}\right)$ and $w_{uf}=-\phi$ and the other with $p_{uf} < u\left(0,x_{uf}\right)$ and $w_{uf}=\pi\left[u\left(0,x_{uf}\right)-p_{uf}\right]-\phi$. But now note that no commitment *always* dominates the first of these strategies. Indeed, when feasible, the latter yields profits equal to $\max_x\left\{u\left(M,x\right)^2/4-C\left(x\right)-M\phi\right\}$, which is strictly lower than (9). This is understood in the following way: if the platform is to subsidize sellers, it is better not to commit, in which case it maintains the flexibility to charge a higher price to buyers, which in turn allows it to also charge a higher price to sellers ($w=\pi u\left(0,x\right)/2-\phi$ instead of $w=-\phi$).

Consequently, the only relevant strategy with commitment has $p_c=\left[u\left(M,x\right)-M\pi\right]/2 < u\left(0,x\right)$, yielding profits equal to:

$$\max_x \underbrace{M\pi\left[u\left(0,x\right)-\frac{u\left(M,x\right)-M\pi}{2}\right]}_{\substack{\text{rents extracted}\\ \text{from sellers}}} + \underbrace{\frac{u\left(M,x\right)^2-\left(M\pi\right)^2}{4}}_{\substack{\text{rents extracted}\\ \text{from buyers}}} - C\left(x\right)-M\phi \tag{10}$$

Comparing (10) with (9), it is not clear whether the platform prefers to commit or not (assuming it has the choice). The two expressions above make it clear that for a platform facing unfavorable expectations, commitment involves giving up the ability to extract higher rents from the buyer side (since the platform must commit to a low buyer price to convince sellers to join) in order to extract higher rents from the seller side. This can only be profitable if the surplus that can be extracted from the seller side is sufficiently large relative to the surplus that can be extracted on the buyer side. Otherwise, the platform is better off attracting sellers with a low price and maintaining the flexibility to charge a high price to buyers once sellers have adopted (no commitment). This is in stark contrast with the platform facing favorable expectations.

The following lemma characterizes the optimal pricing strategy for the platform facing unfavorable expectations as a function of its level of investment in first-party content.

Lemma 2 *Given x, a platform facing unfavorable expectations and sequential entry commits to its buyer price in stage 1 if and only if $u(M,x) \geq u(0,x) + M\pi/2$. Its optimization problem is equivalent to solving $\max_x \Pi(x)$, where:*

$$\Pi(x) = \begin{cases} M\pi\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4} - C(x) - M\phi & \text{if } \; u(M,x) \geq u(0,x) + \frac{M\pi}{2} \\ \\ \frac{[u(M,x)-M\pi]^2}{4} + M\pi u(0,x) - C(x) - M\phi & \text{if } \; u(M,x) \leq u(0,x) + \frac{M\pi}{2} \end{cases} \tag{11}$$

The resulting solution is either x_{uf3} defined by (9) or x_{uf2} defined in (5).

Proof. In the appendix. ■

The choice of optimal pricing strategies for a given x defined in Lemma 2 is similar to the one defined in Lemma 1, but with two important differences. First, the Seller Subsidy Strategy has been replaced by the no commitment strategy. Although these strategies yield different total profits, they both rely on subsidizing the participation of the seller side[8] and making more profits on the buyer side). Second, the Buyer Attraction Strategy is chosen *less* often. Overall, the platform facing unfavorable expectations is better off when the two sides arrive sequentially than when they arrive simultaneously.

As a parallel to proposition 1, the following proposition compares the levels of investments in first-party content made by platforms facing favorable, respectively unfavorable expectations, in the case with sequential entry (assuming second order conditions are satisfied):

Proposition 2 *If content and seller participation are complements then:*

$$x_{uf2} \leq x_{uf3} \leq x_f$$

[8] In the case with sequential entry, the no commitment strategy involves $w = u(0,x)/2 - \phi$, which may not be a subsidy strictly speaking if $u(0,x)$ is sufficiently large. What we mean by subsidization is charging one side a price lower than the price which would maximize profits conditional on participation by the other side.

If content and seller participation are substitutes then:

$$x_f \leq x_{uf3} \leq x_{uf2}$$

Proof. In the appendix. ■

Proposition 2 confirms part of the result from Proposition 1: a platform facing unfavorable expectations invests less (more) in content relative to a platform facing favorable expectations whenever content and seller participation are complements (substitutes). Note however that the result in Proposition 2 is stronger, in the sense that it does not depend on whether the platform chooses the Seller Subsidy (no commitment) or the Buyer Attraction (commitment) strategy. Thus, with sequential entry, the table summarizing the first-party content strategies for the platform facing unfavorable expectations (relative to the platform facing favorable expectations) becomes:

	Complements	Substitutes
Buyer Attraction (commitment)	*underinvest*	*overinvest*
Seller Subsidy (no-commitment)	*underinvest*	*overinvest*

Furthermore, whether the platform facing unfavorable expectations invests more or less in first-party content when it commits (x_{uf2}) relative to when it does not commit (x_{uf3}) also depends on whether content and seller participation are complements or substitutes. This can be interpreted in the following way. When the platform facing unfavorable expectations chooses to commit, it has no choice but to commit to a low buyer price, therefore it chooses to give up extracting rents from buyers in order to extract rents from sellers. Conversely, when it does not commit, it has to charge a low seller price, but then it has maintained the option of charging a high price to buyers. Thus, the profits of the platform facing unfavorable expectations are more aligned with maximizing (and extracting) buyer surplus when it chooses not to commit. Consequently, when seller participation and first party content are complements in buyers' utility, the platform facing unfavorable expectations has stronger incentives to invest in first party content when it does not commit. The conclusion is reversed when seller participation and first party content are substitutes in buyer demand.

Let us now return to the three examples used in the previous section.

Example 1 *Let* $u(m,x) = sm + x$ *and* $C(x) = c\frac{x^2}{2}$*, where* $s > 0$ *and* $2c > 1$*. Then the platform facing unfavorable expectations chooses to commit and sets* $x_{uf2} = \frac{M(s+\pi)}{2c-1} = x_f$ *if* $\pi \geq 2s$*; it chooses not to commit and sets* $x_{uf3} = \frac{M(s+\pi)}{2c-1} = x_f$ *if* $\pi \leq 2s$*.*

Example 2 *Let* $u(m,x) = u_0 + Mx$ *and* $C(x) = c\frac{x^2}{2}$*, where* $2u_0 > \pi M$ *and* $2c > M^2$*. Then the platform facing unfavorable expectations chooses to commit and sets* $x_{uf2} = \frac{Mu_0 - M^2\pi}{2c-M^2} < x_f$ *if* $c \geq \frac{Mu_0}{\pi}$*; it chooses not to commit and sets* $x_{uf3} = \frac{Mu_0}{2c-M^2} < x_f$ *if* $c \leq \frac{Mu_0}{\pi}$*.*

Example 3 *Let* $u(m,x) = M(1-x) + M_0 x$ *and* $C(x) = c\frac{x^2}{2}$*, where* $M_0 > M$ *and* $2c > M_0(M_0 - M) + \pi M(M_0 + M)$*. Then the platform facing unfavorable expectations chooses to commit and sets* $x_{uf2} = \frac{M[M_0 - M + \pi(M_0+M)]}{2c-(M_0-M)^2} > x_f$ *if* $c(2-\pi) \leq M_0\left[M_0 - M + \pi\left(2M - \frac{M_0}{2}\right)\right]$*; it chooses not to commit and sets* $x_{uf3} = \frac{M(M_0 - M + \pi M_0)}{2c-(M_0-M)^2} > x_f$ *if* $c(2-\pi) \geq M_0\left[M_0 - M + \pi\left(2M - \frac{M_0}{2}\right)\right]$*.*

The way in which the cost c of first-party content impacts the choice of pricing strategy in examples 2 and 3 is the same as with simultaneous entry, replacing the Seller Subsidy strategy by the no-commitment strategy (both rely on subsidizing sellers and making more profits on buyers). In example 2, no commitment is chosen when the cost of providing first-party content is *low*. By contrast, in example 3 no commitment is chosen when the cost of providing first-party content is *high* (assuming $\pi < 2$).

