Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger Policy*

PRELIMINARY AND INCOMPLETE

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Abstract

We study optimal merger policy in a dynamic model in which the presence of scale economies imply that firms can reduce costs through either internal investment in building capital or through mergers. The model, which we solve computationally, allows firms to invest or propose mergers according to the relative profitability of these strategies. An antitrust authority is able to block mergers at some cost. We examine the optimal policy when the antitrust can commit to a policy rule and when it cannot commit, and consider both consumer value and aggregate value as possible objectives fo the antitrust authority. We find that optimal policy can differ substantially from what would be optimal considering only welfare in the period the merger is proposed. We also find that the ability to commit can lead to a significant welfare improvement. In general, firms' optimal investment behavior can be greatly affected by the antitrust policy, and the optimal policy (absent commitment) can in turn be greatly affected by firms' investment behavior.

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1 Introduction

Most analyses of optimal horizontal merger policy are static.[1] But many real-world mergers occur in markets in which dynamic issues are a central feature of competition among firms. In this paper, we analyze merger policy in the context of a model in which the presence of economies of scale presents firms with the opportunity to lower their average and marginal costs through capital accumulation. These scale economies are also the source of merger-related efficiencies, as a merger of two equal-sized firms lowers average and marginal cost at the merging firms' pre-merger joint output level. Thus, in such settings, merger decisions must weigh any extra cost reductions gained by allowing the merger (compared to insisting on internal growth) against the deadweight losses arising from increased market power.

As one example, consider the 2011 attempted merger between AT&T and t-Mobile. The merger would have combined the network infrastructure of the two firms. Proponents of the merger argued that this combination would greatly improve the quality of both firms' service. Opponents countered that the merger would increase market power, and that absent the merger the two firms would each have incentives to independently increase their networks. Thus, the FCC and Department of Justice faced the question of whether any efficiency gains from increased infrastructure scale due to the merger (which in this case were realized on the demand side through enhanced quality) were sufficient to justify the increase in market power.

We study these issues computationally in a dynamic industry model in which in each period active firms compete and also make investments to increase their capital stock. Economies of scale in production imply that mergers generate efficiencies. The model is similar to Pakes and McGuire (1994), but with some important differences. Most significantly, we modify the investment technology to make it *merger neutral*; that is, so that mergers do not change the investment opportunities that are available in the market. Our investment technology also allows for significantly richer investment dynamics than do most computational dynamic models, as firms can increase their capital stocks by multiple units, and new entrants can choose endogenoeously how many units of capital to build when entering.

In addition, we introduce the possibility for firms to merge, as well as an antitrust authority who can block proposed mergers. The decision to propose a merger is endogeneous and determined through a bargaining process. In general, bargaining over mergers involves externalities and the theory literature currently has few satisfying general solutions for such settings. For this reason, the present paper restricts attention to industries in which there are at most two active firms in any period. Doing so allows us to use the familiar Nash bargaining solution, and makes clear how the prospect of bargaining over mergers impacts investment incentives. Our approach to modeling the antitrust authority considers both the case in which the authority can commit to a policy rule and the case in which commitment is impossible. We also consider both maximization of consumer surplus and of aggregate surplus as possible objectives of the authority.

We begin in Section 2 by describing our model. Section 3 formally defines a Markov perfect equlibrium and discusses how we identify these equilibria computationally.

Section 4 analyzes firm behavior under two extreme antitrust policies: one in which no mergers are allowed, and the other in which all are. We examine firms' investment behavior

[1] For example, see the classic papers by Williamson (1968) and Farrell and Shapiro (1990).

and merger decisions in two different markets: a large one and a small one. We first describe firm behavior and industry dynamics for these two markets when no mergers are allowed. In the large market steady state, the industry spends most of its time in states with relatively symmetric capital levels for the firms, although should the industry through chance depreciation end up at a highly asymmetric position, it stays there for a long time. In contrast, the small market steady state is highly asymmetric, with one firm often dominating the industry.

We then explore the impact of merger policy by studying the Markov perfect equilibria when instead all mergers are allowed. We find a number of striking features. Overall, the steady state when all mergers are allowed has a much lower average total capital level than when no mergers are allowed. This is true for two reasons. First, mergers move the industry into states in which investments are lower. Second, investment behavior changes when all mergers are allowed. In states in which a subsequent merger is highly likely, the possibility of mergers introduces a hold-up problem: firms sink investments that increase the value of the merged firm prior to bargaining. Nonetheless, firms still have significant incentives to invest due to the incentive that comes through effects on the disagreement payoffs in bargaining. Particularly striking is a significantly greater investment by small firms in states where one firm is very dominant, a form of "entry for buyout" [Rasmusen (1988)]. Consumer value, producer value, and aggregate value all decline when all mergers are allowed. The surprising reduction in producer value seems driven by the behavior, just noted, of new entrants after a merger. This investment is done at high investment costs and dissipates a great deal of producer value.

In Section 5 we examine optimal merger policy, considering as objectives both discounted expected consumer surplus ("consumer value") and discounted expected aggregate surplus ("aggregate value"). We begin by looking at the static benchmark, examining what mergers would be approved from a myopic perspective; that is, considering only the effect on welfare in the period the merger is proposed. From that perspective, very few mergers are consumer surplus enhancing in the large market and none are in the small market, while many mergers increase aggregare surplus in both markets.

We then identify both the Markov perfect policy (the no commitment policy outcome) and the optimal commitment policy under both objectives in each of the markets. With a consumer value objective, the Markov perfect policy and the no commitment policy are almost identical and basically allow no mergers, just as with the static consumer surplus criterion.

With an aggregate value objective, however, the no commitment and commitment policies differ substantially from what is statically optimal. The reason is that, as seen in Section 4, merger policy has substantial effects on firms' investment behavior. In both markets, the optimal policy under either commitment or no commitment is much more restrictive than what is statically optimal. Indeed, with commitment, the optimal policy is to essentially allow no mergers in the large market.

The no commitment (Markov perfect) policies are less restrictive than these commitment policies. In examining the antitrust authority's incentives absent commitment, we find that the interaction between policy and investment behavior runs both ways: there is a substantial effect of firm behavior on the antitrust authority's incentives to approve mergers. Given firm behavior when mergers are not possible, many mergers should be approved. But if many mergers are approved and firms' investment behavior therefore looks more like that seen in

Section 4 when all mergers are allowed, the antitrust authority should approve few mergers. The Markov perfect policy ends up between these two extremes. Strikingly, it results in a much lower level of aggregate value than if the antitrust authority could commit: consumers bear the brunt of this welfare loss, a result of both more monopoly pricing and a lower average aggregate capital level, while industry profit is almost unchanged. Indeed, in our model endowing the antitrust authority with a consumer value objective achieves a substantial welfare gain when the antitrust authority cannot commit to its approval policy. This finding is consistent with the suggestion of Lyons (2002), but for a different reason: in Lyons (2002) this gets the firms to propose more attractive mergers, while here it induces much better investment behavior.

In terms of the trade-off between internal and external growth we see several things. First, the very nature of this trade-off depends on whether we are taking the perspective of an antitrust authority that cannot commit and must decide what to do about a given proposed merger, or the perspective of identifying an optimal commitment policy. From the former persective, we see that the desirability of approving a merger can indeed depend importantly on the investment behavior that will follow if it is or is not approved. However, this involves more than just the behavior of the merging firms, as the investment behavior of outsiders to the merger (here, new entrants) can have significant welfare effects. Moreover, these investment behaviors can be importantly influenced by firms' beliefs about future merger policy. From the perspective of identifying an optimal commitment policy, these potential effects on investment behavior can make the optimal commitment policy differ substantially from the policy that emerges when the antitrust authority instead considers mergers on a case-by-case basis without commitment.

Section 6 (TBA) considers some extensions, while Section 7 (TBA) concludes.

The paper is related to several strands of literature. The first is theoretical work on dynamic merger policy. Most of this work examines models in which two mergers between two non-overlapping pairs of firms can take place sequentially in static models of competition [e.g., Nilssen and Sorgard (1998), Motta and Vasconcelos (2005), and Matushima (2001)]. An exception is Nocke and Whinston (2010). They study a many-period dynamic model in which mergers become feasible stochastically through time and establish conditions under which the optimal dynamic (commitment) policy of an antitrust authority who maximizes consumer value is the fully myopic policy that approves a merger if and only if it would raise consumer surplus in the period it is proposed. The model in this paper departs from Nocke and Whinston (2010) in a number of ways, most notably in introducing investment by firms and in locating the efficiency gains from merger in the achievement of scale economies through capital acquisition.[2]

The second strand of literature is dynamic models of industry equilibrium with investment, most notably Pakes and McGuire (1994). Some of this literature has examined the effects of one-time mergers on industry evolution [e.g., Berry and Pakes (1993), Cheong and Judd (2000), Benkard, Bodoh-Creed, and Lazarez (2010)]. Closest to our work are Gowrisankaran (1997) and (1999). Gowrisankaran (1999) introduces an endogeneous merger bargaining game into the Pakes-McGuire model and examines industry evolution when firms can choose whether, when, and with whom to merge. Our model differs in a number of respects: First, as mentioned

[2] The model here also differs from Nocke and Whinston (2010) in that firms that do not merge in a given period may consummate a merger with different efficiencies (i.e. with different capital levels) in future periods.

above, we modify the Pakes-McGuire investment technology to make it merger neutral, and give entrants the same technology as incumbents with zero capital. Second, we locate the efficiency effects of mergers in scale economies achieved through capital acquisition, rather than in randomly drawn synergy gains. Third, we focus on settings with just two active firms and use the Nash bargaining solution over mergers. While restrictive, we do this because it allows us to examine a case in which the bargaining model is well accepted and easily understood. In unpublished work, Gowrisankaran (1997) introduces antitrust policy into the Gowrisankaran (1999) model. Specifically, he examines the effect of commitments to Herfindahl-based policies that block mergers if they result in a Herfindahl index above some maximum threshold and finds little effect of varying the threshold on welfare. We differ in considering a broader range of possible policy commitments and in examining the equilibrium policy when the antitrust authority cannot commit. We also find quite different results, with policy having significant effects. In both papers, optimal policy differs substantially from what would be myopically (i.e., statically) optimal.

2 The Model

We study a dynamic industry model in which firms may invest in capacity, or alternatively merge, to increase their capital stocks and harness scale economies. The model follows in broad outline Pakes and McGuire (1994) [see also Ericson and Pakes (1995)], but with some important differences in its investment technology, as well as in the introduction of mergers and merger policy.

2.1 Static demand, costs, and competition

In each period, active firms produce a homogeneous good in a market in which demand is given by the function $Q(P) = B(A-p)^{\gamma}$. As noted in the Introduction, we restrict attention to situations in which there are at most two active firms. The production technology, which requires capital and labor, is described by the production function $F(K,L) = K^{\beta\theta}L^{(1-\beta)\theta}$, where the capital share parameter is $\beta \in (0,1)$ and the scale economy parameter is $\theta > 1$. Normalizing the price of labor to be 1, for a fixed level of capital K, this production function gives rise to the short-run cost function

$$C(Q|K) = \frac{Q^{1/(1-\beta)\theta}}{K^{\beta/(1-\beta)}}$$

with marginal cost

$$C_Q(Q|K) = \left(\frac{1}{(1-\beta)\theta}\right)\frac{Q^{[1/(1-\beta)\theta]-1}}{K^{\beta/(1-\beta)}}.$$

With this technology, a merger of two identical firms reduces both average and marginal cost if their joint output remains unchanged. This effect will be the source of merger-related efficiencies in our model. Letting R measure the extent of this cost reduction, we have

$$R \equiv \frac{C_Q(2Q|2K)}{C_Q(Q|K)} = \frac{C(2Q|2K)/Q}{C(Q|K)/Q} = 2^{\left(\frac{1}{1-\beta}\right)\left(\frac{1-\theta}{\theta}\right)}.$$

Note in particular that the marginal cost reduction depends on the scale economy parameter θ and capital share β, but is independent of the output level (and hence demand). In our computations we will focus on a case in which $\beta = 1/3$. For this value of β, the magnitude of R for various values of θ is shown in the table below.

$\boldsymbol{\theta}$	1.05	1.1	1.15	1.2	1.3	1.4
R	0.95	0.91	0.87	0.84	0.79	0.74

In each period, active firms engage in Cournot competition given their capital stocks.

2.2 Investment and Depreciation

In Pakes and McGuire (1994) a firm chooses in each period how much money to invest, with the probability of successfully adding one unit of capital increasing in the investment level. We depart from this technology because in a model of mergers it would impose a significant inefficiency to mergers, as each merger between two firms would remove an investment possibility from the market.[3] Instead, we specify an investment technology that is *merger neutral* at a market level. By that we mean that a planner who controlled the firms and wanted to achieve at least cost any fixed increase in the market's aggregate capital stock would be indifferent about whether the firms merge. With this assumption we isolate the efficiency effects of mergers fully in the scale economies of the production function.[4] Specifically, we imagine that there are two ways that a firm can invest.

The first is *capital augmentation*: each unit j of capital that a firm owns can be doubled at some cost $c_j \in [\underline{c}, \overline{c}]$ drawn from a distribution F. The draws for different units of capital are independent and identically distributed. Thus, for a firm that has N_K units of capital, there are N_K cost draws. Given these draws, if the firm decides to augment m units of capital it will do so for the capital units with the cheapest cost draws. Note that capital augmentation is completely merger neutral: when two firms merge, their collective investment possibilities do not change.

The second is *greenfield investment*: a firm can build as many capital units as it wants at a cost $c_g \in [\overline{c}, \overline{c}_g]$ drawn from a distribution G. Greenfield investment allows a firm whose capital stock has depreciated to zero to invest, albeit at a cost that exceeds that of capital augmentation. We also choose the range of greenfield costs $[\overline{c}, \overline{c}_g]$ to be small so that this investent technology is approximately merger neutral. (It would be fully merger neutral if $\overline{c}_g = \overline{c}$; in our computations we introduce uncertain greenfield investment costs to ensure existence of equilibrium.)

As we will discuss shortly, our model will allow for entry. In contrast to Pakes and McGuire (1994), we endow an entrant with the same investment technology as incumbents. The entrant, however, starts with no capital, so it must initially do greenfield investment.

Put together, the capital augmentation and greenfield investment processes allow for significantly richer investment dynamics than in the typical dynamic industry model. Firms can

[3] Alternatively, if the merged firm kept both investment processes we would need to keep track, as a separate state variable, of how many investment processes a firm possesses, which has no natural bound.

[4] Of course, because of noncooperative investment behavior, there could be efficiency benefits of the merger in actually achieving a given amount of market-wide capital growth at least cost.

expand their capital by multiple units at a time through either investment method. And firms with no capital, including new entrants, can decide endogenously how far to "jump" up in their capital stock.

A state is a pair (K_1, K_2). In our computations firms will be restricted to some number S of possible capital levels, with this number chosen to be non-binding.

Capital can also depreciate: in each period a unit of capital has a probably $d > 0$ of becoming worthless (including for any future capital augmentation). Depreciation realizations are independent across units of capital. This depreciation process is also merger neutral, in contrast to the depreciation process in Pakes and McGuire (1994). Finally, the firms discount the future according to discount factor $\delta < 1$.

2.3 Mergers and Bargaining

In each period, firms can propose a merger. Following a merger, a new entrant appears in the market with zero capital.[5] We assume that the new entrant is owned by the manager of the acquired firm. Thus, the post-merger continuation values used to evaluate the profitability of a merger include the continuation value of this manager in his new role as an entrant. That is, there are just two managers who possess the know-how required to run a firm in this market. This assumption also justifies our restriction to a maximum of two active firms.[6]

Proposing a merger involves a cost $\phi \in [\underline{\phi}, \overline{\phi}]$ drawn each period in an iid fashion from distribution Φ. Firms engage in Nash bargaining to decide whether to merge. Thus, they propose their merger provided the expected gain in their joint continuation value, taking into account the likelihood the merger will be approved, exceeds ϕ. If they do agree to propose, they make a side transfer to split evenly the joint value gain from the merger. The disagreement values in this bargaining reflect the two firms' continuation values in the event they do not merge. For example, when $e = 1$, if the initial capital stocks are (K_1, K_2) and $V(K_1, K_2)$ is the value of a firm with K_1 units of capital when its rival has K_2 units of capital, this joint value gain is

$$R \equiv \{[V(K_1 + K_2, 0) + V(0, K_1 + K_2)] - [\overline{V}(K_1, K_2) + \overline{V}(K_2, K_1)]\},$$

where the "disagreement payoff" $\overline{V}(K_1, K_2)$ is the value of a firm that has K_1 units of capital when its rival has K_2 units of capital if no merger occurs (in the timing given below, this would be its value at the start of stage 5).

2.4 Merger Policy

The antitrust authority has the ability to block mergers. Blocking a merger involves a cost $b \in [\underline{b}, \overline{b}]$ drawn each period in an iid fashion from a distribution H. We will consider two possible scenarios. In one, we suppose that the antitrust authority can commit to a deterministic policy

[5] In an extension we consider the case in which the entrant enters with probability $e \in (0, 1)$ in each subsequent period.