In example 1 however, first-party content has no effect on the choice between the two strategies, which is solely determined by π and s, the respective benefits that each side derives from the presence of the other side. Specifically, the platform chooses to commit if the surplus derived by sellers is sufficiently large relative to the surplus derived by buyers ($\pi \geq 2s$). This is intuitive: the no commitment strategy aims to extract relatively more surplus from buyers, therefore it is more attractive when s is larger and π is smaller. As first-party content and third-party products are neither complements nor substitutes, it turns out that c has no impact on the comparison between the two strategies (they both result in the same level of first-party content).

All of the other comparative statics (effects of π, s, u_0 and M_0) and corresponding discussion from the case with simultaneous entry go through unchanged.

4 Competition between homogeneous platforms

In this section we turn to the analysis of platform competition with simultaneous entry of the two sides. There are two identical platforms, which we denote I (incumbent) and E (entrant). Each platform chooses (p_k, w_k, x_k) where $k = \{I, E\}$. We assume that the incumbent faces favorable expectations while the entrant faces unfavorable expectations. The way in which these expectations determine market outcomes is the same as in Caillaud and Jullien (2003) and Hagiu (2006): market participants always coordinate on the equilibrium which maximizes adoption on both sides for I and minimizes adoption for E.

We analyze three scenarios: in the first one sellers single-home (i.e. join at most one platform) and buyers multi-home (i.e. can join both platforms); in the second one buyers single-home and sellers multi-home and in the third scenario both sides single-home.

4.1 Sellers single-home and buyers multi-home

Suppose first that sellers can only join one platform at most, whereas buyers may join both. In this context, buyer i joins platform $k \in \{I, E\}$ if and only if $u(m_k^e, x_k) - p_k - i \geq 0$, where m_k^e is the expected number of sellers who join k and, as in the monopoly section, i is uniformly distributed over $[0,1]$. A seller who joins platform $k \in \{I, E\}$ derives payoff $\pi n_k^e - w_k - \phi$, where $n_k^e = u(m_k^e, x_k) - p_k$ is the expected number of buyers who join k. All M sellers are identical, therefore they all make the same platform adoption decision: they join the platform $k \in \{I, E\}$ which offers the higher payoff (we assume they join I if indifferent).

At a minimum, even if no sellers join E, the latter still attracts $\max[u(0, x_E) - p_E, 0]$ buyers, in which case it can make profits:

$$\Pi_0^E \equiv \max_{p_E, x_E} \{p_E [u(0, x_E) - p_E] - C(x_E)\} = \max_{x_E} \left\{ \frac{u(0, x_E)^2}{4} - C(x_E) \right\}$$

The only way in which E *might* be able to do better is by attracting the M sellers. Since E faces unfavorable expectations, this requires:

$$\pi \max[u(0, x_E) - p_E, 0] - w_E - \phi \geq \pi [u(M, x_I) - p_I] - w_I - \phi \equiv \pi_I$$

By setting its prices and investment in first party content to just satisfy this constraint, E can obtain profits equal to:

$$\max_{p,x} \{p[u(M, x) - p] + M\pi \max[u(0, x) - p, 0] - C(x) - M\phi - M\pi_I\} \equiv \Pi^E - M\pi_I$$

which is almost identical to the expression of monopoly platform profits under unfavorable expectations (3). The only difference is the constant factor $(-M\pi_I)$ which has no impact on the choices of (p, x). The solution for E is therefore the one characterized in Lemma 1.

In order to render this stratey by E unprofitable, the incumbent must set (p_I, x_I, w_I) such that[9] $M\pi_I \geq \Pi^E - \Pi_0^E$, which ensures that E prefers to focus on attracting buyers only or to stay out of the market. This results in the following optimization problem for I[10]:

$$\max_{p_I, x_I} \left\{ (p_I + M\pi) [u(M, x_I) - p_I] - C(x_I) - \left(\Pi^E - \Pi_0^E\right) \right\}$$

which is identical to the optimization problem (1) for a monopoly platform facing favorable expectations. Again, the difference is a constant factor which has no impact on the equilibrium choices

[9] We have assumed at the outset that ϕ is small enough so that $\Pi^E - \Pi_0^E > 0$, which implies that sellers derive non-negative payoffs from joining I (i.e. $\pi_I \geq 0$) whenever $M\pi_I \geq \Pi^E - \Pi_0^E$.

[10] It is easily verified that the resulting profits for I are positive.

of (p, x)). We have thus proven:

Proposition 3 *When sellers single-home and buyers multi-home, the incumbent makes the same investment in first-party content and chooses the same buyer price as if it were a monopolist facing favorable expectations.*

This result is due to the fact that here competition between the entrant and the incumbent focuses on attracting sellers (buyers multihome), whose payoffs from joining a given platform are not affected by that platform's investments in first-party content. This has an interesting policy implication. Suppose that the incumbent firm were to merge with or acquire the prospective entrant, making the incumbent a monopolist. In evaluating such a merger or acquisition, the proper comparison would be between competition and a monopolist that faced favorable expectations. The merger or acquisition would have no effect on the incumbent's investment in first-party content or the buyer price. So, when sellers single-home and buyers multi-home, such a merger or acquisition need not raise antitrust concerns. The outcome differs when competition focuses on buyers as the next section shows.

4.2 Buyers single-home and sellers multi-home

Suppose now that buyers single-home whereas sellers multi-home. Buyer i's utility from joining platform k is $u(m_k^e, x_k) - p_k - i$. A seller who joins platform $k \in \{I, E\}$ obtains payoffs $\pi n_k^e - w_k - \phi$, whereas a seller who multihomes obtains payoffs $\pi(n_E^e + n_I^e) - w_E - w_I - 2\phi$.

In this context, since E faces unfavorable expectations, it will see 0 adoption from buyers and sellers whenever $w_E > -\phi$ *and* $u(0, x_E) - p_E < \max\{0, u_I\}$, where we denote $u_I \equiv u(M, x_I) - p_I$ the net utility offered to buyers by I. Thus, the only ways in which E can break into the market are to set[11] $w_E \leq -\phi$ or $p_E \leq u(0, x_E) - u_I$.

If E charges $w_E = -\phi$ (the Seller Subsidy Strategy) then it ensures that all sellers will join E regardless of whether they join I or not. In this case, E can attract all buyers by charging $p_E \leq u(M, x_E) - u_I$, so that its resulting profits are:

$$\Pi_1^E = \max_{p_E, x_E} \{p_E [u(M, x_E) - p_E] - M\phi - C(x_E)\} \quad (12)$$
$$\text{subject to } u(M, x_E) - p_E \geq u_I$$

[11] Indeed, in equilibrium it will always be the case that $u_I \geq 0$, otherwise I cannot get any buyers.