[6] Absent this assumption we could not evaluate the profitability of entry for a third firm without having a solution for the multi-firm bargaining with externalities problem that would arise after its entry.

$a(K_1, K_2) \in \{0,1\}^{S^2}$ which specifies whether a proposed merger would be approved ($a = 1$) or not ($a = 1$) in each state (K_1, K_2). These commitment policies will be restricted further to two classes of policies described in Section 5. We also consider cases in which the antitrust authority cannot commit to its policy. In that case, in any state (K_1, K_2) it will decide whether to block a merger by comparing the increase in its welfare criterion from blocking (we will consider both consumer value and aggregate value) to its blocking cost realization b. In that case, a Markovian strategy for the antitrust authority is a state contingent threshold $\widehat{b}(K_1, K_2)$ describing the highest blocking cost at which the authority will block a merger in a given state (K_1, K_2). Equivalently, this can be translated into a merger acceptance probability $a(K_1, K_2) \in [0,1]$. We call the equilibrium policy that emerges a "Markov perfect policy." Identifying this policy is of interest for both positive and normative reasons. First, on a positive level, the antitrust authority may well lack an ability to commit to its future approval policy. For example, while both the DOJ and FTC in the U.S. periodically issue *Horizontal Merger Guidelines*, which may serve to partially commit these agencies, it is also true that over time their actual policy often comes to deviate substantially from the *Guidelines'* prescriptions. On a normative level, if there is a Markov perfect policy whose welfare performance is close to that of the optimal commitment policy, the fact that this policy will be one that the agencies won't have an incentive to deviate from may well make it the most desirable choice as it will reduce firms' incentives to lobby for exceptions to stated policy.

2.5 Timing

In each period, the timing of the model is as follows:

1. Firms observe each others' capital stocks.

2. The firms observe their proposal cost ϕ and bargain over whether to propose a merger.

3. If a merger is proposed, the antitrust agency observes its blocking cost b and decides whether to block it. (This is when commitment is not possible; the antitrust authority simply follows its commitment strategy when commitment is possible.) If a merger is consummated in state (K_1, K_2), the merged firm's capital stock will be $K_1 + K_2$.

4. If a merger occurred, an entrant enters with no capital.

5. Firms choose their output levels simultaneously and the market price is determined.

6. Firms privately observe their capital augmentation and greenfield cost draws and decide on their investments.

7. (Stochastic) depreciation occurs, resulting in the capital levels at which firms begin the next period.

3 Equilibrium and Computation

In this section we initially explain our notation, our definition of Markov perfect equilibrium, and the algorithm we use for numerically computing equilibria in terms of the simplest model that we consider: an industry with two firms, potential entry from monopoly to duopoly, possible merger from duopoly to monopoly, a random proposal cost whenever merger occurs, and a fixed merger policy that the antitrust authority (AA) imposes with commitment. We then relax the commitment assumption, introduce into the model the AA's costs of considering and blocking a proposed merger, and define our notion of a Markov perfect merger policy.

3.1 Fixed Merger Policy

Fix the industry's fundamentals: demand, production function, investment costs, discount factor, and depreciation rate. Let $\pi(K_1, K_2)$ be the single-period Cournot profit in state (K_1, K_2). Also fix the AA's merger policy. In particular suppose the AA is able to *ex ante* commit to a symmetric merger policy $a : \{1, 2, \ldots, S\}^2 \to \{0, 1\}$ where $a(K_1, K_2) = 1$ indicates that the AA will approve a merger proposal in state (K_1, K_2) and $a(K_1, K_2) = 0$ indicates that it will reject without hope of appeal. Symmetry means, for any $(K', K'') \in \{1, 2, \ldots, S\}^2$, $a(K', K'') = a(K'', K')$.[7] Let $\mathcal{S} = \{0, 1, \cdots, S\}$ be the possible levels of capital a firm may possess within the industry. A Markov perfect equilibrium consists of (i) a value function $V_i(K_1, K_2) : \mathcal{S}^2 \to \mathbb{R}$ and a policy function $u_i(K_1, K_2) : \mathcal{S}^2 \to [0, 1] \times [0, 1]^{S-K_1+1}$ for each firm. The firms that we model are *ex ante* symmetric and we solve only for symmetric equilibria. Therefore, given that the merger policy is also symmetric, for any $(K', K'') \in \mathcal{S}^2$, $V_1(K', K'') = V_2(K'', K')$ and $u_1(K', K'') = u_2(K'', K')$. Consequently we can simplify the presentation of the model and its notation by presenting it in terms of firm 1 alone.

The value $V_1(K_1, K_2)$ is firm 1's beginning of period expected net present value (ENPV) when the industry is in state (K_1, K_2). The policy function $u_1(K_1, K_2)$ gives the vector $(\psi, \xi_{1,0}, \xi_{1,1}, \cdots, \xi_{1,S-K_1})$ where:

1. ψ is the *ex ante* probability that firm 1's optimal strategy in state (K_1, K_2) is to merge with firm 2

2. $\xi_{1,k}$ is the probability that, if it does not merge, firm 1's optimal investment in state (K_1, K_2) is k additional units of capital.

This policy function solves a somewhat complicated maximization problem. We first discuss the expected gains from merger. We then outline (i) how we compute the policy functions given firms' value functions and (ii) how we compute value functions given firms' policy functions. We then outline the algorithm we use to find the fixed point in both policy and value functions that define an equilibrium.

Gains from merger. Fix estimates of the two firms' value functions $V_1^{(\ell)}$ and $V_2^{(\ell)}$. Suppose the industry is in state (K_1, K_2) at the beginning of period t. Just before firm 1 makes the decision whether to merge or not, it draws a random cost ϕ that firms 1 and 2 jointly incur if they proceed to proposing a merger. This cost has distribution Φ and support $\left[\underline{\phi}, \overline{\phi}\right]$. Merging, provided it is not forbidden by the AA's merger policy, is individually rational for the two firms if[8]

$$V_1^{(\ell)}(K_1 + K_2, 0) + V_2^{(\ell)}(K_1 + K_2, 0) - \phi > V_1^{(\ell)}(K_1, K_2) + V_2^{(\ell)}(K_1, K_2). \quad (1)$$

[7] We assume that merger takes place only in states in which $K_1, K_2 \geq 1$.

[8] Firms 1 and 2 draw the identical value of ϕ; consequently they never disagree on whether a merger is individual rational.

Let

$$\Delta_G(K_1,K_2) = \left[V_1^{(\ell)}(K_1+K_2,0)+V_2^{(\ell)}(K_1+K_2,0)\right] - \left[V_1^{(\ell)}(K_1,K_2)+V_2^{(\ell)}(K_1,K_2)\right]; \tag{2}$$

it is the gain from merging gross of the proposal cost. Therefore merger is only *individually rational* if $\phi < \Delta_G(K_1,K_2)$. This implies that the *ex ante* probability of merger in state (K_1,K_2) is

$$\psi_1(K_1,K_2) = \Phi^{-1}(\Delta_G(K_1,K_2)) \tag{3}$$

The expected value of ϕ conditional on merger at (K_1,K_2) being individually rational is

$$\mathcal{E}[\phi|K_1,K_2] = \frac{\int_{\underline{\phi}}^{\Phi^{-1}(\psi(K_1,K_2))}\phi\Phi'(\phi)d\phi}{\psi(K_1,K_2)} \tag{4}$$

where Φ' is the density of Φ. Therefore, conditional on $V_1^{(\ell)}$ and $V_2^{(\ell)}$, an estimate of the *ex ante* gains from merger net of expected proposal costs is

$$\Delta_N\left(K_1,K_2|V_1^{(\ell)},V_2^{(\ell)},u_1^{(\ell)},u_2^{(\ell)}\right) = \Delta_G(K_1,K_2) - \mathcal{E}[\phi|K_1,K_2]. \tag{5}$$

As discussed in Section 2, the net gains from merger are split evenly between the two firms through Nash bargaining.

Policy Functions. Once firm 1 has decided whether or not to merge, it draws a vector c of cost draws that has length $S-K_1$. Its first K_1 elements are the capital augmentation costs $c_j \in [\underline{c},\bar{c}]$, each of which is drawn independently with distribution F. The remaining $\max\{0,S-K_1\}$ elements are identically the greenfield cost, $c_g \in [\bar{c},\bar{c}_g]$, which is drawn independently from distribution G. Define $c(k)$ to be the sum of the k smallest elements of c.

Four cases must now be considered. First we analyze period t behavior when the two firms do not merge. On the equilibrium path merger does not occur in states (K_1,K_2) in which (i) the AA has prohibited merger or (ii) merger fails the individual rationality test (1). In such a state the firm earns static profits $\pi(K_1,K_2)$, invests in capital so as to maximize its ENPV, suffers depreciation, and transits to its next period state. The ENPV of the cash flows this behavior creates is the firm's *value* $V_1(K_1,K_2)$.

Second, a firm at a state (K_1,K_2) in which merger is the equilibrium action will be forced to deviate from the equilibrium path if bargaining breaks down over how to split the gains with the other firm. When a breakdown occurs we assume that the firm makes a one-shot deviation and behaves as if merger were prohibited in the state (K_1,K_2) for the current period. For firm 1 the ENPV of the future cash flows this behavior creates is its *disagreement point,* $\overline{V}_1(K_1,K_2)$, that plays a crucial role in splitting the gains from merger. While the disagreement point $\overline{V}_1(K_1,K_2)$ and the value $V_1(K_1,K_2)$ when no merger occurs in the current period's equilibrium path are distinct conceptually, they share the same calculation.

Third, we analyze behavior and value creation if in state (K_1,K_2) merger occurs. On the equilibrium path this is straightforward because we have already shown how to calculate each firm's disagreement point payoff. The two firms pay a random proposal cost and merge, moving the market immediately to state $(K_1+K_2,0)$. The firm earns static profits $\pi(K_1+K_2)$, invests in capital so as to maximize its ENPV, suffers depreciation, and transits to its next period state. The ENPV of the cash flows this behavior creates is the firm's *value* $V_1(K_1, K_2)$. Notice that the firm in state (K_1, K_2) that merges at the beginning of period t behaves the remainder of the period exactly like a firm that began period t in state $(K_1 + K_2, 0)$.

Finally, if merger occurs off the equilibrium path, then we assume that the aberration is a one-shot deviation after which the merged firm resumes playing the equilibrium behavior associated with whatever state in which it finds itself.

No merger case. Fix estimates $V_1^{(\ell)}$ and $V_2^{(\ell)}$ of both firms' value functions, a draw of c, and an estimate $u_2^{(\ell)}$ of firm 2's policy function. Consider state (K_1, K_2) and suppose a merger does not occur in that state. An estimate of firm 1's value at the beginning of period t if it purchases k_1 additional units of capital is

$$\begin{aligned} v_{1k}(k_1, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}) &= \pi_i(K_1, K_2) - c(k_1) \\ &\quad + \delta \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} \hat{\tau}_{1,i}(k_1|K_1)\, \tau_{2,j}\left(K_1, K_2, u_2^{(\ell)}\right) V_1^{(\ell)}(i, j) \end{aligned} \tag{6}$$

where:

1. $\hat{\tau}_{1,i}(k|K_1)$ is the probability that firm 1 ends the period with i units of capital given that it starts with capital K_1, purchases k_1 additional units of capital, and suffers depreciation on each unit of capital with probability d.

2. $\tau_{2,j}\left(K_1, K_2, u_2^{(\ell)}\right)$ is the probability that firm 2 ends the period with j units of capital given that it starts with capital K_2, purchases k_2 units of additional capital with probability ξ_{2,k_2}, k_2' units of additional capital with probability $\xi_{2,k_2'}$, etc., and suffers depreciation on each unit of capital with probability d.[9]

In state (K_1, K_2), given the realization c of cost draws, firm 1's optimal purchase of capital is the $k_1^* \in \{0, 1, \cdots, S - K_1\}$ that satisfies $v_{1k}(k_1^*, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}) \geq v_{1k}(k_1, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)})$ for all $k_1 \in \{0, 1, \cdots, S - K_1\}$. Define $\omega\left(k_1, c, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}\right)$ to be the indicator function with value 1 if $k_1 = k_1^*$ and 0 otherwise.

Let the probability density of the vector c of cost draws over its domain $\mathcal{C}$ be h; this density is determined by the distributions F and G and the independence of the draws. For a given $k_1 \in \{0, 1, \ldots S - K_1\}$, an updated estimate of the probability $\xi_{1,k_1}(K_1, K_2)$ in firm 1's policy function is therefore

$$\xi_{1,k_1}^{(\ell+1)}\left(K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}\right) = \int_{\mathcal{C}} \omega\left(k_1, c, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}\right) h(c)\, dc. \tag{7}$$

This is a difficult integral to evaluate because it is intractable to partition the region of integration $\mathcal{C}$ into the $S - K + 1$ regions $\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{S-K}$ such that, for $k \in \{0, 1, \ldots S - K_1\}$, $c \in \mathcal{C}_{k_1}$ if and only if $\omega\left(k_1, c, K_1, K_2|u_2^{(\ell)}, V_1^{(\ell)}\right) = 1$. We sidestep this difficulty by using Monte Carlo integration. While this is effective, it is computationally intensive and does introduce error into our policy function calculations.

Merger case. In state (K_1, K_2), at the beginning of period t, firm 1's policy function is $u_1(K_1, K_2) = (\psi, \xi_{1,0}, \xi_{1,1}, \cdots, \xi_{1,S-K_1})$. Firm 1 therefore knows ψ, the *ex ante* probability

[9] $\tau_{1,j}(K_1, K_2, u_2)$, which is used below, is defined analogously.

that it will merge, but does not yet know its proposal cost ϕ or its investment cost vector c. If $\psi = 1$ or if the value of ϕ that it realized just after the period begins is sufficiently small to make merger individually rational, then firm 1 opens negotiations with firm 2. We assume they follow an alternating offer bargaining protocol under full information with only short periods between offers and responses. Consequently the firms' discount factor during the negotiations is very close to one: it is not the $\delta = 0.8$ discount factor that applies to the capital accumulation game the firms play. Binmore, Rubinstein, and Wolinsky (1986) show that the equilibrium outcome of such alternate offering bargaining is essentially immediate agreement on the Nash bargaining solution.

Thus, if merger is individually rational and allowed, then on the equilibrium path merger occurs without delay at the beginning of the period. No discounting of the gains occur. It occurs with firm 1 buying out firm 2's owner, and the industry immediately moving to state $(K_1 + K_2, 0)$.[10] In that state the merged firm is a monopoly so a second merger in period t is impossible. Firm 1 adopts the policy function $u_1(K_1 + K_2, 0)$, realizes a vector c of cost draws, and invests so as to maximize its value. The bought-out owner of firm 2 founds a new firm that he leads into the industry. It adopts the policy function $u_2(K_1 + K_2, 0)$, realizes its greenfield cost draw, and invests to maximize its value.

Off the equilibrium path—for example if the merger is individually rational and allowed, but negotiations break down anyway—firm 1 retains the policy function $u_1(K_1, K_2)$ and invests in new capacity in accordance with the probabilities $\left(\xi_{1,0}, \xi_{1,1}, \cdots, \xi_{1,S-K_1}\right)$.

Value Function. In state (K_1, K_2) firm 1's value $V_1(K_1, K_2)$ is its equilibrium beginning of period ENPV. The expectation is taken before any information is revealed within the period. To emphasize, $V_1(K_1, K_2)$ is firm 1's value when the period begins, before it learns its proposal cost ϕ, before it learns whether merging is individually rational, and before it realizes its investment cost vector c. The firm, however, does know the *ex ante* probability $\psi(K_1, K_2)$ that it will merge with firm 2.

The value $V_1(K_1, K_2)$ must satisfy its Bellman equation. Fix estimates of both firms' value functions, $V_1^{(\ell)}$ and $V_2^{(\ell)}$, and both firms policy functions, $u_1^{(\ell)}$ and $u_2^{(\ell)}$. Calculate for state (K_1, K_2) firm 1's beginning-of-period expected value. It has two components: (i) firm 1's expected value if merger does not occur and (ii) the expected additional value if the expected gains from merger exceed the minimum proposal cost $\underline{\phi}$. Let

$$\begin{aligned} w(K_1, K_2) &= \pi_1(K_1, K_2) - \mathcal{E}c(K_1, K_2 | u_1^{(\ell)}) \\ &\quad + \delta \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} \tau_{1,i}\left(K_1, K_2, u_1^{(\ell)}\right) \tau_{2,j}\left(K_1, K_2, u_2^{(\ell)}\right) V_1^{(\ell)}(i, j) \end{aligned} \tag{8}$$

where

$$\mathcal{E}c\left(K_1, K_2 | u_1^{(\ell)}\right) = \int_{\mathcal{C}} \sum_{k_1 \in \{0,1,\ldots,S-K_1\}} \omega\left(k_1, c, K_1, K_2 | u_2^{(\ell)}, V_1^{(\ell)}\right) c(k_1)\, h(c)\, dc.$$

is the expected investment cost at (K_1, K_2). Then

$$\begin{array}{l} V_1^{(\ell+1)}(K_1, K_2) = w(K_1, K_2) \text{ if } \Delta_G(K_1, K_2) - \underline{\phi} \leq 0 \\ \overline{V}_1^{(\ell+1)}(K_1, K_2) = w(K_1, K_2) \text{ if } \Delta_G(K_1, K_2) - \underline{\phi} > 0 \end{array}. \tag{9}$$

[10] That firm 1 buys out firm 2 is arbitrary. Just as well it could be that firm 2 buys out firm 1.