If E charges $p_E \leq u(0, x_E) - u_I$ (the Buyer Attraction Strategy) then it ensures all buyers who enter choose E *even if* all sellers were to join I exclusively. In this case, E can charge sellers $w_E = \pi [u(M, x_E) - p_E] - \phi$, so that its resulting profits are:

$$\Pi_2^E = \max_{p_E, x_E} \{(p_E + \pi M)[u(M, x_E) - p_E] - M\phi - C(x_E)\} \tag{13}$$
$$\text{subject to } u(0, x_E) - p_E \geq u_I$$

It is easily seen that u_I must be such that the constraints corresponding to these two strategies are binding, otherwise E would be able to profitably enter. Consequently, the profits that E can attain can be re-written as follows:

$$\begin{aligned} \Pi_1^E &= \max_{x_E} \{u_I [u(M, x_E) - u_I] - M\phi - C(x_E)\} \\ \Pi_2^E &= \max_{x_E} \{(u(0, x_E) + \pi M - u_I)[u(M, x_E) - u(0, x_E) + u_I] - M\phi - C(x_E)\} \end{aligned} \tag{14}$$

With either one of these options, E attracts all sellers and all buyers (the latter exclusively). E chooses the best of the two options, so that its profits are:

$$\Pi^E = \max\left(\Pi_1^E, \Pi_2^E\right)$$

It is easily seen that Π^E is non-increasing in u_I (since both Π_1^E and Π_2^E are non-increasing in u_I). And, since $C(.)$ is increasing and $u(.,.)$ is increasing in both of its arguments, there exists $\overline{u} \geq 0$ such that $\Pi^E \leq 0$ if and only if $u_I \geq \overline{u}$. Therefore, platform I solves:

$$\max_{p_I, x_I} \{(p_I + M\pi)[u(M, x_I) - p_I] - C(x_I) - M\phi\}$$
$$\text{subject to } u(M, x_I) - p_I \geq \overline{u}$$

where the constraint is designed to make it unprofitable for E to enter the market. One can then re-write I's optimization programme as:

$$\max_{u_I, x_I} \{(u(M, x_I) - u_I + M\pi) u_I - C(x_I) - M\phi\} \tag{15}$$
$$\text{subject to } u_I \geq \overline{u}$$

which leads to:

Proposition 4 *When buyers single-home and sellers multi-home, the incumbent firm facing entry invests more in first-party content than when it faces no competition.*

Proof. Denote by $x_I(u)$ the incumbent's optimal choice of x_I given its choice of u_I:

$$x_I(u_I) \equiv \arg\max_x \{(u(M,x) - u_I + M\pi) u_I - C(x)\}$$

so that $x_I(u_I)$ solves:

$$u_I \times u_x(M,x) = C'(x)$$

Clearly, $x_I(u)$ is increasing in u (since $u(.,.)$ is concave in its second argument and $C(.)$ is convex). If I was an unconstrained monopolist, its choice of u_I would be:

$$u_I^* = \arg\max_u \{(u(M, x_I(u)) - u + M\pi) u - C(x_I(u))\}$$

By contrast, when I is constrained by E's presence, its choice u_I^C is the solution in u_I to (15), which implies $u_I^C \geq u_I^*$ and therefore $x_I\left(u_I^C\right) \geq x_I(u_I^*)$. ■

Thus, in contrast to the previous scenario, competitive pressure from the entrant induces higher investments in first-party content by the incumbent. This is because competition is for buyers (sellers multihome), which means the incumbent has to offer higher net utility to buyers than it would if it were a monopolist. Since buyer net utility is increasing in x, the result follows. Note that it is independent of whether first-party content and seller participation are complements or substitutes. For public policy purposes, competition between the incumbent and the entrant should be compared with a hypothetical monopolist facing favorable expectations. If the incumbent firm were to merge with or acquire the entrant, the effect would be to lower content provided to buyers. Overall, when buyers single-home and sellers multi-home, buyer benefits are increased by the effects of competition. This suggests that if the incumbent firm were to merge with or acquire the entrant, buyers would be made worse off.

To illustrate, in the appendix we provide a detailed derivation of the solutions corresponding to each of the three examples used in the two previous sections. For the sake of concision, here we summarize the results for example 1.

Example 1 *Let $u(m,x) = sm + x$; $C(x) = c\frac{x^2}{2}$ and $\phi = 0$, where $s > 0$ and $2c > 1$. Then the incumbent's equilibrium profits and investments in first-party content are:*

$$\Pi^I = \tfrac{2c}{2c-1}\pi s M^2 \text{ and } x_I = \tfrac{2Ms}{2c-1} \qquad \text{if} \quad s\left(1 - \tfrac{1}{2c}\right) \geq \pi$$

$$\Pi^I = \left(\pi + \tfrac{s}{2c}\right) s M^2 \text{ and } x_I = \tfrac{2M\pi + Ms/c}{2c-1} \quad \text{if} \quad s\left(1 - \tfrac{1}{2c}\right) \leq \pi$$

It is easily seen that this example confirms the general claims in proposition 4.

4.3 Buyers and sellers single-home

Suppose now that both sides single-home. In this context, since E faces unfavorable expectations, it will see 0 adoption from buyers and sellers whenever $-w_E - \phi < \max(\pi u_I - w_I - \phi, 0)$ *and* $u(0, x_E) - p_E < \max(0, u_I)$, where $u_I \equiv u(M, x_I) - p_I$. Thus, the only ways in which E can break into the market are to set $w_E \leq w_I - \pi u_I$ (very low price for sellers) or $p_E \leq u(0, x_E) - u_I$ (very low price for buyers).[12]

If E charges $w_E = w_I - \pi u_I$ (the Seller Subsidy Strategy) then it ensures that all sellers will join E. In this case, E can attract all buyers by charging p_E such that $u(M, x_E) - p_E \geq u(0, x_I) - p_I$, so that its resulting profits can be written:

$$\Pi_1^E = \max_{p_E, x_E} \{p_E [u(M, x_E) - p_E] - C(x_E) + M w_I - M \pi u_I\}$$
$$\text{subject to } u(M, x_E) - p_E \geq u_I + u(0, x_I) - u(M, x_I)$$

If E charges $p_E \leq u(0, x_E) - u_I$ (the Buyer Attraction Strategy) then it ensures all buyers who enter choose E *even if* all sellers were to join I. In this case, E can charge sellers w_E such that $\pi [u(M, x_E) - p_E] - w_E \geq \max(-w_I, \phi)$, so that its resulting profits are:

$$\Pi_2^E = \max_{p_E, x_E} \{(p_E + \pi M) [u(M, x_E) - p_E] - C(x_E) - M \max(-w_I, \phi)\}$$
$$\text{subject to } u(0, x_E) - p_E \geq u_I$$

With either one of these options, E attracts all sellers and all buyers exclusively. E chooses the best of the two options, so that its profits are:

$$\Pi^E(u_I, x_I, w_I) = \max\left(\Pi_1^E, \Pi_2^E\right)$$

It is easily seen that Π^E is (weakly) decreasing in u_I and weakly increasing in w_I and x_I (since both Π_1^E and Π_2^E are weakly decreasing in u_I and weakly increasing in w_I and x_I). Therefore, platform I solves:

$$\max_{u_I, x_I, w_I} \{u_I [u(M, x_I) - u_I] + w_I M - C(x_I)\}$$
$$\text{subject to } \Pi^E(u_I, x_I, w_I) < 0 \text{ and } \pi u_I - w_I - \phi \geq 0$$

where the first constraint is designed to make it unprofitable for E to enter the market.

This optimization program cannot be easily solved for the general case. In the appendix we

[12] Indeed, in equilibrium it will always be the case that $u_I \geq 0$ and $\pi u_I - w_I \geq \phi$, otherwise I cannot attract any buyers or sellers.

solve it for our example 1: the resulting incumbent choice of first-party content and profits are summarized below.