Thus, if merger does not occur at (K_1, K_2) because the gains from merger are insufficient to cover $\underline{\phi}$, then $w(K_1, K_2)$ is an estimate of firm 1's value $V_1^{(\ell+1)}(K_1, K_2)$. On the other hand, if at (K_1, K_2) merger may occur with positive probability, then $w(K_1, K_2)$ is firm 1's disagreement point $\overline{V}_1^{(\ell+1)}(K_1, K_2)$, i.e., it is the ENPV of firm 1's cash flows if the two firms' negotiations over the terms of a merger break down.[11]

If the two firms may possibly merge at (K_1, K_2), then the expected gain to firm 1's owners from merging must be added to $\overline{V}_1^{(\ell+1)}(K_1, K_2)$ in order to obtain an estimate of the firm's value $V_1^{(\ell+1)}(K_1, K_2)$. As discussed above, the expected total gain to both firms' owners is $\Delta_N\left(K_1, K_2|V_1^{(\ell+1)}, V_2^{(\ell+1)}, u_1^{(\ell+1)}, u_2^{(\ell+1)}\right)$. Nash bargaining splits this gain evenly between two firms' owners. From a beginning-of-period perspective the probability of merger is $\psi(K_1, K_2)$. This means that the *ex ante* expected gain to firm 1's owner from the possibility of merger is

$$\frac{1}{2}\psi(K_1, K_2)\,\Delta_N\left(K_1, K_2|V_1^{(\ell)}, V_2^{(\ell)}, u_1^{(\ell)}, u_2^{(\ell)}\right). \tag{10}$$

Therefore an updated estimate of firm 1's value is

$$\begin{aligned} V_1^{(\ell+1)}(K_1, K_2|u_1^{(\ell)}, u_2^{(\ell)}, V_1^{(\ell)}) &= \overline{V}_1^{(\ell+1)}(K_1, K_2|u_1^{(\ell)}, u_2^{(\ell)}, V_1^{(\ell)}) \\ &\quad + \frac{1}{2}\psi_1(K_1, K_2)\,\Delta_N\left(K_1, K_2|V_1^{(\ell)}, V_2^{(\ell)}, u_1^{(\ell)}, u_2^{(\ell)}\right). \end{aligned} \tag{11}$$

This is firm 1's Bellman equation for the merger case. If $V_1^{(\ell+1)}(K_1, K_2|u_1^{(\ell)}, u_2^{(\ell)}, V_1^{(\ell)}) = V_1^{(\ell)}(K_1, K_2)$ for all $(K_1, K_2) \in \mathcal{S}^2$, then the equation is satisfied and, conditional on $u_1^{(\ell)}$ and $u_2^{(\ell)}$, the value function $V_1^{(\ell+1)} \equiv V_1^{(\ell)}$ is a fixed point. We emphasize that, at all states (K_1, K_2) for which merger is a possibility, $V_1(K_1, K_2)$ and $V_2(K_1, K_2)$ incorporate the *ex ante* expected gains from merger. These gains increase the value of reaching (K_1, K_2) and therefore provide both firms with incentives to invest in order to get there.

Computation of Equilibrium. The functions (V_1, V_2, u_1, u_2) are an equilibrium for the industry if they are a fixed point for this system of equations:

1. Given u_1 and u_2, the value functions V_1 and V_2 satisfy their Bellman equations (equation (11) for V_1 and the analogous equation for V_2.
2. Given V_1 and V_2, each component of the policy functions u_1 and u_2 satisfies equation (7).

The algorithm that we use to numerically solve for equilibria is a version of the well-known Pakes-McGuire algorithm (1994). It is a straightforward iterative process. We only have to find V_1 and u_1 since we are seeking symmetric equilibria. Given an initial guess for $V_1^{(0)}$ and $u_1^{(0)}$use equations (6–7) to compute an updated estimate $u_1^{(1)}$ of firm 1's policy function. Plug $u_1^{(1)}$ into equation (11) and use an iterative method to find its fixed point $V_1^{(1)}$. Plug this revised value function estimate into equations (6–7) to obtain $u_1^{(2)}$. Continue this iterative process until $\|V_1^{(\ell+1)} - V_1^{(\ell)}\| \leq \varepsilon$ for some small $\varepsilon > 0$. This algorithm is not guaranteed to converge, but in our experience with this model it has converged quite successfully whenever the support of each firm's cost draws for ϕ is not too narrow.

[11] In case of a breakdown in negotiations no proposal cost ϕ is incurred because no merger is proposed.

3.2 Merger Policy without Commitment

The discussion up to this point had the AA fix a merger policy without making any explicit reference to the AA's objective function. Given a merger policy, the equilibrium calculations are driven solely by each firm's objective of maximizing the ENPV of its future cash flows. Modeling merger policy without commitment makes the AA an active player in the game we are modeling and therefore requires specification of what objective function it will seek to maximize dynamically in its interaction with the industry's firms. We consider two objectives: maximizing the ENPV of either consumer surplus or aggregate surplus (the sum of consumer surplus and all firms' producer surplus). In computing these objectives we adopt $\delta = 0.8$, the discount factor the firms use in computing their ENPVs, as the discount factor for calculating the ENPVs of aggregate value and consumer surplus. Ee seek a policy in which (i) the policy never rejects a merger that as a one-shot deviation would increase consumer or aggregate value and (ii) never allows a merger whose rejection as a one-shot deviation would increase consumer or aggregate value.

The formal setup is as follows:

Model. The three additions that we must make to the model are: add the AA as a formally modeled player, introduce the blocking cost b, and define the merger policy $a(K_1, K_2)$ to be mixed. If the firms in state (K_1, K_2) propose a merger, then at cost b the AA can block the merger without possibility of appeal on the firms' part. The blocking cost is a random variable with distribution function H and support $\left[\underline{b}, \bar{b}\right]$. The AA's draw of b of its blocking cost is independent of the firms' draw of the proposal cost ϕ. In the presence of random blocking costs, the AA makes deterministic decisions based on its realizations of the blocking costs. To the firms, however, the policy appears randomized at the time they propose a merger: $a(K_1, K_2) \in [0, 1]$ is the probability of approval. Finally, the AA's objective in each state is to maximize the ENPV of either aggregate surplus or consumer surplus, which we refer to as simply the aggregate value AV or consumer value CV, respectively.

Consumer Value and Aggregate Value. Let $CS(K_1, K_2)$ be the static consumer surplus that the market generates in a single period when production and sales take place with firms having capital levels K_1 and K_2.; $AS(K_1, K_2) = CS(K_1, K_2) + \pi(K_1, K_2) + \pi(K_2, K_1)$ is the aggregate surplus. Let $CV(K_1, K_2)$ be the dynamic consumer value the market produces if it begins the period in state (K_1, K_2). More specifically, $CV(K_1, K_2)$ is the equilibrium ENPV of static consumer surplus that the industry generates if it starts at state (K_1, K_2) and moves through the state space according to equilibrium's transition probabilities. Aggregate value is the sum of the two firms' value and consumer value: $AV(K_1, K_2) = CV(K_1, K_2) + V_1(K_1, K_2) + V_2(K_1, K_2)$. We describe the AA's decisions here in terms of maximizing aggregate value; the analysis is exactly parallel for the objective of maximizing consumer value alone.

In order to write down the Bellman equation that generates CV for all $(K_1, K_2) \in \mathcal{S}$ we need three probabilities. First, the probability that the firms will merge in state (K_1, K_2) is $\rho_M(K_1, K_2) = \psi(K_1, K_2)\, a(K_1, K_2)$, the product of the probability the firms will propose a merger and the probability the AA will approve it. Second, the probability that the firms will propose a merger but the AA will block it is $\rho_B(K_1, K_2) = \psi(K_1, K_2)(1 - a(K_1, K_2))$. Last, the probability that the firms will not propose a merger is $\rho_{NP}(K_1, K_2) = 1 - \rho_M(K_1, K_2) -$

$\rho_B(K_1, K_2)$, the complement of the other two events. Calculation of ψ is defined in equation (3) and calculation of a is defined below. We also need the expected value of b conditional on the AA being willing to block at (K_1, K_2).

$$\mathcal{E}[b|K_1, K_2] = \frac{\int_{\underline{b}}^{H^{-1}(a(K_1,K_2))} bH'(b)db}{1 - a(K_1, K_2)} \tag{12}$$

where H' is the density of H. Observe that the AA only blocks at (K_1, K_2) if b is sufficiently small.

Given these definitions, the Bellman equation for CV is

$$\begin{aligned} CV(K_1, K_2) &= (1 - \rho_M(K_1, K_2))CV(K_1, K_2) + \rho_M(K_1, K_2) CV(K + K_2, 0) \\ &\quad -\rho_B(K_1, K_2) \mathcal{E}[b|K_1, K_2] \\ &\quad +\delta(1 - \rho_M(K_1, K_2)) \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} \tau_{1,i}(K_1, K_2, u_1) \tau_{2,j}(K_1, K_2, u_2) CV(i, j) \\ &\quad +\delta\rho_M(K_1, K_2) \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} \tau_{1,i}(K_1 + K_2, 0, u_1) \tau_{2,j}(K_1 + K_2, 0, u_2) CV(i, j) \end{aligned}$$

where (i) $\tau_{1,i}(K_1, K_2, u_1)$ and $\tau_{2,j}(K_1, K_2, u_2)$ are the transition probabilities that firms 1 and 2 follow if they do not merge at (K_1, K_2) and (ii) $\tau_{1,i}(K_1 + K_2, 0, u_1)$ and $\tau_{2,j}(K_1 + K_2, 0, u_2)$ are the transition probabilities that the merged firm and the new entrant follow if the firms do merge at (K_1, K_2) and immediately transit to state $(K_1 + K_2, 0)$. Notice that in each period expected blocking costs are subtracted out. Aggregate value is therefore

$$AV(K_1, K_2) = V_1(K_1, K_2) + V_2(K_1, K_2) + CV(K_1, K_2). \tag{13}$$

Firm 1's Proposal Decision. Suppose at the beginning of period t at state (K_1, K_2) firms 1 and 2 consider merging. At that point in time, they do not know the AA's realized b, but do know the *ex ante* probability $a(K_1, K_2)$ that the AA will approve the merger. Doing so has positive gains net of the proposal cost ϕ if:

$$\begin{aligned} a(K_1, K_2) \Delta_G(K_1, K_2) - \phi &= a(K_1, K_2) \{[V_1(K_1 + K_2, 0) + V_2(K_1 + K_2, 0)] \\ &\quad - [V_1(K_1, K_2) + V_2(K_1, K_2)]\} - \phi > 0. \end{aligned}$$

Before they know the realization of ϕ, the expected gain from the possibility of merger is $\widetilde{\Delta}_N(K_1, K_2) = [a(K_1, K_2) \Delta_G(K_1, K_2) - \mathcal{E}[\phi|K_1, K_2]$, where $\mathcal{E}[\phi|K_1, K_2]$ are the expected proposal costs, ddefined as in (4) but with $a(K_1, K_2) \Delta_G(K_1, K_2)$ replacing $\Delta_G(K_1, K_2)$ in (3). Then firm 1's Bellman equation is

$$\begin{aligned} V_1^{(\ell+1)}(K_1, K_2|u_1^{(\ell)}, u_2^{(\ell)}, V_1^{(\ell)}) &= \overline{V}_1^{(\ell+1)}(K_1, K_2|u_1^{(\ell)}, u_2^{(\ell)}, V_1^{(\ell)}) + \frac{1}{2}\psi_1(K_1, K_2) \\ &\quad \times\widetilde{\Delta}_N\left(K_1, K_2|V_1^{(\ell)}, V_2^{(\ell)}, u_1^{(\ell)}, u_2^{(\ell)}\right). \end{aligned} \tag{14}$$

where the expression $\overline{V}_1^{(\ell+1)}$ is given in equations (8-9).

Observe that the introduction of blocking costs into the model does not change the formulas for calculating firm 1's policy function u_1 though, of course, it does change the values in which those formulas result.

AA's Blocking Decision and Bellman Equation. We now turn to deriving the AA's policy and value functions. Suppose the industry is in state (K_1, K_2) and the gains from merger are sufficiently positive that the two firms with probability $\psi(K_1, K_2) > 0$ will propose a merger. If the firms do propose a merger, then the AA knows $\Delta_G(K_1, K_2)$, the gains from merger gross of proposal costs, and $\mathcal{E}[\phi|K_1, K_2]$, the expected proposal cost conditional on the merger being proposed. It also knows the dynamic aggregate value if the merger is blocked:

$$\begin{aligned} AV^B(K_1, K_2) &= AS(K_1, K_2) \\ &+\delta \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} \tau_{1,i}(K_1, K_2, u_1)\, \tau_{2,j}(K_1, K_2, u_2)\, AV(i, j) \end{aligned}$$

where $\tau_{1,i}(K_1, K_2, u_1)$ and $\tau_{2,j}(K_1, K_2, u_2)$ are the transition probabilities that firms 1 and 2 follow if they do not merge at (K_1, K_2). Additionally it knows the dynamic aggregate value if it approves and the merger proceeds:

$$\begin{aligned} AV^A(K_1, K_2) &= AS(K_1 + K_2, 0) \\ &+\delta \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} \tau_{1,i}(K_1 + K_2, 0, u_1)\, \tau_{2,j}(K_1 + K_2, 0, u_2)\, AV(i, j) \end{aligned}$$

where $\tau_{1,i}(K_1 + K_2, 0, u_1)$ and $\tau_{2,j}(K_1 + K_2, 0, u_2)$ are the transition probabilities that the merged firm and the new, potential entrant follow if they do merge at (K_1, K_2) and immediately transit to state $(K_1 + K_2, 0)$.

Let $\Delta_{AV}(K_1, K_2) = AV^B(K_1, K_2) - AV^A(K_1, K_2)$ be the decrease in dynamic aggregate value that the merger causes. If the merger decreases expected dynamic aggregate value, then the firm compares the decrease to its realized blocking cost b and only blocks the merger (incurring the realized b) if the decrease is greater than the blocking cost: block if and only if $b < \Delta_{AV}(K_1, K_2)$. Given that blocking cost b has distribution function H, the firms perceive a mixed strategy: blocking occurs with probability $1 - a(K_1, K_2)$ and approval occurs with probability $a(K_1, K_2)$ where[12]

$$a(K_1, K_2) = 1 - H(\Delta_{AV}(K_1, K_2)).$$

If the merger increases expected dynamic aggregate value $(\Delta_{AV}(K_1, K_2) < 0)$, then the AA approves the merger and sets $a(K_1, K_2) = 1$. The function $a(K_1, K_2)$ is the policy function for the AA.

The Bellman equation for the AA is the recursive formulation of aggregate value given by equation (13).

Equilibrium and Computation. An equilibrium to this extended model with an aggregate value (resp. consumer value) objective for the AA consists of the three value functions V_1, V_2, and AV (resp. CV) and the three policy functions u_1, u_2, and a. Numerically finding an equilibrium involves finding a fixed point in all six of these functions for the system of equations set out above. We have had good success in computing equilibria to this no commitment model using the same variant of the Pakes-McGuire algorithm that we have used to compute fixed policy equilibria of the basic model.

[12] $H^{-1}(1 - a(K_1, K_2))$ is therefore the cut-off value of b above which the AA does not block.

4 Investment and Merger Incentives under Fixed Merger Policies

In this section we have three goals. First, we describe the specific parameterization of the model that we employ and discuss the properties of the static monopoly and Cournot equilibria that this parameterization implies. Second, we consider the Markov perfect equilibrium when mergers are prohibited—the no-mergers case. We report its long run steady distribution over the state space $\mathcal{S}^2$, the producer and consumer values it generates, and the investment incentives it creates. Third, we consider the Markov perfect equilibrium when firms are permitted to merge in any state where it is profitable for them to do so—the all-mergers case. We report its steady state, measures of the consumer and producer values it generates, and the investment incentives it provides the firms. The all-mergers case is very different than the no-mergers case in structure, incentives, and welfare measures. Merging causes the industry to be much more concentrated than in the no-merger state. Not surprisingly, expected consumer value is on average substantially reduced. But surprisingly expected producer value is also reduced, though not by nearly the same amount. Comparison of the investment incentives and resulting dynamics in the no- and all-mergers cases shows the mechanisms behind this outcome.[13]

The Market. We examine two markets that are identical except that in the large market at any given price the quantity demanded is 50% larger than in the small market. Demand's functional form is $Q(p) = B(\alpha - p)^{\gamma}$ with parameter values $\alpha = 3, \gamma = 1$, and $B = 30$ for the large market and $B = 20$ for the small market. The firms share an identical production technology: Cobb-Douglas with functional form $Q = \left(K^{\beta} L^{(1-\beta)}\right)^{\theta}$, capital share parameter $\beta = 1/3$, and scale parameter $\theta = 1.1$. The wage rate of labor (L) is 1 per period. Firms have integer valued capital stocks. If an active firm has K units of capital at the beginning of a period, then each of the K units, $j \in \{1, 2, \ldots, K\}$, can be doubled at an idiosyncratic cost of c_j where c_j is independently drawn from the uniform distribution on the interval $[3, 6]$. Each firm, if it chooses to add less than K units of capital, naturally chooses to double those units that are cheapest to double. This is the capital augmentation process. In addition, a firm can more than double its capital level within a period by purchasing further units of capital at a uniform "greenfield" price c_g drawn from the uniform distribution on the interval $[6, 7]$. A potential entrant has no capital, so it is only able to purchase greenfield capital at the price c_g per unit where c_g is the result of an independent draw.

The discount factor firms use is $\delta = 0.8$; this corresponds to a period length of about 5 years. Each unit of capital depreciates independently with probability $d = 0.2$ per period. Since the state space in our model is $\{0, 1, \ldots, 20\}^2$, each active firm can accumulate up to 20 units of capital. This upper limit is artificial but, given the parameterization of demand, production, and capital costs, the firms never come close to accumulating 20 units in equilibrium.

The table below gives a sense of the large market's fundamental properties with its strong economies of scale and linear demand. It shows the static Cournot equilibrium outcomes for three different states: $(1, 0), (10, 0)$, and $(5, 5)$. The first two states, obviously, are monopoly states since the second firm has zero units of capital. The comparison between the two

[13]Future versions of the paper will include the benchmark cases of a social planner prescribing the investment behavior of the firms and, at the opposite end of the scale, an unregulated monopolist is exploiting the market dynamically.

monopoly states shows the substantial effects of the scale economies on marginal cost. It also shows, for state (1,0) monopoly, the effect of linear demand when price is high and quantity small: demand is quite elastic causing a small price-cost markup.

The (5, 5) state shows the static Cournot equilibrium when each firm has 5 units of capital. Splitting the 10 units of capital between two firms instead of concentrating them in one firm creates offsetting inefficiencies. The monopoly in state (10,0) exerts its market power to restrict output and raise price to 2.19 compared to the duopoly's 2.06. Per period consumer surplus as a consequence falls from 13.17 to 9.88, a change of 3.3. But the market's strong scale economies gives the monopolist a low marginal cost of 1.38 compared to the duopolist's marginal cost of 1.59. This results in total profits in the (10, 0) monopoly being 28.7 instead of 25.1 in the (5, 5) duopoly, an increase of 3.6. Thus aggregate static surplus in the (10, 0) state is almost identical to that in the $(5, 5)$ state.

Large Market Static Equilibrium

State	$(1, 0)$	$(10, 0)$	$(5, 5)$
Marginal Cost (MC)	2.60	1.38	1.59
Price (P)	2.80	2.19	2.06
$P \div MC$	1.08	1.59	1.29
Total Quantity	5.93	24.3	28.1
Total Profit	5.30	28.7	25.1
Consumer Surplus	0.586	9.88	13.17
Aggregate Static Surplus	5.89	38.6	38.3

As is always the case for static Cournot equilibrium, in every state where both firms are active, static incentives for the two firms to merge exist. The increase in total profits in going from state $(5, 5)$ to $(10, 0)$ is an example. Even with the market's strong scale economies the only state in which merger increases static consumer surplus is $(1, 1)$, and there the increase is trivial. Merger increases aggregate static surplus for all states in which $K_1 + K_2 \leq 11$, but unless the state is quite asymmetric the gain turns negative at higher total capital levels.[14]

Equilibrium with No Mergers Permitted. For the large market solving for the Markov perfect equilibrium without mergers being allowed gives an outcome that is not surprising if one is familiar with static Cournot equilibrium. Two firms tend to be active at almost all times and they tend towards approximately equal capital levels. The easiest way to see this is to examine the steady state distribution of states that the industry goes to in the long run. Figure 1 shows this distribution for the large market.[15] The mass in state $(4, 3)$ is 2.5%. This means that if the industry ran for many periods undisturbed and an observer then picked a period at random to check in what state the industry is, then the probability that it would be in state $(4, 3)$ is 0.025. A monopoly is of course any state when there is one active firm. Let a near monopoly be any state in which only one firm has greater than 1 unit of capital. The probability that the industry is a monopoly is 0.022 and the probability it is a near monopoly is 0.056. But the probability that it lies in the box of states $\{3, 4, \ldots 7\}^2$ is 0.760. The long run, steady state,

[14] When merger occurs the costs of both firms' capital levels are sunk.

[15] For space reasons, we only show the distribuiton for states $\{0, 1, \ldots, 10\}$. In fact, 99.8% of the distribution's mass lies within this truncated space.

Steady State	0	1	2	3	4	5	6	7	8	9	10
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.2%	0.2%	0.2%
1	0.0%	0.0%	0.0%	0.1%	0.2%	0.4%	0.4%	0.3%	0.2%	0.1%	0.0%
2	0.0%	0.0%	0.2%	0.5%	1.0%	1.4%	1.3%	0.9%	0.4%	0.1%	0.0%
3	0.0%	0.1%	0.5%	1.3%	2.5%	3.2%	2.8%	1.7%	0.7%	0.2%	0.0%
4	0.0%	0.2%	1.0%	2.5%	4.3%	5.1%	4.3%	2.4%	0.8%	0.2%	0.0%
5	0.1%	0.4%	1.4%	3.2%	5.1%	5.8%	4.5%	2.3%	0.7%	0.1%	0.0%
6	0.1%	0.4%	1.3%	2.8%	4.3%	4.5%	3.3%	1.6%	0.5%	0.1%	0.0%
7	0.2%	0.3%	0.9%	1.7%	2.4%	2.3%	1.6%	0.7%	0.2%	0.0%	0.0%
8	0.2%	0.2%	0.4%	0.7%	0.8%	0.7%	0.5%	0.2%	0.0%	0.0%	0.0%
9	0.2%	0.1%	0.1%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%
10	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 1: Steady state distribution of large market. The entry for a given state (K_1, K_2) reports the probability that in the long run the market will be in state (K_1, K_2).

expected total capital is 9.6 units. Expected consumer value in the steady state – the ENPV of per period consumer surplus – is 61.4. This value is slightly less than the consumer value of 65.8 that the static Cournot equilibrium would generate if the market remained stationary in state $(5, 5)$. Finally, the two firms in the equilibrium steady state have substantial market power: expected producer value is 81.1. Summing producer value and consumer value gives an aggregate value of 142.4.

While most of the steady state involves fairly symmetric capital positions, there is a low probability that the industry will switch from unconcentrated to quite concentrated. Specifically, if the market is currently in state $(5, 5)$—it's modal state—then in 10 periods there is a 0.04 probability the industry will have become a near monopoly as a result of catastrophic depreciation to one of the firms. But once it is in a monopoly state, it may a take a substantial time to escape. If the firm 1 is in state $(7, 0)$ currently, there is a 0.40 probability that firm 1 will still dominate in 10 periods with the market remaining a monopoly. There are two cost reasons it is so hard to catch up. First, the firm with capital 7 pays much less per unit of capital purchased: the firm (who chooses to add at most 4 units of capital) can engage in capital accumulation, using the lowest of its 7 cost draws from the uniform distribution on $[3, 6]$, whereas the entrant (who chooses to add at most 1 unit) has to engage in greenfield investment using a cost draw from the uniform on $[6, 7]$. Second, its scale economies are great: with capital level of 7 its marginal cost as a monopolist is 1.57 while setting a price of 2.29. If the potential entrant should enter with 2 units of capital, then at state $(7, 2)$ the dominant firm sells quantity 19.1 with a marginal cost of 1.51. The entering firm sells 6.7 units with a marginal cost of 1.92. Profits are 19.8 and 4.9 respectively. In equilibrium the entering firm finds the odds difficult: in state $(7, 0)$ it purchases one unit of capital with probability 0.21.

Convergence to the long run steady state is also slow and, as a corollary, catching up with a competing firm that has a lead in capital accumulation is both uncertain and slow. If the industry starts in period 0 at state $(0, 0)$, then in two periods the modal states for the industry is $(3, 2)$ and $(2, 3)$, each with probability 0.10. If the industry is at $(3, 2)$, then firm 2 in expectation only very slowly catches up with firm 1. As the table below shows, after five periods the lagging firm has in expectation increased its capital to 4.31 units from 2 units. In

Steady State	0	1	2	3	4	5	6	7	8	9	10
0	0.0%	0.3%	1.8%	5.7%	10.1%	9.7%	5.1%	1.2%	0.1%	0.0%	0.0%
1	0.3%	0.3%	0.6%	0.8%	0.8%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%
2	1.8%	0.6%	1.4%	1.9%	1.8%	1.1%	0.3%	0.0%	0.0%	0.0%	0.0%
3	5.7%	0.8%	1.9%	2.6%	2.3%	1.2%	0.3%	0.0%	0.0%	0.0%	0.0%
4	10.1%	0.8%	1.8%	2.3%	1.8%	0.8%	0.2%	0.0%	0.0%	0.0%	0.0%
5	9.7%	0.5%	1.1%	1.2%	0.8%	0.3%	0.1%	0.0%	0.0%	0.0%	0.0%
6	5.1%	0.2%	0.3%	0.3%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
7	1.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
8	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
10	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 2: Steady state distribution of small market with no mergers.

expectation firm 2 stays behind, but there is quite a bit of variance and after 5 periods it has a 0.30 probability of passing firm 2. The reason the lagging firm 'tends to catch up' is that in any Cournot industry the lagging firm has greater marginal revenue that gives it a greater incentive to invest.

Expected Capital levels from $(3, 2)$

Start	1 period	5 periods	10 periods
3	3.75	4.78	4.85
2	2.77	4.31	4.70

The small market with demand parameter $B = 20$ behaves quite differently because its lower demand causes it to get trapped into a monopoly state more frequently than is the case with the big market. Figure 2 shows the small market's steady state distribution. Its probability of being a monopoly in the long run is 0.678, which is enormously higher than the large market's 0.022 probability. When the small market is not caught in a monopoly its modal state is $(3, 3)$. In this modal state it does not take too serious of a depreciation catastrophe to reduce one of the firms to a zero capital level. Specifically, if the market is at $(3, 3)$, then 10 periods later there is a 0.32 probability that it is in a monopoly state. Further once it is in the monopoly state, it is unlikely to leave: starting in state $(6, 0)$ the probability is only 0.06 that it will escape monopoly within ten periods.

Equilibrium with All Mergers Allowed. Equilibrium with all mergers allowed is quite different. Figure 3 shows the steady state distribution that this equilibrium generates in the large market. Figure 4 shows the probability that a merger occurs in the different states. The probability that the industry is a monopoly is 0.300 and the probability it is a near monopoly is 0.687, up from 0.022 and 0.056 respectively in the no mergers case. This change in structure leads to substantial negative changes in the welfare measures as shown in the table immediately below. These numbers are striking because they uniformly hurt the three welfare measures of consumer value, producer value, and aggregate value. This is despite the success that unrestricted mergers have from the firms' point in view in raising expecting price, reducing

Steady State	0	1	2	3	4	5	6	7	8	9	10
0	0.0%	0.0%	0.1%	0.4%	1.1%	2.1%	2.9%	3.3%	2.7%	1.6%	0.6%
1	0.0%	***0.1%***	***0.3%***	***1.0%***	2.6%	4.3%	4.7%	3.7%	1.9%	0.6%	0.1%
2	0.1%	***0.3%***	***0.5%***	1.1%	2.1%	3.0%	2.8%	1.7%	0.6%	0.1%	0.0%
3	0.4%	***1.0%***	1.1%	0.9%	0.9%	0.9%	0.6%	0.3%	0.1%	0.0%	0.0%
4	1.1%	2.6%	2.1%	0.9%	0.5%	0.3%	0.1%	0.1%	0.0%	0.0%	0.0%
5	2.1%	4.3%	3.0%	0.9%	0.3%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
6	2.9%	4.7%	2.8%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
7	3.3%	3.7%	1.7%	0.3%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
8	2.7%	1.9%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
9	1.6%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
10	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 3: Steady state distribution of large market when mergers are permitted at every state. A darker shade of gray indicates a higher likelihood of merger occuring.

gers Happen	0	1	2	3	4	5	6	7	8	9	10
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1	0.0%	55.0%	42.4%	23.1%	8.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	0.0%	42.4%	54.0%	48.6%	40.2%	33.2%	28.1%	24.7%	22.5%	19.6%	17.4%
3	0.0%	23.1%	48.6%	54.2%	52.3%	49.2%	46.6%	43.1%	38.8%	35.2%	32.6%
4	0.0%	8.5%	40.2%	52.3%	55.0%	54.6%	51.4%	43.4%	37.2%	32.0%	25.4%
5	0.0%	0.0%	33.2%	49.2%	54.6%	54.5%	42.7%	33.6%	24.3%	12.6%	0.0%
6	0.0%	0.0%	28.1%	46.6%	51.4%	42.7%	29.9%	23.6%	15.1%	6.4%	0.0%
7	0.0%	0.0%	24.7%	43.1%	43.4%	33.6%	23.6%	17.9%	11.0%	4.3%	0.0%
8	0.0%	0.0%	22.5%	38.8%	37.2%	24.3%	15.1%	11.0%	6.3%	1.7%	0.0%
9	0.0%	0.0%	19.6%	35.2%	32.0%	12.6%	6.4%	4.3%	1.7%	0.0%	25.6%
10	0.0%	0.0%	17.4%	32.6%	25.4%	0.0%	0.0%	0.0%	0.0%	25.6%	87.4%

Figure 4: Probability of merger occuring whenever the large market enters state (K_1, K_2).

expected quantity, and limiting total capital. This not surprisingly reduces consumer value, but surprisingly it also significantly reduces producer value.

Expected Value[16]	No Mergers	All Mergers
Consumer Value	61.4	41.5
Producer Value	81.1	75.1
Aggregate Value	142.4	116.7
Price	2.10	2.26
Quantity	27.0	22.2
Total Capital	9.6	7.0

It is interesting to explore further the reasons behind these results. Consider, first, the reduction in total capital. Allowing mergers does two things. First, it changes the states in which investments are taking place by moving the market to monopoly positions. The average capital addition in the no mergers steady state is 2.40. If we keep firms' investment behavior fixed at their no mergers equilibrium levels but change the weighting over states to be that in the all-mergers-allowed steady state the average capital addition drops to 0.96.[17] Second, it introduces a hold-up-like aspect to investment. In states where mergers are very likely to occur, investments made today increase the value of the soon-to-be merged firm but are sunk before bargaining over the merger occurs. If this were the only effect, each firm would see half of the gain in joint value as its return on investment. However, investments also affect disagreement payoffs. Examining Figure 5 which shows equilibrium producer values for the all-mergers-allowed case, we can see that this disagreement effect is significant. For example, a change in the state from $(3,3)$ to $(3,4)$ lowers the value of the row firm from 35.08 to 34.63 despite the hold-up effect. Still, this reduction in the row firm's value is much less than in the same state when no mergers are allowed - see Figure 6 which shows equilibrium producer values for the no-mergers case. This effect lowers the row firm's incentives to invest even though the effect of this increase in capital on producer value is nearly identical in the two cases (5.38 with all mergers allowed versus 5.26 with no mergers allowed).

Why is average producer value so much lower? Allowing mergers puts the market in monopoly states with high probabilities. In these states, when all mergers are allowed, the firm with zero capital frequently invests with the hope of being bought out. Indeed, it invests much more than a similarly positioned firm would when no mergers are allowed. For example, as we noted earlier, in the no mergers equilibrium, the firm with no capital in state $(7,0)$ invests with probability 0.21. When all mergers are allowed this probability jumps to 1. This fact can also be seen in the dramatically higher value that this firm has when all mergers are allowed:

[16] These expected values are weighted averages across the full state space using the steady state probabilities that the market is in a particular state.