Example 1 *Let* $u(m,x) = sm + x$*;* $C(x) = c\frac{x^2}{2}$ *and* $\phi = 0$*, where* $s > 0$ *and* $2c > 1$*. Then the incumbent's equilibrium profits and investments in first-party content are:*

- $x_I = \frac{M(s+\pi)}{2c-1}$ *and* $\Pi^I = \frac{cM^2(2\pi s+\pi^2)}{2(2c-1)}$ *if* $\pi \leq \frac{c-1}{c}s$ *or (*$c \leq 1+\frac{1}{\sqrt{3}}$ *and* $\pi \in \left[\frac{c-1}{c}s, \frac{\sqrt{1+4c^2}-1}{4c}s\right]$*) or (*$c \leq \frac{9+\sqrt{17}}{16}$ *and* $\pi \in \left[\frac{3c-2}{2c}s, \frac{2(2c-1)^2}{c(8c-3)}s\right]$*)*
- $x_I = \frac{M(s+2c\pi)}{c(2c-1)}$ *and* $\Pi^I = \frac{sM^2(s+2c\pi)}{2c}$ *if* $\pi \geq \frac{2c-1}{c}s$
- $x_I = \frac{M(s+2c\pi)}{c(2c-1)}$ *and* $\Pi^I = \frac{M^2(s^2+2c\pi)}{2c-1}\left(\frac{2c-1}{2c}s - \pi\right)$ *if (*$c \geq 1+\frac{1}{\sqrt{3}}$ *and* $\pi \in \left[\frac{c-1}{c}s, \frac{3c-2}{2c}s\right]$*) or (*$c \leq 1+\frac{1}{\sqrt{3}}$ *and* $\pi \in \left[\frac{\sqrt{1+4c^2}-1}{4c}s, \frac{3c-2}{2c}s\right]$*) or (*$c \geq \frac{9+\sqrt{17}}{16}$ *and* $\pi \in \left[\frac{3c-2}{2c}s, \frac{2c-1}{c}s\right]$*)*
- $x_I = \frac{sM(2s+\pi)}{2[s(2c-1)-\pi c]}$ *and* $\Pi^I = \frac{sM}{2}\left[2sM\left(1-\frac{1}{2c}\right) - \pi M\right]$ *if (*$c \leq \frac{9+\sqrt{17}}{16}$ *and* $\pi \in \left[\frac{2(2c-1)^2}{c(8c-3)}s, \frac{2c-1}{c}s\right]$*)*

For this particular example, the incumbent platform still invests more in first-party content then it would in the absence of competition. But unlike the previous two competition scenarios, this is not a general result here.

5 Conclusion

This paper studies the incentives that two-sided platforms have to invest in first-party content in order to coordinate adoption by their two sides. A first key result is that, for a platform facing unfavorable expectations, the effect of first-party content on its optimal choice of pricing strategy depends crucially on whether first-party content and seller participation are complements or substitutes in buyer demand. If they are substitutes (complements) then larger investments in first-party content decrease (increase) the benefits derived by buyers from the presence of third-party sellers, so that the platform finds it optimal to subsidize buyer participation (seller participation) and make higher profits on sellers (buyers). Second, the relationship between first-party content and third-party seller participation also determines the relative incentives to invest in first-party content between platforms facing favorable expectations and platforms facing unfavorable expectations. The latter may invest more in first-party content if and only if first-party content is a substitute for seller participation. This is because when expectations are unfavorable platforms have to make up for the greater difficulty in attracting sellers by offering more first-party content.

All of these results hold for monopoly platforms, both under simultaneous and under sequential entry of the two sides (i.e. when sellers arrive before buyers). In the latter scenario, our analysis has also shown that the conditions under which a platform facing unfavorable expectations commits to the price charged to the side arriving later (buyers) depend once again on whether first-party content and seller participation are complements or substitutes. The reason that no commitment may be profitable for such a platform is that it allows the flexibility to charge a higher price to the side arriving later, *after* unfavorable expectations have been overcome. This is in stark contrast with platforms facing favorable expectations, which *always* find it profitable to commit.

We have also analyzed three scenarios with two homogeneous competing platforms, one facing favorable expectations, the other facing unfavorable expectations. Comparing the competitive outcome with the monopolist that faces favorable expectations has implications for antitrust policies toward mergers and acquisitions among platform firms. When sellers single-home and buyers multi-home, such a merger or acquisition need not raise antitrust concerns because it would have no effect on the incumbent's investment in first-party content or the buyer price. When buyers single-home and sellers multi-home, however, such a merger or acquisition would raise concerns because it would lower first-party content as well as lowering buyer benefits. When both buyers and sellers single home, the policy implications of a merger or acquisition among platform firms are less clear. This suggests the need for additional study of competition among platforms when buyers and sellers commit to single platforms.

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6 APPENDIX

6.1 Proofs of lemmas and propositions not included in the text

Proof. of Lemma 1.

Fix x. For $p \geq u(0,x)$, the expression of platform profits is:

$$\Pi_1(p,x) = p\left[u(M,x) - p\right] - M\phi - C(x)$$

which attains its maximum in p at $p_1(x) = u(M,x)/2$.

For $p \leq u(0,x)$, the expression of platform profits is:

$$\Pi_2(p,x) = p\left[u(M,x) - p\right] + M\pi\left[u(0,x) - p\right] - M\phi - C(x)$$

which attains its maximum in p at $p_2(x) = \left[u(M,x) - M\pi\right]/2 < p_1(x)$.

There are thus three possibilities:

- for $u(M,x)/2 < u(0,x)$ the platform's optimal choice of p is $p_2(x)$
- for $u(M,x)/2 > u(0,x) + M\pi/2$ the platform's optimal choice of p is $p_1(x)$
- for $u(0,x) \leq u(M,x)/2 \leq u(0,x) + M\pi/2$ the platform's optimal choice of p is $p_1(x)$ if $\Pi_1(p_1(x),x) \geq \Pi_2(p_2(x),x)$ and $p_2(x)$ otherwise.

But $\Pi_1(p_1(x),x) \geq \Pi_2(p_2(x),x)$ is equivalent to $u(M,x)/2 \geq u(0,x) + M\pi/4$. This leads to the expression of $\Pi(x)$ in the text of the lemma.

The only thing left to verify is that the corner solution x_0 defined by $u(M,x_0)/2 = u(0,x_0) + M\pi/4$ cannot maximize $\Pi(x)$ unless x_0 is a maximizer of $\Pi_1(p_1(x),x)$ or $\Pi_2(p_2(x),x)$, i.e. $x_0 = x_{uf1}$ (defined in 4) or $x_0 = x_{uf2}$ (defined in 5). Suppose by contradiction that x_0 maximizes $\Pi(x)$ but $x_0 \neq x_{uf1}$ and $x_0 \neq x_{uf2}$. Then there are only two possibilities:

- either:

$$\frac{u_x(M,x_0)\,u(M,x_0)}{2} - C'(x_0) < 0 < \frac{u_x(M,x_0)\,[u(M,x_0) - M\pi]}{2} + M\pi u_x(0,x_0) - C'(x_0)$$

  which implies $u_x(M,x_0)/2 - u_x(0,x_0) < 0$. But this means that for all x smaller than but sufficiently close to x_0, we have $u(M,x)/2 > u(0,x) + M\pi/4$ and therefore $\Pi(x) = \Pi_1(p_1(x), x)$. Since x_0 is a maximizer of $\Pi(x)$, this implies in turn that the derivative of $\Pi_1(p_1(x), x)$ in x evaluated at x_0 must be non-negative, i.e. $\frac{u_x(M,x_0)u(M,x_0)}{2} - C'(x_0) \geq 0$, which is a contradiction.

- or:

$$\frac{u_x(M,x_0)\,u(M,x_0)}{2} - C'(x_0) > 0 > \frac{u_x(M,x_0)\,[u(M,x_0) - M\pi]}{2} + M\pi u_x(0,x_0) - C'(x_0)$$

  which implies $u_x(M,x_0)/2 - u_x(0,x_0) > 0$. But this means that for all x larger than but sufficiently close to x_0, we have $u(M,x)/2 > u(0,x) + M\pi/4$ and therefore $\Pi(x) = \Pi_1(p_1(x), x)$. Since x_0 is a maximizer of $\Pi(x)$, this implies in turn that the derivative of $\Pi_1(p_1(x), x)$ in x evaluated at x_0 must be non-positive, i.e. $\frac{u_x(M,x_0)u(M,x_0)}{2} - C'(x_0) \leq 0$, which is a contradiction.

■

Proof. of Lemma 2.

If x is such that $u(M,x)/2 \geq u(0,x) + M\pi/4$ then we know from Lemma 1 and expression (9) that the platform's profits with commitment are strictly lower than the profits without commitment. Suppose then that x is such that $u(M,x)/2 \leq u(0,x) + M\pi/4$. Then the profits obtained by the platform when it commits are higher than when it does not commit if and only if:

$$\frac{[u(M,x) - M\pi]^2}{4} + M\pi u(0,x) \geq M\pi\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4}$$

which is equivalent to:

$$\frac{M\pi}{2} \geq u(M,x) - u(0,x)$$

Note that this last inequality holds only if $u(M,x)/2 \leq u(0,x) + M\pi/4$ holds, which leads to expression (11) in the text of the lemma.