[17] The average capital addition in the all-mergers-allowed steady state is 1.75. Keeping investment behavior fixed at the all-mergers-allowed equilibrium behavior and reweighting by the steady state probabilities in the no mergers equilibrium, the average capital addition would be 2.71.

roducer Value	0	1	2	3	4	5	6	7	8	9	10
0	8.07	8.65	8.64	8.06	7.53	7.16	6.92	6.75	6.64	6.55	6.47
1	21.09	20.39	20.15	19.79	19.44	19.15	18.91	18.73	18.59	18.46	18.32
2	32.50	29.82	28.79	28.29	27.98	27.74	27.57	27.42	27.31	27.23	27.11
3	42.20	37.97	35.97	35.08	34.63	34.37	34.19	34.05	33.99	33.89	33.73
4	50.39	45.10	42.26	40.91	40.23	39.86	39.62	39.51	39.42	39.26	39.02
5	57.41	51.41	47.90	46.12	45.19	44.68	44.32	44.22	44.10	43.89	43.59
6	63.58	57.07	53.05	50.91	49.75	49.07	48.59	48.36	48.10	47.76	47.29
7	69.11	62.24	57.81	55.36	53.87	52.93	52.35	52.00	51.65	51.23	50.68
8	74.16	67.01	62.22	59.40	57.70	56.60	55.93	55.47	55.03	54.53	53.94
9	78.84	71.45	66.30	63.23	61.37	60.16	59.41	58.85	58.32	57.75	57.13
10	83.22	75.58	70.13	66.88	64.91	63.65	62.85	62.21	61.57	60.92	60.40

Figure 5: Each entry reports the row firm's producer value at state (K_1, K_2) for the large market's all-mergers allowed case.

roducer Value	0	1	2	3	4	5	6	7	8	9	10
0	4.58	4.58	2.47	1.51	0.84	0.42	0.23	0.13	0.09	0.08	0.07
1	18.36	17.40	14.61	13.09	12.03	11.24	10.61	10.11	9.70	9.36	9.07
2	32.51	30.24	25.74	23.34	21.69	20.46	19.49	18.71	18.06	17.51	17.02
3	43.74	39.82	34.81	31.89	29.86	28.32	27.11	26.12	25.30	24.59	23.99
4	53.99	47.88	42.56	39.28	36.96	35.19	33.80	32.66	31.70	30.87	30.14
5	64.14	54.90	49.38	45.82	43.27	41.33	39.79	38.53	37.46	36.55	35.67
6	72.99	61.33	55.49	51.71	48.99	46.91	45.26	43.89	42.74	41.77	40.75
7	80.94	67.23	61.08	57.11	54.25	52.05	50.30	48.86	47.63	46.61	45.46
8	88.11	72.65	66.24	62.12	59.13	56.84	55.01	53.49	52.20	51.10	49.88
9	94.47	77.70	71.07	66.81	63.72	61.34	59.43	57.86	56.52	55.32	54.05
10	99.47	82.43	75.61	71.23	68.06	65.61	63.64	62.01	60.63	59.30	57.97

Figure 6: Each entry reports the row firm's producer value at state (K_1, K_2) for the large market's no-mergers case.

6.75 versus only 0.13 in the no-mergers-allowed equilibrium. Unfortunately for producer value, these investments are made, on average, at very high cost and greatly reduce the dominant firm's value.[18]

Worse yet for joint profit, while a merger doesn't occur at state $(7, 1)$, with probability 0.98 the next transition (prior to depreciation) is to $(7, 2)$, where a merger occurs with probability 0.247. If after depreciation the state remains at $(7, 2)$, the next transition (prior to depreciation) goes to state $(7, 3)$ with probability 0.73 where a merger happens with probability 0.431. And, once a merger happens, we are back to a monopoly state at which the cycle starts again.

5 Optimal Merger Policy

In this section, we analyze the antitrust authority's optimal merger approval policy. We again focus on the two markets analyzed in the previous section. We consider both expected discounted consumer surplus and expected discounted aggregate surplus as objective functions for the antitrust authority, which we refer to as consumer value (CV) and aggregate value (AV) respectively.

5.1 Feasible Policies

We allow mergers to occur provided that the total capital of the two firms is no greater than 20. (This restriction ensures that we know the continuation value of the merged firm.) In practice, states outside of this range have close to a zero probability of occurring in the markets we are studying. We consider two different types of settings, depending upon whether or not the authority can commit to its decision in a given state:

No Commitment In this setting, we assume that the antitrust authority cannot commit to its policy. Like each of the firms, the authority is thus a player in a dynamic game. If the authority chooses to block a proposed merger in state (K_1, K_2), it has to pay the blocking cost b which it privately observes prior to making its decision. Recall that a Markovian strategy for the antitrust authority is a state-contingent threshold $\widehat{b}(K_1, K_2)$ specifying the highest blocking cost at which the authority will block a merger in a given state (K_1, K_2). Appealing to the one-stage deviation principle, strategy $\widehat{b}(\cdot) \in [\underline{b}, \overline{b}]^{S^2}$ is a Markov-perfect merger policy if the authority has no incentive to deviate from $\widehat{b}(\cdot)$ at any decision node, assuming it follows $\widehat{b}(\cdot)$ in the continuation game. In the computations we have conducted so far, we assume that b is drawn from a uniform distribution on $[0, 1]$.[19]

Commitment In this setting, we assume that the antitrust authority can commit ex ante to a pure action $a(K_1, K_2) \in \{0, 1\}$ for each state (K_1, K_2), where $a = 1$ if the merger is approved when proposed and $a = 0$ if it is blocked. Observe that there are 2^{100}

[18] By affecting the value of the merged firm, these reductions in producer value at monopoly states also affect investment incentives at non-monopoly states at which mergers are very likely.

[19] Part of our motivation for introducing the blocking cost is to insure existence of equilibrium, by smoothing out the antitrust authority's behavior. At present, the blocking cost, which has the same distribution as the proposal cost, may be too large. We plan to explore reducing the upper bound of its support.

possible deterministic symmetric merger policies. Thus, for computational reasons, we restrict the space of feasible commitment policies, focusing on two classes of deterministic commitment policies:[20]

Herfindahl-based Policy Under this type of policy, a proposed merger in state (K_1, K_2) is approved if and only if the capital stock-based Herfindahl index in that state, denoted $H(K_1, K_2)$, satisfies the inequality $H(K_1, K_2) \geq \underline{H}$, where $\underline{H} \in [0, 1]$ is the authority's policy variable.[21] Note that because there are only two firms, the post-merger Herfindahl index always equals one: $H(K_1 + K_2, 0) = H(0, K_1 + K_2) = 1$. So, this policy of setting a lower bound on the pre-merger Herfindahl index is equivalent to a policy where a merger is approved if and only if the increase in the Herfindahl index, $\Delta H(K_1, K_2) \equiv 1 - H(K_1, K_2)$, is below some threshold: $\Delta H(K_1, K_2) \leq \underline{\Delta H} \equiv 1 - H(K_1, K_2)$. It can also be thought of as permitting a merger with a "failing firm."

Capital-stock-based Policy Under this type of policy, a proposed merger in state (K_1, K_2) is approved if and only if (i) $K_1 + K_2 \leq \overline{K}$ and $\min\{K_1, K_2\} \geq \underline{K}_i$, or (ii) $K_1 + K_2 \geq \underline{K}$ and $\min\{K_1, K_2\} \geq \underline{K}_i$, where $\overline{K}$, $\underline{K}$ and $\underline{K}_i$ are the authority's policy variables.[22] Figure 7(a), for example depicts the policy $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 10, 1)$, where states with $a(K_1, K_2) = 1$ are shaded (only states with $\max\{K_1, K_2\} \leq 10$ are shown), while Figure 7(b) shows the policy $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 10, 3)$.

Observe that under a deterministic commitment policy, such as those outlined above, the antitrust authority never incurs any blocking costs since if it commits to block a merger in state (K_1, K_2), the merger will not be proposed in the first place (the firms will not want to incur a proposal cost).

5.2 Static Benchmarks

As a benchmark, and to understand some of the forces behind the optimal merger policy, Figures 8 and 9 show, for the large and small markets respectively, the static change in consumer surplus [panel (a)] and aggregate surplus [panel (b)] from allowing a merger (the figures show only states with $\max\{K_1, K_2\} \leq 10$; other states are rarely reached). This is the change in CV or AV due to production and consumption in the period the merger occurs.

In the large market, only in state $(K_1, K_2) = (1, 1)$ does a merger generate a static increase in consumer surplus, and there the gain is only 0.1.[23] There are no states in which the merger generates a static gain in consumer surplus in the small market.

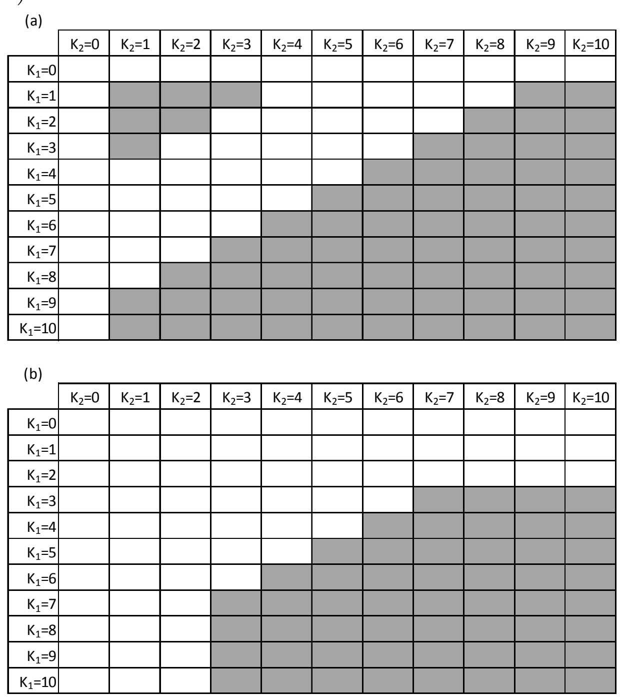
[20] The particular form these simple commitment policies take is motivated by which mergers are AV-increasing as one-shot deviations. We intend to explore additional policies, although we think our findings are unlikely to change much.

[21] In the computations we have conducted so far, we restrict attention to $\underline{H} \in \{0.6, 0.6+\Delta, 0.6+2\Delta, ..., 0.925-\Delta, 0.925\}$, where $\Delta = 0.25$.

[22] In the computations we have conducted so far, we restrict attention to $\overline{K} \in \{2, 4, ..., 10, 12\}$, $\underline{K} \in \{6, 8, ..., 18, 20\}$ and $\underline{K}_i \in \{1, 2, ..., 6, 7\}$.

[23] For a merger among symmetrically-positioned firms to increase consumer surplus, the marginal cost reduction at the pre-merger output Q of the merging firms, $c_Q(Q|K) - c_Q(2Q|2K)$, must exceed the pre-merger price cost margin, $P(2Q) - c_Q(Q|K)$ [see Farrell and Shapiro (1990) or Nocke and Whinston (2010)].

Figure 7: Capital-stock-based merger policy: (a) is $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 10, 1)$ and (b) is $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 10, 3)$



In contrast, many mergers increase aggregate surplus. In general, these tend to be states in which the total capital in the industry is not too large. For example, in the large market there is a static gain in aggregate surplus in any state in which total capital is less than 12. In the small market, there is a static gain in aggregate surplus whenever total capital is less than 9. (In both size markets there are also very asymmetric states with total capital above those levels in which a merger creates a static aggregate surplus gain.) The gains in aggregate surplus are generally smaller the larger is the total capital in the industry. This effect of larger aggregate capital on aggregate surplus arises because, as Figures 11 and 12 show, in those states the ratio of price to marginal cost is greater and so is the reduction in total output from the merger.

To understand why fewer mergers are aggregate surplus-enhancing in the small market, consider the effect of a merger in state $(5, 5)$, a state in which the merger improves static aggregate surplus in the large market but reduces it in the small market. In the small market both the ratio of price to marginal cost and the percentage reduction in output resulting from the merger are larger than in the large market. Thus, the percentage losses due to increased market power are greater in the small market, while the percentage reduction in average cost is the same.

Holding total capital fixed, asymmetry of capital positions has varying effects on the static gains in aggregate surplus from a merger. This gain gets smaller with increased asymmetry at low levels of total capital, but grows larger with increased asymmetry at greater levels of total capital. Since holding total capital fixed the final monopoly position remains the same as we vary the premerger capital positions, this is reflecting the fact that more asymmetry in

Figure 8: Static change in consumer surplus (a) and aggregate surplus (b) from a merger in the large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.1	(0.1)	(0.3)	(0.5)	(0.6)	(0.7)	(0.8)	(0.8)	(0.9)	(0.9)
K_1=2	-	(0.1)	(0.5)	(0.9)	(1.2)	(1.5)	(1.7)	(1.8)	(1.9)	(2.0)	(2.1)
K_1=3	-	(0.3)	(0.9)	(1.5)	(1.9)	(2.2)	(2.5)	(2.7)	(2.9)	(3.0)	(3.1)
K_1=4	-	(0.5)	(1.2)	(1.9)	(2.4)	(2.8)	(3.1)	(3.4)	(3.6)	(3.8)	(3.9)
K_1=5	-	(0.6)	(1.5)	(2.2)	(2.8)	(3.3)	(3.7)	(4.0)	(4.2)	(4.4)	(4.6)
K_1=6	-	(0.7)	(1.7)	(2.5)	(3.1)	(3.7)	(4.1)	(4.4)	(4.7)	(4.9)	(5.1)
K_1=7	-	(0.8)	(1.8)	(2.7)	(3.4)	(4.0)	(4.4)	(4.8)	(5.1)	(5.4)	(5.6)
K_1=8	-	(0.8)	(1.9)	(2.9)	(3.6)	(4.2)	(4.7)	(5.1)	(5.5)	(5.7)	(6.0)
K_1=9	-	(0.9)	(2.0)	(3.0)	(3.8)	(4.4)	(4.9)	(5.4)	(5.7)	(6.0)	(6.3)
K_1=10	-	(0.9)	(2.1)	(3.1)	(3.9)	(4.6)	(5.1)	(5.6)	(6.0)	(6.3)	(6.6)

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.8	1.9	1.7	1.4	1.2	1.1	1.0	0.8	0.8	0.7
K_1=2	-	1.9	2.0	1.8	1.5	1.3	1.1	0.9	0.8	0.6	0.6
K_1=3	-	1.7	1.8	1.5	1.2	1.0	0.7	0.5	0.4	0.3	0.2
K_1=4	-	1.4	1.5	1.2	0.9	0.6	0.4	0.1	(0.0)	(0.2)	(0.3)
K_1=5	-	1.2	1.3	1.0	0.6	0.3	0.0	(0.2)	(0.4)	(0.6)	(0.7)
K_1=6	-	1.1	1.1	0.7	0.4	0.0	(0.3)	(0.6)	(0.8)	(1.0)	(1.1)
K_1=7	-	1.0	0.9	0.5	0.1	(0.2)	(0.6)	(0.8)	(1.1)	(1.3)	(1.5)
K_1=8	-	0.8	0.8	0.4	(0.0)	(0.4)	(0.8)	(1.1)	(1.3)	(1.6)	(1.8)
K_1=9	-	0.8	0.6	0.3	(0.2)	(0.6)	(1.0)	(1.3)	(1.6)	(1.8)	(2.0)
K_1=10	-	0.7	0.6	0.2	(0.3)	(0.7)	(1.1)	(1.5)	(1.8)	(2.0)	(2.2)

capital stocks is good at low levels of total capital, but bad at higher ones.

Firms, always have a static profit gain from merging, as a merger creates a monopoly in the period in which it occurs.

5.3 The Large Market ($B = 30$)

We first examine the optimal merger approval policies in the large market where $B = 30$.

5.3.1 No commitment

We first examine the Markov perfect merger policy. To do so, we start with the policy of allowing no mergers and the associated equilibrium strategies for the firms (discussed in Section 4). Given these firm strategies, we next identify for each state (K_1, K_2) the antitrust authority's optimal approval rule given its expectation that its own behavior in the future will be to approve no mergers and that the firms will conform to their equilibrium strategies given that policy. We then identify firms' equilibrium strategies given this new approval policy by the antitrust authority, and continue to iterate until the antitrust authority has no incentive to deviate from its current policy.

Figures 13 and 14 show the first step in this iteration process. Figure 13 shows for each state the gain (before blocking costs) in CV or AV (for states with both firms having less than 10 units of capital) from a one-time merger approval given the expectation that no mergers will

Figure 9: Static change in consumer surplus (a) and aggregate surplus (b) from a merger in the small market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	(0.0)	(0.3)	(0.5)	(0.6)	(0.7)	(0.8)	(0.9)	(0.9)	(1.0)	(1.0)
K_1=2	-	(0.3)	(0.8)	(1.1)	(1.4)	(1.6)	(1.7)	(1.9)	(1.9)	(2.0)	(2.1)
K_1=3	-	(0.5)	(1.1)	(1.6)	(2.0)	(2.2)	(2.4)	(2.6)	(2.7)	(2.8)	(2.9)
K_1=4	-	(0.6)	(1.4)	(2.0)	(2.4)	(2.7)	(3.0)	(3.1)	(3.3)	(3.4)	(3.5)
K_1=5	-	(0.7)	(1.6)	(2.2)	(2.7)	(3.1)	(3.3)	(3.6)	(3.7)	(3.9)	(4.0)
K_1=6	-	(0.8)	(1.7)	(2.4)	(3.0)	(3.3)	(3.7)	(3.9)	(4.1)	(4.3)	(4.4)
K_1=7	-	(0.9)	(1.9)	(2.6)	(3.1)	(3.6)	(3.9)	(4.2)	(4.4)	(4.6)	(4.7)
K_1=8	-	(0.9)	(1.9)	(2.7)	(3.3)	(3.7)	(4.1)	(4.4)	(4.6)	(4.8)	(5.0)
K_1=9	-	(1.0)	(2.0)	(2.8)	(3.4)	(3.9)	(4.3)	(4.6)	(4.8)	(5.1)	(5.2)
K_1=10	-	(1.0)	(2.1)	(2.9)	(3.5)	(4.0)	(4.4)	(4.7)	(5.0)	(5.2)	(5.4)

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.3	1.2	1.0	0.8	0.7	0.6	0.5	0.4	0.4	0.3
K_1=2	-	1.2	1.1	0.9	0.7	0.5	0.3	0.2	0.2	0.1	0.0
K_1=3	-	1.0	0.9	0.6	0.3	0.1	(0.0)	(0.2)	(0.3)	(0.3)	(0.4)
K_1=4	-	0.8	0.7	0.3	0.0	(0.2)	(0.4)	(0.5)	(0.6)	(0.7)	(0.8)
K_1=5	-	0.7	0.5	0.1	(0.2)	(0.5)	(0.7)	(0.8)	(1.0)	(1.1)	(1.2)
K_1=6	-	0.6	0.3	(0.0)	(0.4)	(0.7)	(0.9)	(1.1)	(1.3)	(1.4)	(1.5)
K_1=7	-	0.5	0.2	(0.2)	(0.5)	(0.8)	(1.1)	(1.3)	(1.5)	(1.6)	(1.8)
K_1=8	-	0.4	0.2	(0.3)	(0.6)	(1.0)	(1.3)	(1.5)	(1.7)	(1.8)	(2.0)
K_1=9	-	0.4	0.1	(0.3)	(0.7)	(1.1)	(1.4)	(1.6)	(1.8)	(2.0)	(2.2)
K_1=10	-	0.3	0.0	(0.4)	(0.8)	(1.2)	(1.5)	(1.8)	(2.0)	(2.2)	(2.3)

be approved in the future and that firms' strategies will be the ones that form an equilibrium given that no mergers would be allowed. For both the CV and AV welfare criteria, the set of states in which there is a gain (before blocking costs) from a one-time merger decision that approves a merger is very close to the set of states in which a merger is statically beneficial (in both cases, the static criterion is slightly more stringent). For example, the merger increases CV in state $(1,1)$ where the gain is 1.6, and states $(1,2)$ and $(2,1)$, where the gain is 0.3. So, with a CV criterion, a merger is approved with probability one in these states. In all other states the change in CV is less than -1, so with blocking costs drawn from the uniform distribution on $[0,1]$ a merger is blocked with probability 1 in all of these states. In contrast, every state with total capital no greater than 11 (as well as some others) has an increase in AV from merger approval. We will let $\widehat{b}_1(K_1, K_2)$ denote the policy that emerges from this first step in the iteration. Figure 14 shows the resulting probabilities of merger approval in each state.

In the next step of the iteration process we update firm's investment and proposal strategies to reflect an equilibrium given the merger approval probabilities in Figure 14 [policy $\widehat{b}_1(K_1, K_2)$], and we then determine – given these new firm strategies – the gains from a one-time merger approval in each state. When we do this for a CV criterion, there is no change in the approval probabilities, so the approval policy shown in Figure 14 is in fact a Markov perfect policy (we will discuss this equilibrium below). Figures 15 and 16 show the next step in the iteration process for the AV criterion. Figure 15 shows the firms' proposal behavior, indicating in each state the probability that a merger is proposed, as well as the overall probability that a merger happens in each state (equal to the probability it is proposed times the probability it is approved). Figure 16 shows for each state the gain in AV (before blocking costs) from a one-time merger approval given the expectation that the antitrust authority will follow policy $\widehat{b}_1(K_1, K_2)$ in the future and that firms' strategies will be the ones that form an equilibrium given policy $\widehat{b}_1(K_1, K_2)$. The states in which this one-time merger approval is desirable change dramatically with the AV criterion. Only states with aggregate capital no greater than 4 have gains in AV from merger approval, while the states with a positive probability of merger approval given the blocking costs are those with total capital no greater than 6, plus states $(1, 6)$ and $(6, 1)$.

To understand this dramatic change with the AV criterion, observe that once many mergers are likely to be approved, the investment behavior of the firms, especially of new entrants, changes dramatically. For example, starting at states $(2, 0)$, $(4, 0)$, and $(8, 0)$ the distributions of capital additions when no mergers are allowed ("policy 0") and under policy $\widehat{b}_1(K_1, K_2)$ ("policy 1") are as follows:

Table 1: Investments Starting at $(2, 0)$	Policy 0		Policy 1	
Capital addition	firm 0	firm 2	firm 0	firm 2
0	0%	0%	0%	4%
1	23%	14%	100%	44%
2	77%	86%	0%	45%
3	0%	0%	0%	7%
4	0%	0%	0%	0%

Table 2: Investments Starting at $(4, 0)$	Policy 0		Policy 1	
Capital addition	firm 0	firm 4	firm 0	firm 4
0	11%	0%	0%	0%
1	72%	9%	100%	48%
2	13%	52%	0%	52%
3	0%	36%	0%	0%
4	0%	3%	0%	0%

Table 3: Investments Starting at $(8, 0)$	Policy 0		Policy 1	
Capital addition	firm 0	firm 8	firm 0	firm 8
0	90%	0%	0%	44%
1	10%	7%	100%	54%
2	0%	48%	0%	2%
3	0%	42%	0%	0%
4	0%	3%	0%	0%

When mergers are not allowed, the entrant invests provided the capital stock of the incumbent is not too large. Once the incumbent's capital stock reaches 8, however, the entrant invests only 10% of the time. In contrast, when mergers are allowed, the entrant builds one unit of capital in all three of these states. In state $(8, 0)$, the entrant is doing this in part because of the prospect that he will get bought out. Although a merger almost never happens in the first period after this investment, once he has some capital he can grow through capital augmentation in the following periods and reach a state in which a merger may happen. The cumulative probability that a merger occurs within various numbers of periods starting from state $(0, 8)$ is shown in the table below. Note that the monopolist's investment generally falls when mergers are sometimes allowed under policy $\widehat{b}_1(K_1, K_2)$ compared to when they are not.

Table 4: Likelihood of a Merger Starting in State (8,0)	
Period	Cumulative merger probability
1	0.3%
2	15.7%
3	37.2%
4	56.4%
5	71.0%

Thus, when mergers are allowed under policy $\widehat{b}_1(K_1, K_2)$, at states where the monopolist has a lot of capital the investment of the entrant increases. At states where the monopolist has only a little capital, both firms' investments are lower under policy $\widehat{b}_1(K_1, K_2)$. However, as Figure XXX shows [the figure reports for each state the difference between the row firm's value gain from increasing its capital stock by one unit and the gain in aggregate surplus from this change under policy $\widehat{b}_1(K_1, K_2)$], the entrant has excess incentives when the monopolist has a lot of capital and he has insufficient incentives when the monopolist has only a little capital, while the monopolist always has insufficient investment incentives. These changes in firms' investment behavior make mergers that lead to these states less desirable.

As noted above, the second iteration is in fact a Markov perfect policy under the CV criterion. As states $(1, 1)$, $(2, 1)$, and $(1, 2)$ have almost no chance of being reached, the steady state of this policy is almost identical to the steady state when no merges are allowed. Summary statistics are:

Table 5: Markov Perfect Policy Steady State Averages; CV Criterion, Large Market	
CV	61.2
PV	81.0
AV	142.2
Price	2.10
Quantity	27.0
Total K	9.6
Merger Probability	0.000

The Markov perfect policy with the AV criteria lies between the policies discussed above for the first two iterations. Figure 18 shows the merger acceptance probabilities, merger proposal probabilities, and the probabilities a merger actually occurs in various states. Figure 19 shows the steady state distribution over the states.

Under the optimal approval policy without commitment, for states in which both firms have less than 10 units of capital, the antitrust authority approves a proposed merger with positive probability in any state (K_1, K_2) such that $K_1 + K_2 \leq 9$ or $\min\{K_1, K_2\} = 1$ or $(\min\{K_1, K_2\} = 2$ and $\max\{K_1, K_2\} = 8)$. Mergers are proposed in all of these states. A merger proposal is more likely when the firms are symmetrically positioned. Mergers are most likely to happen in state $(3, 3)$ (the state with the highest merger probability), and states surrounding that state.

In the steady state induced by this policy, the industry spends quite some time in a monopoly state (40.6%) and, more generally, in asymmetric states. The modal states are $(6, 1)$ and $(1, 6)$. So, compared to the steady state induced when no mergers are allowed the economy spends much more time in asymmetric states. In addition, the average aggregate capital level is lower. To understand why, consider the state transitions (including depreciation) from state $(5, 5)$, which are shown in Figure 20 for both the no mergers allowed policy and the Markov-perfect policy. With no mergers allowed, each firm may see its capital increase after investment and depreciation 33% of the time (7% of the time it increases by two units). In contrast, this happens only 12% of the time with the Markov-perfect policy. The reason for this difference is that, under the latter policy, mergers are only allowed in small states. As these mergers are profitable for the merger partners, firms do not have great incentives to invest.

Summary statistics for this steady state are:

Table 6: Markov Perfect Policy Steady State Averages; AV Criterion, Large Market	
CV	43.4
PV	80.2
AV	123.6
Price	2.24
Quantity	22.9
Total K	7.4
Merger Probability	0.200

Most strikingly, the Markov perfect policy equilibrium with the AV criterion results in a level of steady state AV which is much lower than what we saw from the policy of no mergers allowed: AV is 123.6 compared to 142.2 when no mergers are allowed. Firms are slightly worse off while consumers are much worse off: CV is 43.4 (vs. 61.2) and producer value is 80.2 (vs. 81.0). Consumers are harmed both from the monopoly pricing and the reduction in capital, both of which lead to higher prices.

5.3.2 Commitment

We now turn to the optimal commitment policy. By this we mean the policy that leads to the largest steady state level of expected welfare, either CV or AV depending on the welfare criterion.

In the large market, the optimal commitment policy – for either a CV or AV standard – is essentially to allow no mergers. Strictly speaking, within the set of feasible commitment policies, for both welfare measures the optimal policy is the capital-stock-based policy with

$(\overline{K}, \underline{K}, \underline{K}_i) = (4, 20, 7)$, which is the "most restrictive" policy, short of blocking all mergers, in the set of (capital-stock-based) commitment policies we outlined above. Essentially no mergers ever occur with this policy, and the steady state and the performance measures are (almost) the same as under the simpler commitment policy where no mergers are ever allowed: aggregate value is 142.6, consumer value is 61.5, and producer value 81.1).[24] Indeed, the differences in these values appear to be within the margin of error of our computations.[25,26] The most restrictive Herfindahl policy, which has $\overline{H} = 0.925$, also yields essentially the same outcome.

Why is the optimal commitment policy so much less permissive under the AV criterion than the Markov perfect policy under no commitment? The most obvious reason is that the optimal commitment policy considers the effects of the policy on firms' investment incentives, which we have already seen are substantial. More permissive policies like the one for the AV criterion in Figure 18 increase an entrants' incentives to invest in states in which its investment incentives are too high and places the market in those states much more often [one firm has zero capital 29.6% of the time; another 38.1% of the time one firm has just one unit of capital].

A less obvious reason is that, under commitment, the antitrust authority considers the impact its policy has on proposal costs, while under no commitment those costs are considered to be sunk.[27]

While full commitment to policy may be difficult to achieve, an alternative is to endow the antitrust authority with an objective that may not be the true social objective. In this regard, note that the steady-state level of AV under the Markov perfect merger policy when the antitrust authority has a CV objective is essentially the same as no mergers allowed – thus, when the antitrust authority cannot commit, a CV-focused antitrust authority is better for AV (indeed, maximizes it) in this market than an AV-focused authority. This is consistent with a suggestion of Lyons (2002), but arises because of the policy's effect on investment, rather than inducing more desirable merger proposals.

5.3.3 The Small Market ($B = 20$)

We now turn to the optimal merger approval policies in the small market where $B = 20$.

5.3.4 No commitment

We first analyze the Markov perfect merger policy. We proceed as in the case of the large market. Starting with the policy of allowing no mergers [which corresponds to the authority

[24] The steady state for policy $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 20, 7)$ spends very little time (about 2.2%) in "monopoly states" (states in which $\min\{K_1, K_2\} = 0$ and $\max\{K_1, K_2\} > 0$). Most of the time, both firms have fairly large capital levels: the modal state is $(5, 5)$; the economy spends 49.6% of the time in states (K_1, K_2) with $4 \leq K_i \leq 6$, $i = 1, 2$, and 81.2% of the times in states (K_1, K_2) with $3 \leq K_i \leq 7$, $i = 1, 2$. Thus, it is essentially the same as the no mergers allowed policy.

[25] For example, we calculate the steady state welfare level under policy $(\overline{K}, \underline{K}, \underline{K}_i) = (3, 20, 7)$, which is identical to policy $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 20, 7)$ in the set of states in which mergers are alllowed, to be 142.3.

[26] An alternative to examining steady state expected welfare is to look at the best commitment policy starting in particular states. One natural choice is state $(0, 0)$ in which case we are maximizing welfare at the start of the industry. In fact, for both CV and AV allowing no mergers [or, essentially equivalently, policy $(4, 20, 7)$] is best among all of the policies we examone.

[27] A similar point arises in Besanko and Spulber (1992).

using a blocking cost cutoff $\widehat{b}_0(K_1, K_2) \geq \overline{b}$ in every state (K_1, K_2)] and the associated equilibrium policies by firms, we check in every state whether the antitrust authority has a profitable one-shot deviation and update the cutoff policy $\widehat{b}_0(\cdot)$ accordingly. Iterating in this fashion until the antitrust authority has no more incentive to deviate from its current policy, yields a Markov-perfect merger policy $\widehat{b}(K_1, K_2)$.

Figure 21 shows that starting from the equilibrium when there are no mergers allowed, there are no CV-enhancing mergers. Thus, allowing no mergers is a Markov perfect policy with a CV welfare criterion.

Figure 22 illustrates the first step in this iteration process for the AV criterion. It indicates in each state (K_1, K_2) with $\max\{K_1, K_2\} \leq 10$ the gain (or loss) in AV from a one-time merger approval, given that no mergers will be allowed in the future and firms' associated equilibrium policies. As in the case of the large market, the set of states in which there is a gain (before blocking costs) from a one-time merger approval is similar to (but slightly larger than) that in which a merger increases aggregate surplus statically. These gains tend to be larger in states with less total capital. For example, the gain from approving a merger in state $(3, 3)$ is 0.8, implying that the merger would be approved independently of the realization of blocking costs, whereas the loss from approving a merger in state $(4, 4)$ is 0.1, implying that the merger would be blocked, provided the realization of the blocking cost b is not larger than 0.1. Figure 23 shows the corresponding merger approval probabilities given this policy, $\widehat{b}_1(\cdot)$.

For the second step in the iteration process, we update firms' investment and proposal strategies to reflect the merger approval probabilities given by merger approval policy $\widehat{b}_1(\cdot)$. Given these updated strategies by firms, we then compute again for each state the gain in AV from approving a one-time merger, holding fixed the approval policy and firms' equilibrium strategies in the continuation game. Figure 24 shows these gains (before blocking costs) for every state (K_1, K_2) with $\max\{K_1, K_2\} \leq 10$. As in the case of the large market, the set of states in which a merger raises AV (before blocking costs) is dramatically smaller than in the previous iteration. In fact, these gains are positive only in states $(1, 1)$, $(2, 1)$ and $(1, 2)$, whereas the set of states in which a merger would be approved with positive probability once blocking costs are taken into account is contained in the set of states with $K \equiv K_1 + K_2 \leq 7$.

The reason for this dramatic change from the first step of the iteration process to the second is again that firms' investment behavior changes significantly. And this holds in particular for new entrants.

Iterating this process until we can find no more one-shot deviations (holding fixed the approval policy and firms' equilibrium strategies in the continuation game) for the AV-maximizing antitrust authority, yields the Markov perfect merger policy for an AV standard. Figure 25 shows the Markov perfect policy. The Markov perfect merger policy is such that the antitrust authority approves a proposed merger in a state (K_1, K_2) such that $\min\{K_1, K_2\} = 1$ or $K_1 + K_2 \leq 7$ or $(\min\{K_1, K_2\} = 2$ and $\max\{K_1, K_2\} \in \{6, 7\})$, provided blocking costs are sufficiently small.

Figure 26 shows the resulting steady state. In the steady state induced by that Markov perfect merger policy, monopoly states are very frequently visited (63.3% of the time), the modal states being $(4, 0)$ and $(0, 4)$. Of this 63.3%, 19.4% of the time this occurs because of a merger. In fact, monopoly is less frequent than when no mergers are allowed. But it is also true that both firms having a significant amount of capital is less likely than when no mergers are allowed (e.g., with no mergers allowed both firms have at least three units of capital 14.2% of the time, but this happens in the Markov perfect policy only 2% of the time). Under the Markov perfect policy, it is much more likely that a firm has just one unit of capital. Table 7 gives some summary statistics for this steady state.

Table 7: Markov Perfect Policy Steady State Averages; AV Criterion, Small Market	
CV	23.7
PV	47.2
AV	70.9
Price	2.55
Quantity	13.6
Total K	4.6
Merger Probability	0.197

As before, if the antitrust authority adopts an AV standard, the Markov perfect merger policy results in a much lower steady state level of AV than the policy of never allowing a merger: AV is 70.9 vs. 77.8 when no mergers are ever allowed. Now, however, both firms and consumers are considerably worse off: CV is 23.7 (vs. 26.3) while PV is 47.2 (vs. 51.4).