As in Lemma 1, we need to verify that the corner solution x_0 defined by $u(M,x_0) = u(0,x_0) + M\pi/2$ cannot maximize $\Pi(x)$ unless x_0 is a maximizer of $\frac{[u(M,x)-M\pi]^2}{4} + M\pi u(0,x) - C(x)$ or of $M\pi\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4} - C(x)$, i.e. $x_0 = x_{uf2}$ (defined in 5) or $x_0 = x_{uf3}$ (defined in **??**). Suppose by contradiction that x_0 maximizes $\Pi(x)$ but $x_0 \neq x_{uf2}$ and $x_0 \neq x_{uf3}$. Then there are only two possibilities:

- either

$$\frac{u_x(M,x_0)\left[u(M,x_0)-M\pi\right]}{2}+M\pi u_x(0,x_0)-C'(x_0)<0<\frac{M\pi u_x(0,x_0)}{2}+\frac{u_x(M,x_0)\,u(M,x_0)}{2}-C'(x_0)$$

  which implies $u_x(M,x_0)-u_x(0,x_0)>0$. But this means that for all x larger than but sufficiently close to x_0, we have $u(M,x)>u(0,x)+M\pi/2$ and therefore $\Pi(x)=\frac{M\pi u(0,x)}{2}+\frac{u(M,x)^2}{4}-C(x)$. Since x_0 is a maximizer of $\Pi(x)$, this implies in turn that the derivative of $\frac{M\pi u(0,x)}{2}+\frac{u(M,x)^2}{4}-C(x)$ in x evaluated at x_0 must be non-positive, i.e. $\frac{M\pi u_x(0,x_0)}{2}+\frac{u_x(M,x_0)u(M,x_0)}{2}-C'(x_0)\leq 0$, which is a contradiction.

- or

$$\frac{u_x(M,x_0)\left[u(M,x_0)-M\pi\right]}{2}+M\pi u_x(0,x_0)-C'(x_0)>0>\frac{M\pi u_x(0,x_0)}{2}+\frac{u_x(M,x_0)\,u(M,x_0)}{2}-C'(x_0)$$

  which implies $u_x(M,x_0)-u_x(0,x_0)<0$. But this means that for all x smaller than but sufficiently close to x_0, we have $u(M,x)>u(0,x)+M\pi/2$ and therefore $\Pi(x)=\frac{M\pi u(0,x)}{2}+\frac{u(M,x)^2}{4}-C(x)$. Since x_0 is a maximizer of $\Pi(x)$, this implies in turn that the derivative of $\frac{M\pi u(0,x)}{2}+\frac{u(M,x)^2}{4}-C(x)$ in x evaluated at x_0 must be non-negative, i.e. $\frac{M\pi u_x(0,x_0)}{2}+\frac{u_x(M,x_0)u(M,x_0)}{2}-C'(x_0)\geq 0$, which is a contradiction.

■

Proof. of Proposition 2

Recalling (2) and taking the first order conditions of (9), and (10), we obtain the following equations which implicitly define x_f, x_{uf2} and x_{uf3}:

$$\frac{u_x(M,x_f)\left[u(M,x_f)+M\pi\right]}{2}-C'(x_f)=0$$

$$\frac{u_x(M,x_{uf2})\left[u(M,x_{uf2})-M\pi\right]}{2}+M\pi u_x(0,x_{uf2})-C'(x_{uf2})=0$$

$$\frac{u_x(M,x_{uf3})\,u(M,x_{uf3})}{2}+\frac{M\pi u_x(0,x_{uf3})}{2}-C'(x_{uf3})=0$$

Assuming the second order conditions are satisfied (so that each of the equations above has a unique solution), we have:

- $x_{uf2}>x_{uf3}$ if and only if $u_x(0,x_{uf3})-u_x(M,x_{uf3})>0$
- $x_f>x_{uf2}$ if and only if $u_x(M,x_{uf2})-u_x(0,x_{uf2})>0$
- $x_f>x_{uf3}$ if and only if $u_x(M,x_{uf3})-u_x(0,x_{uf3})>0$

Consequently:

- if $\frac{\partial^2 u(m,x)}{\partial m \partial x} < 0$ then $x_{uf2} > x_{uf3} > x_f$
- if $\frac{\partial^2 u(m,x)}{\partial m \partial x} > 0$ then $x_f > x_{uf3} > x_{uf2}$

■

6.2 Competition equilibrium when buyers singlehome and sellers multihome

6.2.1 Example 1

We follow the derivation of the incumbent's profits outlined in section 4.2. Consider the entrant's first strategy:

$$\Pi_1^E = \max_{p_E, x_E} \left\{ p_E \left(sM + x_E - p_E \right) - c \frac{x_E^2}{2} \right\}$$
$$\text{subject to } sM + x_E - p_E \geq u_I$$

The constraint must be binding, which requires:

$$u_I \geq \frac{csM}{2c-1} \tag{16}$$

In this case, we have:

$$\Pi_1^E = \max_{x_E} \left\{ u_I \left(sM + x_E - u_I \right) - c \frac{x_E^2}{2} \right\}$$

so that $x_1^E(u_I) = \frac{u_I}{c}$ and $\Pi_1^E \leq 0$ if and only if:

$$u_I \geq \frac{2csM}{2c-1} \tag{17}$$

Thus, since (17) is stronger than (16), it suffices that u_I satisfies (17) for I to render E's strategy 1 unprofitable.

Consider now E's second strategy:

$$\Pi_2^E = \max_{p_E, x_E} \left\{ \left(p_E + \pi M \right) \left[sM + x_E - p_E \right] - c \frac{x_E^2}{2} \right\}$$
$$\text{subject to } x_E - p_E \geq u_I$$

The constraint must be binding, which requires:

$$u_I \geq \frac{cM(\pi - s) + Ms}{2c-1} \tag{18}$$

In this case, we have:

$$\Pi_2^E = \max_{x_E} \left\{ (x_E + \pi M - u_I)(sM + u_I) - c\frac{x^2}{2} \right\}$$

which yields $x_2^E(u_I) = \frac{sM+u_I}{c}$ and $\Pi_2^E \leq 0$ if and only if:

$$u_I > \frac{2cM\pi + Ms}{2c-1} \tag{19}$$

Thus, since (19) is stronger than (18), it suffices that u_I satisfies (19) for I to render E's strategy 1 unprofitable.

Comparing the resulting Π_2^E with Π_1^E, it is straightforward to show that E chooses strategy 1 if $c\pi < (2c-1)s$ and $u_I \geq \frac{Ms(s+2c\pi)}{2[(2c-1)s-c\pi]}$ (otherwise it chooses strategy 2). The interpretation of this condition is intuitive: strategy 1 relies on subsidizing sellers, therefore it is more attractive when π is small (sellers are harder to attract) and u_I are large (buyers are harder to attract because of competition from I).