5.3.5 Commitment

In the small market, the optimal commitment policy for an AV standard is not quite as restrictive as in the large market. Within the set of feasible commitment policies, the optimal policy is the capital-stock-based policy with $(\overline{K}, \underline{K}, \underline{K}_i) = (4, 20, 1)$. Since total capital almost never exceeds 20, this is basically a policy that allows a merger when total capital is 4 or less.[28] In the steady state induced by this policy, mergers occur on average in 4.3% of the periods. In contrast to the case of the large market, the optimal commitment policy for an AV standard results in a considerably higher welfare level than the policy of never allowing a merger: AV is 82.5 compared to 77.9 when no mergers are ever allowed, while PV is 54.0 (vs. 51.4) and CV is 28.5 (vs. 26.5). Compared to the steady state induced by the policy of never allowing any mergers, the economy spends much more time in highly asymmetric states: a monopoly state is reached in 65.4% of the time, and the modal states are $(6, 0)$ and $(0, 6)$. Average total capital is also higher (5.6 versus 5.0).

Table 8: Optimal Commitment Policy Steady State Averages; AV Criterion, Small Market	
CV	28.5
PV	54.0
AV	82.5
Price	2.50
Quantity	15.0
Total K	5.6
Merger Probability	0.043

As in the large market, the optimal commitment policy is much more restrictive under the AV criterion than the Markov perfect merger policy. The reasons are the same as before:

[28] In fact, total capital rarely exceeds 12, so among our feasible policies, all policies of the form $(4, x, 1)$ are basically equivalent.

the optimal commitment policy considers the impact of the policy on, first, firms' investment incentives and, second, proposal costs.

6 Extensions

6.1 Other Market Parameters

To be added.

6.2 Delayed Entry

To be added.

7 Conclusion

To be added.

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Figure 10: Ratio of price to marginal cost of row firm (a), change in output from a merger (b), and % change in output (c); large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	1.08	1.07	1.06	1.06	1.06	1.06	1.05	1.05	1.05	1.05	1.05
K_1=2	1.16	1.15	1.14	1.13	1.13	1.12	1.12	1.12	1.11	1.11	1.11
K_1=3	1.23	1.22	1.20	1.20	1.19	1.18	1.18	1.18	1.17	1.17	1.17
K_1=4	1.29	1.28	1.27	1.26	1.25	1.24	1.24	1.23	1.23	1.22	1.22
K_1=5	1.35	1.34	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27
K_1=6	1.40	1.39	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32
K_1=7	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.38	1.37	1.37	1.36
K_1=8	1.50	1.49	1.47	1.45	1.44	1.43	1.43	1.42	1.41	1.41	1.40
K_1=9	1.55	1.53	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44
K_1=10	1.59	1.57	1.55	1.54	1.53	1.52	1.51	1.50	1.49	1.49	1.48

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.4	(0.2)	(0.5)	(0.7)	(0.9)	(1.0)	(1.0)	(1.1)	(1.1)	(1.1)
K_1=2	-	(0.2)	(0.9)	(1.4)	(1.8)	(2.0)	(2.1)	(2.2)	(2.3)	(2.3)	(2.4)
K_1=3	-	(0.5)	(1.4)	(2.1)	(2.5)	(2.8)	(3.0)	(3.1)	(3.2)	(3.3)	(3.3)
K_1=4	-	(0.7)	(1.8)	(2.5)	(3.0)	(3.4)	(3.6)	(3.8)	(3.9)	(4.0)	(4.1)
K_1=5	-	(0.9)	(2.0)	(2.8)	(3.4)	(3.8)	(4.1)	(4.3)	(4.4)	(4.6)	(4.6)
K_1=6	-	(1.0)	(2.1)	(3.0)	(3.6)	(4.1)	(4.4)	(4.6)	(4.8)	(5.0)	(5.1)
K_1=7	-	(1.0)	(2.2)	(3.1)	(3.8)	(4.3)	(4.6)	(4.9)	(5.1)	(5.3)	(5.4)
K_1=8	-	(1.1)	(2.3)	(3.2)	(3.9)	(4.4)	(4.8)	(5.1)	(5.4)	(5.5)	(5.7)
K_1=9	-	(1.1)	(2.3)	(3.3)	(4.0)	(4.6)	(5.0)	(5.3)	(5.5)	(5.8)	(5.9)
K_1=10	-	(1.1)	(2.4)	(3.3)	(4.1)	(4.6)	(5.1)	(5.4)	(5.7)	(5.9)	(6.1)

(c)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	3.4%	-1.2%	-3.0%	-3.8%	-4.1%	-4.3%	-4.3%	-4.3%	-4.3%	-4.2%
K_1=2	-	-1.2%	-5.3%	-7.2%	-8.1%	-8.5%	-8.6%	-8.7%	-8.6%	-8.5%	-8.4%
K_1=3	-	-3.0%	-7.2%	-9.4%	-10.5%	-11.0%	-11.3%	-11.4%	-11.4%	-11.4%	-11.3%
K_1=4	-	-3.8%	-8.1%	-10.5%	-11.8%	-12.5%	-12.9%	-13.1%	-13.2%	-13.2%	-13.2%
K_1=5	-	-4.1%	-8.5%	-11.0%	-12.5%	-13.4%	-13.9%	-14.2%	-14.4%	-14.5%	-14.5%
K_1=6	-	-4.3%	-8.6%	-11.3%	-12.9%	-13.9%	-14.5%	-14.9%	-15.2%	-15.3%	-15.4%
K_1=7	-	-4.3%	-8.7%	-11.4%	-13.1%	-14.2%	-14.9%	-15.4%	-15.8%	-16.0%	-16.1%
K_1=8	-	-4.3%	-8.6%	-11.4%	-13.2%	-14.4%	-15.2%	-15.8%	-16.1%	-16.4%	-16.6%
K_1=9	-	-4.3%	-8.5%	-11.4%	-13.2%	-14.5%	-15.3%	-16.0%	-16.4%	-16.7%	-16.9%
K_1=10	-	-4.2%	-8.4%	-11.3%	-13.2%	-14.5%	-15.4%	-16.1%	-16.6%	-16.9%	-17.2%

Figure 11:

Figure 12: Ratio of price to marginal cost of row firm (a), change in output from a merger (b), and % change in output (c); small market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	1.14	1.16	1.17	1.19	1.20	1.21	1.21	1.22	1.22	1.23	1.23
K_1=2	1.28	1.29	1.30	1.31	1.31	1.32	1.33	1.33	1.34	1.34	1.34
K_1=3	1.39	1.40	1.41	1.41	1.42	1.43	1.43	1.43	1.44	1.44	1.44
K_1=4	1.49	1.50	1.51	1.51	1.52	1.52	1.52	1.53	1.53	1.53	1.54
K_1=5	1.59	1.59	1.60	1.60	1.60	1.61	1.61	1.61	1.62	1.62	1.62
K_1=6	1.67	1.68	1.68	1.68	1.69	1.69	1.69	1.69	1.70	1.70	1.70
K_1=7	1.75	1.75	1.76	1.76	1.76	1.77	1.77	1.77	1.77	1.77	1.78
K_1=8	1.83	1.83	1.83	1.83	1.83	1.84	1.84	1.84	1.84	1.84	1.85
K_1=9	1.90	1.90	1.90	1.90	1.90	1.90	1.91	1.91	1.91	1.91	1.91
K_1=10	1.96	1.96	1.96	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.98

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	(0.1)	(0.5)	(0.8)	(0.9)	(1.0)	(1.0)	(1.0)	(1.1)	(1.1)	(1.1)
K_1=2	-	(0.5)	(1.2)	(1.5)	(1.8)	(1.9)	(2.0)	(2.0)	(2.1)	(2.1)	(2.1)
K_1=3	-	(0.8)	(1.5)	(2.0)	(2.3)	(2.5)	(2.6)	(2.7)	(2.8)	(2.8)	(2.8)
K_1=4	-	(0.9)	(1.8)	(2.3)	(2.7)	(2.9)	(3.1)	(3.2)	(3.3)	(3.3)	(3.3)
K_1=5	-	(1.0)	(1.9)	(2.5)	(2.9)	(3.2)	(3.4)	(3.5)	(3.6)	(3.7)	(3.7)
K_1=6	-	(1.0)	(2.0)	(2.6)	(3.1)	(3.4)	(3.6)	(3.7)	(3.9)	(4.0)	(4.0)
K_1=7	-	(1.0)	(2.0)	(2.7)	(3.2)	(3.5)	(3.7)	(3.9)	(4.1)	(4.2)	(4.3)
K_1=8	-	(1.1)	(2.1)	(2.8)	(3.3)	(3.6)	(3.9)	(4.1)	(4.2)	(4.3)	(4.4)
K_1=9	-	(1.1)	(2.1)	(2.8)	(3.3)	(3.7)	(4.0)	(4.2)	(4.3)	(4.5)	(4.6)
K_1=10	-	(1.1)	(2.1)	(2.8)	(3.3)	(3.7)	(4.0)	(4.3)	(4.4)	(4.6)	(4.7)

(c)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	-0.6%	-4.4%	-5.6%	-5.9%	-6.0%	-6.0%	-5.9%	-5.8%	-5.6%	-5.5%
K_1=2	-	-4.4%	-8.4%	-9.9%	-10.5%	-10.7%	-10.7%	-10.6%	-10.4%	-10.3%	-10.1%
K_1=3	-	-5.6%	-9.9%	-11.8%	-12.7%	-13.1%	-13.2%	-13.3%	-13.2%	-13.1%	-13.0%
K_1=4	-	-5.9%	-10.5%	-12.7%	-13.9%	-14.4%	-14.7%	-14.9%	-14.9%	-14.9%	-14.8%
K_1=5	-	-6.0%	-10.7%	-13.1%	-14.4%	-15.2%	-15.6%	-15.9%	-16.0%	-16.0%	-16.0%
K_1=6	-	-6.0%	-10.7%	-13.2%	-14.7%	-15.6%	-16.2%	-16.5%	-16.7%	-16.8%	-16.8%
K_1=7	-	-5.9%	-10.6%	-13.3%	-14.9%	-15.9%	-16.5%	-16.9%	-17.2%	-17.3%	-17.4%
K_1=8	-	-5.8%	-10.4%	-13.2%	-14.9%	-16.0%	-16.7%	-17.2%	-17.5%	-17.7%	-17.9%
K_1=9	-	-5.6%	-10.3%	-13.1%	-14.9%	-16.0%	-16.8%	-17.3%	-17.7%	-18.0%	-18.2%
K_1=10	-	-5.5%	-10.1%	-13.0%	-14.8%	-16.0%	-16.8%	-17.4%	-17.9%	-18.2%	-18.4%

Figure 13: Change in CV from one time merger (a) and change in AV from a one time merger (b); large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.6	0.3	(1.4)	(3.3)	(4.9)	(6.3)	(7.4)	(8.4)	(8.7)	(9.0)
K_1=2	-	0.3	(3.0)	(5.5)	(7.4)	(9.1)	(10.6)	(11.8)	(12.4)	(12.8)	(13.2)
K_1=3	-	(1.4)	(5.5)	(8.2)	(10.4)	(12.2)	(13.7)	(14.5)	(15.1)	(15.6)	(16.0)
K_1=4	-	(3.3)	(7.4)	(10.4)	(12.7)	(14.6)	(15.6)	(16.4)	(17.1)	(17.7)	(18.0)
K_1=5	-	(4.9)	(9.1)	(12.2)	(14.6)	(15.9)	(17.1)	(17.9)	(18.7)	(19.1)	(19.4)
K_1=6	-	(6.3)	(10.6)	(13.7)	(15.6)	(17.1)	(18.2)	(19.2)	(19.7)	(20.1)	(20.5)
K_1=7	-	(7.4)	(11.8)	(14.5)	(16.4)	(17.9)	(19.2)	(19.9)	(20.5)	(20.8)	(21.3)
K_1=8	-	(8.4)	(12.4)	(15.1)	(17.1)	(18.7)	(19.7)	(20.5)	(21.0)	(21.4)	(22.0)
K_1=9	-	(8.7)	(12.8)	(15.6)	(17.7)	(19.1)	(20.1)	(20.8)	(21.4)	(21.9)	(22.6)
K_1=10	-	(9.0)	(13.2)	(16.0)	(18.0)	(19.4)	(20.5)	(21.3)	(22.0)	(22.6)	(23.2)

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.7	0.7	0.5	1.3	2.1	2.8	3.3	3.8	3.7	3.6
K_1=2	-	0.7	0.4	0.9	1.5	2.1	2.5	2.9	2.8	2.8	2.7
K_1=3	-	0.5	0.9	1.2	1.5	1.8	2.0	1.8	1.6	1.5	1.3
K_1=4	-	1.3	1.5	1.5	1.5	1.5	1.1	0.8	0.6	0.3	0.3
K_1=5	-	2.1	2.1	1.8	1.5	0.9	0.4	(0.0)	(0.4)	(0.5)	(0.6)
K_1=6	-	2.8	2.5	2.0	1.1	0.4	(0.2)	(0.8)	(1.0)	(1.2)	(1.4)
K_1=7	-	3.3	2.9	1.8	0.8	(0.0)	(0.8)	(1.2)	(1.5)	(1.8)	(2.1)
K_1=8	-	3.8	2.8	1.6	0.6	(0.4)	(1.0)	(1.5)	(1.9)	(2.3)	(2.6)
K_1=9	-	3.7	2.8	1.5	0.3	(0.5)	(1.2)	(1.8)	(2.3)	(2.7)	(3.2)
K_1=10	-	3.6	2.7	1.3	0.3	(0.6)	(1.4)	(2.1)	(2.6)	(3.2)	(3.7)

Figure 14: First policy iteration according to CV criterion (a) and AV criterion (b); large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.00	1.00	-	-	-	-	-	-	-	-
K_1=2	-	1.00	-	-	-	-	-	-	-	-	-
K_1=3	-	-	-	-	-	-	-	-	-	-	-
K_1=4	-	-	-	-	-	-	-	-	-	-	-
K_1=5	-	-	-	-	-	-	-	-	-	-	-
K_1=6	-	-	-	-	-	-	-	-	-	-	-
K_1=7	-	-	-	-	-	-	-	-	-	-	-
K_1=8	-	-	-	-	-	-	-	-	-	-	-
K_1=9	-	-	-	-	-	-	-	-	-	-	-
K_1=10	-	-	-	-	-	-	-	-	-	-	-

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=2	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=3	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=4	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=5	-	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.56	0.51	0.41
K_1=6	-	1.00	1.00	1.00	1.00	1.00	0.76	0.20	0.01	-	-
K_1=7	-	1.00	1.00	1.00	1.00	0.97	0.20	-	-	-	-
K_1=8	-	1.00	1.00	1.00	1.00	0.56	0.01	-	-	-	-
K_1=9	-	1.00	1.00	1.00	1.00	0.51	-	-	-	-	-
K_1=10	-	1.00	1.00	1.00	1.00	0.41	-	-	-	-	-

Figure 15: First iteration probability merger is proposed (a) and merger happens (b); AV criterion, large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.55	0.42	0.23	0.08	-	-	-	-	-	-
K_1=2	-	0.42	0.53	0.48	0.39	0.32	0.27	0.24	0.22	0.19	0.17
K_1=3	-	0.23	0.48	0.53	0.51	0.48	0.45	0.42	0.38	0.35	0.31
K_1=4	-	0.08	0.39	0.51	0.54	0.53	0.50	0.43	0.37	0.31	0.23
K_1=5	-	-	0.32	0.48	0.53	0.54	0.42	0.33	0.13	0.06	-
K_1=6	-	-	0.27	0.45	0.50	0.42	0.23	0.04	0.00	-	-
K_1=7	-	-	0.24	0.42	0.43	0.33	0.04	-	-	-	-
K_1=8	-	-	0.22	0.38	0.37	0.13	0.00	-	-	-	-
K_1=9	-	-	0.19	0.35	0.31	0.06	-	-	-	-	-
K_1=10	-	-	0.17	0.31	0.23	-	-	-	-	-	-

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.55	0.42	0.23	0.08	-	-	-	-	-	-
K_1=2	-	0.42	0.53	0.48	0.39	0.32	0.27	0.24	0.22	0.19	0.17
K_1=3	-	0.23	0.48	0.53	0.51	0.48	0.45	0.42	0.38	0.35	0.31
K_1=4	-	0.08	0.39	0.51	0.54	0.53	0.50	0.43	0.37	0.31	0.23
K_1=5	-	-	0.32	0.48	0.53	0.54	0.42	0.31	0.07	0.03	-
K_1=6	-	-	0.27	0.45	0.50	0.42	0.17	0.01	0.00	-	-
K_1=7	-	-	0.24	0.42	0.43	0.31	0.01	-	-	-	-
K_1=8	-	-	0.22	0.38	0.37	0.07	0.00	-	-	-	-
K_1=9	-	-	0.19	0.35	0.31	0.03	-	-	-	-	-
K_1=10	-	-	0.17	0.31	0.23	-	-	-	-	-	-