Overall, I must set:

$$u_I > \max\left(\frac{2csM}{2c-1}, \frac{2cM\pi + Ms}{2c-1}\right) \tag{20}$$

in order to render any entry strategy by E unprofitable. Its profits are then:

$$\begin{aligned} \Pi^I &= \max_{u_I, x_I} \left\{ [M(s+\pi) + x_I - u_I] u_I - c\frac{x_I^2}{2} \right\} \\ &\quad \text{subject to (20)} \end{aligned}$$

In any event we must have $x_I(u_I) = \frac{u_I}{c}$. Suppose (20) is not binding. Then $u_I = \frac{cM(\pi+s)}{2c-1}$ which cannot be higher than both $\frac{2csM}{2c-1}$ and $\frac{2cM\pi+Ms}{2c-1}$, leading to a contradiction. Thus, (20) must be binding. There are then two cases:

- if $s\left(1 - \frac{1}{2c}\right) \geq \pi$ then $u_I = \frac{2csM}{2c-1}$ and $\Pi^I = \frac{2c}{2c-1}\pi s M^2$
- if $s\left(1 - \frac{1}{2c}\right) \leq \pi$ then $u_I = \frac{2cM\pi+Ms}{2c-1}$ and $\Pi^I = \left(\pi + \frac{s}{2c}\right) sM^2$

6.3 Competition equilibrium when both sides singlehome

6.3.1 Example 1

We follow the derivation of the incumbent's profits outlined in the text. Consider the entrant's first strategy:

$$\Pi_1^E = \max_{p_E, x_E} \left\{ p_E \left[sM + x_E - p_E \right] - c\frac{x_E^2}{2} + Mw_I - M\pi u_I \right\}$$
$$\text{subject to } sM + x_E - p_E \geq u_I - sM$$

The constraint is binding if and only if $u_I - sM \geq \frac{csM}{2c-1}$ which is equivalent to:

$$u_I \geq \frac{sM(3c-1)}{2c-1}$$

In this case, we have:

$$\begin{aligned} \Pi_1^E &= \max_{x_E} \left\{ (u_I - sM)(sM + x_E - u_I + sM) - c\frac{x_E^2}{2} + Mw_I - M\pi u_I \right\} \\ &= (u_I - sM) \left[sM - (u_I - sM)\left(1 - \frac{1}{2c}\right) \right] + Mw_I - M\pi u_I \end{aligned}$$

If the constraint is not binding then we have:

$$\Pi_1^E = \frac{cs^2M^2}{2(2c-1)} + Mw_I - M\pi u_I$$

Consider now E's second strategy:

$$\Pi_2^E = \max_{p_E, x_E} \left\{ (p_E + \pi M)\left[sM + x_E - p_E \right] - c\frac{x_E^2}{2} - M\max(-w_I, 0) \right\}$$
$$\text{subject to } x_E - p_E \geq u_I$$

The constraint is binding if and only if:

$$u_I \geq \frac{cM(\pi - s) + Ms}{2c-1}$$

In this case, we have:

$$\Pi_2^E = \left[\frac{sM}{2c} + \pi M - \left(1 - \frac{1}{2c}\right) u_I \right] (sM + u_I) - M\max(-w_I, 0)$$

If the constraint is not binding then we have:

$$\Pi_2^E = \frac{c\left(s+\pi\right)^2 M^2}{2\left(2c-1\right)} - M\max\left(-w_I, 0\right)$$

Note that in this last case we must have:

$$Mw_I \le -\frac{c\left(s+\pi\right)^2 M^2}{2\left(2c-1\right)}$$

which implies:

$$\begin{aligned}\Pi^I &= u_I\left[sM + x_I - u_I\right] + w_I M - \frac{cx_I^2}{2} \\ &\le \max_{u_I, x_I}\left\{u_I\left[sM + x_I - u_I\right] - \frac{cx_I^2}{2}\right\} - \frac{c\left(s+\pi\right)^2 M^2}{2\left(2c-1\right)} \\ &< 0\end{aligned}$$

so I will never allow this case to arise.

The incumbent therefore solves:

$$\max_{u_I, x_I, w_I}\left\{u_I\left[sM + x_I - u_I\right] + w_I M - \frac{cx_I^2}{2}\right\}$$

SUBJECT TO:

$$u_I \ge \frac{sM\left(3c-1\right)}{2c-1} \text{ (a) and } M\pi u_I - Mw_I \ge \left(u_I - sM\right)\left[sM - \left(u_I - sM\right)\left(1 - \frac{1}{2c}\right)\right] \text{ (b)}$$

OR:

$$u_I \le \frac{sM\left(3c-1\right)}{2c-1} \text{ (c) and } M\pi u_I - Mw_I \ge \frac{cs^2M^2}{2\left(2c-1\right)} \text{ (d)}$$

AND:

$$u_I \ge \frac{cM\left(\pi - s\right) + Ms}{2c-1} \text{ (e) and } M\min\left(w_I, 0\right) \le -\left(sM + u_I\right)\left[\frac{sM}{2c} + \pi M - \left(1 - \frac{1}{2c}\right)u_I\right] \text{ (f)}$$

AND:

$$\pi u_I \ge w_I \text{ (g)}$$

Note that none of the constraints (a) through (g) depends on x_I, therefore I always sets $x_I = \frac{u_I}{c}$, so that its optimization problem reduces to:

$$\max_{u_I, w_I}\left\{u_I\left[sM - u_I\left(1 - \frac{1}{2c}\right)\right] + w_I M\right\}$$

subject to constraints (a) through (g) above. As it turns out, there are six cases to consider.

Case I: $\pi \leq \frac{c-1}{c}$s

In this case, we have:

$$\frac{sM\left(3c-1\right)}{2c-1} > \frac{M\left(s+2c\pi\right)}{2c-1} > \frac{cM\left(\pi-s\right)+Ms}{2c-1}$$

For $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$, constraint (d) implies (g) so I can set w_I to satisfy (d) and (f), i.e.:

$$Mw_I = \min\left\{M\pi u_I - \frac{cs^2M^2}{2\left(2c-1\right)}, -\left(sM+u_I\right)\left[\frac{sM}{2c}+\pi M-\left(1-\frac{1}{2c}\right)u_I\right]\right\}$$

Denote then:

$$F\left(u_I\right) \equiv M\pi u_I - \frac{cs^2M^2}{2\left(2c-1\right)} + \left(sM+u_I\right)\left[\frac{sM}{2c}+\pi M-\left(1-\frac{1}{2c}\right)u_I\right]$$

We have $F\left(\frac{cM(\pi-s)+Ms}{2c-1}\right) > 0$ and $F'\left(u_I\right) = 2M\pi + sM\left(\frac{1}{c}-1\right) - 2\left(1-\frac{1}{2c}\right)u_I$ is decreasing in u_I. Thus, $F\left(.\right)$ is either positive or positive then negative on the interval $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$.

If $F\left(u_I\right) \geq 0$ then $Mw_I = -\left(sM+u_I\right)\left[\frac{sM}{2c}+\pi M-\left(1-\frac{1}{2c}\right)u_I\right]$ and therefore I's profits are:

$$\begin{aligned}\Pi_1^I\left(u_I\right) &\equiv u_I\left[sM-u_I\left(1-\frac{1}{2c}\right)\right]-\left(sM+u_I\right)\left[\frac{sM}{2c}+\pi M-\left(1-\frac{1}{2c}\right)u_I\right]\\ &= -sM\left(\frac{sM}{2c}+\pi M\right)+u_I\left[2sM\left(1-\frac{1}{2c}\right)-\pi M\right]\end{aligned}$$

which is increasing in u_I.

If $F\left(u_I\right) \leq 0$ then $Mw_I = M\pi u_I - \frac{cs^2M^2}{2(2c-1)}$ and therefore I's profits are:

$$\begin{aligned}\Pi_2^I\left(u_I\right) &\equiv u_I\left[sM-u_I\left(1-\frac{1}{2c}\right)\right]+M\pi u_I-\frac{cs^2M^2}{2\left(2c-1\right)}\\ &= u_I\left[\left(s+\pi\right)M-u_I\left(1-\frac{1}{2c}\right)\right]-\frac{cs^2M^2}{2\left(2c-1\right)}\end{aligned}$$

which is also increasing in u_I on the interval $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$ because $\frac{cM(s+\pi)}{2c-1} \geq \frac{M(s+2c\pi)}{2c-1}$.

Thus, any choice of $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$ is dominated by $u_I = \frac{M(s+2c\pi)}{2c-1}$.

Suppose now $u_I \in \left[\frac{M(s+2c\pi)}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$. Then (f) is automatically satisfied and $Mw_I = M\pi u_I -$

$\frac{csM^2}{2(2c-1)}$, so that I's profits are:

$$\Pi_2^I(u_I) = u_I\left[(s+\pi)M - u_I\left(1-\frac{1}{2c}\right)\right] - \frac{cs^2M^2}{2(2c-1)}$$

and are maximized for $u_I^* = \frac{cM(s+\pi)}{2c-1} \in \left[\frac{M(s+2c\pi)}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$.

For $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{sM(4c-1)}{2c-1}\right]$, (f) is satisfied and (b) implies (g) because $sM > (u_I - sM)\left(1-\frac{1}{2c}\right)$. Thus, $Mw_I = M\pi u_I - (u_I - sM)\left[sM - (u_I - sM)\left(1-\frac{1}{2c}\right)\right]$, so that I's profits are:

$$\begin{aligned}\Pi_3^I(u_I) &\equiv u_I\left[sM - u_I\left(1-\frac{1}{2c}\right)\right] + M\pi u_I - (u_I - sM)\left[sM - (u_I - sM)\left(1-\frac{1}{2c}\right)\right] \\ &= s^2M^2\left(2-\frac{1}{2c}\right) + u_I\left[M\pi - 2sM\left(1-\frac{1}{2c}\right)\right]\end{aligned}$$

which is decreasing in u_I. This means that any $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{sM(4c-1)}{2c-1}\right]$ is dominated by $u_I = \frac{sM(3c-1)}{2c-1}$.

Finally, for all $u_I \geq \frac{sM(4c-1)}{2c-1}$, (f) is satisfied and (g) implies (b) so that I's profits are:

$$u_I\left[sM - u_I\left(1-\frac{1}{2c}\right)\right] + M\pi u_I = u_I\left[(s+\pi)M - u_I\left(1-\frac{1}{2c}\right)\right]$$

which is decreasing in u_I because $\frac{cM(s+\pi)}{2c-1} < \frac{sM(3c-1)}{2c-1}$. Thus, any $u_I \geq \frac{sM(4c-1)}{2c-1}$ is dominated by $u_I = \frac{sM(4c-1)}{2c-1}$, which is in turn dominated by $u_I = \frac{sM(3c-1)}{2c-1}$.

We can therefore conclude that for this case, the optimal choice of u_I is $u_I^* = \frac{cM(s+\pi)}{2c-1}$, yielding $x_I^* = \frac{M(s+\pi)}{2c-1} = x_f$ and $\Pi^I = \frac{cM^2(2\pi s+\pi^2)}{2(2c-1)}$.

Case II: $\pi \in \left[\frac{c-1}{c}s, \frac{3c-2}{2c}s\right]$

What changes relatively to the previous case is that now:

$$\frac{cM(s+\pi)}{2c-1} \leq \frac{M(s+2c\pi)}{2c-1}$$

For $u_I \geq \frac{sM(3c-1)}{2c-1}$, the analysis is exactly the same as in Case I, so we know that any $u_I \geq \frac{sM(3c-1)}{2c-1}$ is dominated by $u_I = \frac{sM(3c-1)}{2c-1}$.

Furthermore, for $u_I \in \left[\frac{M(s+2c\pi)}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$, I's profits are (just like in Case I):

$$\Pi_2^I(u_I) = u_I\left[(s+\pi)M - u_I\left(1-\frac{1}{2c}\right)\right] - \frac{csM^2}{2(2c-1)}$$

which are now strictly decreasing in u_I because $\frac{cM(s+\pi)}{2c-1} \leq \frac{M(s+2c\pi)}{2c-1}$.

Thus, any $u_I \geq \frac{M(s+2c\pi)}{2c-1}$ is dominated by $u_I = \frac{M(s+2c\pi)}{2c-1}$.
Suppose $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$. Using the definition of $F(.)$ above, we have:

$$
\begin{aligned}
F\left(\frac{M(s+2c\pi)}{2c-1}\right) &\geq 0 \Longleftrightarrow 4c\pi^2 + 2\pi s - cs^2 \geq 0 \Longleftrightarrow \pi \geq \frac{\sqrt{1+4c^2}-1}{4c}s \\
F\left(\frac{cM(\pi+s)}{2c-1}\right) &\geq 0 \Longleftrightarrow 2c\pi s + \frac{3}{2}c\pi^2 - \frac{s^2}{c}(c-1)^2 - cs^2 \geq 0
\end{aligned}
$$

It is then easily verified that:

$$
F\left(\frac{M(s+2c\pi)}{2c-1}\right) \leq 0 \Longrightarrow F\left(\frac{cM(\pi+s)}{2c-1}\right) \leq 0
$$

Recalling that $F\left(\frac{cM(\pi-s)+Ms}{2c-1}\right) > 0$ and $F'(u_I)$ is decreasing in u_I, we can conclude that if $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \leq 0$ then the optimal choice of u_I is $u_I^* = \frac{cM(\pi+s)}{2c-1}$, yielding $x_I^* = \frac{M(\pi+s)}{2c-1}$ and $\Pi^I = \frac{cM^2\left(2\pi s+\pi^2\right)}{2(2c-1)}$.

Suppose now $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \geq 0$. Then, for all $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$, $F(u_I) \geq 0$, which in turn implies $\Pi^I(u_I) = \Pi_1^I(u_I)$, which is increasing in u_I, so the optimal choice of u_I is $u_I^* = \frac{M(s+2c\pi)}{2c-1}$, yielding $x_I^* = \frac{M(s+2c\pi)}{c(2c-1)}$ and $\Pi^I = \frac{M(s+2c\pi)}{2c-1}\left(\frac{2c-1}{2c}s - \pi\right)$.

Finally, it is easily verified that for $c \geq \frac{1}{\sqrt{3}} + 1$, we have $\frac{\sqrt{1+4c^2}-1}{4c} \leq \frac{c-1}{c}$, so $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \geq 0$ for all $\pi \in \left[\frac{c-1}{c}s, \frac{3c-2}{2c}s\right]$. If on the other hand $c \geq \frac{1}{\sqrt{3}} + 1$ then $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \geq 0$ for $\pi \in \left[\frac{\sqrt{1+4c^2}-1}{4c}s, \frac{3c-2}{2c}s\right]$ and $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \leq 0$ for $\pi \in \left[\frac{c-1}{c}s, \frac{\sqrt{1+4c^2}-1}{4c}s\right]$.

Case III: $\pi \in \left[\frac{3c-2}{2c}s, \frac{2c-1}{c}s\right]$
What changes relatively to the previous case is that now:

$$
\frac{M(s+2c\pi)}{2c-1} \geq \frac{sM(3c-1)}{2c-1} \geq \frac{cM(s+\pi)}{2c-1}
$$

The same analysis as in the previous case shows that any $u_I \geq \frac{M(s+2c\pi)}{2c-1}$ is dominated by $u_I = \frac{M(s+2c\pi)}{2c-1}$.
For $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$, we have:

- If $F(u_I) \geq 0$ then I's profits are $\Pi_1^I(u_I) = -sM\left(\frac{sM}{2c} + \pi M\right) + u_I\left[2sM\left(1 - \frac{1}{2c}\right) - \pi M\right]$, increasing in u_I

- If $F(u_I) \leq 0$ then I's profits are $\Pi_2^I(u_I) = u_I\left[(s+\pi)M - u_I\left(1 - \frac{1}{2c}\right)\right] - \frac{cs^2M^2}{2(2c-1)}$.

For $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$, w_I must satisfy (b) and (f), so that we have:

$$Mw_I = \min \left\{ \begin{array}{c} M\pi u_I - (u_I - sM)\left[sM - (u_I - sM)\left(1 - \frac{1}{2c}\right)\right], \\ -(sM + u_I)\left[\frac{sM}{2c} + \pi M - \left(1 - \frac{1}{2c}\right)u_I\right] \end{array} \right\}$$

Let then:

$$\begin{aligned} G(u_I) &\equiv M\pi u_I - (u_I - sM)\left[sM - (u_I - sM)\left(1 - \frac{1}{2c}\right)\right] + (sM + u_I)\left[\frac{sM}{2c} + \pi M - \left(1 - \frac{1}{2c}\right)u_I\right] \\ &= sM(2sM + \pi M) + 2u_I\left[M\pi - 2sM\left(1 - \frac{1}{2c}\right)\right] \end{aligned}$$

which is decreasing in u_I in this case.