Figure 16: Change in AV from a merger given firm's behavior after the first policy iteration; AV criterion, large market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.6	0.9	0.3	(0.3)	(0.6)	(0.9)	(1.2)	(1.3)	(1.5)	(1.6)
K_1=2	-	0.9	0.4	(0.2)	(0.8)	(1.3)	(1.6)	(1.9)	(2.1)	(2.4)	(2.5)
K_1=3	-	0.3	(0.2)	(0.8)	(1.4)	(1.9)	(2.3)	(2.5)	(2.8)	(3.1)	(3.4)
K_1=4	-	(0.3)	(0.8)	(1.4)	(2.0)	(2.5)	(2.8)	(3.0)	(3.4)	(3.8)	(4.1)
K_1=5	-	(0.6)	(1.3)	(1.9)	(2.5)	(2.9)	(3.0)	(3.4)	(3.9)	(4.3)	(4.8)
K_1=6	-	(0.9)	(1.6)	(2.3)	(2.8)	(3.0)	(3.3)	(3.9)	(4.6)	(5.2)	(5.8)
K_1=7	-	(1.2)	(1.9)	(2.5)	(3.0)	(3.4)	(3.9)	(4.7)	(5.4)	(6.2)	(6.9)
K_1=8	-	(1.3)	(2.1)	(2.8)	(3.4)	(3.9)	(4.6)	(5.4)	(6.3)	(7.1)	(7.8)
K_1=9	-	(1.5)	(2.4)	(3.1)	(3.8)	(4.3)	(5.2)	(6.2)	(7.1)	(7.9)	(8.4)
K_1=10	-	(1.6)	(2.5)	(3.4)	(4.1)	(4.8)	(5.8)	(6.9)	(7.8)	(8.4)	(9.0)

Marg Benefit	0	1	2	3	4	5	6	7	8	9	10
0	(1.9)	(2.3)	(0.1)	1.6	2.6	3.4	3.9	4.3	4.6	4.6	4.6
1	(4.1)	(3.7)	(2.3)	(1.0)	0.0	0.8	1.4	1.9	2.1	2.2	2.5
2	(2.7)	(2.9)	(2.5)	(1.8)	(1.1)	(0.6)	(0.1)	0.2	0.3	0.6	0.7
3	(2.2)	(2.4)	(2.2)	(1.9)	(1.5)	(1.2)	(0.8)	(0.7)	(0.5)	(0.3)	(0.3)
4	(2.0)	(2.0)	(2.0)	(1.8)	(1.6)	(1.4)	(1.2)	(1.0)	(0.8)	(0.7)	(0.6)
5	(1.8)	(1.8)	(1.8)	(1.7)	(1.5)	(1.3)	(1.4)	(1.4)	(1.4)	(1.4)	(1.5)
6	(1.6)	(1.6)	(1.6)	(1.6)	(1.8)	(1.9)	(1.9)	(1.8)	(1.8)	(1.7)	(1.7)
7	(1.5)	(1.5)	(1.6)	(1.9)	(1.9)	(1.8)	(1.8)	(1.8)	(1.7)	(1.7)	(1.6)
8	(1.4)	(1.5)	(1.8)	(1.8)	(1.8)	(1.7)	(1.7)	(1.7)	(1.6)	(1.6)	(1.5)
9	(1.4)	(1.7)	(1.7)	(1.7)	(1.6)	(1.6)	(1.5)	(1.5)	(1.5)	(1.4)	(1.2)
10											

Figure 17: The figure shows for each state the difference between the row firm's value gain from increasing its capital stock by one unit and the gain in aggregate value from this change after the first policy iteration [policy $\widehat{b}_1(K_1, K_2)$] in the large market.

Figure 18: Markov perfect policy probability mergers are allowed (a), proposed (b), and happen (c); AV criterion, large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.00	1.00	1.00	0.95	0.82	0.74	0.66	0.57	0.50	0.43
K_1=2	-	1.00	1.00	1.00	0.91	0.76	0.59	0.38	0.17	-	-
K_1=3	-	1.00	1.00	0.95	0.78	0.58	0.29	-	-	-	-
K_1=4	-	0.95	0.91	0.78	0.59	0.30	-	-	-	-	-
K_1=5	-	0.82	0.76	0.58	0.30	-	-	-	-	-	-
K_1=6	-	0.74	0.59	0.29	-	-	-	-	-	-	-
K_1=7	-	0.66	0.38	-	-	-	-	-	-	-	-
K_1=8	-	0.57	0.17	-	-	-	-	-	-	-	-
K_1=9	-	0.50	-	-	-	-	-	-	-	-	-
K_1=10	-	0.43	-	-	-	-	-	-	-	-	-

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.44	0.30	0.15	0.11	0.21	0.30	0.38	0.43	0.45	0.46
K_1=2	-	0.30	0.40	0.37	0.45	0.53	0.60	0.62	0.41	-	-
K_1=3	-	0.15	0.37	0.62	0.58	0.67	0.62	-	-	-	-
K_1=4	-	0.11	0.45	0.58	0.85	0.66	-	-	-	-	-
K_1=5	-	0.21	0.53	0.67	0.66	-	-	-	-	-	-
K_1=6	-	0.30	0.60	0.62	-	-	-	-	-	-	-
K_1=7	-	0.38	0.62	-	-	-	-	-	-	-	-
K_1=8	-	0.43	0.41	-	-	-	-	-	-	-	-
K_1=9	-	0.45	-	-	-	-	-	-	-	-	-
K_1=10	-	0.46	-	-	-	-	-	-	-	-	-

(c)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.44	0.30	0.15	0.10	0.17	0.22	0.25	0.25	0.22	0.20
K_1=2	-	0.30	0.40	0.37	0.41	0.41	0.36	0.24	0.07	-	-
K_1=3	-	0.15	0.37	0.59	0.45	0.38	0.18	-	-	-	-
K_1=4	-	0.10	0.41	0.45	0.50	0.20	-	-	-	-	-
K_1=5	-	0.17	0.41	0.38	0.20	-	-	-	-	-	-
K_1=6	-	0.22	0.36	0.18	-	-	-	-	-	-	-
K_1=7	-	0.25	0.24	-	-	-	-	-	-	-	-
K_1=8	-	0.25	0.07	-	-	-	-	-	-	-	-
K_1=9	-	0.22	-	-	-	-	-	-	-	-	-
K_1=10	-	0.20	-	-	-	-	-	-	-	-	-

Figure 19: Markov perfect equilibrium steady state at the time of production; large market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10	K_2=11	K_2=12
K_1=0	-	-	0.001	0.003	0.009	0.019	0.037	0.048	0.046	0.027	0.009	0.002	-
K_1=1	-	0.001	0.003	0.009	0.019	0.029	0.033	0.030	0.021	0.011	0.004	0.001	-
K_1=2	0.001	0.003	0.004	0.008	0.012	0.016	0.017	0.015	0.011	0.005	0.001	-	-
K_1=3	0.003	0.009	0.008	0.005	0.007	0.007	0.008	0.006	0.003	0.001	-	-	-
K_1=4	0.009	0.019	0.012	0.007	0.004	0.005	0.004	0.002	0.001	-	-	-	-
K_1=5	0.019	0.029	0.016	0.007	0.005	0.002	0.001	-	-	-	-	-	-
K_1=6	0.037	0.033	0.017	0.008	0.004	0.001	-	-	-	-	-	-	-
K_1=7	0.048	0.030	0.015	0.006	0.002	-	-	-	-	-	-	-	-
K_1=8	0.046	0.021	0.011	0.003	0.001	-	-	-	-	-	-	-	-
K_1=9	0.027	0.011	0.005	0.001	-	-	-	-	-	-	-	-	-
K_1=10	0.009	0.004	0.001	-	-	-	-	-	-	-	-	-	-
K_1=11	0.002	0.001	-	-	-	-	-	-	-	-	-	-	-
K_1=12	-	-	-	-	-	-	-	-	-	-	-	-	-

Figure 20: Investment and depreciation transition probabilities from state (5,5) for no mergers allowed (a) and for the Markov perfect equilibrium (b); AV criterion, large market

(a)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	-	-	-	-	-	-	-	-	-	-
K_1=2	-	-	-	-	-	0.01	-	-	-	-	-
K_1=3	-	-	-	0.01	0.02	0.03	0.02	0.01	-	-	-
K_1=4	-	-	-	0.02	0.05	0.08	0.06	0.02	-	-	-
K_1=5	-	-	0.01	0.03	0.08	0.12	0.09	0.02	-	-	-
K_1=6	-	-	-	0.02	0.06	0.09	0.07	0.02	-	-	-
K_1=7	-	-	-	0.01	0.02	0.02	0.02	0.01	-	-	-
K_1=8	-	-	-	-	-	-	-	-	-	-	-
K_1=9	-	-	-	-	-	-	-	-	-	-	-
K_1=10	-	-	-	-	-	-	-	-	-	-	-

(b)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	-	-	-	-	-	-	-	-	-	-
K_1=2	-	-	-	0.01	0.01	0.01	-	-	-	-	-
K_1=3	-	-	0.01	0.02	0.05	0.05	0.02	-	-	-	-
K_1=4	-	-	0.01	0.05	0.11	0.12	0.04	-	-	-	-
K_1=5	-	-	0.01	0.05	0.12	0.12	0.04	-	-	-	-
K_1=6	-	-	-	0.02	0.04	0.04	0.01	-	-	-	-
K_1=7	-	-	-	-	-	-	-	-	-	-	-
K_1=8	-	-	-	-	-	-	-	-	-	-	-
K_1=9	-	-	-	-	-	-	-	-	-	-	-
K_1=10	-	-	-	-	-	-	-	-	-	-	-

Figure 21: Change in CV from a one-time merger; small market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	(6.0)	(7.8)	(8.2)	(8.3)	(8.4)	(8.4)	(8.2)	(7.9)	(7.7)	(7.5)
K_1=2	-	(7.8)	(9.7)	(10.7)	(11.2)	(11.6)	(11.6)	(11.4)	(11.3)	(11.2)	(11.0)
K_1=3	-	(8.2)	(10.7)	(12.0)	(12.8)	(13.2)	(13.2)	(13.2)	(13.2)	(13.1)	(13.1)
K_1=4	-	(8.3)	(11.2)	(12.8)	(13.6)	(13.9)	(14.0)	(14.2)	(14.3)	(14.3)	(14.3)
K_1=5	-	(8.4)	(11.6)	(13.2)	(13.9)	(14.4)	(14.5)	(14.8)	(14.9)	(15.1)	(15.2)
K_1=6	-	(8.4)	(11.6)	(13.2)	(14.0)	(14.5)	(14.8)	(15.3)	(15.6)	(15.9)	(16.2)
K_1=7	-	(8.2)	(11.4)	(13.2)	(14.2)	(14.8)	(15.3)	(15.8)	(16.2)	(16.6)	(17.0)
K_1=8	-	(7.9)	(11.3)	(13.2)	(14.3)	(14.9)	(15.6)	(16.2)	(16.8)	(17.2)	(17.6)
K_1=9	-	(7.7)	(11.2)	(13.1)	(14.3)	(15.1)	(15.9)	(16.6)	(17.2)	(17.8)	(18.2)
K_1=10	-	(7.5)	(11.0)	(13.1)	(14.3)	(15.2)	(16.2)	(17.0)	(17.6)	(18.2)	(18.8)

Figure 22: Change in AV from a one-time merger; small market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	2.8	2.5	2.0	1.7	1.4	1.3	1.3	1.4	1.4	1.4
K_1=2	-	2.5	2.0	1.5	1.1	0.8	0.7	0.8	0.8	0.8	0.8
K_1=3	-	2.0	1.5	0.8	0.3	0.1	0.1	0.1	0.0	0.0	(0.0)
K_1=4	-	1.7	1.1	0.3	(0.1)	(0.3)	(0.4)	(0.6)	(0.7)	(0.8)	(0.9)
K_1=5	-	1.4	0.8	0.1	(0.3)	(0.6)	(0.8)	(1.1)	(1.3)	(1.5)	(1.7)
K_1=6	-	1.3	0.7	0.1	(0.4)	(0.8)	(1.2)	(1.6)	(1.9)	(2.2)	(2.4)
K_1=7	-	1.3	0.8	0.1	(0.6)	(1.1)	(1.6)	(2.1)	(2.4)	(2.8)	(3.1)
K_1=8	-	1.4	0.8	0.0	(0.7)	(1.3)	(1.9)	(2.4)	(2.9)	(3.3)	(3.6)
K_1=9	-	1.4	0.8	0.0	(0.8)	(1.5)	(2.2)	(2.8)	(3.3)	(3.7)	(4.1)
K_1=10	-	1.4	0.8	(0.0)	(0.9)	(1.7)	(2.4)	(3.1)	(3.6)	(4.1)	(4.6)

Figure 23: First iteration merger approval probability; AV criterion, small market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=2	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
K_1=3	-	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
K_1=4	-	1.00	1.00	1.00	0.91	0.72	0.59	0.45	0.32	0.21	0.12
K_1=5	-	1.00	1.00	1.00	0.72	0.40	0.15	-	-	-	-
K_1=6	-	1.00	1.00	1.00	0.59	0.15	-	-	-	-	-
K_1=7	-	1.00	1.00	1.00	0.45	-	-	-	-	-	-
K_1=8	-	1.00	1.00	1.00	0.32	-	-	-	-	-	-
K_1=9	-	1.00	1.00	1.00	0.21	-	-	-	-	-	-
K_1=10	-	1.00	1.00	0.98	0.12	-	-	-	-	-	-

Figure 24: Change in AV from a merger in the first policy iteration equilibrium; AV criterion, small market

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	0.7	0.3	(0.1)	(0.4)	(0.7)	(0.9)	(1.0)	(1.1)	(1.2)	(1.3)
K_1=2	-	0.3	(0.0)	(0.4)	(0.8)	(1.1)	(1.3)	(1.6)	(1.8)	(2.0)	(2.2)
K_1=3	-	(0.1)	(0.4)	(0.7)	(0.9)	(1.3)	(1.7)	(2.1)	(2.4)	(2.7)	(3.0)
K_1=4	-	(0.4)	(0.8)	(0.9)	(1.4)	(1.9)	(2.4)	(2.8)	(3.3)	(3.7)	(4.1)
K_1=5	-	(0.7)	(1.1)	(1.3)	(1.9)	(2.5)	(3.1)	(3.7)	(4.3)	(4.8)	(5.3)
K_1=6	-	(0.9)	(1.3)	(1.7)	(2.4)	(3.1)	(3.9)	(4.6)	(5.3)	(5.8)	(6.3)
K_1=7	-	(1.0)	(1.6)	(2.1)	(2.8)	(3.7)	(4.6)	(5.4)	(6.1)	(6.7)	(7.2)
K_1=8	-	(1.1)	(1.8)	(2.4)	(3.3)	(4.3)	(5.3)	(6.1)	(6.8)	(7.5)	(8.0)
K_1=9	-	(1.2)	(2.0)	(2.7)	(3.7)	(4.8)	(5.8)	(6.7)	(7.5)	(8.1)	(8.7)
K_1=10	-	(1.3)	(2.2)	(3.0)	(4.1)	(5.3)	(6.3)	(7.2)	(8.0)	(8.7)	(9.2)

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	-	-	-	-	-	-	-	-	-	-	-
K_1=1	-	1.00	1.00	1.00	0.93	0.77	0.66	0.57	0.54	0.50	0.45
K_1=2	-	1.00	1.00	0.87	0.58	0.33	0.22	0.12	0.02	-	-
K_1=3	-	1.00	0.87	0.52	0.13	-	-	-	-	-	-
K_1=4	-	0.93	0.58	0.13	-	-	-	-	-	-	-
K_1=5	-	0.77	0.33	-	-	-	-	-	-	-	-
K_1=6	-	0.66	0.22	-	-	-	-	-	-	-	-
K_1=7	-	0.57	0.12	-	-	-	-	-	-	-	-
K_1=8	-	0.54	0.02	-	-	-	-	-	-	-	-
K_1=9	-	0.50	-	-	-	-	-	-	-	-	-
K_1=10	-	0.45	-	-	-	-	-	-	-	-	-

Figure 25: Small market Markov perfect policy probability of merger being allowed

Figure 26: Small market Markov perfect policy steady state at time of production

	K_2=0	K_2=1	K_2=2	K_2=3	K_2=4	K_2=5	K_2=6	K_2=7	K_2=8	K_2=9	K_2=10
K_1=0	0.001	0.006	0.025	0.058	0.089	0.082	0.044	0.011	0.001	-	-
K_1=1	0.006	0.010	0.020	0.031	0.035	0.026	0.011	0.002	-	-	-
K_1=2	0.025	0.020	0.012	0.014	0.014	0.008	0.002	-	-	-	-
K_1=3	0.058	0.031	0.014	0.007	0.004	0.001	-	-	-	-	-
K_1=4	0.089	0.035	0.014	0.004	0.001	-	-	-	-	-	-
K_1=5	0.082	0.026	0.008	0.001	-	-	-	-	-	-	-
K_1=6	0.044	0.011	0.002	-	-	-	-	-	-	-	-
K_1=7	0.011	0.002	-	-	-	-	-	-	-	-	-
K_1=8	0.001	-	-	-	-	-	-	-	-	-	-
K_1=9	-	-	-	-	-	-	-	-	-	-	-
K_1=10	-	-	-	-	-	-	-	-	-	-	-