For $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$, we have therefore:

- If $G(u_I) \geq 0$ then I's profits are $\Pi_1^I(u_I) = -sM\left(\frac{sM}{2c} + \pi M\right) + u_I\left[2sM\left(1 - \frac{1}{2c}\right) - \pi M\right]$, increasing in u_I
- If $G(u_I) \leq 0$ then I's profits are:

$$\begin{aligned} \Pi_3^I(u_I) &\equiv u_I\left[sM - u_I\left(1 - \frac{1}{2c}\right)\right] + M\pi u_I - (u_I - sM)\left[sM - (u_I - sM)\left(1 - \frac{1}{2c}\right)\right] \\ &= s^2M^2\left(2 - \frac{1}{2c}\right) + u_I\left[M\pi - 2sM\left(1 - \frac{1}{2c}\right)\right] \end{aligned}$$

  which is decreasing in u_I.

It is then easily shown that:

$$F\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0 \Longleftrightarrow G\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0 \Longleftrightarrow \pi \geq \frac{2(2c-1)^2}{c(8c-3)}s$$

If $c \geq \frac{9+\sqrt{17}}{16}$ then $\frac{2(2c-1)^2}{c(8c-3)}s \leq \frac{3c-2}{2c}s$ so that $\pi \geq \frac{2(2c-1)^2}{c(8c-3)}s$ for all $\pi \in \left[\frac{3c-2}{2c}s, \frac{2c-1}{c}s\right]$. This implies that $F\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0$ and $G\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0$. Furthermore, $c \geq \frac{9+\sqrt{17}}{16}$ implies $G\left(\frac{M(s+2c\pi)}{2c-1}\right) \geq 0$. Thus, $\Pi^I(u_I)$ is increasing on $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$ so that the optimal choice of u_I is $u_I^* = \frac{M(s+2c\pi)}{2c-1}$, yielding $x_I^* = \frac{M(s+2c\pi)}{c(2c-1)}$ and $\Pi^I = \frac{M(s+2c\pi)}{2c-1}\left(\frac{2c-1}{2c}s - \pi\right)$.

Suppose now $c \leq \frac{9+\sqrt{17}}{16}$. Then, for $\pi \in \left[\frac{3c-2}{2c}s, \frac{2(2c-1)^2}{c(8c-3)}s\right]$, $G\left(\frac{sM(3c-1)}{2c-1}\right) \leq 0$ and $F\left(\frac{sM(3c-1)}{2c-1}\right) \leq 0$. $G\left(\frac{sM(3c-1)}{2c-1}\right) \leq 0$ implies that all $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$ is dominated by $u_I = \frac{sM(3c-1)}{2c-1}$. $F\left(\frac{sM(3c-1)}{2c-1}\right) \leq 0$ implies that $F\left(\frac{M(s+2c\pi)}{2c-1}\right) \leq 0$, which implies $F\left(\frac{cM(\pi+s)}{2c-1}\right) \leq 0$. Consequently, the optimal choice of u_I is $u_I^* = \frac{cM(\pi+s)}{2c-1}$, yielding $x_I^* = \frac{M(\pi+s)}{2c-1}$ and $\Pi^I = \frac{cM^2\left(2\pi s+\pi^2\right)}{2(2c-1)}$.

For $\pi \in \left[\frac{2(2c-1)^2}{c(8c-3)}s, \frac{2c-1}{c}s\right]$, $F\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0$ and $G\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0 \geq G\left(\frac{M(s+2c\pi)}{2c-1}\right)$. This implies that the optimal choice of u_I is $u_I^0 \in \left[\frac{sM(3c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$ such that $G\left(u_I^0\right) = 0$. Using the expression of $G(.)$ above, we conclude that the solution is $u_I^* = u_I^0 = \frac{csM(2s+\pi)}{2[s(2c-1)-\pi c]}$, yielding $x_I^* = \frac{sM(2s+\pi)}{2[s(2c-1)-\pi c]}$ and $\Pi_I^* = \frac{sM}{2}\left[2sM\left(1-\frac{1}{2c}\right) - \pi M\right] > 0$.

Case IV: $\pi \in \left[\frac{2c-1}{c}s, \frac{4c-2}{c}s\right]$

Note first that $\pi \geq \frac{2c-1}{c}s$ implies $F\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0$, which means that I's profits for $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$ are $\Pi_1^I(u_I)$, which is decreasing in u_I. Furthemore, $\Pi_1^I\left(\frac{cM(\pi-s)+Ms}{2c-1}\right) < 0$.

Also, $G(u_I)$ is now increasing and positive for all u_I so that for $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{sM(4c-1)}{2c-1}\right]$, I's profits are once again equal to $\Pi_1^I(u_I)$, decreasing and negative. The same holds for $u_I \in \left[\frac{sM(4c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$.

Finally, for $u_I \geq \frac{M(s+2c\pi)}{2c-1}$ the only constraint left is $Mw_I \leq M\pi u_I$, so that I's profits are equal to $u_I\left[M(s+\pi) - u_I\left(1-\frac{1}{2c}\right)\right]$. Given that $\frac{M(s+2c\pi)}{2c-1} > \frac{cM(s+\pi)}{2c-1}$, the optimal solution for this case is is $u_I^* = \frac{M(s+2c\pi)}{2c-1}$, yielding $x_I^* = \frac{M(s+2c\pi)}{c(2c-1)}$ and $\Pi^I = \frac{sM^2(s+2c\pi)}{2c}$.

Case V: $\pi \in \left[\frac{4c-2}{c}s, \frac{5c-2}{c}s\right]$

In this case, $\frac{sM(4c-1)}{2c-1} \geq \frac{cM(\pi-s)+Ms}{2c-1} \geq \frac{sM(3c-1)}{2c-1}$ so that the only relevant constraints are (b), (e), (f) and (g).

Suppose first $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{sM(4c-1)}{2c-1}\right]$. Like in Case IV, $G(u_I) > 0$ so I's profits are equal to $\Pi_1^I(u_I)$, decreasing and negative. The same holds for $u_I \in \left[\frac{sM(4c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$.

Note first that $\pi \geq \frac{2c-1}{c}s$ implies $F\left(\frac{sM(3c-1)}{2c-1}\right) \geq 0$, which means that I's profits for $u_I \in \left[\frac{cM(\pi-s)+Ms}{2c-1}, \frac{sM(3c-1)}{2c-1}\right]$ are $\Pi_1^I(u_I)$, which is decreasing in u_I. Furthemore, $\Pi_1^I\left(\frac{cM(\pi-s)+Ms}{2c-1}\right) < 0$.

Also, $G(u_I)$ is now increasing and positive for all u_I so that for $u_I \in \left[\frac{sM(3c-1)}{2c-1}, \frac{sM(4c-1)}{2c-1}\right]$, I's profits are once again equal to $\Pi_1^I(u_I)$, decreasing and negative. The same holds for $u_I \in \left[\frac{sM(4c-1)}{2c-1}, \frac{M(s+2c\pi)}{2c-1}\right]$.

Finally, for $u_I \geq \frac{M(s+2c\pi)}{2c-1}$ the only constraint left is $Mw_I \leq M\pi u_I$, so that I's profits are equal to $u_I\left[M(s+\pi) - u_I\left(1-\frac{1}{2c}\right)\right]$. Given that $\frac{M(s+2c\pi)}{2c-1} > \frac{cM(s+\pi)}{2c-1}$, the optimal solution for this case is is $u_I^* = \frac{M(s+2c\pi)}{2c-1}$, yielding $x_I^* = \frac{M(s+2c\pi)}{c(2c-1)}$ and $\Pi^I = \frac{sM^2(s+2c\pi)}{2c}$.

Case VI: $\pi \geq \frac{5c-2}{c}\mathrm{s}$

The analysis is exactly the same as in Case V, except that now $\frac{cM(\pi-s)+Ms}{2c-1} \geq \frac{sM(4c-1)}{2c-1}$. The same solution prevails.